Accepting Blame: Expressive Checked Exceptions

Yizhou Zhang* Guido Salvaneschi† Quinn Beightol*
Barbara Liskov‡ Andrew C. Myers*

*Cornell University †TU Darmstadt ‡MIT

yizhou@cs.cornell.edu salvaneschi@cs.tu-darmstadt.de qeb2@cornell.edu
liskov@csail.mit.edu andru@cs.cornell.edu

October, 2017

Abstract

Unhandled exceptions crash programs, so a compile-time check that exceptions are handled should in principle make software more reliable. But designers of some recent languages have argued that the benefits of statically checked exceptions are not worth the costs. We introduce a new statically checked exception mechanism that addresses the problems with existing checked-exception mechanisms. In particular, it interacts well with higher-order functions and other design patterns. The key insight is that whether an exception should be treated as a "checked" exception is not a property of its type but rather of the context in which the exception propagates. Statically checked exceptions can "tunnel" through code that is oblivious to their presence, but the type system nevertheless checks that these exceptions are handled. Further, exceptions can be tunneled without being accidentally caught, by expanding the space of exception identifiers to identify the exception-handling context. The resulting mechanism is expressive and syntactically light, and can be implemented efficiently. We demonstrate the expressiveness of the mechanism using significant codebases and evaluate its performance. We have implemented this new exception mechanism as part of the new Genus programming language, but the mechanism could equally well be applied to other programming languages.

1 Introduction

Exceptions make code more reliable by helping programmers handle abnormal or unusual run-time conditions. The core idea is to transfer control in a nonlocal way to handler code that can be factored out from common-case code. This separation of concerns simplifies code and prompts programmers not to forget about exceptional conditions.

There has been disagreement since the 1970's about how or whether exceptions should be subject to static checking [Goodenough 1975; MacLaren 1977]. This disagreement continues to the present day [Goetz 2004]. Some currently popular languages—Java [Gosling et al. 2005] and Swift [Swift 2014]—offer *checked exceptions* that the compiler statically ensures are handled. However, exceptions are not part of type checking in other popular languages such as C++ [Stroustrup 1987], C# [Hejlsberg et al. 2003b], Scala [Odersky 2014], and Haskell [Peyton Jones 2003].

Proponents of static checking argue that exceptions represent corner cases that are easy to forget. The evidence suggests they have a point. One study of a corpus of C# code [Cabral and Marques 2007b] determined that 90% of the possible exceptions are undocumented. Undocumented exceptions make it hard to know whether all exceptions are handled, and these unhandled exceptions percolate up through abstraction layers, causing unexpected software failures. Statically checked exceptions help programmers build more robust code [Venners 2003].

Opponents of static checking argue that the annotation burden of statically checked exceptions does not pay off—that statically checked exceptions are too rigid to support common design patterns and common ways in which software evolves [Waldhoff 2003; Eckel 2003]. They, too, have a point. The problems with statically checked exceptions have become more apparent in recent years as object-oriented (OO) languages like C#, Scala, and Java have acquired lambda expressions and the use of higher-order functions has become more common. As a result, the promise of exceptions to help make software more reliable has been partly lost.

Studies of the effectiveness of exception mechanisms have concluded that existing mechanisms do not satisfy the appropriate design criteria [Buhr and Mok 2000; Cabral and Marques 2007a]. C# does not statically check exceptions because its designers did not know how to design such an exception mechanism well, saying "more thinking is needed before we put some kind of checked exceptions mechanism in place" [Hejlsberg et al. 2003a]. It seems the long-running conflict between checked and unchecked exceptions can be resolved only by a new exception mechanism.

This paper aims to provide a better exception mechanism, one that combines the benefits of static checking with the flexibility of unchecked exceptions. The new mechanism gives programmers static, compile-time guidance to ensure exceptions are handled, but works well with higher-order functions and design patterns. It adds little programmer burden and even reduces that burden. The run-time overhead of the mechanism is low, because exception handling does not require stack-trace collection in common use cases and avoids the need to wrap checked exceptions inside unchecked ones.

Two main insights underlie the new design. The first is that the distinction between "checked" and "unchecked" should not be a property of the type of the exception being raised, as it is in Java, but rather a property of the context in which the exception propagates. In contexts that are *aware* of an exception, the exception should be checked statically to ensure that it is handled. To handle higher-order functions and design patterns, however, some contexts must be *oblivious* to exceptions propagating through them; exceptions should *tunnel* uncaught through oblivious contexts, effectively *unchecked*.

This principle implies that the same exception may be *both* checked and unchecked at different points during its propagation. To prevent oblivious code from *accidentally* catching exceptions, a second insight is needed: exceptions can be distinguished by expanding the space of exception identifiers with an additional label that describes the exception-aware context in which this exception can be caught. These labels can be viewed as an extension of the notion of *blame labels* found in previous work on gradual typing [Wadler and Findler 2009]. Unlike with gradual typing, this sort of "blame" is not a programmer error; it is instead a way to indicate that exceptions should tunnel through the oblivious code until they arrive at the right exception-aware context.

We start the rest of the paper by exploring requirements for a good exception mechanism in Section 2. Sections 3–6 present our new exception mechanism informally in the context of a Java-like language. Section 7 defines a core language whose semantics show more precisely how the mechanism works. Using this core language, we prove the key theorem that all exceptions are handled explicitly. Section 8 describes our implementation of the new exception mechanism in the context of the Genus programming language [Zhang et al. 2015]. The effectiveness of the mechanism is evaluated in Section 9, using code drawn from real-world codebases. Related work is explored more fully in Section 10; we conclude in Section 11.

2 Design Principles for Exceptions

The goal of an exception mechanism is to simplify and regularize the handling of exceptional events, making programs more reliable. However, there are two quite different classes of exceptional events, with different design goals:

- **Failures.** Some events cannot reasonably be expected to be handled correctly by the program—especially, events that arise because of programmer mistakes. Other events such as running out of memory also fall into this category.
- **Unusual conditions.** Other events arise during correct functioning of the program, in response to an unusual but planned-for state of the environment, or even just an unusual case of an algorithm.

These two classes place different requirements on the exception mechanism. For failures, efficiency is not a concern because the program is not expected to recover. However, programmers need the ability to debug the (stopped) program to discover why the failure occurred, so it is important to collect a stack trace. Furthermore, since failures imply violation of programmer assumptions, having to declare them as part of method signatures or write handler code for them is undesirable. Nonetheless, there are cases where the ability to catch failure exceptions is useful, such as when building frameworks for executing code that might fail.

For the second class of exceptions, unusual conditions, the design goals are different. Now efficiency matters! Because exceptions are slow in many common languages, programmers have learned to avoid using them. One insight is that because unusual conditions are part of the correct functioning of the program, the overhead of collecting a stack trace is unnecessary.

Unfortunately, existing languages tend not to support the distinction between these two exception classes well. For example, typical Java usage always leads to stack-trace collection, making exceptions very expensive. At the same time, code is cluttered with handlers for impossible exceptions.

2.1 Higher-Order Functions and Exceptions

Current exception mechanisms do not work well in code that uses higher-order functions. An example is an ML-style `map` method that applies a function argument to each element of a list, returning a new list. Callers of the higher-order function may wish to provide as an argument a function that produces exceptions to report an unusual condition that was encountered, such as an I/O exception. Of course, we want these exceptions to be handled. But the implementation of `map` knows nothing about these exceptions, so if such exceptions do occur, they should be handled by the caller of `map`. It is unreasonable for `map` either to handle or to declare them—its code should be *oblivious* to the exceptions.

This example illustrates why otherwise statically typed functional programming languages such as ML and Haskell do not try to type-check exceptions statically [Leroy and Pessaux 2000]. The problem has also become more prominent in modern OO languages that have added *lambda expressions* and are increasingly relying on libraries that use them (e.g., JavaFX, Apache Spark). The problem is encountered even without lambda expressions, though; for example, Java's `Iterator` interface declares no exceptions on its methods, which means that implementations of `Iterator` are not allowed to generate exceptions—unless they are made "unchecked".

Our goal is to achieve the notational convenience of the functional programming languages along with the assurance that exceptions are handled, which is offered by languages such as Java, Swift [Swift 2014], and Modula-3 [Nelson 1991]. We propose that a higher-order function like `map` should be implementable in a way that is *oblivious* to the exceptions possibly generated by its argument. The possible exceptions of the argument should not be declared in the signature of `map`; nor should the code of `map` have to say anything about these exceptions.

A subtle problem arises when a higher-order function like `map` uses exceptions inside its own implementation. If the exceptions of the argument and the internal exceptions collide, the `map` code could then *accidentally* catch exceptions that are not intended for it—an effect we call *exception capture* by analogy with the problem of variable capture in dynamically scoped languages [Steele 1990]. For modularity, a way is needed to *tunnel* such exceptions through the intermediate calling context. In fact, accidentally-caught exceptions are a real source of serious bugs [FindBugs bug descriptions; CWE].

An alternative to our oblivious-code approach that has been suggested previously [van Dooren and Steegmans 2005; Goetz 2010; Rytz et al. 2012; Leijen 2014] is to parameterize higher-order code like `map` over the unknown exceptions of its argument. This *exception polymorphism* approach requires writing annotations on oblivious code yet still permits accidental exception capture.

2.2 Our Approach

Backed by common sense and some empirical evidence, we believe that code is more reliable when compile-time checking guides programmers to handle exceptional cases. It is disappointing that recent language designs such as C# and Scala have backed away from statically declared exceptions.

We propose a new statically checked exception mechanism that addresses the weaknesses of prior exception mechanisms:

- It supports static, local reasoning about exceptions. Local reasoning is efficient, but more importantly, it aids programmer understanding. A code context is required to handle only the exceptions it knows about statically.
- The mechanism is cheap when it needs to be: when exceptions are used for nonlocal control flow rather than failures.
- In the failure case, however, the mechanism collects the stack trace needed for debugging.
- It supports higher-order functions whose arguments are other functions that might throw exceptions to which the higher-order code is oblivious.
- It avoids the exception-capture problem both for higher-order functions and for failures.

3 The Exception Mechanism

We use Java as a starting point for our design because it is currently the most popular language with statically checked exceptions. Our design is presented as a version of the Genus language, a variant of Java with an expressive constrained genericity mechanism [Zhang et al. 2015]. The essential ideas should apply equally well to other languages, such as Java, C# [Microsoft Corporation 2001], Scala [Odersky 2014], and ML [Milner et al. 1990]. Since exception-oblivious code (like `map`) is often generic, it is important to study how exceptions interact with sophisticated generics.[1]

Previous languages have either had entirely “checked” or “unchecked” exceptions (in Java’s terminology), or, as in Java, have assigned exception types to one of these two categories. Our insight is that “checked” vs. “unchecked” is a property of the context of the exception rather than of its type. Any exception should be “checked” in a context that is not oblivious to the exception and can therefore handle it. But in a context that is oblivious to the exception, the exception should be treated as “unchecked”.

Genus requires that a method handle or explicitly propagate all exceptions it knows can arise from operations it uses or methods it calls. If the implementation of a method explicitly throws an unhandled exception whose type (or the supertype thereof) is not listed in the method’s header, the program is rejected.

Like in Java, exceptions can be handled using the `try–catch` construct, and a `finally` block that is always executed may be provided. (The `try`-with-resources statement of Java 7 is easily supported; it is orthogonal to the new features.)

[1] Genus uses square brackets rather than angle brackets for generic type arguments: `List[T]` rather than `List<T>`. For brevity, we use Genus syntax and the Genus equivalents of Java core classes without further explanation.

A method that wishes merely to propagate an exception to its caller can simply place a throws clause in its signature. We say such an exception propagates in *checked mode*. Unlike in Java, exceptions in checked mode do not cause stack-trace collection.

3.1 Failures

Unlike Java, our mechanism makes it easy for the programmer to indicate that an exception should not happen. The programmer ordinarily does this by putting a fails clause in the method header. Any caller of the method is then oblivious to the exception, meaning that the exception will be treated as unchecked as it propagates further. When code fails because of an exception, the exception propagates in a special mode, the *failure mode*.

For example, a programmer who is certain that the Object class can be loaded successfully can write

```
Class loadObject() fails ClassNotFoundException {
  return Class.forName("genus.lang.Object");
}
```

where the method forName in Class declares ClassNotFoundException in its throws clause. Note that a fails clause is really part of the implementation rather than part of the method signature or specification. We write it in the header just for convenience.

Exceptions propagating in failure mode also differ in what happens at run time. Programmers need detailed information to debug the stopped program to discover how the failure occurred. Therefore, failure exceptions collect a stack trace. This is relatively slow (as slow as most Java exceptions!) but efficiency is not a concern for failures.

3.2 Avoiding Exception Capture

Since exceptions propagating in failure mode do not appear in method signatures, it is important to avoid catching them accidentally. For example, consider the following code that calls two functions g() and h():

```
void f() {
  try {
    g(); // signature says "throws MyExc"
    h(); // signature doesn't say "throws MyExc"
  } catch (MyExc e) {...}
}
```

Suppose that because of a programmer mistake, the call to h() unexpectedly fails with exception MyExc. If this exception were caught by the catch clause, f() would execute code intended to compensate for something that happened in g(). We prevent this undesirable exception capture by ensuring that failure exceptions cannot be caught by any ordinary catch clauses: failure exceptions *tunnel* past all ordinary catch clauses.

Although exceptions in failure mode are not normally handled, there may be value in catching them at the top level of the program or at the boundary between components, to allow for more graceful exit. Genus supports this with a rarely used catch all construct that catches all exceptions of a given type, regardless of propagation mode. For example, if the try statement above were extended to include a second clause catch all (MyExc e) {...}, the first catch clause would catch the expected MyExcs thrown by g, and the second catch all clause would catch failure-mode MyExcs tunneled through h.

3.3 Fail-by-Default Exceptions

Java has a commonly accepted set of exceptions that usually correspond to programmer mistakes: the *built-in* subclasses of RuntimeException or Error. To reduce annotation burden for the programmer, our mecha-

```
List[R] map[T,R](Function[T,R] f, List[T] src) {
    List[R] dest = new ArrayList[R]();
    for (T t : src) dest.add(f.apply(t));
    return dest;
}
```

Figure 1: A higher-order function written in Genus

```
List[String] x = ...;     List[Class] y;
try { y = map(Class::forName, x); }
catch (ClassNotFoundException e) {...}
```

Figure 2: Passing a function that throws extra exceptions

nism does not ordinarily require writing a `fails` clause in order to convert such exceptions to failure mode. We say these exceptions *fail by default*.

Fail-by-default exceptions are different from Java's unchecked exceptions. Unchecked exceptions conflate failures with ordinary exceptions that are tunneling through oblivious code but that still ought to be subject to static checking.

In contrast, fail-by-default exceptions remain in checked mode until they reach a method-call boundary; they convert from checked mode to failure mode only if the current method does not declare the exception in its `throws` clause. A fail-by-default exception collects a stack trace only if and when it does fail, so code can still handle exceptions like `NoSuchElementException` efficiently. It is therefore reasonable and useful to write code that handles such exceptions.

Genus also permits the set of fail-by-default exceptions to be changed on a per-class basis by adding a `throws` or `fails` clause to the class definition.

4 Higher-Order Abstractions and Tunneling

As discussed in Section 2.1, higher-order functions pose a problem for statically checked exception mechanisms. The same problem arises for many common object-oriented design patterns, which are essentially higher-order functions. Our solution is to tunnel exceptions through oblivious code.

