Online Learning of Prediction Suffix Trees

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Introduction

Sequential prediction

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Prediction Suffix Trees

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Learning algorithm

Motivation

Monitoring of applications in a computer system

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System calls

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Prediction model

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Some assumptions

Outline

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Algorithm and Properties

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What is the next item in the sequence?

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$+1, -1, +1, -1, +1, \ldots$
all,your,base,...

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Markovian Assumption

Markov Models

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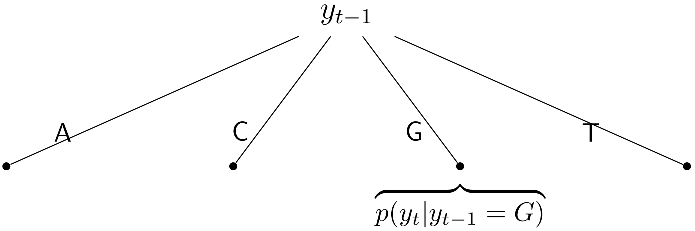
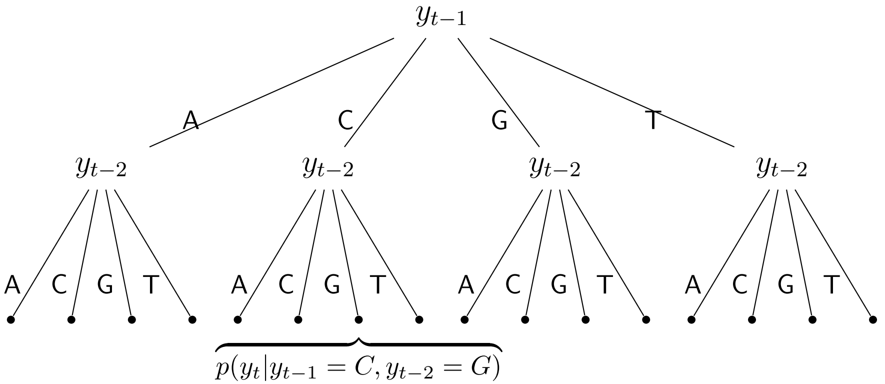


Markov Models

Second order:



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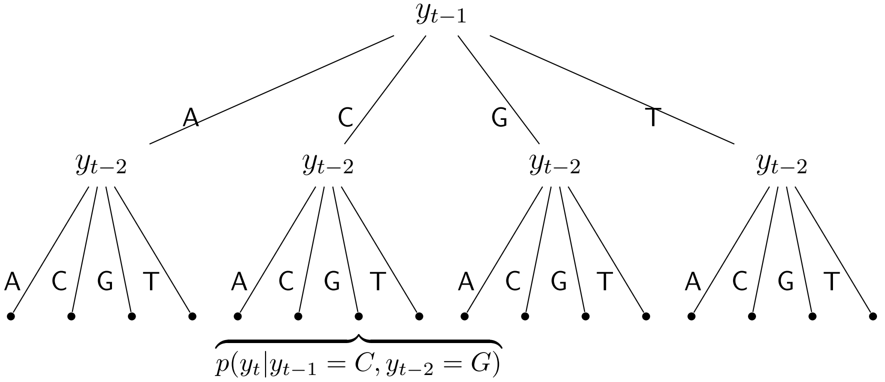

Third order, k-th order etc.

Markov Models — Problems

How to select the best k?

