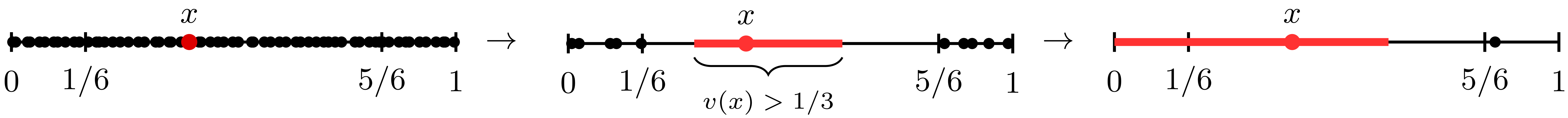

The Moderating Effect of Instant Runoff Voting

AAAI ‘24

Kiran Tomlinson
Cornell University

Johan Ugander
Stanford University

Jon Kleinberg
Cornell University

How do we elect a winner given the preferences of voters?

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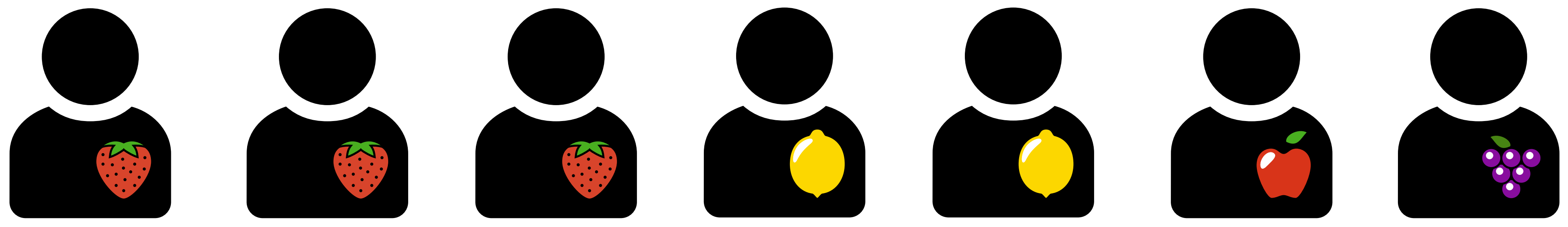