For example, Genus allows the programmer to pass to the higher-order function `map` (Figure 1) an implementation of `Function` that throws exceptions, even though the signature of `Function` does not mention any exceptions. If the passed `Function` throws an exception, that exception is tunneled through the exception-oblivious code of `map` to the caller of the exception-aware code that called `map`.

In the code of Figure 2, the method `forName` in class `Class` is passed to `map`. This call is legal even though `forName` is declared to throw an exception `ClassNotFoundException`. Since `map` is oblivious to this exception, it cannot be expected to handle it. By contrast, the caller of `map` is aware that an object that throws exceptions is used at a type (`Function`) that does not declare any exceptions. Because it is aware of the exception, the caller is responsible for the exception.

If `ClassNotFoundException` arises at run time, it tunnels through the code of `map` and is caught by the `catch` clause. Alternatively, the caller could have explicitly converted the exception to a failure (via a `fails` clause) or explicitly allowed it to propagate (via a `throws` clause). In any case, exception handling is enforced statically.

```
interface Visitor {                         class IfTree implements Tree {
  void visit(IfTree t);                       void accept(Visitor v)
  ...                                           {v.visit(this);} ...
}                                           }
class Pretty implements Visitor { ...
  void visit(IfTree t) {
    try {...} // pretty-print IfTree
    catch (IOException e) { throw new UncheckedIO(e); }
  } // wraps IOException
}
void printTree(Tree t, Pretty v) throws IOException {
  try { t.accept(v); }
  catch (UncheckedIO u) { throw u.getCause(); }
} // unwraps UncheckedIO
class UncheckedIO extends RuntimeException {...}
```

Figure 3: The pretty-printing visitor in `javac` (simplified).
Code for exception wrapping and unwrapping is highlighted.

```
weak interface Visitor{                     class IfTree implements Tree {
  void visit(IfTree t);                       void accept(Visitor v)
  ...                                           {v.visit(this);} ...
}                                           }
class Pretty implements Visitor { ...
  void visit(IfTree t) throws IOException {...} // OK
}
void printTree(Tree t, Pretty v) throws IOException {
  t.accept(v);
}
```

Figure 4: Exception tunneling in `javac`, ported to Genus.

4.1 Exception Tunneling is Safe and Light

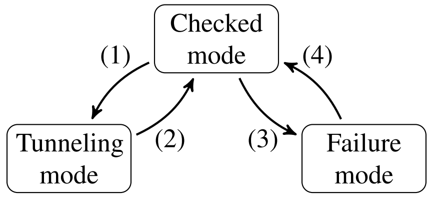
In Java, the rigidity of checked exceptions has led to some verbose and dangerous idioms, especially when higher-order functions and design patterns are used. Exception tunneling helps avoid these undesirable programming practices.

In particular, Java programmers often abandon static checking of exceptions to make it possible for their exceptions to pass through exception-oblivious higher-order code. They either define their own exceptions as unchecked, or they cope with preexisting checked exceptions by calling unsafe APIs like `sun.misc.Unsafe::throwException` [Mastrangelo et al. 2015], or by wrapping checked-exception objects inside unchecked-exception objects. Wrapping with unchecked exceptions is the safest of these workarounds because it makes exception capture less likely, but wrappers are verbose and expensive.

Figure 3 shows an example of this idiom taken from the `javac` compiler [OpenJDK javac], which contains a number of visitors for the Java AST. In order to conform to the `Visitor` interface, the `visit` methods in the pretty-printing visitor (`Pretty`) wrap the checked `IOException` into unchecked wrappers, which are then unwrapped as shown in method `printTree`. This programming pattern is verbose, abandons static checking, and is likely to be slow due to stack-trace collection.

When written in Genus, the same visitor pattern (Figure 4) does not require exception wrapping or unwrapping to achieve tunneling. The modifier `weak` on the interface `Visitor` (see Section 4.4) makes it legal for its implementations to declare new exceptions (the interface `Function` is annotated similarly). In `printTree`, the call `t.accept(v)` passes a visitor object that throws the additional exception `IOException`.

	Checked	Tunneling	Failure
Local, static checking	Yes	No	No
Stack-trace collection	No	No	Yes



(1)(2): §4.2 and §4.3 (3)(4): §3.1 and §3.3

Figure 5: Three exception propagation modes

```
List[R] map[T,R](Function[T,R] f,
                 List[T] src) {
  List[R] dst = new ArrayList[R]();
  mapImpl(f, src, dst);
  return dst;
}
void mapImpl[T,R](Function[T,R] f,
    List[T] s, List[R] d) {
  if (d.size() >= s.size()) return;
  d.add(f.apply(s.get(d.size())));
  mapImpl(f, s, d);
}
```

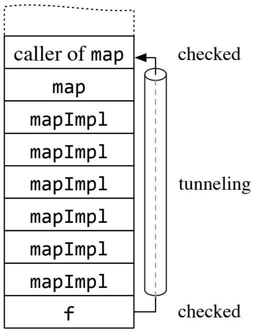


Figure 6: A recursive implementation of map (left) and a stack snapshot showing propagation of an exception caused by f (right). The stack grows downwards.

If this exception is thrown by the visitor, it tunnels until it reaches printTree. Thus, the antipattern of exception wrapping becomes unnecessary.

4.2 Tunneling Checked Exceptions

Earlier we discussed two modes of exception propagation: checked mode and failure mode. Tunneling mode is a third mode of propagation. The relationships between the three modes are summarized in Figure 5. In tunneling mode, as in checked mode, static checking enforces handling of exceptions. As in failure mode, these exceptions do not need to be declared in signatures of the methods they tunnel through.

A given exception may propagate in more than one mode. Consider the alternative, slightly contrived implementation of map in Figure 6. It calls a helper method mapImpl, which then recursively calls itself to traverse the list. Suppose an exception arises when function f is applied to the sixth element in the list. Figure 6 shows a snapshot of the current call stack. Since f is the place where the exception is generated, the exception first propagates in *checked mode* within the function f. Because its caller mapImpl is oblivious to the exception, the exception then switches to *tunneling mode* and propagates through all the mapImpl stack frames. Finally, the caller of map knows about the exception and thus can handle it. The exception returns to checked mode when it reaches this caller. From there, it can be either caught, rethrown in checked mode, or turned into failure.

4.3 Tunneling, Exception Capture, and Blame

Tunneling avoids the phenomenon of exception capture discussed in Section 3. In OO languages like Java and C#, exception capture occurs because of an unexpected collision in the space of *exception identifiers*;

an exception identifier in these languages is simply the exception type. We avoid exception capture by augmenting the identity of a thrown exception to include a notion of "blame".

To ensure that every exception is eventually either handled or treated as a failure, a method must *discharge* every exception it is aware of statically. There are three ways to discharge an exception: 1) handle it with a `catch` clause, 2) propagate it to its caller as a checked exception via a `throws` clause, or 3) convert it to a failure via a `fails` clause (which is implicit for fail-by-default exceptions).

Each of these three ways discharges exceptions from a set of program points that are *statically* known to give rise to these exceptions. We say these program points can be *blamed* for the exception. With each such program point, we associate a *blame label* that identifies where responsibility lies for the exception.

At run time, then, a thrown exception is identified both by its exception type, and by its blame label. An exception is discharged (e.g., caught by a `catch` clause, or converted to failure by a `fails` clause), only if the blame label of the exception lies within the lexical scope covered by that particular discharging point. Otherwise, the exception is one that the discharging point is oblivious to.

We use the word "blame" because this mechanism is related to the notion of blame used in work on behavioral contracts [Findler et al. 2001; Findler and Felleisen 2002] and gradual typing [Wadler and Findler 2009]. A compiler for a gradually typed language might label program points with unique identifiers wherever there is a *mismatch* between expected and actual types. When a cast failure happens at run time, blame can be attributed to the program point at fault.

Here, mismatch occurs analogously when the type of an actual argument declares exceptions not encompassed by those declared by the formal parameter type. Any exception mismatch in the parameters passed to a method call causes blame to be assigned to the program point of the method call. However, unlike in gradual typing, exceptions arising from a program point assigned blame do not imply mistakes,[2] since the programmer must discharge the exceptions.

For example, in Figure 2, the program point where `Class::forName` is used at type `Function` is in the scope of the ensuing `catch` clause at line 3. Because it creates a mismatch with the signature of `Function`, this program point can be blamed for the exception. Similarly, in Figure 4, the shaded program point (line 2) where a `Pretty` object is used at type `Visitor` is in the scope of the clause `throws IOException` (line 1). Because there is a mismatch between `Pretty` and `Visitor`, the shaded program point can be blamed for the exception. Throughout the paper, we highlight program points that are assigned blame and their matching discharging points.

4.4 Weak Types

Some supertypes, usually interfaces, abstract across families of otherwise unrelated subtypes. Such interfaces often arise with design patterns like `Iterator` and `Visitor`. The intention of such types is to capture only a fragment of the behavior—a core set of methods—so that various implementations can have a common interface other software components can use.

Frequently there is utility in allowing subtypes of these interfaces to throw new exceptions. For example, suppose a lexer breaks input files into tokens; an `Iterator` might be used to deliver the tokens to code that consumes them. However, reading from a file can cause an `IOException`; the exception cannot reasonably be handled by the iterator, so should be propagated to the client code. In Java, programmers cannot throw a checked exception like `IOException` in such an implementation; they must either resort to unchecked exceptions, abandoning static checking, or define their own interfaces that compose poorly with existing components.

Genus addresses this need for flexibility. Methods that allow their overridings to throw new exceptions are declared with the `weak` modifier. The `weak` modifier can also be applied to a type definition to con-

[2] Findler et al. [2001] use the term "blame" to mean "the programmer should be held accountable for shoddy craftsmanship". At the risk of confusion, we reuse the term to mean there is an exception to be discharged in this context.

```
  weak interface Iterator[E] {
    E next() throws NoSuchElementException;
        // The exception indicates iteration is over
    ...
  }
  class Tokenizer implements Iterator[Token] {
    Token next() throws IOException,
                        NoSuchElementException {...}
    ...
  }
```

Figure 7: The `Iterator` interface and an inexact subtype

```
Iterator[Token] iter = new Tokenizer(reader);
while (true) {
  Token t;
  try { t = iter.next(); }
  catch (NoSuchElementException e) { break; }
  catch (IOException e) { log.write(...); continue; }
  ...
}
```

Figure 8: Using a `Tokenizer` as an `Iterator` generates blame.

veniently indicate that all methods in that type are intended to be `weak`; see the definition of `Iterator` in Figure 7 for an example.

A subtype of a weak type can be *inexact*; for example, the `Tokenizer` class in Figure 7 is inexact with respect to its weak interface since its `next` method adds `IOException`. A `Tokenizer` can be used as an `Iterator` but this generates blame, forcing `IOException` to be handled, as in Figure 8.

Behavioral subtyping and conformance. Behavioral subtyping [Liskov and Wing 1994] is based on the idea that the allowed uses of an object should be known based on its apparent type. Therefore, an overriding method cannot add new exceptions to the supertype's signature for the method.

Our mechanism relaxes this requirement for weak types. Methods of an inexact subtype must obey the supertype specification—except that they can throw more exceptions. This implies that their additional exceptional conditions must be signaled with different types than those in the supertype method—`Tokenizer` indicates an I/O problem by throwing `IOException`, not `NoSuchElementException`—and that the exceptional conditions the supertype defines must not be signaled in other ways—`Tokenizer` cannot issue a failure or return `null` when the iteration has no more elements.

5 Generics and Exceptions

We have also used Genus to explore the important interaction between exceptions and mechanisms for constrained parametric polymorphism. Various languages constrain generic type arguments in various ways: for example, Java and C# use subtyping constraints, whereas Haskell and Genus use the more flexible mechanism of type classes [Wadler and Blott 1989].

Genus provides constrained parametric polymorphism via *constraints* and *models* [Zhang et al. 2015]. Like type classes, Genus constraints are predicates describing type features required by generic code. Genus models show how types can satisfy constraints, like type class instances in Haskell. Unlike Haskell instances, models are explicitly part of the instantiation of a generic abstraction. For example, the two instantiations `Set[String]` and `Set[String with CaseInsensEq]` are different types distinguished by the use of the

```
class ObjectPool<T> {
 Factory<T> f;
 T borrow() throws Exception
   {... f.make() ...}
 ...
}
interface Factory<T> {
 T make() throws Exception;
}
class ConnFactory implements
    Factory<Connection> {
 Connection make()
   throws SQLException {...}
}
```

```
class ObjectPool[T]
    where Factory[T] {
 T borrow()
   {... T.make() ...}
 ...
}
weak constraint Factory[T]{
 static T make();
}
model ConnFactory for
    Factory[Connection] {
 static Connection make()
   throws SQLException {...}
}
```

Figure 9: Object pool in Java (left) and in Genus (right)

model `CaseInsensEq` in place of default string equality. This distinction is helpful for precisely reasoning about exceptions.

As with interfaces, we would like the flexibility to instantiate generic abstractions with types whose operations throw additional exceptions not provided for by the constraint. Thus, similar to interfaces, Genus constraints can be weak; models may be inexact with respect to the weak constraints they witness.

The example in Figure 9 shows the utility of this feature, comparing Java and Genus code for an object pool abstraction. The left side of the figure shows an example adapted from the Apache Commons project [Apache Commons]. The abstract factory type `Factory` defines a method `make` with a signature that declares "`throws Exception`". This idiom is common in Java libraries, because it permits subtypes to refine the actual exceptions to be thrown.

However, such declarations are a source of frustration for Java programmers [Waldhoff 2003]. Consider that method `make` is called by the method `borrow` in `ObjectPool` to create a new object in case there is no idle one, so `borrow` must also declare "`throws Exception`". The precise exception (in the example, `SQLException`) is therefore lost. Clients of `ObjectPool<Connection>` must handle `Exception`, which is no better, and perhaps worse, than having an unchecked exception. Further, client code that handles the overly general exception is more likely to suffer from exception capture.

The right side of Figure 9 shows a reasonable way to implement the example in Genus. A constraint `Factory[T]` is used to express the requirement that objects of type `T` can be made in some way; because it does not declare any exceptions, method `borrow` need not either. However, a model like `ConnFactory` can add its exceptions such as `SQLException` to the method `make`. Client code can then use the type `ObjectPool[Connection with ConnFactory]` to recycle `Connection` objects. Because the model is part of the type, it is statically apparent that the exception `SQLException` may be thrown; the client code will be required to handle this exception—but only this exception.

6 Exactness Analysis

The new exception mechanism poses new challenges for type checking. One challenge is that the identity of an exception includes a blame label, so blame should not be allowed to escape its scope. Otherwise, an exception might not be handled.

For example, consider the first definition of the `PeekingIterator` class in Figure 10. It decorates a wrapped `Iterator` in field `inner` to support one-element lookahead. Its `next` and `peek` methods call `inner.next()`. Per Section 4.4, it is possible to pass an object of some inexact subtype of `Iterator` to the constructor, to be assigned to `inner`. However, `Iterator` is a weak type, so the methods of the actual

```
1  class PeekingIterator[E] implements Iterator[E] {
2    Iterator[E] inner;
3    PeekingIterator(Iterator[E] it) { inner = it; ...}
4    E peek() throws NoSuchElementException
5      {... inner.next() ...}
6    ...
7  }

8  class PeekingIterator[I extends Iterator[E], E]
9      implements Iterator[E] {
10   I inner;
11   PeekingIterator(I it) { inner = it; ...}
12   E peek() throws NoSuchElementException
13     {... inner.next() ...}
14   ...
15 }
```

Figure 10: Two definitions of `PeekingIterator`. The top one allows blame to escape, but a warning is issued for the constructor. The bottom one uses dependent exactness to soundly avoid the warning.

object being passed may throw exceptions not declared by `Iterator`. If the assignment to `inner` were allowed, the same exceptions would propagate when the `next` method of the `PeekingIterator` object were called. And this call could be delayed until outside the context that is aware of the mismatch with `Iterator`. In this case, the exceptions would not be guaranteed to be handled.

