Designing efficient market pricing mechanisms

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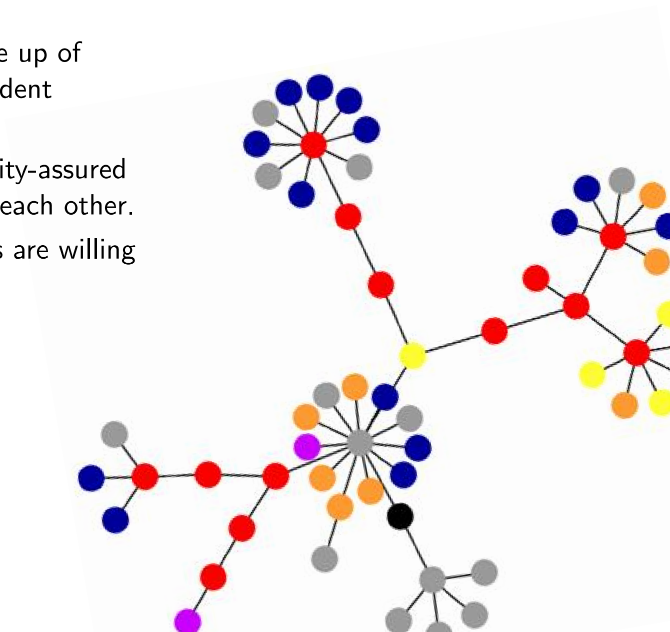
If we could design an economy,
which market rules would result
in the most efficient use of resources?

If we could design a network,
which protocols would result
in the most efficient use of bandwidth?

Engineering application in networking

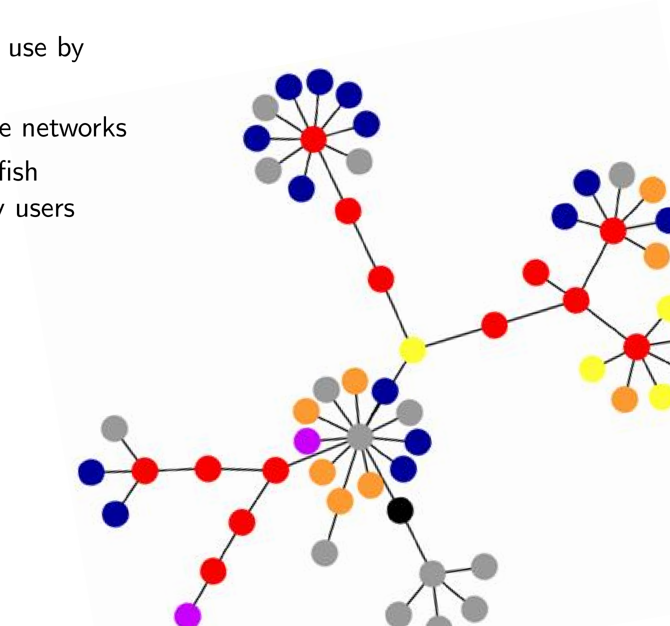
- Internet is made up of smaller independent networks.
- They need quality-assured connectivity to each other.
- Network owners are willing to sell transit.



Example companies

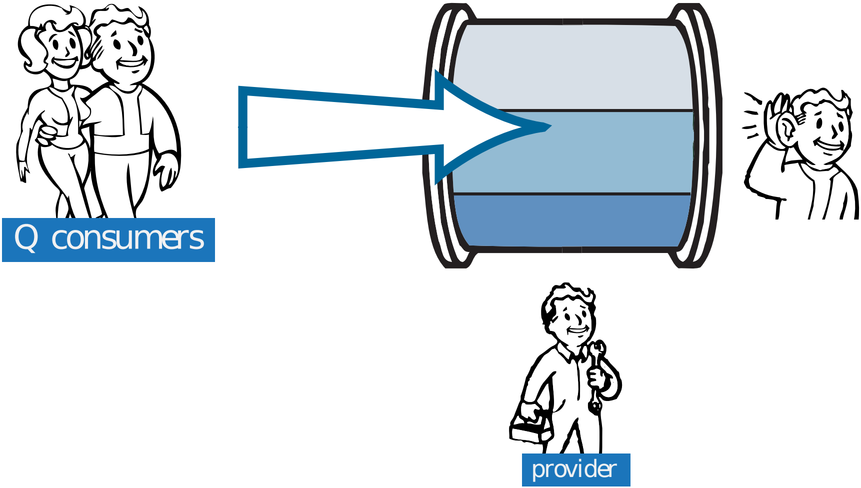
Mechanism requirements

- Easy enough to use by actual people
- Scalable to large networks
- Resistant to selfish manipulation by users