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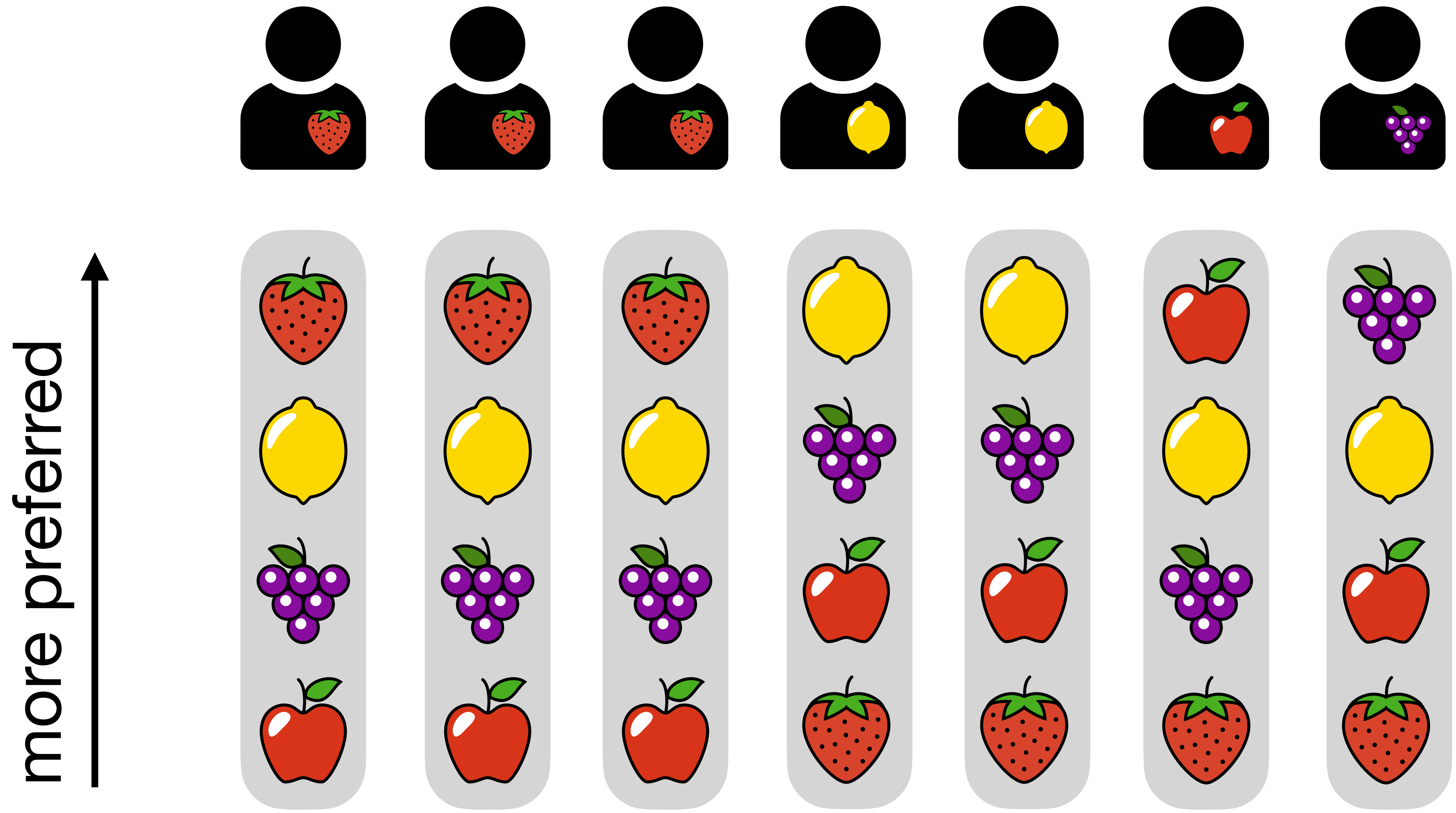