Therefore, we want to detect statically that the assignment to `inner` lets the blame from the inexact object escape. Storing an inexact object into a data structure at a weak type, or even returning an inexact object from a method, may permit such an escape of blame.

A second challenge for the new exception mechanism, and indeed for exception mechanisms generally, is that programmers should not be forced to write handlers for program points where exceptions cannot happen. To address this challenge, the location of blame should be precise. Figure 8 offers an example of this problem. The method call `iter.next()` might appear (to the compiler) not to throw any exceptions because `iter` has type `Iterator[Token]`, yet we know that it may throw an `IOException` because `iter` is initialized to a `Tokenizer`. A safe, conservative solution would be to require all of the code below this initialization to be wrapped in a `try–catch`. But this solution would make it difficult to continue the iteration after an `IOException` is raised.

Genus addresses these two challenges using an intraprocedural program analysis that assigns *exactness* to uses of weak types, with little annotation effort by the programmer.

6.1 Exactness Annotations and Exactness Defaults

Indicating intolerance of inexactness at use sites. When a formal parameter or local variable is assigned a weak type, the programmer can annotate the type with `@exact` to indicate that an exact subtype must be used. For example, if the programmer adds `@exact` to the formal parameter of the constructor at line 3 in Figure 10,

```
PeekingIterator(@exact Iterator[E] it) {inner = it;...}
```

it becomes a static error to pass an inexact implementation of `Iterator` to the constructor. So it is guaranteed that using `inner` will not generate unexpected exceptions. By contrast, without `@exact`, Genus issues a warning about the assignment to the escaping pointer `inner` at line 3. If the exception is actually thrown at run time, it is converted into failure and the usual stack trace is collected.

Although `@exact` might appear to add notational burden, our experience with porting existing Java code into Genus (Section 9) suggests that this escape hatch is rarely needed. We have not seen the need to use the `@exact` annotation to dismiss such warnings in existing code ported from Java. It seems that programmers use weak types differently from other types—weak types provide functionality rather than structure. Further, exactness defaults and exactness inference (Section 6.2) reduce annotation overhead, and exactness-dependent types (Section 6.3) provide more expressiveness.

Exactness defaults. Exactness defines what exceptions that are not declared by the weak type might nevertheless be generated by the term being typed. Exactness $\mathcal{E}$ is formally a mapping from methods to sets of exceptions. These mappings form a lattice ordered as follows:

$$\mathcal{E}_1 \leq \mathcal{E}_2 \Leftrightarrow \text{dom}(\mathcal{E}_1) \subseteq \text{dom}(\mathcal{E}_2) \ \wedge \ \forall m.\, \mathcal{E}_1(m) \subseteq \mathcal{E}_2(m)$$

The bottom lattice element is strict exactness, denoted by $\varnothing$.

To avoid the need for programmers to write the annotation `@exact` for most uses of weak types, the compiler determines exactness using a combination of exactness defaults and automatic exactness inference. To see how these mechanisms work, consider the code in Figure 11.

1. Weak types used as return types or field types are exact, as these are the channels through which pointers can escape. For these types, we have $\mathcal{E} = \varnothing$.

2. Methods and constructors are implicitly polymorphic with respect to exactness. That is, they can be viewed as parameterized by the exactness of their argument and receiver types.

3. Weak types in a local context are labeled by exactness variables: for example, x and y in Figure 11, at lines 2 and 11 respectively.

   A unification-based inference engine solves for these variables, inferring exactness from the local variable uses (Section 6.2).

4. For a procedure call, an exactness variable is generated for each argument and/or receiver whose formal parameter type is weak. Exactness variables of this kind in Figure 11 are z (line 9) and w (line 12).

5. Recall that Genus supports constrained parametric polymorphism via type constraints [Zhang et al. 2015]. Subtype constraints and where-clause constraints can also specify exactness. Their default exactness is deduced in ways similar to those listed above. For example, in Figure 11 the use of `Runnable` in `g`'s signature (line 1) constrains the type parameter `T`, so its exactness is resolved to a fresh name e, with respect to which `g` is polymorphic.

6.2 Solving Exactness Constraints

At each assignment (including variable initialization and argument passing) to a variable with weak type τ, inexact values must not escape into variables with exact types. Therefore, constraints of form $\tau\langle\mathcal{E}_r\rangle \leq \tau\langle\mathcal{E}_l\rangle$ are generated, where $\mathcal{E}_l$ is the exactness of the left-hand side expected type τ in an assignment, resolved as described in Section 6.1, and $\mathcal{E}_r$ is the exactness of the right-hand side actual type with respect to τ. For example, the code of Figure 11 then generates the following constraints with line numbers attached:

$$\begin{array}{rcll}
\tau\langle\mathcal{E}_r\rangle & \leq & \tau\langle\mathcal{E}_l\rangle & \\
\texttt{Runnable}\langle\{\texttt{run} \mapsto \texttt{IOException}\}\rangle & \leq & \texttt{Runnable}\langle x\rangle & 4 \\
\texttt{Runnable}\langle\{\texttt{run} \mapsto \texttt{EOFException}\}\rangle & \leq & \texttt{Runnable}\langle x\rangle & 7 \\
\texttt{Runnable}\langle x\rangle & \leq & \texttt{Runnable}\langle z\rangle & 9 \\
\texttt{Runnable}\langle e\rangle & \leq & \texttt{Runnable}\langle y\rangle & 11 \\
\texttt{Runnable}\langle y\rangle & \leq & \texttt{Runnable}\langle w\rangle & 12 \\
\texttt{Iterator[T]}\langle\varnothing\rangle & \leq & \texttt{Iterator[T]}\langle v\rangle & 11
\end{array}$$

```
weak interface Runnable { void run(); }
```

```
 1 void g[T extends Runnable⟨e⟩](List[T] l) {
 2   Runnable⟨x⟩ r0;
 3   if (new Random().nextBoolean())
 4     r0 = new Runnable()
 5           { void run() throws IOException {...} };
 6   else
 7     r0 = new Runnable()
 8           { void run() throws EOFException {...} };
 9   try {r0.run();}   // Runnable⟨z⟩
10   catch (IOException ex) {...}
11   for (Runnable⟨y⟩ r : l)
12     r.run();        // Runnable⟨w⟩
13 }
```

Figure 11: Running example for exactness analysis. Uses of weak types are tagged by exactness variables, to be fed into the solver. (`EOFException` is a subtype of `IOException`.)

The last constraint is due to desugaring of the loop at lines 11–12:

```
for (Iterator[T]⟨v⟩ i = l.iterator();;) {
    Runnable⟨y⟩ r;
    try { r = i.next(); }
    catch (NoSuchElementException ex) { break; }
    r.run();
}
```

Note that enhanced `for` loops are translated differently by Java and Genus. Genus exceptions are fast enough that it is often faster to call `next()` and catch a final exception, rather than calling `hasNext()` on every iteration.

The compiler solves these constraints by finding the least upper bounds of x, y, z, w, and v:

$$\begin{aligned} x = z\ &= \{\texttt{run} \mapsto \{\texttt{IOException}, \texttt{EOFException}\}\} \\ y = w &= e \\ v &= \varnothing \end{aligned}$$

The fact that a solution exists implies that inexact pointers will not escape to the heap, addressing the first challenge of escaping blame.

The solution also reveals precisely where exception handling is required, addressing the second challenge of blame precision. Specifically, the solution to each variable generated via the fourth case of exactness defaults (Section 6.1) tells what exception can be produced by the method call. In our running example, the solution to z is that the method call `r0.run()` may throw `IOException` or `EOFException`, which are handled by the `catch` block. Notice that although mismatches happen at lines 4 and 7, the blame does not take effect until the method call at line 9.

The solution to w is the polymorphic label e, which means that the current context is oblivious to whatever exceptions correspond to e when `g` is called, as discussed in Section 4.

6.3 Exactness-Dependent Types

It is possible to write a type-safe `PeekingIterator` using the `@exact` annotation, but we might want a peeking iterator that throws the same exceptions as its underlying iterator does. This expressiveness can be obtained by making `PeekingIterator` talk about the potential mismatch, as in the bottom definition of `PeekingIterator` in Figure 10. The class is now parameterized by the type of the iterator it decorates. In the using code below, PeekingIterator is instantiated on an inexact subtype of Iterator, so the compiler requires the exception to be handled:

```
try {
  PeekingIterator[Tokenizer,Token] pi =
    new PeekingIterator[Tokenizer,Token](...);
  while (pi.hasNext()) {... pi.peek() ...}
} catch (IOException e) {...}
```

Parameterized by a weak constraint, the Genus version of ObjectPool in Figure 9 is similarly exactness-dependent.

The special this keyword, when referring to the current object of a weak class, also has an exactness-dependent type—it is dependent on the exactness of the run-time class of the object with respect to the enclosing weak class. Since the run-time class is statically unknown, the compiler must assume that it can add arbitrary exceptions. Thus it results in a compiler warning to use this in ways that generate blame. However, we expect this to be rare: most weak types are interfaces, which do not normally use this.

7 Formalization

We formalize the new exception mechanism using a core calculus, CBC (for Checked Blame Calculus).

7.1 Syntax and Notations

Figure 12 defines both a *surface* and a *kernel* syntax for CBC. Programs are written in the surface syntax, and rewritten to and evaluated in the kernel calculus. Applications in surface terms are tagged by unique *blame labels* (ranged over by ℓ) representing lexical positions where blame can arise. Rewriting propagates these labels from the surface language to the kernel language, for blame tracking during kernel evaluation.

The surface syntax assumes a fixed set of exception names ranged over by E. The kernel syntax allows them to be labeled to form new names; E and E^ℓ are different names.

Surface types (τ) include weak types and strong types. A strong type (σ) is either the base type B or a function type $\tau \xrightarrow{\perp} [\sigma]_{\overline{E}}$ that does not allow mismatch against $\overline{E}$. (An overline denotes a (possibly empty) set.) A weak type $\tau \xrightarrow{\top} [\sigma]_{\overline{E}}$, on the other hand, tolerates mismatch. Notice that function return types must be strong to prevent blame from escaping. The same can be said about kernel types. Since there is an obvious injection (syntactic identity) from surface types to kernel types, we abuse notation by using τ and σ in places where kernel types T and S are expected.

Environments Γ contain mappings from variables to their types and exactness. Exactness is represented by sets of exceptions. Strong types enforce strict exactness, so their exactness is represented by $\varnothing$. The auxiliary function $\mathsf{K}(\cdot)$ returns $\perp$ for strong types and $\top$ for weak types; values of strong types must not escape. $\Gamma_\perp$ retains only the strongly-typed variables in Γ.

7.2 Semantics

Surface-to-kernel rewriting. Figure 13 defines the rewriting rules. The judgment $\Gamma; K \vdash f \rightsquigarrow e : [\tau]_{\overline{E}}$ translates the surface term f to the kernel term e, assigns f the type τ, and infers the exceptions $\overline{E}$ that evaluating e might raise.

Typing is dependent on K, which indicates whether the term in question is guaranteed not to escape. For example, the left-hand-side term in an application is type-checked with $K = \top$ as in R-APP-E and R-APP-I, while the body of an abstraction is type-checked with $K = \perp$ as in R-ABS. R-VAR-W denies first-class citizenship to weakly typed variables, and augments the return type if the environment indicates inexactness.

$$
\begin{array}{llcl}
\text{Surface terms} & f & ::= & b \mid x \mid \lambda x{:}\tau\,.\,f \mid \{f_1\ f_2\}^{\ell} \mid \\
 & & & \mathsf{let}\ x{:}\tau \leftarrow f_1\ \mathsf{in}\ f_2 \mid \\
 & & & \mathsf{throw}\ E \mid \mathsf{try}\ f_1\ \mathsf{catch}\ E \triangleright f_2 \\
\text{Surface types} & \tau & ::= & \sigma \mid \tau \xrightarrow{\top} [\sigma]_{\overline{E}} \\
\text{Surface strong types} & \sigma & ::= & B \mid \tau \xrightarrow{\bot} [\sigma]_{\overline{E}} \\
\text{Kernel terms} & e & ::= & b \mid x \mid \lambda x{:}T\,.\,e \mid e_1\ e_2 \mid \\
 & & & \mathsf{let}\ x{:}T \leftarrow e_1\ \mathsf{in}\ e_2 \mid \\
 & & & \mathsf{throw}\ U \mid \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U \triangleright e_2} \\
\text{Kernel types} & T & ::= & S \mid T \xrightarrow{\top} [S]_{\overline{U}} \\
\text{Kernel strong types} & S & ::= & B \mid T \xrightarrow{\bot} [S]_{\overline{U}} \\
\text{Kernel exceptions} & U & ::= & E \mid E^{\ell} \\
\text{Environments (surface)} & \Gamma & ::= & \varnothing \mid \Gamma,\ x{:}\tau\langle\overline{E}\rangle \\
\text{Environments (kernel)} & \Delta & ::= & \varnothing \mid \Delta,\ x{:}T\langle\overline{U}\rangle \\
\text{Escape kind} & K & ::= & \top \mid \bot
\end{array}
$$

$$
\begin{array}{ll}
\mathsf{K}\,(\sigma) = \bot & \Gamma_{\bot} = \{\, x{:}\sigma\langle\varnothing\rangle \mid x{:}\sigma\langle\varnothing\rangle \in \Gamma \,\} \\
\mathsf{K}\left(\tau \xrightarrow{\top} [\sigma]_{\overline{E}}\right) = \top & \Gamma_{\top} = \Gamma \\
\mathsf{K}\,(S) = \bot & \Delta_{\bot} = \{\, x{:}S\langle\varnothing\rangle \mid x{:}S\langle\varnothing\rangle \in \Delta \,\} \\
\mathsf{K}\left(T \xrightarrow{\top} [S]_{\overline{U}}\right) = \top & \Delta_{\top} = \Delta
\end{array}
$$

Figure 12: Syntax of CBC

Exception mismatch is computed using the subtyping relation $\lesssim_{\overline{E}}$, as in R-APP-I and R-LET. But only with R-APP-I can blame take effect, and its translation is consequently less obvious. To avoid exception capture, we need to give new names to exceptions involved in the mismatch. Therefore, the translation wraps the argument in an abstraction, which, when applied, catches any exceptions in the mismatch ($\overline{E}$) and rethrows their labeled versions ($\overline{E^{\ell}}$). The caller is aware of the exceptions $\overline{E}$, so it catches the labeled exceptions tunneled to its context and strips off their labels.

Semantics of the kernel calculus. Figure 14 defines the static semantics of the kernel, and Figure 15 gives the dynamic semantics. The static semantics is largely similar to the typing induced by rewriting, except that the kernel need not worry about exception capture. Hence, exception propagation happens in the usual way.

7.3 Type Safety

The type system guarantees that if a program can be typed without exceptional effects, it cannot get stuck when evaluated, or terminate in an exception. This guarantee easily follows from two other standard results.

