Efficient Generic Search Heuristics within the EMBP framework

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Abstract

Accurately estimating the distribution of solutions to a problem, should such solutions exist, provides efficient search heuristics. We propose new ways of computing such estimates, with different degrees of accuracy and complexity. We build on the Expectation-Maximization Belief-Propagation (EMPB) framework proposed by Hsu et al. to solve Constraint Satisfaction Problems (CSPs). We propose two general approaches within the EMBP framework: we firstly derive update rules at the constraint level while enforcing domain consistency and then derive update rules globally, at the problem level. The contribution is two-fold: first, we derive new generic update rules suited to tackle any CSP; second, we propose an efficient EMBP-inspired approach, thereby improving this method and making it competitive with the state of the art.

Running Example & Motivation

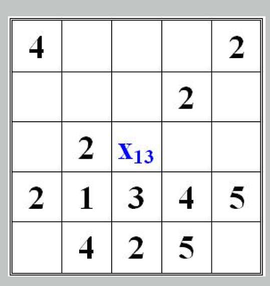


Figure: A choice point in a QWH instance

Then, if we were enumerating all the solutions, we could deduce:

- $P(x_{13}) = \begin{cases} P(x_{13}=1) = 2/5 \\ P(x_{13}=4) = 2/5 \\ P(x_{13}=5) = 1/5 \end{cases}$

Instead, we try to approximate this probability distribution:

- $\theta_{x_{13}} = \begin{cases} \theta_{x_{13}}(1) = 0.391 \\ \theta_{x_{13}}(4) = 0.472 \\ \theta_{x_{13}}(5) = 0.137 \end{cases}$

The EMBP Framework

Definitions. The EMBP framework introduces the following definitions:

- Θ Probability distribution of the variables
- $\mathbf{y}$ Binary-vector random variable indicating whether the constraints are satisfied
- $\mathbf{z}$ Satisfying configurations of the constraints

Essentially, EMBP aims at maximizing the loglikelihood of $P(\mathbf{y},\mathbf{z}|\Theta)$.

Intuition. We observe a solution of the CSP ($\mathbf{y}$) but we do not really know how the constraints are satisfied ($\mathbf{z}$). Thus, we are asking EMBP to figure out how the variables are set (Θ).

Additional definition. Let $Q(\mathbf{z})$ be the distribution function $P(\mathbf{z}|\mathbf{y},\Theta)$, representing each solution probability given the biases Θ and given the observation $\mathbf{y}$ that the constraints are satisfied.

EMBP *(Hsu et al.)* iteratively adjusts Θ in a two-step process:

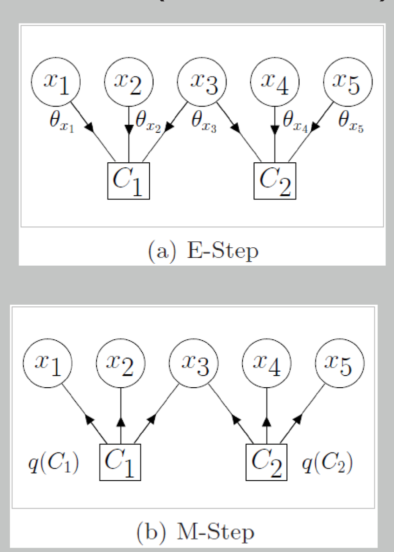


Figure: EMBP Steps

In the E-Step, we compute the probability of the satisfying configurations given the variable distribution:

$Q(\mathbf{z}) = \prod_{i=1}^{m}(q(C_i))$,

where $q(C_i)$ is the probability of a given configuration for C_i.