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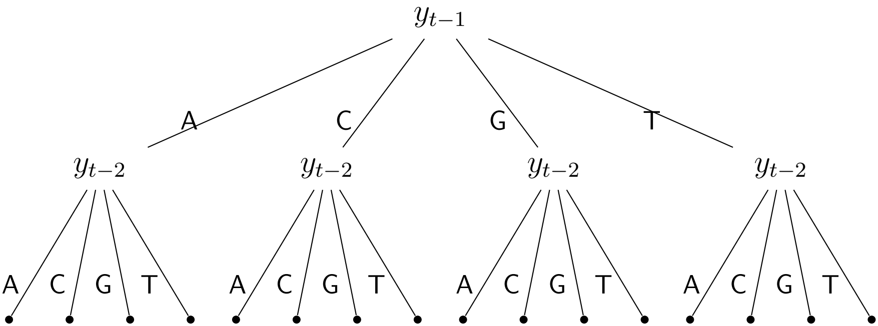
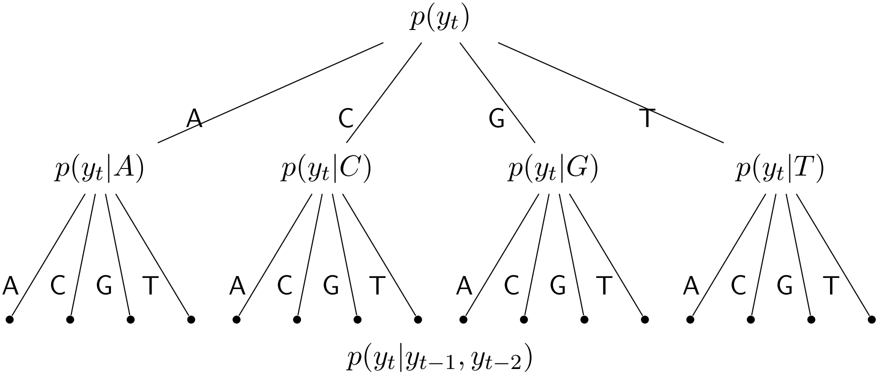
Prediction Suffix Trees address these problems.
[Willems et al., 1995, Ron et al., 1996, Helmbold & Schapire, 1997, Pereira & Singer, 1999]

Prediction Suffix Trees



Prediction Suffix Trees

Keep a model for each k. Use a weighted sum.



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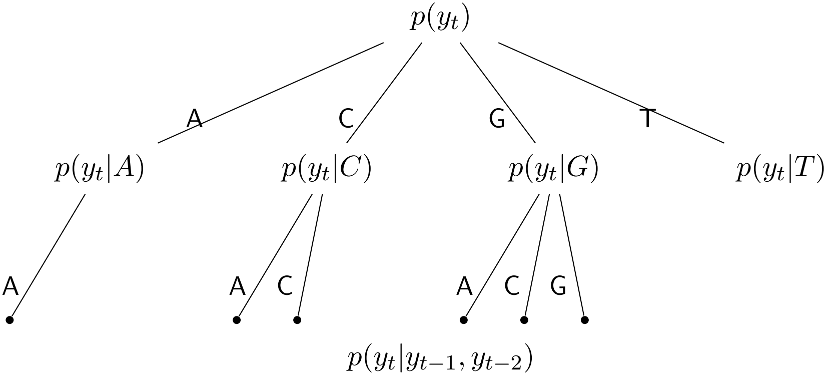
Prune useless branches and subtrees.



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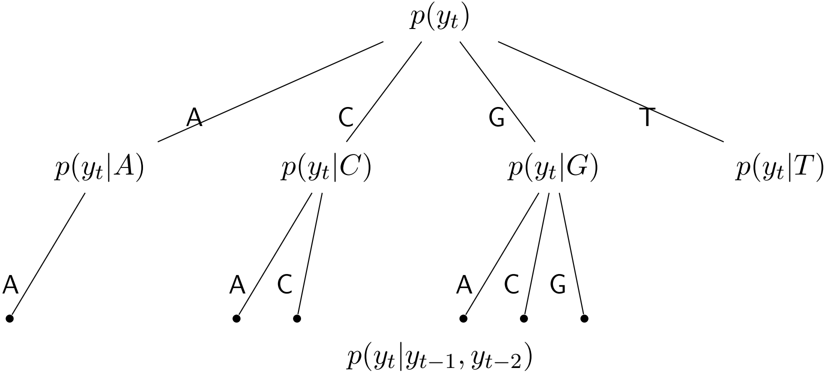

Assumptions: $p(y_t|y_{t-1} = T, y_{t-2}) = p(y_t|y_{t-1} = T)$,
$p(y_t|y_{t-1} = A, y_{t-2} = \neg A) = p(y_t|y_{t-1} = A)$.