First of all, the kernel type system is sound:

Theorem 1 (Kernel soundness: preservation and progress)

- *If* $\varnothing; K \vdash e : [T]_{\overline{U}}$ *and* $e \longrightarrow e'$ *then* $\varnothing; K \vdash e' : [T]_{\overline{U}}$.
- *If* $\varnothing; K \vdash e : [T]_{\overline{U}}$ *then either*
  1. $\exists v.\, e = v$, *or*
  2. $\exists U_0 \in \overline{U}.\, e = \mathsf{throw}\ U_0$, *or*
  3. $\exists e'.\, e \longrightarrow e'$.

$$\boxed{\Gamma; K \vdash f \ \rightsquigarrow\ e : [\tau]_{\overline{E}}}$$

[R-Const]

$$\frac{}{\Gamma; K \vdash b \ \rightsquigarrow\ b : [B]_{\varnothing}}$$

[R-Var-s]

$$\frac{x\!:\!\sigma\langle\varnothing\rangle \in \Gamma_K}{\Gamma; K \vdash x \ \rightsquigarrow\ x : [\sigma]_{\varnothing}}$$

[R-Var-w]

$$\frac{x\!:\!\tau \xrightarrow{\top} [\sigma]_{\overline{E_1}}\langle\overline{E_2}\rangle \in \Gamma_K}{\Gamma; K \vdash x \ \rightsquigarrow\ x : \left[\tau \xrightarrow{\top} [\sigma]_{\overline{E_1} \cup \overline{E_2}}\right]_{\varnothing}}$$

[R-Abs]

$$\frac{\Gamma_K,\ x\!:\!\tau\langle\varnothing\rangle; \bot \vdash f \ \rightsquigarrow\ e : [\sigma]_{\overline{E}}}{\Gamma; K \vdash \lambda x\!:\!\tau\,.\,f \ \rightsquigarrow\ \lambda x\!:\!\tau\,.\,e : \left[\tau \xrightarrow{\bot} [\sigma]_{\overline{E}}\right]_{\varnothing}}$$

[R-Let]

$$\frac{\Gamma; \mathsf{K}\,(\tau_1) \vdash f_1 \ \rightsquigarrow\ e_1 : [\tau_3]_{\overline{E_1}} \qquad \tau_3 \ \lesssim_{\overline{E}}\ \tau_1 \qquad \Gamma,\ x\!:\!\tau_1\langle\overline{E}\rangle; K \vdash f_2 \ \rightsquigarrow\ e_2 : [\tau_2]_{\overline{E_2}}}{\Gamma; K \vdash \mathsf{let}\ x\!:\!\tau_1 \leftarrow f_1\ \mathsf{in}\ f_2 \ \rightsquigarrow\ \mathsf{let}\ x\!:\!\tau_1 \leftarrow e_1\ \mathsf{in}\ e_2 : [\tau_2]_{\overline{E_1} \cup \overline{E_2}}}$$

[R-App-e]

$$\frac{\Gamma; \top \vdash f_1 \ \rightsquigarrow\ e_1 : \left[\tau_1 \xrightarrow{\top} [\sigma]_{\overline{E_3}}\right]_{\overline{E_1}} \qquad \Gamma; \mathsf{K}\,(\tau_1) \vdash f_2 \ \rightsquigarrow\ e_2 : [\tau_2]_{\overline{E_2}} \qquad \tau_2 \ \lesssim_{\emptyset}\ \tau_1}{\Gamma; K \vdash \{f_1\ f_2\}^{\ell} \ \rightsquigarrow\ e_1\ e_2 : [\sigma]_{\overline{E_1} \cup \overline{E_2} \cup \overline{E_3}}}$$

[R-App-i]

$$\frac{\Gamma; \top \vdash f_1 \ \rightsquigarrow\ e_1 : \left[\tau_1 \xrightarrow{\top} [\sigma_1]_{\overline{E_3}}\right]_{\overline{E_1}} \qquad \Gamma; \mathsf{K}\,(\tau_1) \vdash f_2 \ \rightsquigarrow\ e_2 : [\tau_2]_{\overline{E_2}} \qquad \tau_2 \ \lesssim_{\overline{E}}\ \tau_1 \qquad \tau_2 = \tau_{21} \xrightarrow{\top} [\sigma_{22}]_{\overline{E_4}}}{\Gamma; K \vdash \{f_1\ f_2\}^{\ell} \ \rightsquigarrow\ \mathsf{try}\ e_1\ \left(\begin{array}{l}\mathsf{let}\ x\!:\!\tau_2 \leftarrow e_2\ \mathsf{in} \\ \lambda y\!:\!\tau_{21}\,.\,\mathsf{try}\ x\ y\ \mathsf{catch}\ \overline{E \triangleright \mathsf{throw}\ E^{\ell}}\end{array}\right)\ \mathsf{catch}\ \overline{E^{\ell} \triangleright \mathsf{throw}\ E} : [\sigma_1]_{\overline{E_1} \cup \overline{E_2} \cup \overline{E_3} \cup \overline{E}}}$$

[R-Try-Catch]

$$\frac{\Gamma; K \vdash f_1 \ \rightsquigarrow\ e_1 : [\tau]_{\overline{E_1}} \qquad \Gamma; K \vdash f_2 \ \rightsquigarrow\ e_2 : [\tau]_{\overline{E_2}} \qquad E \in \overline{E_1}}{\Gamma; K \vdash \mathsf{try}\ f_1\ \mathsf{catch}\ E \triangleright f_2 \ \rightsquigarrow\ \mathsf{try}\ e_1\ \mathsf{catch}\ E \triangleright e_2 : [\tau]_{\left(\overline{E_1} \setminus \{E\}\right) \cup \overline{E_2}}}$$

[R-Throw]

$$\frac{}{\Gamma; K \vdash \mathsf{throw}\ E \ \rightsquigarrow\ \mathsf{throw}\ E : [\sigma]_{E}}$$

[R-Subsume]

$$\frac{\Gamma; K \vdash f \ \rightsquigarrow\ e : [\tau_1]_{\overline{E_1}} \qquad \tau_1 \ \lesssim_{\varnothing}\ \tau_2 \qquad \overline{E_1} \subseteq \overline{E_2}}{\Gamma; K \vdash f \ \rightsquigarrow\ e : [\tau_2]_{\overline{E_2}}}$$

$$\boxed{\tau_1 \ \lesssim_{\overline{E}}\ \tau_2}$$

[SS-B]

$$\frac{}{B \ \lesssim_{\varnothing}\ B}$$

[SS-S]

$$\frac{\begin{array}{c}\tau_2 \ \lesssim_{\varnothing}\ \tau_1 \qquad \sigma_1 \ \lesssim_{\varnothing}\ \sigma_2 \\ \overline{E_1} \subseteq \overline{E_2}\end{array}}{\tau_1 \xrightarrow{\bot} [\sigma_1]_{\overline{E_1}} \ \lesssim_{\varnothing}\ \tau_2 \xrightarrow{\bot} [\sigma_2]_{\overline{E_2}}}$$

[SS-W]

$$\frac{\tau_2 \ \lesssim_{\varnothing}\ \tau_1 \qquad \sigma_1 \ \lesssim_{\varnothing}\ \sigma_2}{\tau_1 \xrightarrow{K} [\sigma_1]_{\overline{E_1}} \ \lesssim_{\overline{E_1} \setminus \overline{E_2}}\ \tau_2 \xrightarrow{\top} [\sigma_2]_{\overline{E_2}}}$$

Figure 13: Surface-to-kernel rewriting

$$\boxed{\Delta; K \vdash e : [T]_{\overline{U}}}$$

[T-SUBSUME]

$$\frac{\Delta; K \vdash e : [T_1]_{\overline{U_1}} \qquad T_1 \lesssim_{\varnothing} T_2 \qquad \overline{U_1} \subseteq \overline{U_2}}{\Delta; K \vdash e : [T_2]_{\overline{U_2}}}$$

[T-CONST]

$$\frac{}{\Delta; K \vdash b : [B]_{\varnothing}}$$

[T-VAR-S]

$$\frac{x : S\langle \varnothing \rangle \in \Delta_K}{\Delta; K \vdash x : [S]_{\varnothing}}$$

[T-VAR-W]

$$\frac{x : T \xrightarrow{\top} [S]_{\overline{U_1}} \langle \overline{U_2} \rangle \in \Delta_K}{\Delta; K \vdash x : \left[T \xrightarrow{\top} [S]_{\overline{U_1} \cup \overline{U_2}}\right]_{\varnothing}}$$

[T-ABS]

$$\frac{\Delta_K,\ x : T\langle \varnothing \rangle; \bot \vdash e : [S]_{\overline{U}}}{\Delta; K \vdash \lambda x : T \,.\, e : \left[T \xrightarrow{\bot} [S]_{\overline{U}}\right]_{\varnothing}}$$

[T-APP]

$$\frac{\Delta; \top \vdash e_1 : \left[T_1 \xrightarrow{\top} [S]_{\overline{U_3}}\right]_{\overline{U_1}} \qquad \Delta; \mathsf{K}\,(T_1) \vdash e_2 : [T_2]_{\overline{U_2}} \qquad T_2 \lesssim_{\overline{U}} T_1}{\Delta; K \vdash e_1\ e_2 : [S]_{\overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U}}}$$

[T-LET]

$$\frac{\Delta; \mathsf{K}\,(T_1) \vdash e_1 : [T_3]_{\overline{U_1}} \qquad T_3 \lesssim_{\overline{U}} T_1 \qquad \Delta,\ x : T_1 \langle \overline{U} \rangle; K \vdash e_2 : [T_2]_{\overline{U_2}}}{\Delta; K \vdash \mathsf{let}\ x : T_1 \leftarrow e_1\ \mathsf{in}\ e_2 : [T_2]_{\overline{U_1} \cup \overline{U_2}}}$$

[T-THROW]

$$\frac{}{\Delta; K \vdash \mathsf{throw}\ U : [S]_U}$$

[T-TRY-CATCH]

$$\frac{\Delta; K \vdash e_1 : [T]_{\overline{U_1}} \qquad \Delta; K \vdash \overline{e_2} : [T]_{\overline{U_2}} \qquad \overline{U} \subseteq \overline{U_1}}{\Delta; K \vdash \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U \triangleright e_2} : [T]_{(\overline{U_1} \setminus \overline{U}) \cup \overline{U_2}}}$$

$$\boxed{T_1 \lesssim_{\overline{U}} T_2}$$

[KS-B]

$$\frac{}{B \lesssim_{\varnothing} B}$$

[KS-SS]

$$\frac{T_2 \lesssim_{\varnothing} T_1 \qquad S_1 \lesssim_{\varnothing} S_2 \qquad \overline{U_1} \subseteq \overline{U_2}}{T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}} \lesssim_{\varnothing} T_2 \xrightarrow{\bot} [S_2]_{\overline{U_2}}}$$

[KS-*W]

$$\frac{T_2 \lesssim_{\varnothing} T_1 \qquad S_1 \lesssim_{\varnothing} S_2}{T_1 \xrightarrow{K} [S_1]_{\overline{U_1}} \lesssim_{\overline{U_1} \setminus \overline{U_2}} T_2 \xrightarrow{\top} [S_2]_{\overline{U_2}}}$$

Figure 14: Static semantics of the kernel

$$\begin{array}{llcl} \text{Values} & v & ::= & k \mid \lambda x{:}T\,.\,e \\ \text{Evaluation contexts} & \mathcal{C} & ::= & [\cdot] \mid \mathcal{C}\;e \mid v\;\mathcal{C} \mid \mathsf{let}\;x{:}T \leftarrow \mathcal{C}\;\mathsf{in}\;e \mid \mathsf{try}\;\mathcal{C}\;\mathsf{catch}\;\overline{U \triangleright e} \end{array}$$

$$\boxed{e \longrightarrow e'}$$

$$[\textsc{Beta}] \qquad (\lambda x{:}T\,.\,e)\;v \longrightarrow e\{v/x\}$$

$$[\textsc{Let}] \qquad \mathsf{let}\;x{:}T \leftarrow v\;\mathsf{in}\;e \longrightarrow e\{v/x\}$$

$$[\textsc{Try}] \qquad \mathsf{try}\;v\;\mathsf{catch}\;\overline{U \triangleright e} \longrightarrow v$$

$$[\textsc{Catch}] \qquad \mathsf{try}\;\;(\mathsf{throw}\;U)\;\;\mathsf{catch}\;\;\ldots U \triangleright e \ldots \longrightarrow e$$

$$[\textsc{Throw}] \qquad \frac{\mathsf{throw}\;U \curvearrowright \mathcal{C}}{\mathcal{C}[\mathsf{throw}\;U] \longrightarrow \mathsf{throw}\;U}$$

$$[\textsc{Context}] \qquad \frac{e \longrightarrow e'}{\mathcal{C}[e] \longrightarrow \mathcal{C}[e']}$$

$$\boxed{\mathsf{throw}\;U \curvearrowright \mathcal{C}}$$

$$\mathsf{throw}\;U \curvearrowright [\cdot]\;e \qquad \mathsf{throw}\;U \curvearrowright v\;[\cdot] \qquad \mathsf{throw}\;U \curvearrowright \mathsf{let}\;x{:}T \leftarrow [\cdot]\;\mathsf{in}\;e \qquad \mathsf{throw}\;U_0 \curvearrowright \mathsf{try}\;[\cdot]\;\mathsf{catch}\;\overline{U \triangleright e}\;\;(U_0 \notin \overline{U})$$

Figure 15: Dynamic semantics of the kernel

Proof. This is proved in the usual way, by induction on the kernel typing derivation. See the appendix for details of this proof and of other formal results in this paper.

Second, the translation from the surface language to the kernel language is type-preserving:

Lemma 1 (Rewriting preserves types) *If* $\Gamma; K \vdash f \rightsquigarrow e : [\tau]_{\overline{E}}$ *then* $\Gamma; K \vdash e : [\tau]_{\overline{E}}$.

Proof. By induction on the derivation of the translation.

The guarantee that well-typed programs handle their exceptions is a direct corollary of these two previous results:

Corollary 1 (No uncaught exceptions) *If* $\varnothing; \bot \vdash f \rightsquigarrow e : [\tau]_{\varnothing}$ *then* $\exists v.\, e \longrightarrow^{*} v$.

8 Implementation

We have implemented the new exception mechanism for the Genus programming language. The implementation consists of about 5,800 lines of code, extending the compiler for the base Genus language [Zhang et al. 2015]. Genus is implemented using Polyglot [Nystrom et al. 2003], so code generation works by translating to Java code, using a Java compiler as a back end.

The rest of this section focuses on the translation into Java. The translation is guided by two goals: 1) it should prevent accidental capturing of exceptions, and 2) it should add negligible performance overhead to normal control flow.

```
class Blame extends RuntimeException {
  Throwable inner;
  Blame(Throwable t) { inner = t; }
  Throwable fillInStackTrace() { return this; }
} // represents exceptions in tunneling mode

class Failure extends RuntimeException {
  Throwable inner;
  Failure(Throwable t) { inner = t; }
} // represents exceptions in failure mode
```

Figure 16: The `Blame` and `Failure` classes in the Genus runtime

8.1 Representing Non-Checked Exceptions

Unlike exceptions in checked mode, exceptions traveling in tunneling mode or failure mode must acquire new identities to avoid accidental capturing. These identities are implemented by wrapping exceptions into objects of classes `Blame` and `Failure`(Figure 16). Both classes extend `RuntimeException`, but `Blame` does not collect a stack trace.