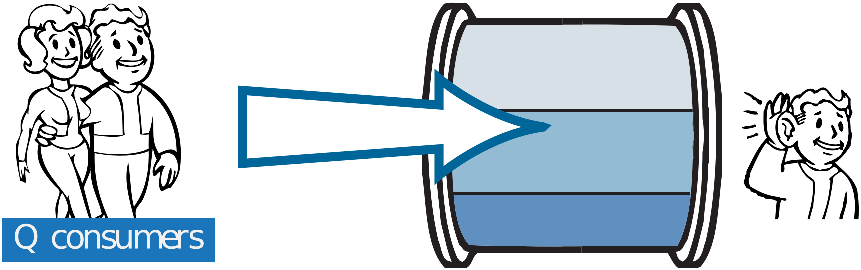


We propose a simple, scalable, and provably economically efficient pricing mechanism.

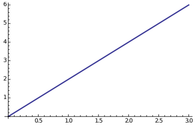
Mechanism definition



Mechanism definition



1. Provider r submits a *pricing function* $p(f) = \gamma f$



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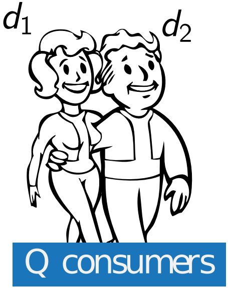

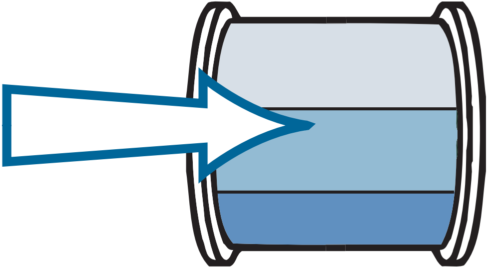

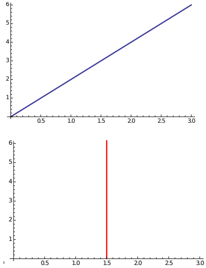

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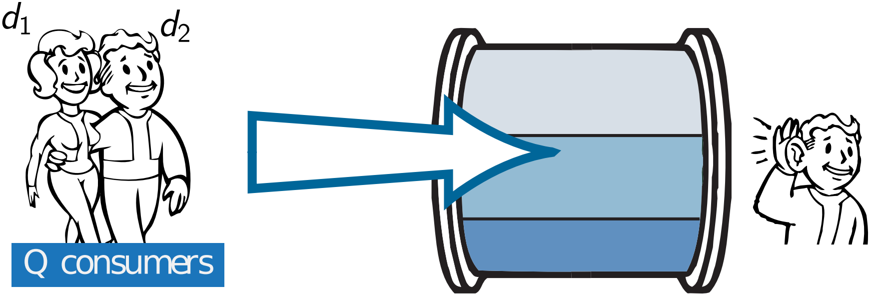
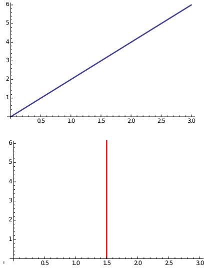
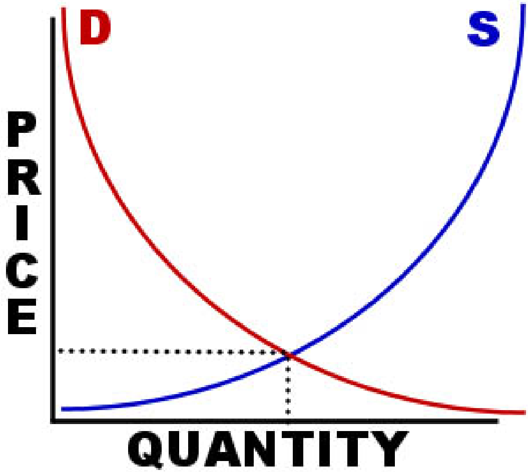
Mechanism definition





1. Provider r submits a *pricing function* $p(f) = \gamma f$
2. User q chooses a rate d_q to transmit
3. The provider receives $p(\sum_i d_i) d_q$ from consumer q.