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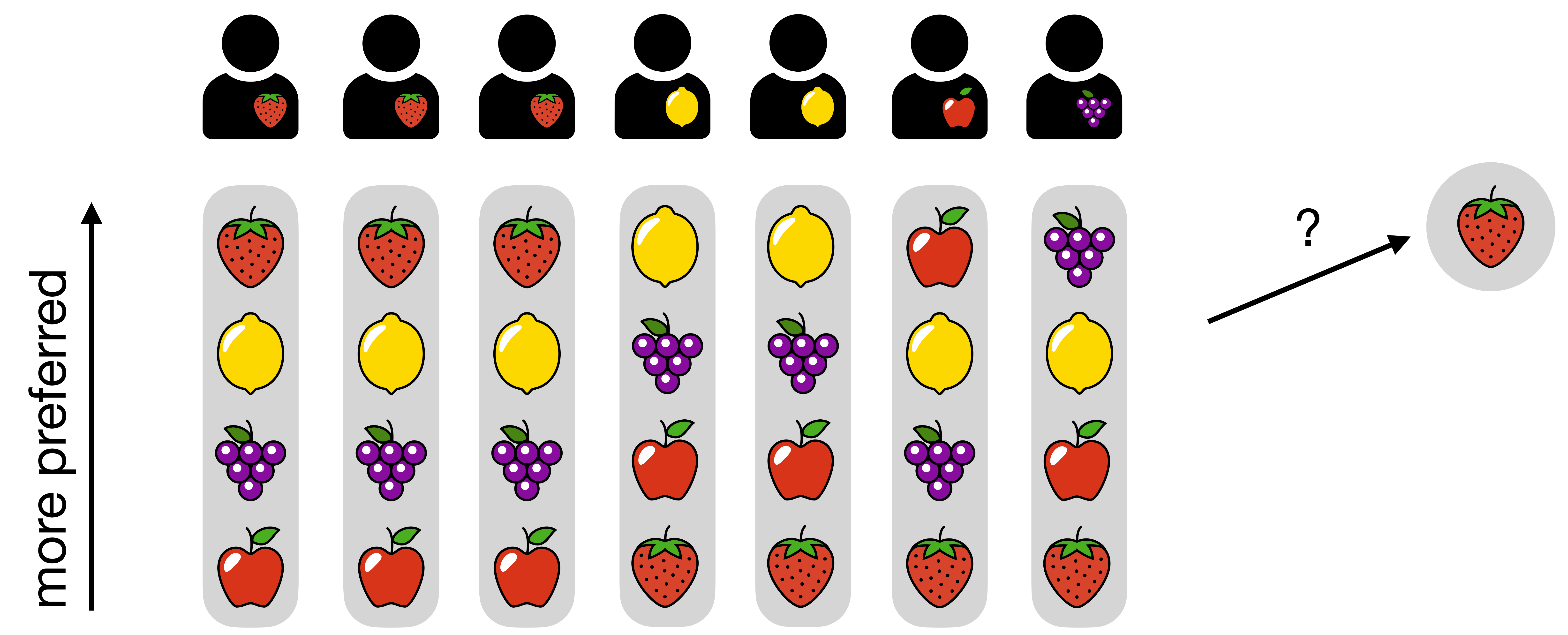