8.2 Translating Exception-Oblivious Code

Methods with weakly typed parameters ignore extra exceptions generated by them. To ensure they are handled in the appropriate context, weakly typed parameters, including the receiver, are accompanied by an additional `Blame` argument that serves as the blame label. When the actual argument is exact, the `Blame` argument is null. For example, the weak type `Iterator` has the following translation:

```
interface Iterator<E> {
 E next$Iterator(Blame b$) throws NoSuchElementException;
 ...
}
```

It receives a `Blame` object from its caller to accompany the weakly typed receiver. If an implementation of `Iterator` throws a mismatched exception, it is wrapped in this `Blame` object and tunneled through the code oblivious to it.

Exception-oblivious procedures are translated in a similar way. For example, the code generated for `map` (Figure 1) looks like the following:

```
<T,R> List<R> map(Function<T,R> f, List<T> src, Blame b$)
{ ... f.apply$Function(t, b$) ... }
```

The extra `Blame` argument `b$` is intended for potential mismatch in the argument function `f`, and is passed down to the method call `f.apply$Function(...)` so that exceptions from `f` have the right blame label.

8.3 Translating Exception-Aware Code

The definition of `Iterator`'s inexact subtype `Tokenizer` is exception-aware. Its translation is shown in the top of Figure 17. Per Java's type-checking rules, the overriding method in `Tokenizer` cannot throw extra checked exceptions. Instead, the overriding method `next$Iterator(Blame)` redirects to method `next()` that does the real work, possibly throwing an `IOException`, and then turns that exception into either a `Blame` or a `Failure`, which is unchecked. If the `Blame` argument is *not* null, there must be a program point ready to handle the exception. So the `IOException` is wrapped in the `Blame` object and is tunneled to that program point. If the `Blame` object *is* null, a `Failure` object wrapping the `IOException` is created and thrown. This might happen, for example, if the programmer chose to disregard the compiler warning reported for passing a `Tokenizer` (Figure 7) into the constructor of `PeekingIterator` (top of Figure 10).

```
class Tokenizer implements Iterator<Token> {
  Token next$Iterator(Blame b$)
        throws NoSuchElementException {
    try { return next(); }
    catch (IOException e) {
      if (b$ != null) { b$.inner = e; throw b$; }
      else throw new Failure(e);
    }
  }
  Token next() throws IOException,
                      NoSuchElementException {...}
  ...
}

Blame b$ = null;
try {
 Iterator<Token> iter = new Tokenizer(reader);
 while (true) {
   Token t;
   try { t = iter.next$Iterator(b$=Thread.borrowB$()); }
   catch (NoSuchElementException e) { break; }
   catch (Blame bCaught$) {
     if (bCaught$ == b$) { log.write(...); continue; }
     else throw bCaught$;
   }
 }
} finally { Thread.handbackB$(b$); }
```

Figure 17: Translating exception-aware code (Tokenizer from Figure 7 and its using code from Figure 8) into Java

The code in Figure 8 is also exception-aware, and Figure 17 (bottom) shows its translation. Instead of creating a new Blame object every time a mismatch happens, each thread maintains a Blame object pool that recycles Blame objects. A Blame object is borrowed from the pool at the blamable program point in the try block, and is returned in the finally block. The catch block catches any Blame, but *only* executes the exception handling code if the Blame caught is indeed the one associated with the blamable program point; otherwise it rethrows the Blame.

Aggressive interweaving of try–catch in method bodies might preclude certain compiler optimizations. Therefore, the translation uses a simple program analysis to identify existing enclosing try blocks onto which these new catch blocks can be attached.

8.4 Translating Failure Exceptions

A method body is wrapped in a try block whose corresponding catch block catches all exceptions that will switch to failure mode after exiting the method. A catch all block is translated into possibly multiple catch blocks to also catch compatible Failure and Blame objects.

9 Evaluation

The aim of our evaluation is to explore the expressiveness of the new exception mechanism, and its overhead with respect to both performance and notational burden.

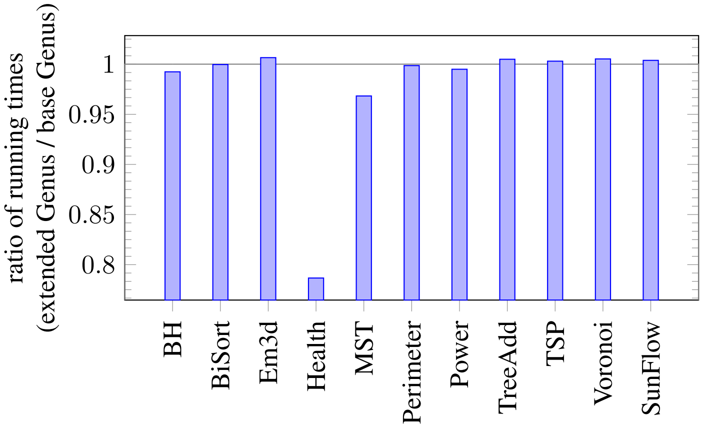


Figure 18: Performance of the exception mechanism on the JOlden benchmarks and SunFlow.

9.1 Porting Java Code to Use Genus Exceptions

To evaluate the expressive power of the new exception mechanism, we ported various existing Java programs and libraries into Genus. Some of this code (`ObjectPool` and `PeekingIterator`) is described earlier, but we examined some larger code samples:

- We ported the Java Collections Framework and found that no `@exact` annotations were needed. In addition, the Genus compiler found unreachable code in `AbstractSequentialList`, thanks to fail-by-default exceptions propagating in checked mode (Section 3.3).
- We ported the `javac` visitor of Figure 3 into Genus, as mentioned earlier in Section 4.1. Conversion to the new exception mechanism allows more than 200 lines of code to be removed from class `Pretty` (~1,000 LOC), and more importantly, restores static checking.
- Using only checked exceptions, we managed to reimplement the `EasyIO` text parsing package[3] that was developed for a Cornell programming course. This codebase (~1,000 LOC) uses exceptions heavily for backtracking.

9.2 Performance

The current Genus implementation targets Java. We explored its performance through several experiments. All data were collected using Java 8 on a 3.4GHz Intel Core i7 processor after warming up the HotSpot JVM.

Performance of normal-case code. Perhaps the most important performance consideration for an exception mechanism is whether it slows down normal-case code. To evaluate this, we ported Java code that only uses exceptions lightly—specifically, the JOlden benchmarks [Cahoon and McKinley 2001] and, representing larger applications, the SunFlow benchmark [SunFlow] from the DaCapo suite [Blackburn et al. 2006]. SunFlow is a ray tracer containing ~200 classes and ~21K LOC.

To evaluate the overhead of the new exception mechanism fairly, we compared the running times of this code between the extended Genus language (Section 8) and the base Genus language. Despite support for reified generics, the performance under base Genus is close to Java: compared to Java, it incurred a slowdown of 0.3%.

[3] `www.cs.cornell.edu/courses/cs2112/2015fa/#Libraries`

Java w/ stack	Genus	Java w/o stack
7.19	1.16	1.16

Table 1: Performance with `EasyIO` (s)

mechanism	exception objects	time (ns)
Java exceptions	new instance	817.7
	cached	124.7
Java unchecked wrappers	cached	826.0
Genus exceptions (tunneled)	new instance	139.8
	cached	128.6

Table 2: Exception tunneling microbenchmarks

Figure 18 reports the results of this comparison. Each reported measurement averages at least 20 runs, with a standard error less than 1.2%. Benchmark parameters were set so that each run took more than 30 seconds.

Overall, the extended compiler generates slightly faster code than the original compiler. The average speedup is 2.4%, though performance varies by benchmark. The speedup is caused largely by the exception-based translation of enhanced `for` loops.

Performance of exception-heavy code. To evaluate the performance improvement that Genus obtains by avoiding stack-trace collection on realistic code, we measured the running times of pattern-matching regular expressions using the `EasyIO` package. It makes heavy use of exceptions to control search. The running times are shown in Table 1. Each number averages 10 runs, with 6.2M exceptions thrown in each run.

Stack-trace collection in Java causes more than 6× overhead compared to Genus exceptions. Genus targets Java, so it is not surprising that similar performance can be achieved with Java, if stack-trace collection is turned off and exception objects are cached.

Exception tunneling performance. We used a microbenchmark to explore the overhead of exception tunneling, the key difference between Genus and Java. The microbenchmark performs method calls on objects passed down from a higher-order function; the method call either throws an exception or returns immediately, and is prevented from being inlined. Since there is no other work done by the method, performance differences are magnified.

The results are shown in Table 2. We compare exception tunneling in Genus to two (unsafe) Java workarounds: typical Java exceptions and unchecked-exception wrappers. We also refine the comparison for typical Java exceptions and tunneled Genus exceptions based on whether a new exception object is created for each `throw`; throwing a single cached instance is reasonable for non-failure exceptions carrying no extra information. Each number averages 20 runs, with a standard error less than 0.6% of the mean.

The rightmost column measures time to exit via an exception. Genus exceptions perform well because they do not collect a stack trace. The slowdown compared to the second row is mostly because exception mismatch requires `borrow` and `handback` calls (Section 8.3) in every loop iteration.

On the other hand, Genus exceptions significantly outperform unchecked wrappers, the safest way to tunnel in Java. Java's performance is poor here because an unchecked-exception object is created for each raised exception, whereas the implementation of Genus recycles `Blame` objects.

The microbenchmark also measures the time to return from a method normally. The average cost of a call and return in Java was 6.0 ns. In the absence of mismatch, our translation adds the overhead of passing a null pointer to the normal return path, increasing the cost slightly to 6.3 ns. The results in Figure 18 suggest this increase is negligible in practice.

10 Related Work

Notable approaches to exceptions. PL/I was the first language with exceptions. It supports user-defined exceptions with exception handlers dynamically bound to exceptions [MacLaren 1977]. Goodenough [1975] and Liskov and Snyder [1979] introduced statically scoped handlers. CLU [Liskov and Snyder 1979] was the first language to support some static checking of exceptions. Exceptions are declared in function signatures, and thrown exceptions must appear in these signatures. If not explicitly resignaled, propagating exceptions automatically convert to failure. Mesa [Mitchell et al. 1978] supports both termination- and resumption-style exceptions but does not check them statically. Ada [Ada 95] attaches handlers to blocks, procedures, or packages. Unhandled exceptions propagate automatically, but exceptions are not declared. Eiffel [Meyer 1992] exceptions originate from the violation of assertions and are raised implicitly. Upon exceptions, programmers can *retry* the execution with different parameters; otherwise, exceptions implicitly propagate to callers. Modula-3 [Nelson 1991] introduced fully statically checked exceptions. Black [1982] and Garcia et al. [2001] present comparative studies of some exception mechanisms.

Empirical findings. An empirical study by Cabral and Marques [2007a] shows that in Java and .NET exception handlers are not specialized enough to allow effective handling, which we believe is partly attributable to a lack of documentation of exceptions in the .NET case [Cabral and Marques 2007b] and the rigidity of checked exceptions in the Java case. Robillard and Murphy [2000] identify the global reasoning required of programmers as a major reason why exceptions are hard to use.

Exception analysis. Function types in functional languages such as ML and Haskell do not include exceptions because they would interfere with the use of higher-order functions. Exception capture can be avoided in SML [Milner et al. 1990] because exception types are generative, but other variants of ML lack this feature. Leroy and Pessaux [2000] observe that uncaught exceptions are the most common failure mode of large ML applications, motivating them and others [Fähndrich et al. 1998] to develop program analyses to infer exceptions. Such analyses can be helpful, especially for usage studies [Weimer and Necula 2008], but they necessarily involve trading off performance and precision, and entail non-local reasoning that does not aid programmers in reasoning about their code. A benefit of our mechanism is that it is likely to lead to more accurate, scalable static analyses, because precise exceptions largely remove the need to approximate exceptional control flow [Sinha and Harrold 2000; Bravenboer and Smaragdakis 2009].

Exception polymorphism. Some recent designs attempt to address the rigidity of checked exceptions through exception polymorphism: anchored exceptions [van Dooren and Steegmans 2005] as a Java extension, polymorphic effects [Rytz et al. 2012] as a Scala extension, row-polymorphic effect types in the Koka language [Leijen 2014], and the `rethrows` clause introduced in Swift 2 [Swift 2014]. These approaches add annotation burden to exception-oblivious code; more fundamentally, they do not address exception capture.

Blame tracking. Our notion of blame is related to that introduced by work on contracts [Findler and Felleisen 2002] and further developed by work on gradual typing [Wadler and Findler 2009], in which blame indicates where fault lies in the event of a contract violation (or cast failure). In our setting, an exception mismatch results in static blame being assigned that indicates where "fault" lies should an exception arise at run time, with the compiler statically checking that the "faulty" program point handles exceptions.

Blame has polarity [Findler and Felleisen 2002; Wadler and Findler 2009]; in our setting, exception mismatch at covariant (or contravariant) positions gives rise to positive (or negative) blame. Existing mechanisms for checked exceptions only consider positive blame; exceptions with negative blame are the missing piece. By contrast, in Genus both kinds of exceptions are subject to static checking, and our implementation and formalization manifest their difference: exceptions with negative blame acquire new identities to achieve safe tunneling.

Type inference. Our exactness inference bears resemblance to work on gradual type inference [Rastogi et al. 2012]. Both inference algorithms encode escape analysis "for free", although the motivation for escape analysis differs. Our approach to default exactness (Section 6.1) is similar to default region annotations in a region-based type system [Grossman et al. 2002].

11 Conclusions

Our new exception mechanism combines the benefits of static checking with the flexibility of unchecked exceptions. We were guided in the design of this mechanism by thinking carefully about the goals of an exception mechanism, by much previous work, and by many discussions found online. Our formal results and experience suggest that our approach improves assurance that exceptions are handled. The evaluation shows that the mechanism works well on real code. It adds negligible cost when exceptions are not being used; exception tunneling comes with a small performance penalty that appears to be more than offset in practice by avoiding the run-time overhead of wrapping exceptions. We hope this work helps programmers use exceptions in a principled way and gives language implementers an incentive to make exceptions more efficient.

Acknowledgments

We thank Owen Arden, Chinawat Isradisaikul, Jed Liu, Michael Hicks, and the anonymous reviewers for discussions about exception mechanisms and for suggestions about this paper. This research has been supported by MURI grant FA9550-12-1-0400, administered by the US Air Force, by NSF grant 1513797, and by the European Research Council, grant No. 321217. This paper does not necessarily reflect the views of any of its sponsors.

References

Ada 95. Ada 95 reference manual: language and standard libraries, 1997.

Apache Commons. The Apache Commons project. `https://commons.apache.org/`.

A. P. Black. *Exception handling: The case against*. PhD thesis, University of Oxford, 1982.

S. M. Blackburn, R. Garner, C. Hoffmann, A. M. Khang, K. S. McKinley, R. Bentzur, A. Diwan, D. Feinberg, D. Frampton, S. Z. Guyer, M. Hirzel, A. Hosking, M. Jump, H. Lee, J. E. B. Moss, A. Phansalkar, D. Stefanović, T. VanDrunen, D. von Dincklage, and B. Wiedermann. The DaCapo benchmarks: Java benchmarking development and analysis. In *21st ACM SIGPLAN Conf. on Object-Oriented Programming, Systems, Languages and Applications (OOPSLA)*, pages 169–190, 2006.

M. Bravenboer and Y. Smaragdakis. Exception analysis and points-to analysis: Better together. In *Proceedings of the Eighteenth International Symposium on Software Testing and Analysis*, ISSTA '09, 2009.