In the M-Step, the variables adjust their distribution taking into account the probability of the valid tuples of the constraints:

$\theta_{x_i}(v) = \frac{1}{\eta}\sum_{C_k\in\mathcal{C}:x_i\in X(C_k)}\left(\sum_{\mathbf{z}\in S_{z:x_i=v}} Q(\mathbf{z})\right)$

Computing EMBP. Within the EMBP framework, the definition of $Q(\mathbf{z})$ remains however unspecified. Computing $Q(\mathbf{z})$ exactly implies expressing every single solution of the problem, which is clearly intractable. Hence, we approximate $Q(\mathbf{z})$ and the following methods gradually improve the accuracy of the estimation of $Q(\mathbf{z})$.

EMBPa for the `alldifferent` constraint

Within the *alldifferent* constraint, the probability that variable x_i is assigned the value v can be approximated by the probability that no other variable in the constraint takes the value v.

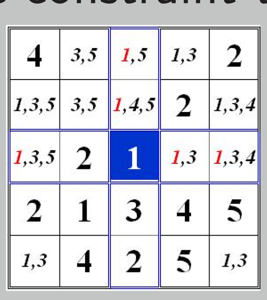


Figure: EMBPa on the running example

$$\theta_{x_i}(v) = \frac{1}{\eta}\sum_{C_k\in\mathcal{C}:x_i\in X(C_k)}\left(\prod_{x_j\in X(C)\setminus x_i}(1-\theta_{x_j}(v))\right)$$

where η is a normalizing constant.

(*Hsu et al., 2007*)

EMBP-Lsup - First Contribution

We propose to derive local X-consistency EMBP methods, which are a fairly natural extension to EMBPa, to improve the accuracy of EMBPa and extend this approach to any constraint.

We consider all assignments $\mathbf{y}=\mathbf{b}$ that are X-consistent with the assignment $\mathbf{x}=\mathbf{a}$ within a constraint to compute $\theta_{\mathbf{x}}(\mathbf{a})$ for a given constraint.

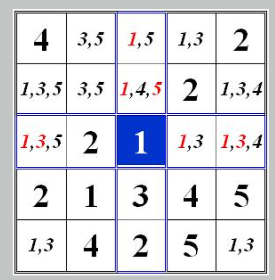


Figure: EMBP-Lsup on the running example

$$\theta_{x_i}(v) = \frac{1}{\eta}\sum_{C_k\in\mathcal{C}:x_i\in X(C_k)}\left(\prod_{x_j\in X(C_k)\setminus x_i}\ \sum_{v'\in\tilde{D}_{x_i=v}(x_j)}\theta_{x_j}(v')\right)$$

where $\tilde{D}_{x_i=v}(x_j)$ represents the reduced domain of the variable x_j after assigning $x_i = v$ and enforcing X-consistency on C_i.

EMBP-Gsup - Second Contribution

We suggest to go one step further in terms of accuracy for the computation of $Q(\mathbf{z})$. With EMBP-Gsup, the problem is considered as a whole and the method directly exploits the dependence between constraints when computing $Q(\mathbf{z})$. EMBP-Gsup improves the quality of the approximation taking into account supports that are X consistent after propagating every constraint of the problem.

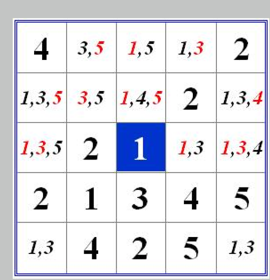


Figure: EMBP-Gsup on the running example

$$\theta_{x_i}(v) = \frac{1}{\eta}\prod_{x_j\in\mathcal{X}\setminus x_i}\ \sum_{v'\in\hat{D}_{x_i=v}(x_j)}\theta_{x_j}(v')$$