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Store a weight instead of a probability

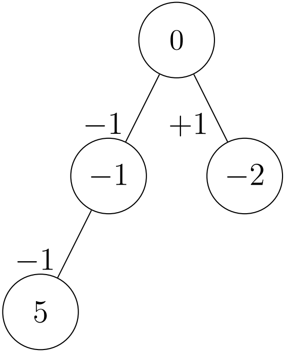
Use the sign of a weighted sum of the values in the appropriate path

Example

True Sequence: $-1, -1, +1, -1, -1, +1, \ldots$

Discounting: A node at depth d is discounted by 2^{-d}

Tree:



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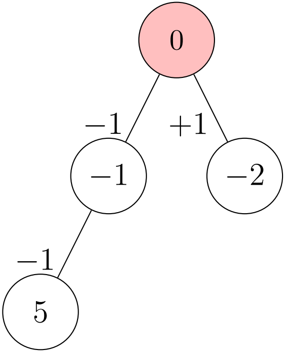

Input Sequence: $\ldots, -1, -1$

Decision: 0

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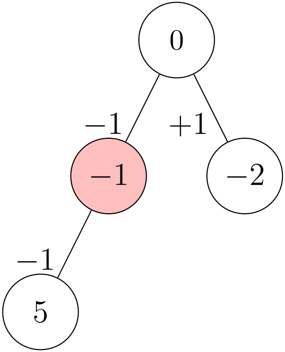

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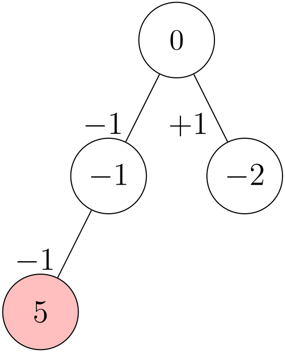

Input Sequence: $\ldots, \mathbf{-1}, -1$

Decision: $0 - \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 5$

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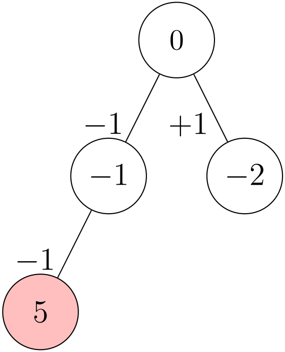

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Decision: $0 - \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 5 = \frac{3}{4} \xrightarrow{sign} +1$

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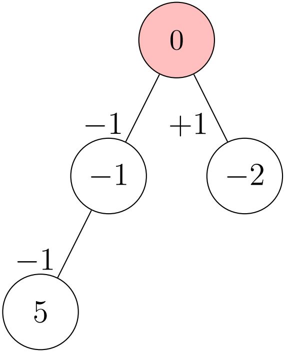

Input Sequence: $\ldots, +1, -1$

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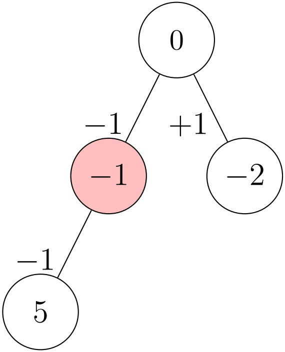

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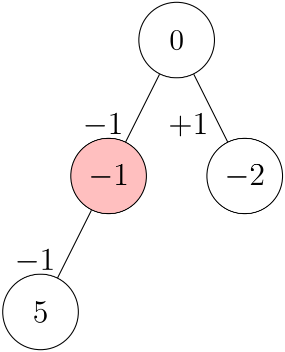

Input Sequence: $\ldots, +1, \mathbf{-1}$

Decision: $0 - \frac{1}{2} \cdot 1 = -\frac{1}{2} \stackrel{sign}{\longrightarrow} -1$

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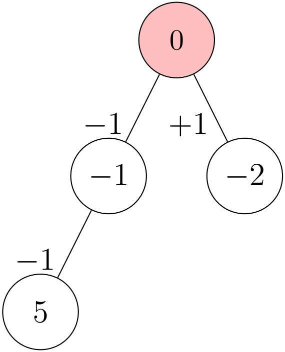

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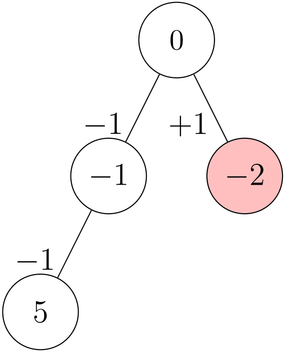

