The Impact of Tribalism on Social Welfare

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joint work with Yuwen Wang and Seunghee Han

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Naive view of PoA

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Tribalism (1)

In the real world, actual altruism may be a tall order.

At best, players care about their tribe:

- Democrats want to maximise the sum utility of Democrats, Republicans want to maximise the sum utility of Republicans...
- Uber cars want to maximise the sum utility of Uber cars...
- ants from anthill A want to maximise the sum utility of ants from anthill A...

Social context

Other extreme: Social context games (player i weighs player j's utility by arbitrary factor p_{ij}). (Bilò et al. (2013)...)

Our model is "in the middle".

Tribalism (2)

Think of tribalism as players playing a game with different payoffs:

Definition

G: game with utility functions u_i. τ: a function that assigns players to tribes. The τ-tribal extension of G is the game G^τ with the same players and strategies, and modified utility functions

$$u_i^\tau(\vec{s}) = \sum_{j \in N : \tau(i) = \tau(j)} u_j(\vec{s}).$$

The modified game can have different equilibria.

However, we still rate them in terms of the original game:

Definition

The Price of Tribalism (PoT) of a class of games $\mathcal{G}$ and partition functions $\mathcal{T}$ is

$$\mathrm{PoT}(\mathcal{T}, \mathcal{G}) = \sup_{G \in \mathcal{G}, \tau \in \mathcal{T}_G} \frac{\sup_{\vec{s} \in \Sigma_1 \times \cdots \times \Sigma_n} \sum_i u_i(\vec{s})}{\inf_{\vec{s} \in S_{G^\tau}} \sum_i u_i(\vec{s})},$$

where S_{G^τ} is the set of pure Nash equilibria of G^τ.

Our results

Game	PoA	Altruistic PoA	PoT
Social grouping (2 cliques)	2	2	3
Social grouping (k cliques)	k	k	$2k-1$
Network contribution (additive rewards)	1	1	2
Network contribution (convex rewards)	2	2	4
Atomic linear routing	5/2	3	4

Social grouping (1)

Players $i \in N$ want to socialise by joining one of two social clubs, say

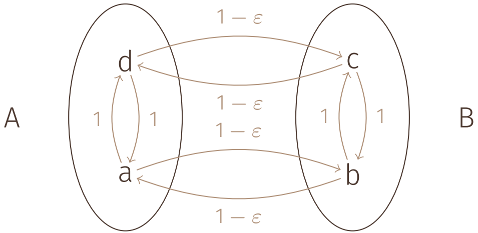
$$\Sigma_i = \{A, B\}.$$

If two players i and j are in the same club, they can be friends and get utility $u_{ij} \geq 0$. Players who are in different clubs don't befriend each other.

Social grouping (2)

Clearly, it's optimal for everyone to be in the same club and befriend each other.

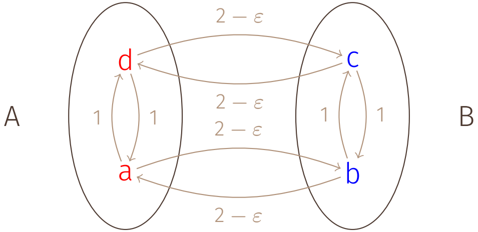
But what if players start out in different clubs?



Here, every player gets a utility of 1.

This lower bound is tight: PoA$= 2$. (Also works for altruism.)

With tribes, however, the following is a Nash equilibrium:



The friends in the other tribe would be twice as valuable, but friends in the same tribe count for twice as much! So nobody wants to switch. This gives $\mathrm{PoT} \geq 3$.

Each player $i \in P$ has a budget B_i they can divide up among their relations. (Money, time...)

For each pair of players $e = \{i, j\} \in P^{(2)}$, have some symmetric function that tells us how much they'd gain from investing in their relationship, depending on each of their investments.

Taking the pairs e with $f_e(a, b) = 0$ to be non-edges, we can think of this as a graph.

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Get different PoA depending on what sorts of functions we allow.

Simple case: all functions are of the form $f_e(x, y) = c_e(x + y)$.

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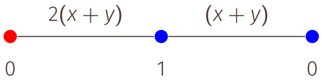
Not so under tribalism!



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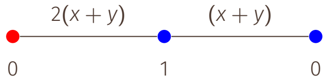


$\Rightarrow \mathrm{PoT} \geq 2$ (=, actually)

Network Contribution Games (coordinate convex rewards)

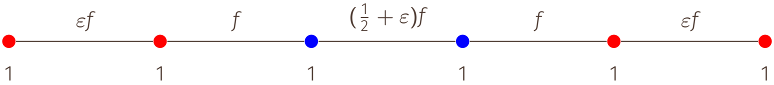
Less simple case: functions f_e are convex in each coordinate. (max, product...)

