**The Local Inconsistency Resolution Algorithm**

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**Key Representation:** Probabilistic Dependency Graphs (PDGs) are directed (hyper) graphs with probabilities and confidences attached to edges.

PDGs can capture:
- inconsistent beliefs, providing a natural way to measure the degree of this inconsistency;
- **graphical models**, such as Bayesian networks and factor graphs;
- **learning settings** and their **loss functions**, e.g.,
  - variational autoencoders (VAEs);
  - statistical divergences $\mathcal{X}$, $\varphi$ and $\gamma$;
- **regularizers** as priors, accuracy, MSE, ….

**Inconsistency semantics.**

- The two variants are equivalent.
-PDGs Add each $\theta_{\alpha}$ as a variable
- Trivial parameterization: every $\theta_{\alpha} = \{0\}$

**PDGs ≈ PPDGs**

**Structural Deficiency with** $(\alpha, \alpha)$

$$E \left[ \log \frac{p(x)}{\lambda(x)} \prod_{a \in A} \frac{\lambda(T_{a})}{\lambda(T_{a})} \right]$$

Degree of inconsistency $m_{\gamma} := \inf_{\mu} \left( \Omega_{\text{mc}}(\mu) + \gamma \text{Def}_{m}(\mu) \right)$

is the smallest possible incompatibility with any $\mu(x)$.

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**Algorithm:** Local Inconsistency Resolution (LIR)

**Input:** context PDG $\mathcal{G}_{x}$, mutable memory $\mathcal{M}(\theta)$.

1. Initialize $\theta^{(0)}$; for $t = 0, 1, 2, \ldots$ do
   - $\mathcal{G}_{x} \leftarrow \text{REFRESH}(\mathcal{G}_{x})$;
   - $\varphi, \chi, \gamma \leftarrow \text{REFOCUSE}()$;
   - Write $\exp(t \cdot X)$ for the path following vector field $X$ for time $t$, starting at 0.
   - Gradient Flow of $f: \Theta \rightarrow \mathbb{R}$, starting at $\theta$:
     - $\theta(t+1) \leftarrow \exp(t \cdot \nabla_{\theta} f(\theta))$;
   - Calculate the inconsistency of the combined context and memory, weighted by attention.

2. $\theta^{(t+1)} \leftarrow \exp(t \cdot \nabla_{\theta} f(\theta))$

   Reduce this inconsistency by an approximation to gradient flow, starting at previous state $\theta^{(t)}$, changing each parameter in proportion to its control of it.

**Focal Points:**

**Attention and Control**

attend only to probabilities of a subset of arcs $A \subseteq \mathcal{A}$

control only parameters of a subset of arcs $C \subseteq \mathcal{A}$

Typical use case: select focus $(\varphi, \chi)$ from a fixed set of focal points $F = \{ \bullet, \square, \triangle, \ldots \}$.

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**More Examples**

- Training Generative Adversarial Networks (GANs).
- Message Passing: Sum-Product Belief Propagation
- Variational Inference, EM algorithm, e.g., VAE training.

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**Why causes changes in beliefs?** Some say it is internal conflict. But identifying inconsistencies is difficult. So in practice, we resolve them **locally**: looking only at a small part of the picture, and changing only another small part at a time.