Input Sequence: $\ldots, \mathbf{+1}$

Decision: $0 - \frac{1}{2} \cdot 2$

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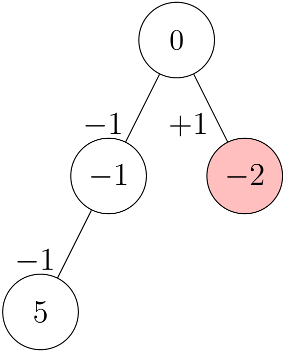

Input Sequence: $\ldots, +\mathbf{1}$

Decision: $0 - \frac{1}{2} \cdot 2 = -1 \overset{sign}{\longrightarrow} -1$

Remarks

Notation for the node values: $g_{t,s}$

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Discounting scheme in this work:

$$x^+_{t,s} = \begin{cases} (1-\epsilon)^{|s|} & \text{if} \quad s = y^{t-1}_{t-i} \qquad i = 1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases}$$

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Decision at time t: $\operatorname{sign}\left(\sum_s g_{t,s} x_{t,s}^{+}\right) = \operatorname{sign}(\langle g_t, x_t^{+} \rangle)$

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SVMs, logistic regression,. . .

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Focus on online setting

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A mistake is made when $y_t \langle w_t, x_t \rangle \leq 0$.

Perceptron

If a mistake is made at round t: $w_{t+1} = w_t + \alpha y_t x_t$

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α is a learning rate.

It is selected so that it optimizes various tradeoffs.

Balanced Winnow

Balanced Winnow [Littlestone, 1989]	Perceptron [Rosenblatt, 1958]
$\theta_1 \leftarrow 0$	$\theta_1 \leftarrow 0$
for $t = 1, 2, \ldots, T$ **do**	**for** $t = 1, 2, \ldots, T$ **do**
$\quad w_{t,i} \leftarrow \frac{e^{\theta_{t,i}}}{\sum_j e^{\theta_{t,j}}}$	$\quad w_t \leftarrow \theta_t$
$\quad \hat{y}_t \leftarrow \langle w_t, x_t \rangle$	$\quad \hat{y}_t \leftarrow \langle w_t, x_t \rangle$
$\quad$ **if** $y_t \hat{y}_t \leq 0$	$\quad$ **if** $y_t \hat{y}_t \leq 0$
$\quad\quad \theta_{t+1} \leftarrow \theta_t + \alpha y_t x_t$	$\quad\quad \theta_{t+1} \leftarrow \theta_t + \alpha y_t x_t$
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if $y_t \hat{y}_t \leq 0$	**if** $y_t \hat{y}_t \leq 0$
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else	**else**
$\theta_{t+1} \leftarrow \theta_t$	$\theta_{t+1} \leftarrow \theta_t$

Important for Balanced Winnow:

$$x_t = [x_t^+, -x_t^+] = [x_{t,1}^+, \ldots, x_{t,d}^+, -x_{t,1}^+, \ldots, -x_{t,d}^+]$$

$$x_{t,s}^+ = \begin{cases} (1-\epsilon)^{|s|} & \text{if} \quad s = y_{t-i}^{t-1} \qquad i = 1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases}$$

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For our application it can track changes better.

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Need to learn $O(T^2)$ weights in T rounds ☹

Need to store $O(T^2)$ numbers ☹

Naively, all weights affect the decision and must be stored.

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Decision $\hat{y}_t = \langle w_t, x_t \rangle = \langle w_t^+ - w_t^-, x_t^+ \rangle =$[1]

$$= \sum_{i=1}^{d} \frac{\sinh(\theta_{t,i}^+)}{\sum_{j=1}^{d} \cosh(\theta_{t,j}^+)} x_{t,i}^+ \propto \sum_{i=1}^{d} \sinh(\theta_{t,i}^+) x_{t,i}^+$$

[1] $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$

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Iff $\theta_{t,i}^+ = 0$ the decision does not depend on feature i.

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As mistakes are made, the tree grows.