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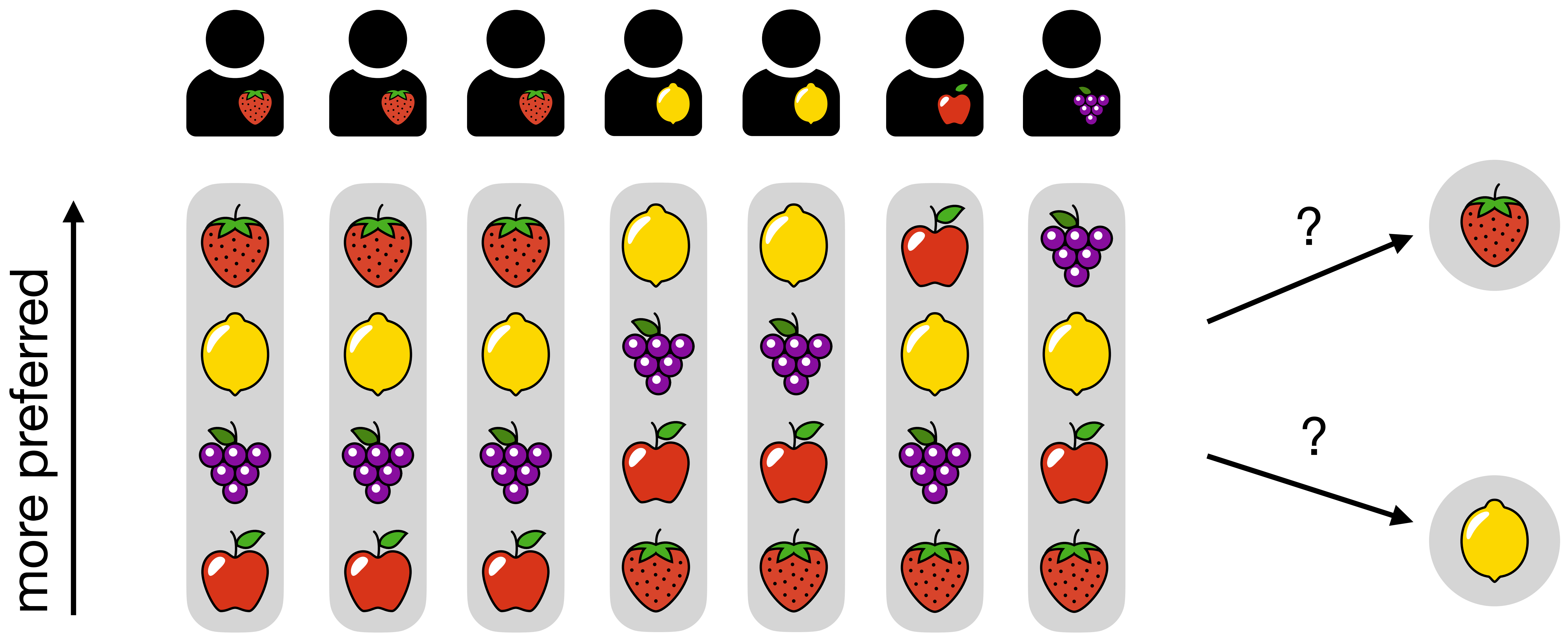
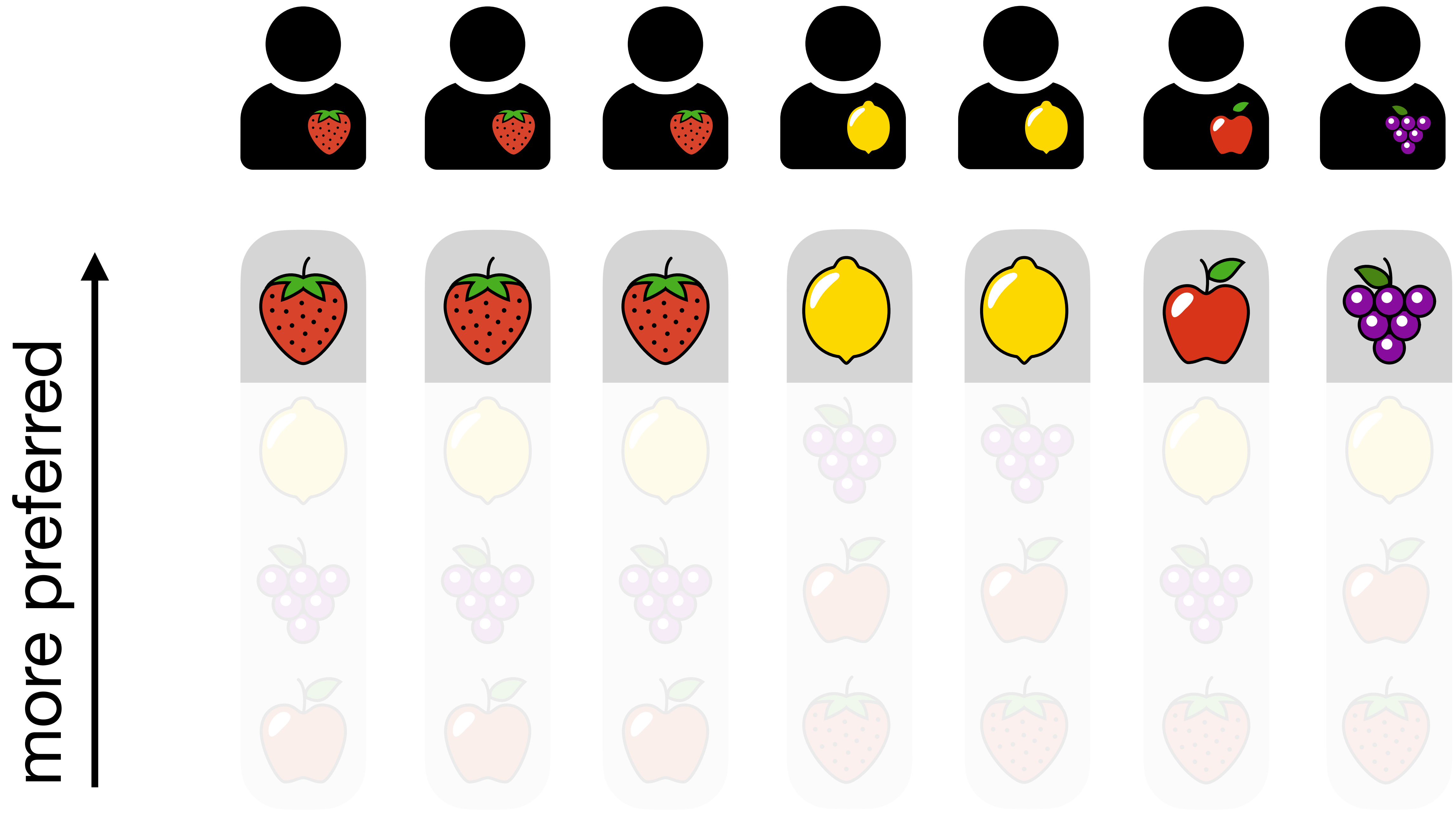