AH ’12: PoA$=2$ under bilateral deviations. (Unbounded for unilateral!)

Same under full altruism.

What about tribes?

Network Contribution Games (coordinate convex rewards)



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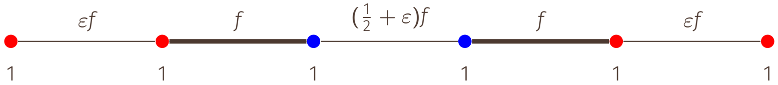

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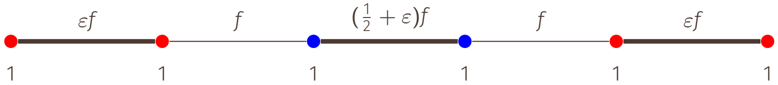

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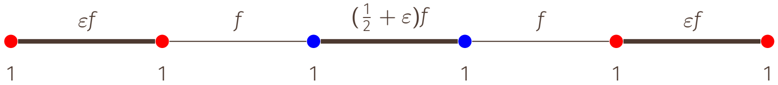

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Stable against bilateral and even whole-tribe deviations!

Notation

$u_i(\mathbf{s})$ is the utility player i gets in $\mathbf{s}$; u_i^T is the same for their tribe.
$w_e(\mathbf{s})$ is the utility edge e pays to its endpoints.

Lemma (AH '12): There exists an optimum $\mathbf{s}^*$ where every player's strategy is tight: the whole budget goes into one edge.

Say player i is witness to the edge e they invest in. Witnesses of e: $W_{\mathbf{s}^*}(e) \subseteq P$.

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Pick a Nash equilibrium $\mathbf{s}$, and consider each edge $e = \{i, j\}$ in turn. If $|W_{\mathbf{s}^*}(e)| = 2$:

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$$u_i^T(\mathbf{s}) \quad \geq \quad u_i^T(\mathbf{s}_i^*; \mathbf{s}_j^*; \mathbf{s}_{-i,j})$$

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Lose at most $2(u_i(\mathbf{s}) + u_j(\mathbf{s}))$, when i, j and their partners in $\mathbf{s}$ are all the same tribe.

$$\begin{aligned} u_i^T(\mathbf{s}) &\geq u_i^T(\mathbf{s}_i^*; \mathbf{s}_j^*; \mathbf{s}_{-i,j}) \\ &\geq u_i^T(\mathbf{s}) - 2(u_i(\mathbf{s}) + u_j(\mathbf{s})) \qquad . \end{aligned}$$

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Gain at least $w_e(\mathbf{s}^*)$, when $\tau(i) \neq \tau(j)$.

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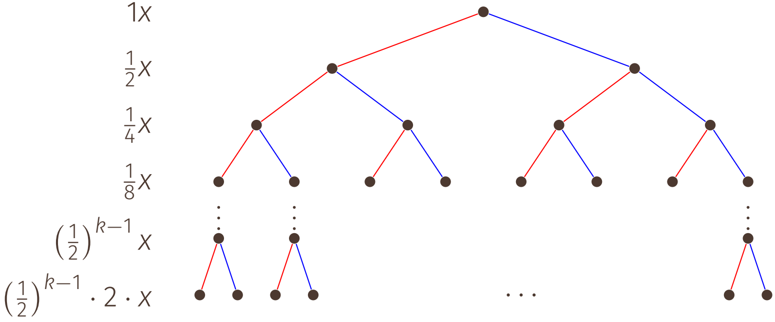
$$U(\mathbf{s}^*) = 2 \sum_e w_e(\mathbf{s}^*) \leq 4 \sum_e \sum_{i \in W(e)} u_i(\mathbf{s}) = 4 \sum_{i \in P} u_i(\mathbf{s}) = 4U(\mathbf{s}).$$

Variation on construction of Caragiannis gives a lower bound of 4:



Can prove matching upper bound via smoothness.

Closing thoughts (1)

We’ve shown some examples where tribalism produces worse equilibria.

In some games (e.g. opinion-forming game of BKO ’12), any form of altruism improves the equilibria. Can we find some interesting condition for this?

(Opinion-forming: it seems relevant that player costs are convex degree-2 polynomials.)

Closing thoughts (2)

Smoothness for tribalism:

$$\sum_{i \in N} \left(c_i^T(\mathbf{s}'_i; \mathbf{s}) - (c_i^T(\mathbf{s}) - c_i(\mathbf{s})) \right) \leq \lambda C(\mathbf{s}') + \mu C(\mathbf{s}).$$

This is actually true for any c_i^T that players optimise for while their actual welfare is c_i.

Can we say something general when players are “confused about their own utility”?

Thanks for listening!