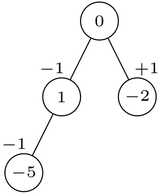
Classic Winnow/Perceptron update quickly destroys sparsity.

Illustration

Input Sequence: $\ldots, -1, +1, -1, +1, -1, ?$

$$x^+_{t,s} = \begin{cases} 2^{-|s|} & \text{if} \quad s = y^{t-1}_{t-i} \qquad i = 1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases}$$

Tree:



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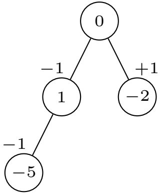

Decision: $\frac{1}{2} \cdot \sinh(1)$

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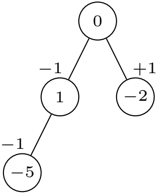

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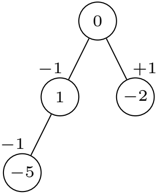

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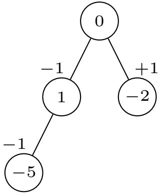

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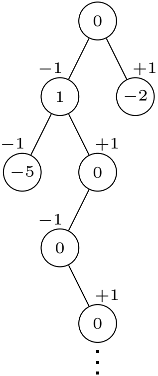

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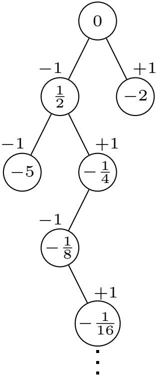

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Summary of the Problem

Mistake at time t: $O(t)$ nodes are inserted in the tree.

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Not all sparse vectors are equal.

Better Update Rule

Adaptive bound d_t on the length of the suffixes.

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Equivalently:

$$\theta_{t+1} = \theta_t + \alpha y_t x_t + \alpha n_t$$

where n_t is a noise vector that cancels part of the update:

$$n_{t,s} = \begin{cases} -y_t x_{t,s} & \text{if } |s| > d_t \\ 0 & \text{otherwise} \end{cases}$$

Setting d_t

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To guarantee that we can set

$$d_t = \max\left\{h_t, \left\lceil \log_{1-\epsilon}\left(\sqrt[3]{P_{t-1}^3 + 2P_{t-1}^{3/2} + 1} - P_{t-1}\right) - 1\right\rceil\right\}$$

Algorithm Properties — Learning

Mistake Bound

If there exists a tree which over the input sequence $y_1, y_2, \ldots, y_T$ correctly predicts all items with confidence $\geq \delta$, our algorithm's mistakes will be at most

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$L = \sum_{t=1}^{T} \ell_t(u)$ where $\ell_t(u) = \max(0, \delta - y_t\langle u, x_t\rangle)$

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Growth Bound

Our algorithm will not grow a tree deeper than $\log_2 M_{T-1} + 4$

How to go about proving the properties

Bounding how fast the tree grows is straightforward.

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We need a goal and measure of progress.

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Potential function: $\Phi(w_t) = D(u||w_t) \geq 0$

Proof Technique

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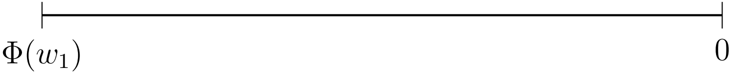
Keep the total effect of noise bounded. [Dekel et al., 2004]

Pictorially

Potential:



Noise P_t :

Example	Correct Prediction
x_1	

Progress due to classic update

Effect of noise

Net progress

Pictorially

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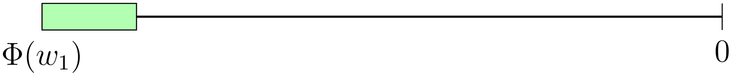

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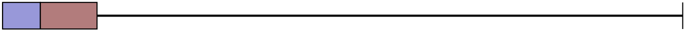

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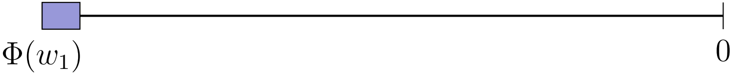

Noise P_t :

Example	Correct Prediction
x_1	✗
x_2	

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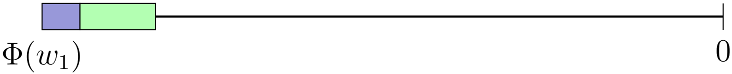