P. A. Buhr and W. Y. R. Mok. Advanced exception handling mechanisms. *IEEE Trans. Softw. Eng.*, 26(9), Sept. 2000.

B. Cabral and P. Marques. Exception handling: A field study in Java and .NET. In *21st European Conf. on Object-Oriented Programming*, pages 151–175, 2007a.

B. Cabral and P. Marques. Hidden truth behind .NET's exception handling today. *IET Software*, 1(6), 2007b.

B. Cahoon and K. S. McKinley. Data flow analysis for software prefetching linked data structures in Java. In *International Conference on Parallel Architectures and Compilation Techniques (PACT)*, Sept. 2001.

CWE. Common weakness enumeration list. `http://cwe.mitre.org/data/`.

B. Eckel. Does Java need checked exceptions? `http://www.mindview.net/Etc/Discussions/CheckedExceptions`, 2003.

M. Fähndrich, J. S. Foster, A. Aiken, and J. Cu. Tracking down exceptions in standard ML programs. Technical report, EECS Department, UC Berkeley, 1998.

FindBugs bug descriptions. Findbugs bug descriptions. `http://findbugs.sourceforge.net/bugDescriptions.html/`.

R. B. Findler and M. Felleisen. Contracts for higher-order functions. In *7th ACM SIGPLAN Int'l Conf. on Functional Programming*, 2002.

R. B. Findler, M. Latendresse, and M. Felleisen. Behavioral contracts and behavioral subtyping. In *ESEC/FSE-9*, 2001.

A. F. Garcia, C. M. Rubira, A. Romanovsky, and J. Xu. A comparative study of exception handling mechanisms for building dependable object-oriented software. *Journal of Systems and Software*, 2001.

B. Goetz. Java theory and practice: The exceptions debate. `http://www.ibm.com/developerworks/library/j-jtp05254`, 2004.

B. Goetz. Exception transparency in Java. `http://blogs.oracle.com/briangoetz/entry/exception_transparency_in_java`, 2010.

J. B. Goodenough. Exception handling: Issues and a proposed notation. *Comm. of the ACM*, 18:683–696, Dec. 1975.

J. Gosling, B. Joy, G. Steele, and G. Bracha. *The Java Language Specification*. Addison Wesley, 3rd edition, 2005. ISBN 0321246780.

D. Grossman, G. Morrisett, T. Jim, M. Hicks, Y. Wang, and J. Cheney. Region-based memory management in Cyclone. In *ACM SIGPLAN Conf. on Programming Language Design and Implementation (PLDI)*, pages 282–293, 2002.

A. Hejlsberg, B. Venners, and B. Eckel. Remaining neutral on checked exceptions. `http://www.artima.com/intv/handcuffs.html`, 2003a.

A. Hejlsberg, S. Wiltamuth, and P. Golde. *The C# Programming Language*. Addison-Wesley, 1st edition, Oct. 2003b. ISBN 0321154916.

D. Leijen. Koka: Programming with row polymorphic effect types. In *5th Workshop on Mathematically Structured Functional Programming*, 2014.

X. Leroy and F. Pessaux. Type-based analysis of uncaught exceptions. *ACM Trans. on Programming Languages and Systems*, 22(2), Mar. 2000.

B. Liskov and A. Snyder. Exception handling in CLU. *IEEE Transactions on Software Engineering*, SE-5 (6):546–558, Nov. 1979.

B. Liskov and J. Wing. A behavioral notion of subtyping. *ACM Trans. on Programming Languages and Systems*, 16(6):1811–1841, Nov. 1994.

M. D. MacLaren. Exception handling in PL/I. In *Proceedings of an ACM Conference on Language Design for Reliable Software*, 1977.

L. Mastrangelo, L. Ponzanelli, A. Mocci, M. Lanza, M. Hauswirth, and N. Nystrom. Use at your own risk: The Java Unsafe API in the wild. In *2015 ACM SIGPLAN Conf. on Object-Oriented Programming, Systems, Languages and Applications (OOPSLA)*, 2015.

B. Meyer. *Eiffel: The Language*. 1992.

Microsoft Corporation. *Microsoft C# Language Specifications*. Microsoft Press, 2001. ISBN 0-7356-1448-2.

R. Milner, M. Tofte, and R. Harper. *The Definition of Standard ML*. MIT Press, Cambridge, MA, 1990.

J. G. Mitchell, W. Maybury, and R. Sweet. Mesa language manual. Technical Report CSL-78-1, Xerox Research Center, Palo Alto, CA, Feb. 1978.

G. Nelson, editor. *Systems Programming with Modula-3*. Prentice-Hall, 1991.

N. Nystrom, M. R. Clarkson, and A. C. Myers. Polyglot: An extensible compiler framework for Java. In *12th Int'l Conf. on Compiler Construction (CC'03)*, pages 138–152, Apr. 2003.

M. Odersky. *The Scala Language Specification*. EPFL, 2014. Version 2.9.

OpenJDK javac. The `javac` compiler. `http://hg.openjdk.java.net/`.

S. Peyton Jones. *Haskell 98 language and libraries: the revised report*. Cambridge University Press, 2003.

A. Rastogi, A. Chaudhuri, and B. Hosmer. The ins and outs of gradual type inference. In *39th ACM Symp. on Principles of Programming Languages (POPL)*, 2012.

M. P. Robillard and G. C. Murphy. Designing robust Java programs with exceptions. In *Proceedings of the 8th ACM SIGSOFT International Symposium on Foundations of Software Engineering (FSE-8)*, 2000.

L. Rytz, M. Odersky, and P. Haller. Lightweight polymorphic effects. In *26th European Conf. on Object-Oriented Programming*, 2012.

S. Sinha and M. J. Harrold. Analysis and testing of programs with exception handling constructs. *IEEE Transactions on Software Engineering*, 26(9), Sept. 2000.

G. L. Steele, Jr. *Common LISP: the Language*. Digital Press, second edition, 1990. ISBN 1-55558-041-6.

B. Stroustrup. *The C++ Programming Language*. Addison-Wesley, 1987.

SunFlow. SunFlow: the open-source rendering engine. Open-source software, 2007.

Swift 2014. Swift programming language. `https://developer.apple.com/swift/resources`, 2014.

M. van Dooren and E. Steegmans. Combining the robustness of checked exceptions with the flexibility of unchecked exceptions using anchored exception declarations. In *20th ACM SIGPLAN Conf. on Object-Oriented Programming, Systems, Languages and Applications (OOPSLA)*, 2005.

B. Venners. Failure and exceptions: A conversation with James Gosling, Part II. `http://www.artima.com/intv/solid.html`, 2003.

P. Wadler and S. Blott. How to make ad-hoc polymorphism less ad hoc. In *16th ACM Symp. on Principles of Programming Languages (POPL)*, 1989.

P. Wadler and R. B. Findler. Well-typed programs can't be blamed. In *European Symposium on Programming*, 2009.

R. Waldhoff. Java's checked exceptions were a mistake. `http://radio-weblogs.com/0122027/stories/2003/04/01/JavasCheckedExceptionsWereAMistake.html`, 2003.

W. Weimer and G. C. Necula. Exceptional situations and program reliability. *ACM Trans. on Programming Languages and Systems*, 30(2), Mar. 2008.

Y. Zhang, M. C. Loring, G. Salvaneschi, B. Liskov, and A. C. Myers. Lightweight, flexible object-oriented generics. In *36th ACM SIGPLAN Conf. on Programming Language Design and Implementation (PLDI)*, pages 436–445, June 2015.

A Proving CBC Type-Safe

This appendix proves the main results of Section 7.

Theorem 1 (Kernel soundness: preservation and progress)

- *If* $\varnothing; K \vdash e : [T]_{\overline{U}}$ *and* $e \longrightarrow e'$ *then* $\varnothing; K \vdash e' : [T]_{\overline{U}}$.
- *If* $\varnothing; K \vdash e : [T]_{\overline{U}}$ *then either*
  1. $\exists v.\, e = v$, *or*
  2. $\exists U_0 \in \overline{U}.\, e = \mathsf{throw}\ U_0$, *or*
  3. $\exists e'.\, e \longrightarrow e'$.

Proof. We prove preservation first.

Case 1 $e = (\lambda x : T_1 \,.\, e_1)\ v$ and $e \longrightarrow e_1\{v/x\}$ via BETA. The typing derivation of e looks like the following:

$$\dfrac{\dfrac{x\!:\!T_1\langle\varnothing\rangle; \bot \vdash e_1 : [S_1]_{\overline{U_1}}}{\varnothing; \top \vdash \lambda x\!:\!T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}\right]_{\varnothing}} \qquad \varnothing; \mathsf{K}\,(T_1) \vdash v : [S_2]_{\varnothing} \qquad S_2 \ \lesssim_{\overline{U_2}}\ T_1}{\varnothing; K \vdash (\lambda x\!:\!T_1 \,.\, e_1)\ v : [S_1]_{\overline{U_1} \cup \overline{U_2}}}$$

Given the above type judgment for e_1, by Lemma 3, we have $x\!:\!T_1\langle\overline{U_2}\rangle; \bot \vdash e_1 : [S_1]_{\overline{U_1}\cup\overline{U_2}}$. By Lemma 2, $\varnothing; \bot \vdash e_1\{v/x\} : [S_1]_{\overline{U_1}\cup\overline{U_2}}$. By Lemma 5, $\varnothing; K \vdash e_1\{v/x\} : [S_1]_{\overline{U_1}\cup\overline{U_2}}$.

Case 2 $e = (\mathsf{let}\ x : T_1 \leftarrow v\ \mathsf{in}\ e_1)$ and $e \longrightarrow e_1\{v/x\}$ via LET. The typing derivation of e looks like the following:

$$\dfrac{\varnothing; \mathsf{K}\,(T_1) \vdash v : [S_1]_{\varnothing} \qquad x\!:\!T_1\langle\overline{U_1}\rangle; K \vdash e_1 : [T]_{\overline{U}} \qquad S_1 \ \lesssim_{\overline{U_1}}\ T_1}{\varnothing; K \vdash \mathsf{let}\ x\!:\!T_1 \leftarrow v\ \mathsf{in}\ e_1 : [T]_{\overline{U}}}$$

By Lemma 2, $\varnothing; K \vdash e_1\{v/x\} : [T]_{\overline{U}}$.

Case 3 $e = \mathsf{try}\ v\ \mathsf{catch}\ \overline{U' \triangleright e_2}$ and $e \longrightarrow v$ via TRY. The typing derivation of e looks like the following:

$$\dfrac{\varnothing; K \vdash v : [T]_{\varnothing} \qquad \varnothing; K \vdash e_2 : [T]_{\overline{U_2}} \qquad \varnothing \subseteq \overline{U'}}{\varnothing; K \vdash \mathsf{try}\ v\ \mathsf{catch}\ \overline{U' \triangleright e_2} : [T]_{\overline{U_2}}}$$

So $\varnothing; K \vdash v : [T]_{\overline{U_2}}$ follows from T-SUBSUME.

Case 4 $e = \mathsf{try}\ \mathsf{throw}\ U'\ \mathsf{catch}\ \ldots U' \triangleright e' \ldots$ and $e \longrightarrow e'$ via CATCH. The typing derivation of e looks like the following

$$\dfrac{\varnothing; K \vdash \mathsf{throw}\ U' : [T]_{U'} \qquad \varnothing; K \vdash e' : [T]_{\overline{U_2}} \qquad \cdots}{\varnothing; K \vdash \mathsf{try}\ \mathsf{throw}\ U'\ \mathsf{catch}\ \ldots U' \triangleright e' \ldots : [T]_{\overline{U_2}}}$$

The result follows.

Case 5 $e = \mathcal{C}[\mathsf{throw}\ U']$ where $\mathsf{throw}\ U' \curvearrowright \mathcal{C}$, and thus $e \longrightarrow \mathsf{throw}\ U'$ via THROW. By Lemma 6, $U' \in \overline{U}$. Additionally, there exists S such that $S \ \lesssim_{\varnothing}\ T$ and $\Delta; K \vdash \mathsf{throw}\ U' : [S]_{U'}$. Thus the result follows from T-SUBSUME.

Case 6 $e = \mathcal{C}[e_0]$ and $e_0 \longrightarrow e_0'$, and thus $e \longrightarrow \mathcal{C}[e_0']$ via CONTEXT. In this case, the result can be obtained by induction on the typing derivation of e.

Next we prove progress, by induction on the typing derivation of e.

Case 1 $e = b$. The result is trivial.

Case 2 $e = x$. This case is impossible because the environment contains no mapping for x.

Case 3 $e = \lambda x : T_1 \,.\, e_1$. The result is trivial.

Case 4 $e = e_1\ e_2$. In this case, since e has a typing derivation, we know there are typing derivations for e_1 and e_2 as well. The induction hypotheses they provide say that each of e_1 and e_2 is either a value, a throw, or a term that is able to take a step.

Case 4.1 e_1 is a value.

Case 4.1.1 e_2 is a value. In this case e_1 must be an abstraction. BETA allows e to step.

Case 4.1.2 e_2 is a throw. THROW allows e to step.

Case 4.1.3 e_2 steps. CONTEXT allows e to step.

Case 4.2 e_1 is a throw. THROW allows e to step.

Case 4.3 e_1 steps. CONTEXT allows e to step.

Case 5 $e = \mathsf{let}\ x : T_1 \leftarrow e_1\ \mathsf{in}\ e_2$. By induction, we know e_1 is either a value, a throw, or a term that is able to take a step.

Case 5.1 e_1 is a value. LET allows e to step.

Case 5.2 e_2 is a throw. THROW allows e to step.

Case 5.3 e_1 steps. CONTEXT allows e to step.

Case 6 $e = \mathsf{throw}\ U_0$. The result trivially holds.

Case 7 $e = \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U' \triangleright e_2}$. By induction, we know e_1 is either a value, a throw, or a term that is able to take a step.

Case 7.1 e_1 is a value. TRY allows e to step.

Case 7.2 $e_2 = \mathsf{throw}\ U''$.

Case 7.2.1 $U'' \in \overline{U'}$. CATCH allows e to step.

Case 7.2.2 $U'' \notin \overline{U'}$. THROW allows e to step.

Case 7.3 e_1 steps. CONTEXT allows e to step.

Lemma 2 (Substitution Lemma)

$$\Delta, x : T\langle\overline{U}\rangle ; K \vdash e : [T']_{\overline{U'}} \ \wedge\ \varnothing ; K' \vdash v : [S]_{\varnothing} \ \wedge\ S \lesssim_{\overline{U}} T \implies \Delta ; K \vdash e\{v/x\} : [T']_{\overline{U'}}$$

Proof. By induction on the type derivation of e.

Case 1 $e = b$. The result is trivial in this case.

Case 2 e is a variable.

Case 2.1 $e = x$. Then $e\{v/x\} = v$.

Case 2.1.1 T is strong. In this case the type derivation of e is obtained via T-VAR-S. So $T' = S$ and $\overline{U'} = \varnothing$. Given the type judgment for v, we can obtain $\varnothing; K \vdash e\{v/x\} : [S]_{\varnothing}$ by Lemma 4. The result then follows by weakening the variable environment.