A simple interpretation



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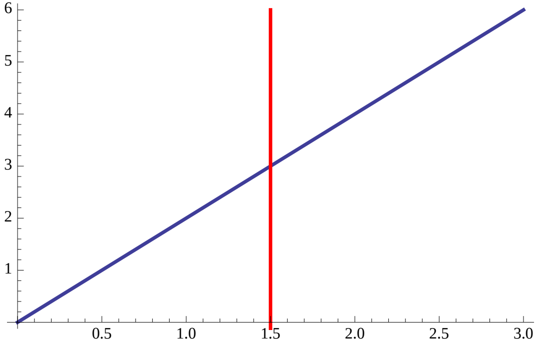

$$\text{At equilibrium, price} = p(\sum_q d_q)$$
$$= \gamma f$$

How do me measure efficiency?

- User q has a utility function of the form

$$U_q(d_q) = \underbrace{V_q(d_q)}_{\text{value}} - \underbrace{pd_q}_{\text{expenses}}$$

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- The *social welfare* is the sum of the utilities:

$$W(\mathbf{d}, \gamma) = \underbrace{\sum_{q \in Q} V_q(d_q)}_{\text{valuations}} - \underbrace{C(f)}_{\text{costs}}$$

How do we measure efficiency?

- Assume that any user's action is always the best he/she can do given what everyone else is doing:
- A combination of actions $(\mathbf{d}^{\mathsf{NE}}, \gamma^{\mathsf{NE}})$ that satisfies this is a *Nash equilibrium*.

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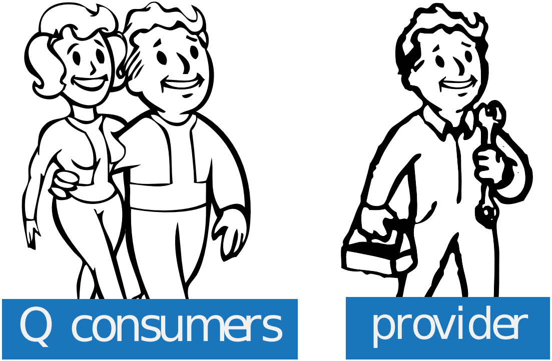
- We measure efficiency using the *price of anarchy*:

$$\frac{\text{welfare at worst Nash equilibrium}}{\text{best possible welfare}}$$

The mechanism, again



There are two sources of inefficiency.

Separating demand and supply side inefficiency

Theorem (Johari and Tsitsiklis, 2005)

When supply is fixed, the price of anarchy on the demand side of the market is $2/3$. □

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Lemma

The price of anarchy of the two-sided market equals

$$\frac{2\rho(\rho-2)}{\rho-4}$$

where $0 \leq \rho \leq 1$ *is an* overcharging parameter. □

Separating demand and supply side inefficiency

Lemma

The worst efficiency occurs with marginal cost functions are linear:

$$c(f) = \beta f$$

$\square$

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□

Lemma

For these cost functions, the overcharging parameter equals

$$\rho = \frac{\beta}{\gamma}$$

where γ *is the slope of* $p(f) = \gamma f$ *(the pricing function).* □

Elasticity of demand

Definition

The elasticity of the flow f with respect to γ is defined to be

$$\epsilon = \frac{\%\Delta f}{\%\Delta \gamma} = \frac{\text{percentage change of f}}{\text{percentage change of } \gamma}$$

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The elasticity of the flow f with respect to γ is defined to be

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Lemma

At equilibrium, the overcharging parameter equals

$$\rho = 2 - \frac{1}{\epsilon}$$

□

Results for one link: user demand

Proposition

When consumer valuations are monomials of degree d:

$$V_q(f) = a_q f^d$$

then

$$\rho = d$$

and the price of anarchy is

$$\frac{2d(2-d)}{4-d}$$

□

Results for one link: supplier competition

Theorem

Suppose there are at least 3 producers. Then the price of anarchy is bounded by a constant. □

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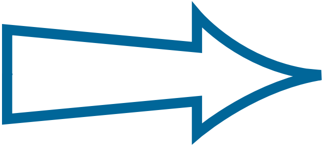
Theorem