Plurality voting

choose the candidate with the most first-place votes



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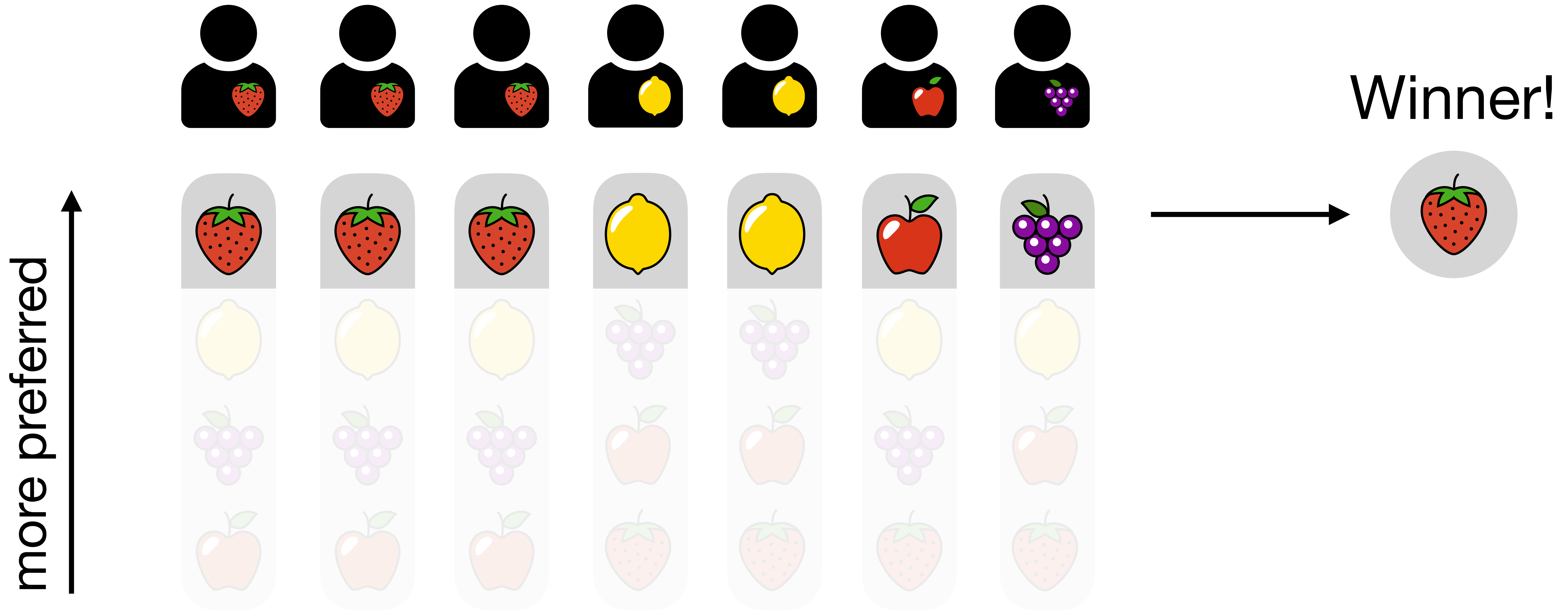