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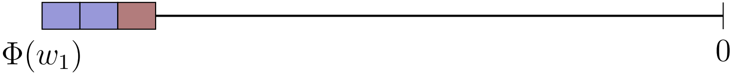

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Noise P_t :

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x_3	

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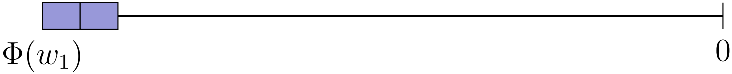

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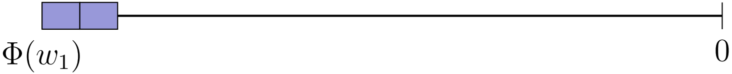

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x_4	

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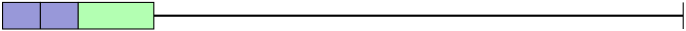

$\Phi(w_1)$ $\qquad$ 0

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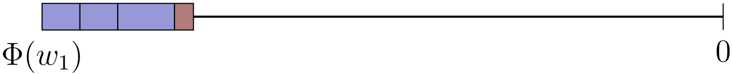

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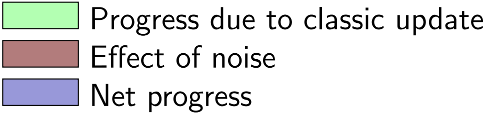

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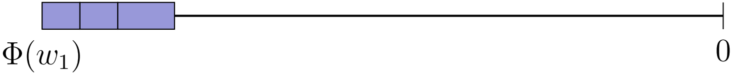

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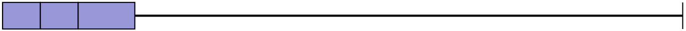
Potential: 

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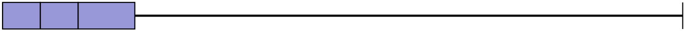
Potential: 

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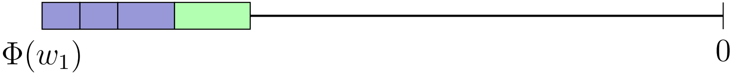

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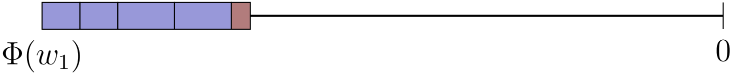

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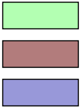

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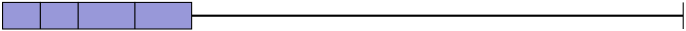
Potential: 

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Multiclass extension

We maintain weights $w^{(1)}, w^{(2)}, \dots, w^{(k)}$

Predict $\hat{y}_t = \mathrm{argmax}_i \langle w^{(i)}, x_t \rangle$

In case of a mistake

$$\begin{aligned} \theta^{(\hat{y}_t)}_{t+1,s} &= \theta^{(\hat{y}_t)}_{t,s} - \alpha x_{t,s} \\ \theta^{(y_t)}_{t+1,s} &= \theta^{(y_t)}_{t,s} + \alpha x_{t,s} \end{aligned}$$

for all s such that $|s| \leq d_t$.

Outline

Prediction Suffix Trees

Algorithm and Properties

Results

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Data

120 sequences of system calls from Outlook, Excel and Firefox (40 each).

The monitoring program records 23 different system calls.

A typical sequence contains hundreds of thousands of system calls.

Results

Averages over the 40 sequences

Outlook	Perceptron	Winnow
% Error	5.1	4.43
PST Size	41239	25679

"Perceptron" is the PST learning algorithm of [Dekel et al., 2004].