Case 2.1.2 T is weak. In this case the type derivation of e looks like the following

$$\frac{x : T_1 \xrightarrow{\top} [S_1]_{\overline{U_1}} \langle \overline{U} \rangle \in \left(\Delta,\ x : T_1 \xrightarrow{\top} [S_1]_{\overline{U_1}} \langle \overline{U} \rangle\right)_K}{\Delta,\ x : T_1 \xrightarrow{\top} [S_1]_{\overline{U_1}} \langle \overline{U} \rangle; K \vdash x : T_1 \xrightarrow{\top} [S_1]_{\overline{U_1} \cup \overline{U}}}$$

So $T = T_1 \xrightarrow{\top} [S_1]_{\overline{U_1}}$, $T' = T_1 \xrightarrow{\top} [S_1]_{\overline{U_1} \cup \overline{U}}$, and $\overline{U'} = \varnothing$.
By $S \lesssim_{\overline{U}} T$ and $\varnothing; K' \vdash v : [S]_{\varnothing}$, we have $\varnothing; K' \vdash v : \left[T_1 \xrightarrow{\top} [S_1]_{\overline{U_1} \cup \overline{U}}\right]_{\varnothing}$.
Thus $\varnothing; K \vdash v : \left[T_1 \xrightarrow{\top} [S_1]_{\overline{U_1} \cup \overline{U}}\right]_{\varnothing}$ by Lemma 4.
The result then follows by weakening the variable environment.

Case 2.2 $e = y \neq x$. Then $e\{v/x\} = e$. Therefore the result trivially holds.

Case 3 $e = \lambda y : T_1 \,.\, e_1$. The type derivation of e looks like the following

$$\frac{\Delta,\ x : T \langle \overline{U} \rangle,\ y : T_1 \langle \varnothing \rangle; K \vdash e_1 : [S_1]_{\overline{U_1}}}{\Delta,\ x : T \langle \overline{U} \rangle; K \vdash \lambda y : T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}\right]_{\varnothing}}$$

So $T' = T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}$ and $\overline{U'} = \varnothing$.

Case 3.1 $x = y$. In this case $e_1\{v/x\} = e_1$ and $\Delta,\ x : T \langle \overline{U} \rangle,\ y : T_1 \langle \varnothing \rangle = \Delta,\ y : T_1 \langle \varnothing \rangle$. So we have

$$\Delta,\ y : T_1 \langle \varnothing \rangle; K \vdash e_1\{v/x\} : [S_1]_{\overline{U_1}}$$

This implies the result via T-ABS.

Case 3.2 $x \neq y$. In this case the type judgment for e_1 gives the following induction hypothesis

$$\Delta,\ y : T_1 \langle \varnothing \rangle; K \vdash e_1\{v/x\} : [S_1]_{\overline{U_1}}$$

This implies the result via T-ABS.

Case 4 $e = e_1\ e_2$. The type derivation of e looks like the following

$$\frac{\Delta,\ x : T \langle \overline{U} \rangle; \top \vdash e_1 : \left[T_1 \xrightarrow{\top} [S']_{\overline{U_3}}\right]_{\overline{U_1}} \qquad \Delta,\ x : T \langle \overline{U} \rangle; \mathsf{K}\,(T_1) \vdash e_2 : [T_2]_{\overline{U_2}} \qquad T_2 \lesssim_{\overline{U''}} T_1}{\Delta,\ x : T \langle \overline{U} \rangle; K \vdash e_1\ e_2 : [S']_{\overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U''}}}$$

So $T' = S'$ and $\overline{U'} = \overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U''}$. The induction hypotheses are

$$\Delta; \top \vdash e_1\{v/x\} : \left[T_1 \xrightarrow{\top} [S']_{\overline{U_3}}\right]_{\overline{U_1}} \quad \text{and} \quad \Delta; \mathsf{K}\,(T_1) \vdash e_2\{v/x\} : [T_2]_{\overline{U_2}}$$

which imply

$$\Delta; K \vdash e_1\{v/x\}\ e_2\{v/x\} : [S']_{\overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U''}}$$

via T-APP.

Case 5 $e = \mathsf{let}\ y\!:\!T_1 \leftarrow e_1\ \mathsf{in}\ e_2$. The type derivation of e looks like the following

$$\frac{T_3 \ \lesssim_{\overline{U''}}\ T_1 \qquad \begin{array}{c}\Delta,\ x\!:\!T\langle\overline{U}\rangle; \mathsf{K}\,(T_1) \vdash e_1 : [T_3]_{\overline{U_1}} \\ \Delta,\ x\!:\!T\langle\overline{U}\rangle,\ y\!:\!T_1\langle\overline{U''}\rangle; \mathsf{K}\,(T_1) \vdash e_2 : [T_2]_{\overline{U_2}}\end{array}}{\Delta,\ x\!:\!T\langle\overline{U}\rangle; K \vdash \mathsf{let}\ y\!:\!T_1 \leftarrow e_1\ \mathsf{in}\ e_2 : [T_2]_{\overline{U_1}\cup\overline{U_2}}}$$

So $T' = T_2$ and $\overline{U'} = \overline{U_1} \cup \overline{U_2}$.
The type judgment for e_1 gives the following induction hypothesis

$$\Delta; \mathsf{K}\,(T_1) \vdash e_1\{v/x\} : [T_3]_{\overline{U_1}}$$

Case 5.1 $x = y$. In this case $e_2\{x/v\} = e_2$ and $\Delta,\ x\!:\!T\langle\overline{U}\rangle,\ y\!:\!T_1\langle\overline{U''}\rangle = \Delta,\ y\!:\!T_1\langle\overline{U''}\rangle$. So we have

$$\Delta,\ y\!:\!T_1\langle\overline{U''}\rangle; \mathsf{K}\,(T_1) \vdash e_2\{v/x\} : [T_2]_{\overline{U_2}}$$

Thus the result follows from T-Let.

Case 5.2 $x \neq y$. In this case the type judgment for e_2 gives the following induction hypothesis

$$\Delta,\ y\!:\!T_1\langle\overline{U''}\rangle; \mathsf{K}\,(T_1) \vdash e_2\{v/x\} : [T_2]_{\overline{U_2}}$$

Thus the result follows from T-Let.

Case 6 $e = \mathsf{throw}\ U$. The result is trivial in this case.

Case 7 $e = \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U_0 \triangleright e_2}$. The type deriviation of e looks like the following

$$\frac{\Delta,\ x\!:\!T\langle\overline{U}\rangle; K \vdash e_1 : \left[T'\right]_{\overline{U_1}} \qquad \Delta,\ x\!:\!T\langle\overline{U}\rangle; K \vdash \overline{e_2} : \left[T'\right]_{\overline{U_2}}}{\Delta,\ x\!:\!T\langle\overline{U}\rangle; K \vdash \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U_0 \triangleright e_2} : \left[T'\right]_{(\overline{U_1}\setminus\overline{U_0})\cup\overline{U_2}}}$$

So $\overline{U'} = \left(\overline{U_1} \setminus \overline{U_0}\right) \cup \overline{U_2}$. Since $e\{v/x\} = \mathsf{try}\ e_1\{v/x\}\ \mathsf{catch}\ \overline{U_0 \triangleright e_1\{v/x\}}$, the result can be obtained by using the induction hypotheses.

Lemma 3 (Exception polymorphism)
Given variable environment Δ and a fixed set of exceptions $\overline{U'}$. Let Δ' be the variable environment that augments the exceptions associated with each variable in Δ with $\overline{U'}$, that is

$$\Delta' = \{\, x\!:\!T\langle\overline{U} \cup \overline{U'}\rangle \mid x\!:\!T\langle\overline{U}\rangle \in \Delta \,\}.$$

Then

(i) $\Delta_0, \Delta; \bot \vdash e_0 : [T_0]_{\overline{U_0}} \implies \Delta_0, \Delta'; \bot \vdash e_0 : [T_0]_{\overline{U_0}\cup\overline{U'}}$

(ii) $\Delta_0, \Delta; \top \vdash e_0 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}}\right]_{\overline{U_0}} \implies \Delta_0, \Delta'; \top \vdash e_0 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_0}\cup\overline{U'}}$

Proof. For any $x\!:\!T\langle\overline{U}\rangle \in \Delta$, if T is a strong type, then $\overline{U}$ and $\overline{U'}$ must both be $\varnothing$, and thus the results hold trivially. So in the following we assume such T's are weak and prove the results by induction on the type derivation of e_0. We will use Δ_{-y} to denote $\{\, x\!:\!T\langle\overline{U}\rangle \mid x\!:\!T\langle\overline{U}\rangle \in \Delta \text{ and } x \neq y \,\}$.

Case 1 $e_0 = b$. The results hold trivially in this case.

Case 2 $e_0 = x$. In this case $\overline{U_0} = \varnothing$.

Case 2.1 $x \in \mathrm{dom}(\Delta)$. Suppose $x\!:\!T\langle\overline{U}\rangle \in \Delta$. Notice that (i) is impossible because $x : T\langle\overline{U}\rangle \notin \left(\Delta, x\!:\!T\langle\overline{U}\rangle\right)_{\bot}$ as T is weak. The type derivation of e_0 is

$$\frac{x\!:\!T_0' \xrightarrow{\top} [S_0']_{\overline{U_0''}}\langle\overline{U}\rangle \in (\Delta_0, \Delta)_{\top}}{\Delta_0, \Delta; \top \vdash x : \left[T_0' \xrightarrow{\top} [S_0']_{\overline{U_0''}\cup\overline{U}}\right]_{\varnothing}}$$

So $T = T_0' \xrightarrow{\top} [S_0']_{\overline{U_0''}}$ and $\overline{U_0'} = \overline{U_0''} \cup \overline{U}$. The result (ii) then can be obtained via T-VAR-W:

$$\frac{x\!:\!T_0' \xrightarrow{\top} [S_0']_{\overline{U_0''}}\langle\overline{U} \cup \overline{U'}\rangle \in \left(\Delta_0, \Delta'\right)_{\top}}{\Delta_0, \Delta'; \top \vdash x : \left[T_0' \xrightarrow{\top} [S_0']_{\overline{U_0''}\cup\overline{U}\cup\overline{U'}}\right]_{\varnothing}}$$

Case 2.2 $x \notin \mathrm{dom}(\Delta)$. The results hold trivially in this case.

Case 3 $e_0 = \lambda y\!:\!T_1 \,.\, e_1$. The type derivation of e_0 looks like the following

$$\frac{(\Delta_0, \Delta)_K, y\!:\!T_1\langle\varnothing\rangle; \bot \vdash e_1 : [S_1]_{\overline{U_1}}}{\Delta_0, \Delta; K \vdash \lambda y\!:\!T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}\right]_{\varnothing}}$$

Case 3.1 $K = \bot$. Then $(\Delta_0, \Delta)_K = (\Delta_0)_K = (\Delta_0, \Delta')_K$ as $\Delta_{\bot} = \Delta'_{\bot} = \varnothing$. The result (i) then follows from T-ABS.

Case 3.2 $K = \top$.

Case 3.2.1 $y \in \mathrm{dom}(\Delta)$. Then $(\Delta_0, \Delta)_K, y\!:\!T_1\langle\varnothing\rangle = (\Delta_0)_K, y\!:\!T_1\langle\varnothing\rangle, \Delta_{-y}$. So the induction hypothesis is

$$(\Delta_0)_K, y\!:\!T_1\langle\varnothing\rangle, \Delta'_{-y}; \bot \vdash e_1 : [S_1]_{\overline{U_1}\cup\overline{U'}}$$

Since $(\Delta_0)_K, y\!:\!T_1\langle\varnothing\rangle, \Delta'_{-y} = (\Delta_0, \Delta')_K, y\!:\!T_1\langle\varnothing\rangle$, we can obtain $\Delta_0, \Delta'; K \vdash \lambda y : T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}\cup\overline{U'}}\right]_{\varnothing}$ via T-ABS. So the result (ii) holds.

Case 3.2.2 $y \notin \mathrm{dom}(\Delta)$. Then we can use the induction hypothesis to obtain $\Delta_0, \Delta'; K \vdash \lambda y : T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}\cup\overline{U'}}\right]_{\varnothing}$. So the result (ii) holds.

Case 4 $e_0 = e_1\ e_2$. The type derivation of e_0 looks like the following

$$\frac{\Delta_0, \Delta; \top \vdash e_1 : \left[T_1 \xrightarrow{\top} [S_1]_{\overline{U_3}}\right]_{\overline{U_1}} \qquad \Delta_0, \Delta; \mathsf{K}\,(T_1) \vdash e_2 : [T_2]_{\overline{U_2}} \qquad T_2 \lesssim_{\overline{U''}} T_1}{\Delta_0, \Delta; K \vdash e_1\ e_2 : [S_1]_{\overline{U_1}\cup\overline{U_2}\cup\overline{U_3}\cup\overline{U''}}}$$

So $\overline{U_0} = \overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U''}$. The type judgment for e_1 gives rise to the following induction hypothesis

$$\Delta_0, \Delta'; \top \vdash e_1 : \left[T_1 \xrightarrow{\top} [S_1]_{\overline{U_3}\cup\overline{U'}}\right]_{\overline{U_1}\cup\overline{U'}}$$

Case 4.1 $\mathsf{K}\,(T_1) = \bot$. In this case the induction hypothesis given by the type judgment for e_2 is

$$\Delta_0, \Delta'; \bot \vdash e_2 : [T_2]_{\overline{U_2}\cup\overline{U'}}$$

So it can be obtained via T-APP that $\Delta_0, \Delta'; K \vdash e_1\ e_2 : [S_1]_{\overline{U_1}\cup\overline{U_2}\cup\overline{U_3}\cup\overline{U''}\cup\overline{U'}}$. The results (i) and (ii) then follow.

Case 4.2 $\mathsf{K}\,(T_1) = \top$. In this case the induction hypothesis given by the type judgment for e_2 is

$$\Delta_0, \Delta'; \top \vdash e_2 : [T_2']_{\overline{U_2}\cup\overline{U'}}$$

where $T_2' = T_{21} \xrightarrow{K_2} [S_{22}]_{\overline{U_{22}}\cup\overline{U'}}$ given that $T_2 = T_{21} \xrightarrow{K_2} [S_{22}]_{\overline{U_{22}}}$. This implies $T_2' \lesssim_{\overline{U''}\cup\overline{U'}} T_1$. So it can be obtained via T-APP that $\Delta_0, \Delta'; K \vdash e_1\ e_2 : [S_1]_{\overline{U_1}\cup\overline{U_2}\cup\overline{U_3}\cup\overline{U''}\cup\overline{U'}}$. The results (i) and (ii) then follow.

Case 5 $e_0 = \mathsf{let}\ y{:}T_1 \leftarrow e_1\ \mathsf{in}\ e_2$. The type derivation of e_0 looks like the following

$$\frac{\Delta_0, \Delta; \mathsf{K}\,(T_1) \vdash e_1 : [T_3]_{\overline{U_1}} \qquad T_3 \lesssim_{\overline{U''}} T_1 \qquad \Delta_0, \Delta, y{:}T_1\langle\overline{U''}\rangle; K \vdash e_2 : [T_2]_{\overline{U_2}}}{\Delta_0, \Delta; K \vdash \mathsf{let}\ y{:}T_1 \leftarrow e_1\ \mathsf{in}\ e_2 : [T_2]_{\overline{U_1}\cup\overline{U_2}}}$$

The type judgment for e_1 gives rise to the following induction hypothesis

$$\Delta_0, \Delta'; \mathsf{K}\,(T_1) \vdash e_1 : [T_3']_{\overline{U_1}\cup\overline{U'}}$$

where $T_3' \lesssim_{\overline{U''}} T_1$ when $\mathsf{K}\,(T_1) = \bot$, and $T_3' \lesssim_{\overline{U''}\cup\overline{U'}} T_1$ when $\mathsf{K}\,(T_1) = \top$.