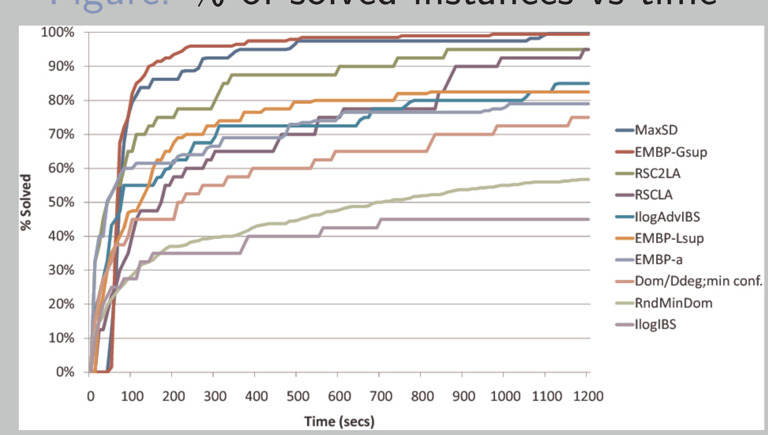
where $\hat{D}_{x_i=v}(x_j)$ represents the reduced domain of the variable x_j after assigning $x_i = v$ and enforcing X-consistency on the CSP.

Experiment Results on 40 hard QWH instances of order 30

Table: Time, bkts and % of solved inst.

heuristics	total time	avg btk	solved
rndMinDom	26328.2	1300056	56.8%
MaxSD	4939.1	3503	**100.0%**
IlogIBS	29017.0	1001570	45.0%
IlogAdvIBS	13795.8	914849	85.0%
RSC-LA	14019.9	856	95.0%
RSC2-LA	7178.7	4880	95.0%
EMBPa	13932.2	82158	79.0%
EMBP-Lsup	12814.2	6642	82.5%
EMBP-Gsup	**3946.9**	**55**	99.5%

Figure: % of solved instances vs time

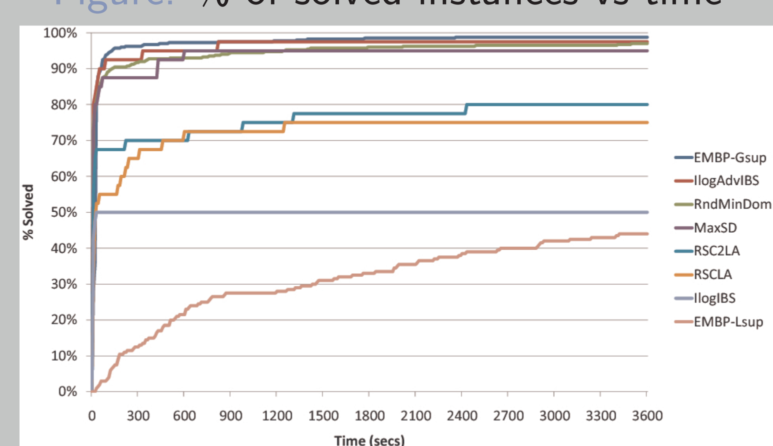


Experiment Results on 40 Magic Square instances

Table: Time, bkts and % of solved inst.

heuristics	total time	avg btk	solved
rndMinDom	7397.0	4018251	97.0%
MaxSD	8895.7	242290	95.0%
IlogIBS	72078.2	22396381	50.0%
IlogAdvIBS	5067.9	2224191	97.5%
RSC-LA	39612.4	48759	75.0%
RSC2-LA	34524.3	1180456	80.0%
EMBP-Lsup	98910.7	20572	43.0%
EMBP-Gsup	**3758.2**	**895**	**98.8%**

Figure: % of solved instances vs time



Summary

- We propose new efficient extensions to the EMBP framework that present better accuracy, due to the propagation at the constraint level (EMBP-Lsup) and at the problem level (EMBP-Gsup).
- The methods are generic and can be applied to any constraint (EMBP-Lsup) or any CSP (EMBP-Gsup).
- EMBP-Gsup tends to be consistent on the problems we experimented and is really competitive with existing approaches.

Future Work

- What is the most efficient way to use biases information?
- How can we make EMBP-Gsup faster?
- Could we exploit Expectation-Maximization Survey Propagation to derive similar EMSP-based update rules?

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