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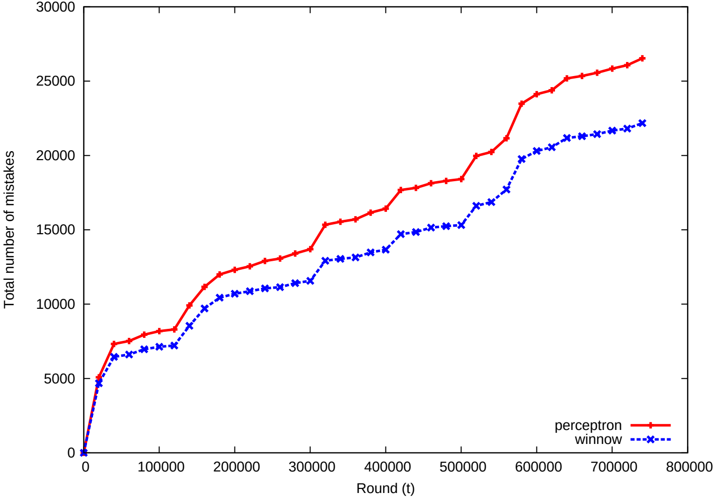
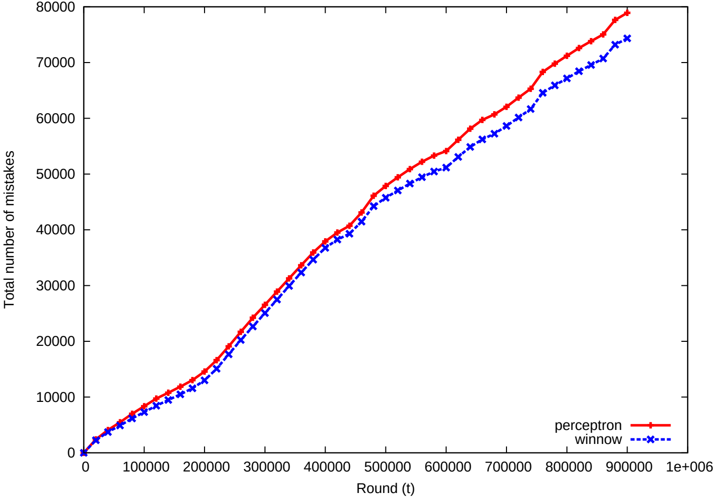
Moreover, Winnow made less mistakes and grew smaller trees for all 120 sequences

"Perceptron" is the PST learning algorithm of [Dekel et al., 2004].

Results

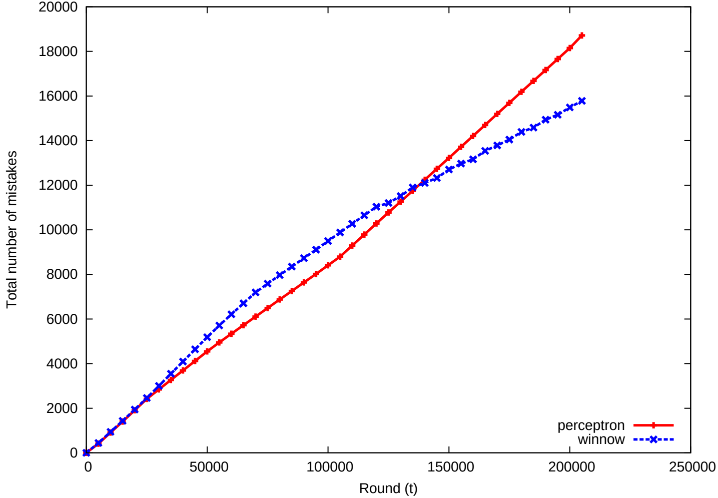

Total number of mistakes
30000
25000
20000
15000
10000
5000
0
0
100000
200000
300000
400000
500000
600000
700000
800000
Round (t)
perceptron
winnow

Results



80000
70000
60000
50000
40000
30000
20000
10000
0
Total number of mistakes
0
100000
200000
300000
400000
500000
600000
700000
800000
900000
1e+006
Round (t)
perceptron
winnow

Results



Total number of mistakes
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
50000
100000
150000
200000
250000
Round (t)
perceptron
winnow

Related Work

Many learning algorithms assume an a priori bound on the tree's depth [Willems et al., 1995], [Ron et al., 1996], [Pereira & Singer, 1999]...

[Dekel et al., 2004] present a perceptron algorithm similar to ours.

[Kivinen & Warmuth, 1997] show how to compete against vectors u with $||u||_1 \leq U$

Sparsifying Winnow is popular (e.g. [Blum, 1997]) but no guarantees.

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On our task, it made less mistakes and grew smaller trees than other state-of-the-art algorithms.

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