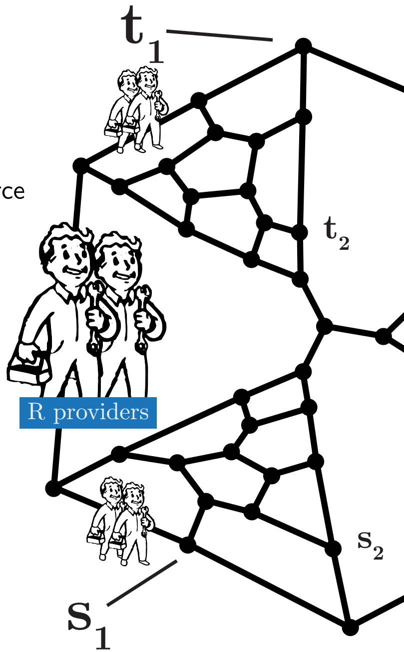
When the number of providers $R \to \infty$, $\rho \to 1$ *and the price of anarchy goes to* $2/3$. □

Markets over general graphs

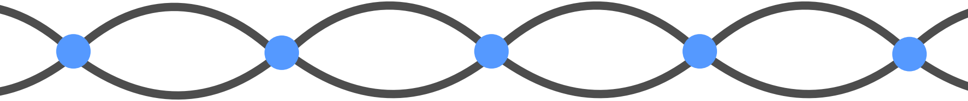
- Each user owns a pair of nodes (s_q, t_q).
- Users buy capacities on edges using a single-resource market at each edge.



- They value the flow they send in the capacitated graph.



Case 1: Route graph



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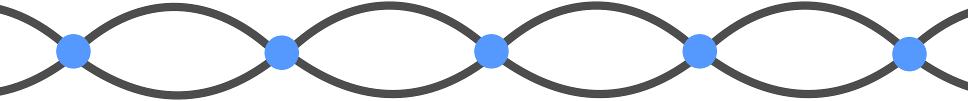

Theorem

Let G be a route with L links and two providers per link. Assume that valuations are linear.

1. *When $L < \infty$, ρ is strictly positive.*
2. *As $L \to \infty$, $\rho \to 0$.*

□

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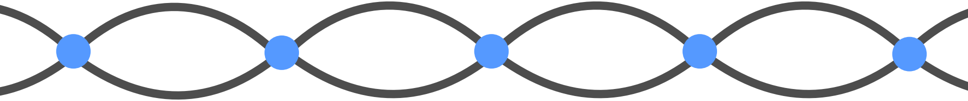

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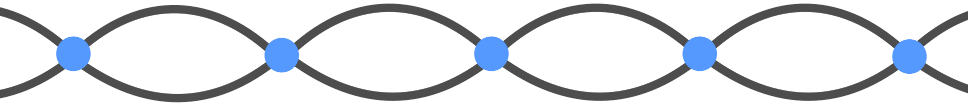
Theorem

Let G be a route with L links and at least three providers per link. Then ρ is always strictly positive □

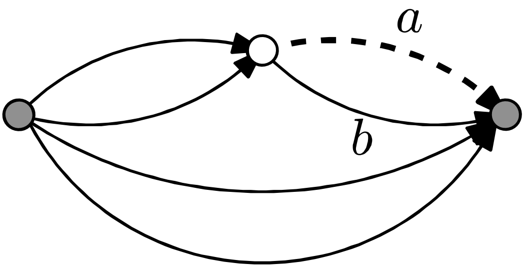
Case 1: Route graph, provider competition



Theorem

Let G be a route with L links and R providers per link.
As $R \to \infty$, $\rho \to 1$, and the price of anarchy tends to $2/3$. □

Case 2: Series-parallel graph



- The price of anarchy in a parallel-serial graphs is bounded by that of a path.
- As the number of paths increases, the effective demand becomes elastic.

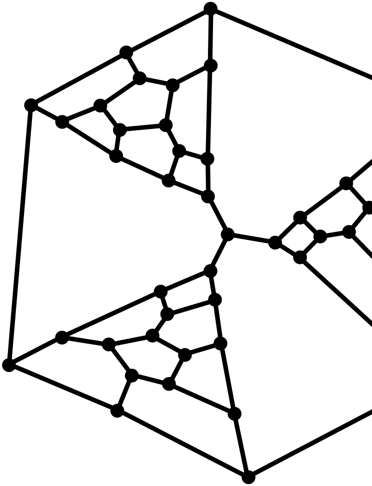
Case 3: Arbitrary graph

Theorem

Let G an arbitrary graph.

- *When there are at least 3 providers on every link, the price of anarchy is non-zero.*
- *As the number of providers goes to infinity $\rho \to 1$, and the price of anarchy goes to $2/3$.*

□



Conclusion

In this work we,

- Analyzed a form of pricing that can be deployed in practice.
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This can be used to

- Guide the design of real-world pricing mechanisms.
- Approximately forecast how market features will affect their performance.