Case 5.1 $K = \bot$.

Case 5.1.1 $y \in \mathrm{dom}(\Delta)$. In this case, $\Delta_0, \Delta, y{:}T_1\langle\overline{U''}\rangle = \Delta_0, y{:}T_1\langle\overline{U''}\rangle, \Delta_{-y}$.

Case 5.1.1.1 $\mathsf{K}\,(T_1) = \bot$. The result (i) can be obtained using the following induction hypothesis

$$\Delta_0, y{:}T_1\langle\overline{U''}\rangle, \Delta'_{-y}; \bot \vdash e_2 : [T_2]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.1.1.2 $\mathsf{K}\,(T_1) = \top$. The result (i) can be obtained using the following induction hypothesis

$$\Delta_0, y{:}T_1\langle\overline{U''}\cup\overline{U'}\rangle, \Delta'_{-y}; \bot \vdash e_2 : [T_2]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.1.2 $y \notin \mathrm{dom}(\Delta)$.

Case 5.1.2.1 $\mathsf{K}\,(T_1) = \bot$. The result (i) can be obtained using the following induction hypothesis

$$\Delta_0, y{:}T_1\langle\overline{U''}\rangle, \Delta'; \bot \vdash e_2 : [T_2]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.1.2.2 $\mathsf{K}\,(T_1) = \top$. The result (i) can be obtained using the following induction hypothesis

$$\Delta_0, \Delta', y{:}T_1\langle\overline{U''}\cup\overline{U'}\rangle; \bot \vdash e_2 : [T_2]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.2 $K = \top$. In this case we know $T_2 = T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}}$.

Case 5.2.1 $y \in \mathrm{dom}(\Delta)$. In this case, $\Delta_0, \Delta, y{:}T_1\langle\overline{U''}\rangle = \Delta_0, y{:}T_1\langle\overline{U''}\rangle, \Delta_{-y}$.

Case 5.2.1.1 $\mathsf{K}\,(T_1) = \bot$. The result (ii) can be obtained using the following induction hypothesis

$$\Delta_0, y{:}T_1\langle\overline{U''}\rangle, \Delta'_{-y}; \top \vdash e_2 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.2.1.2 $\mathsf{K}\,(T_1) = \top$. The result (ii) can be obtained using the following induction hypothesis

$$\Delta_0, y{:}T_1\langle\overline{U''}\cup\overline{U'}\rangle, \Delta'_{-y}; \top \vdash e_2 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_2}\cup\overline{U'}}$$

Case 5.2.2 $y \notin \mathrm{dom}(\Delta)$.

Case 5.2.2.1 $\mathsf{K}\,(T_1) = \bot$. The result (ii) can be obtained using the following induction hypothesis

$$\Delta_0, y\,{:}\,T_1\langle\overline{U''}\rangle, \Delta'; \top \vdash e_2 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_2}\cup\overline{U_0'}}$$

Case 5.2.2.2 $\mathsf{K}\,(T_1) = \top$. The result (ii) can be obtained using the following induction hypothesis

$$\Delta_0, \Delta', y\,{:}\,T_1\langle\overline{U''}\cup\overline{U'}\rangle; \top \vdash e_2 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_2}\cup\overline{U_0'}}$$

Case 6 $e_0 = \mathsf{throw}\ U$. The results (i) and (ii) hold trivially in this case.

Case 7 $e_0 = \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U'' \triangleright e_2}$. The type derivation of e_0 looks like the following

$$\frac{\Delta_0, \Delta; K \vdash e_1 : [T_0]_{\overline{U_1}} \qquad \Delta_0, \Delta; K \vdash \overline{e_2} : [T_0]_{\overline{U_2}} \qquad \overline{U''} \subseteq \overline{U_1}}{\Delta_0, \Delta; K \vdash \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U'' \triangleright e_2} : [T_0]_{(\overline{U_1}\setminus\overline{U''})\cup\overline{U_2}}}$$

Case 7.1 $K = \bot$. The induction hypotheses are $\Delta_0, \Delta'; K \vdash e_1 : [T_0]_{\overline{U_1}\cup\overline{U'}}$ and $\Delta_0, \Delta'; K \vdash \overline{e_2} : [T_0]_{\overline{U_2}\cup\overline{U'}}$. So the result (i) can be obtained via T-TRY-CATCH.

Case 7.2 $K = \top$. The induction hypotheses are $\Delta_0, \Delta'; K \vdash e_1 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_1}\cup\overline{U'}} [T_0]_{\overline{U_1}\cup\overline{U'}}$ and $\Delta_0, \Delta'; K \vdash e_2 : \left[T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}\cup\overline{U'}}\right]_{\overline{U_2}\cup\overline{U'}}$, supposing $T_0 = T_0' \xrightarrow{K_0'} [S_0']_{\overline{U_0'}}$. So the result (ii) can be obtained via T-TRY-CATCH.

Lemma 4 $\varnothing; K \vdash v : [S]_{\overline{U}} \implies \varnothing; K' \vdash v : [S]_{\overline{U}}$.

Proof. The result is trivial if $K = K'$. Below we assume $K \neq K'$.

Case 1 $v = b$. The result is trivial in this case.

Case 2 $v = \lambda x\,{:}\,T_1 \,.\, e_1$. Let S be $T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}$. From the typing derivation of v, we know $x\,{:}\,T_1\langle\varnothing\rangle; \bot \vdash e_1 : [S_1]_{\overline{U_1}}$. Notice that $\varnothing_K = \varnothing_{K'} = \varnothing$. The result then follows from T-ABS.

Lemma 5 $\Delta; \bot \vdash e : [S]_{\overline{U}} \implies \Delta; \top \vdash e : [S]_{\overline{U}}$.

Proof. By induction on the typing derivation of e.

Case 1 $e = b$. The result is trivial in this case.

Case 2 $e = x$. The typing of e must be derived using T-VAR-S. It looks like the following:

$$\frac{x\,{:}\,S\langle\varnothing\rangle \in \Delta_\bot}{\Delta; \bot \vdash x : [S]_\varnothing}$$

So $x\,{:}\,S\langle\varnothing\rangle \in \Delta_\bot \subseteq \Delta_\top$ and the result follows.

Case 3 $e = \lambda x\,{:}\,T_1 \,.\, e_1$. The typing of e looks like the following:

$$\frac{\Delta_\bot, x\,{:}\,T_1\langle\varnothing\rangle; \bot \vdash e_1 : [S_1]_{\overline{U_1}}}{\Delta; \bot \vdash \lambda x\,{:}\,T_1 \,.\, e_1 : \left[T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}\right]_\varnothing}$$

So $S = T_1 \xrightarrow{\bot} [S_1]_{\overline{U_1}}$. The induction hypothesis is $\Delta_\bot, x\,{:}\,T_1\langle\varnothing\rangle; \top \vdash e_1 : [S_1]_{\overline{U_1}}$. Since $\Delta_\bot \subseteq \Delta_\top$, we can weaken the environment and obtain $\Delta_\top, x\,{:}\,T_1\langle\varnothing\rangle; \top \vdash e_1 : [S_1]_{\overline{U_1}}$. The result then follows from T-ABS.

Case 4 $e = e_1\ e_2$. The typing of e looks like the following:

$$\frac{\Delta; \top \vdash e_1 : \left[T_1 \xrightarrow{\top} [S]_{\overline{U_3}}\right]_{\overline{U_1}} \qquad \Delta; \mathsf{K}\,(T_1) \vdash e_2 : [T_2]_{\overline{U_2}} \qquad T_2 \lesssim_{\overline{U'}} T_1}{\Delta; K \vdash e_1\ e_2 : [S]_{\overline{U_1} \cup \overline{U_2} \cup \overline{U_3} \cup \overline{U'}}}$$

which is independent of K. The result holds trivially.

Case 5 $e = \mathsf{let}\ x\!:\!T_1 \leftarrow e_1\ \mathsf{in}\ e_2$. The typing of e looks like the following:

$$\frac{\Delta; \mathsf{K}\,(T_1) \vdash e_1 : [T_3]_{\overline{U_1}} \qquad T_3 \lesssim_{\overline{U}} T_1 \qquad \Delta,\ x\!:\!T_1\langle\overline{U}\rangle; \bot \vdash e_2 : [S]_{\overline{U_2}}}{\Delta; \bot \vdash \mathsf{let}\ x\!:\!T_1 \leftarrow e_1\ \mathsf{in}\ e_2 : [S]_{\overline{U_1} \cup \overline{U_2}}}$$

The induction hypothesis provided by the typing of e_2 is $\Delta,\ x\!:\!T_1\langle\overline{U}\rangle; \top \vdash e_2 : [S]_{\overline{U_2}}$. The result then follows via T-LET.

Case 6 $e = \mathsf{throw}\ U_0$. The result is trivial in this case.

Case 7 $e = \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U' \triangleright e_2}$. The typing of e looks like the following:

$$\frac{\Delta; \bot \vdash e_1 : [S]_{\overline{U_1}} \qquad \Delta; \bot \vdash \overline{e_2} : [S]_{\overline{U_2}} \qquad \overline{U'} \subseteq \overline{U_1}}{\Delta; \bot \vdash \mathsf{try}\ e_1\ \mathsf{catch}\ \overline{U' \triangleright e_2} : [S]_{(\overline{U_1} \setminus \overline{U'}) \cup \overline{U_2}}}$$

The induction hypotheses are $\Delta; \top \vdash e_1 : [S]_{\overline{U_1}}$ and $\Delta; \top \vdash \overline{e_2} : [S]_{\overline{U_2}}$. The result then follows via T-TRY-CATCH.

Lemma 6 $\Delta; K \vdash \mathcal{C}[\mathsf{throw}\ U'] : [T]_{\overline{U}} \ \wedge\ \mathsf{throw}\ U' \curvearrowright \mathcal{C} \implies U' \in \overline{U}$

Proof. By induction on the type derivation of $\mathcal{C}[\mathsf{throw}\ U']$.

Lemma 1 (Rewriting preserves types) *If* $\Gamma; K \vdash f \ \rightsquigarrow\ e : [\tau]_{\overline{E}}$ *then* $\Gamma; K \vdash e : [\tau]_{\overline{E}}$.

Proof. By induction on the derivation of the translation. Most rewriting rules have a corresponding typing rule in the kernel, which causes most cases in the proof to be straightforward constructions of kernel typing derivations. The exception to this pattern is when f is an application that involves mismatch. Below we show that its translation preserves the type.

Let f be $\{f_1\ f_2\}^{\ell}$. Its rewriting derivation has the following form:

$$\frac{\begin{array}{c}\Gamma; \top \vdash f_1 \ \rightsquigarrow\ e_1 : \left[\tau_1 \xrightarrow{\top} [\sigma_1]_{\overline{E_3}}\right]_{\overline{E_1}} \\ \Gamma; \mathsf{K}\,(\tau_1) \vdash f_2 \ \rightsquigarrow\ e_2 : [\tau_2]_{\overline{E_2}} \qquad \tau_2 \lesssim_{\overline{E_\delta}} \tau_1 \qquad \tau_2 = \tau_{21} \xrightarrow{\top} [\sigma_{22}]_{\overline{E_4}}\end{array}}{\Gamma; K \vdash \{f_1\ f_2\}^{\ell} \ \rightsquigarrow\ \mathsf{try}\ e_1 \left(\begin{array}{l}\mathsf{let}\ x\!:\!\tau_2 \leftarrow e_2\ \mathsf{in} \\ \lambda y\!:\!\tau_{21}\,.\, \mathsf{try}\ x\ y\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E_\delta^{\ell}}\end{array}\right)\ \mathsf{catch}\ \overline{E_\delta^{\ell} \triangleright \mathsf{throw}\ E_\delta} : [\sigma_1]_{\overline{E_1} \cup \overline{E_2} \cup \overline{E_3} \cup \overline{E_\delta}}}$$

The induction hypotheses are $\Gamma; \top \vdash e_1 : \left[\tau_1 \xrightarrow{\top} [\sigma_1]_{\overline{E_3}}\right]_{\overline{E_1}}$ and $\Gamma; \mathsf{K}\,(\tau_1) \vdash e_2 : [\tau_2]_{\overline{E_2}}$. They allow the typing derivation in Figure 19, where $\tau_2' \triangleq \tau_{21} \xrightarrow{\top} [\sigma_{22}]_{(\overline{E_4} \setminus \overline{E_\delta}) \cup \overline{E_\delta^{\ell}}}$.

$$
\dfrac{
\Gamma;\top\vdash e_1 : \left[\tau_1 \xrightarrow{\top} [\sigma_1]_{\overline{E_3}}\right]_{\overline{E_1}}
\qquad
\dfrac{
\Gamma;\mathsf{K}\,(\tau_1)\vdash e_2 : [\tau_2]_{\overline{E_2}}
\qquad
\dfrac{
\dfrac{
\dfrac{\cdots}{\Gamma, x\!:\!\tau_2\langle\varnothing\rangle, y\!:\!\tau_{21}\langle\varnothing\rangle;\bot\vdash x\;y : [\sigma_{22}]_{\overline{E_4}}}
\qquad \cdots
}{
\Gamma, x\!:\!\tau_2\langle\varnothing\rangle, y\!:\!\tau_{21}\langle\varnothing\rangle;\bot\vdash \begin{array}{l}\mathsf{try}\ x\ y\\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E^\ell_\delta}\end{array} : [\sigma_{22}]_{(\overline{E_4}\setminus\overline{E_\delta})\cup\overline{E^\ell_\delta}}
}
}{
\Gamma, x\!:\!\tau_2\langle\varnothing\rangle;\mathsf{K}\,(\tau_2)\vdash \lambda y\!:\!\tau_{21}\,.\begin{array}{l}\mathsf{try}\ x\ y\\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E^\ell_\delta}\end{array} : [\tau_2']_{\varnothing}
}
\qquad
\tau_2 \lesssim_{\varnothing} \tau_2
}{
\Gamma;\mathsf{K}\,(\tau_1)\vdash \mathsf{let}\ x\!:\!\tau_2 \leftarrow e_2\ \mathsf{in}\ \lambda y\!:\!\tau_{21}\,.\,\mathsf{try}\ x\ y\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E^\ell_\delta} : [\tau_2']_{\overline{E_2}}
}
\qquad
\tau_2' \lesssim_{\overline{E^\ell_\delta}} \tau_1
}{
\dfrac{
\Gamma;K\vdash e_1\ \left(\mathsf{let}\ x\!:\!\tau_2 \leftarrow e_2\ \mathsf{in}\ \lambda y\!:\!\tau_{21}\,.\,\mathsf{try}\ x\ y\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E^\ell_\delta}\right) : [\sigma_1]_{\overline{E_1}\cup\overline{E_2}\cup\overline{E_3}\cup\overline{E^\ell_\delta}}
\qquad \cdots
}{
\Gamma;K\vdash \mathsf{try}\ e_1\ \left(\mathsf{let}\ x\!:\!\tau_2 \leftarrow e_2\ \mathsf{in}\ \lambda y\!:\!\tau_{21}\,.\,\mathsf{try}\ x\ y\ \mathsf{catch}\ \overline{E_\delta \triangleright \mathsf{throw}\ E^\ell_\delta}\right)\ \mathsf{catch}\ \overline{E^\ell_\delta \triangleright \mathsf{throw}\ E_\delta} : [\sigma_1]_{\overline{E_1}\cup\overline{E_2}\cup\overline{E_3}\cup\overline{E_\delta}}
}
}
$$

Figure 19: The typing derivation the proof of Lemma 1 refers to