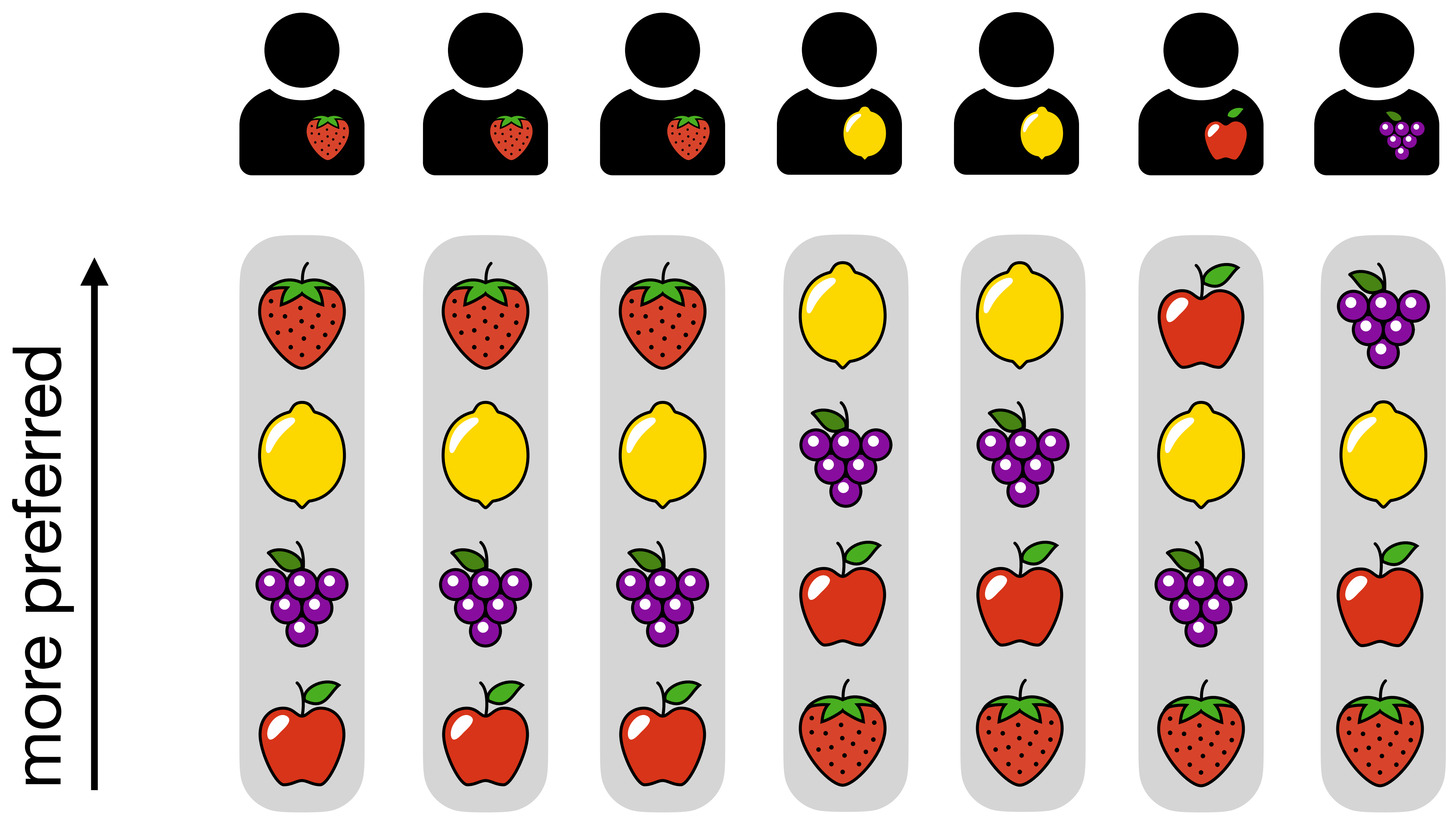


Instant runoff voting (IRV)

repeatedly eliminate candidate w/ fewest first-place votes



Elimination order

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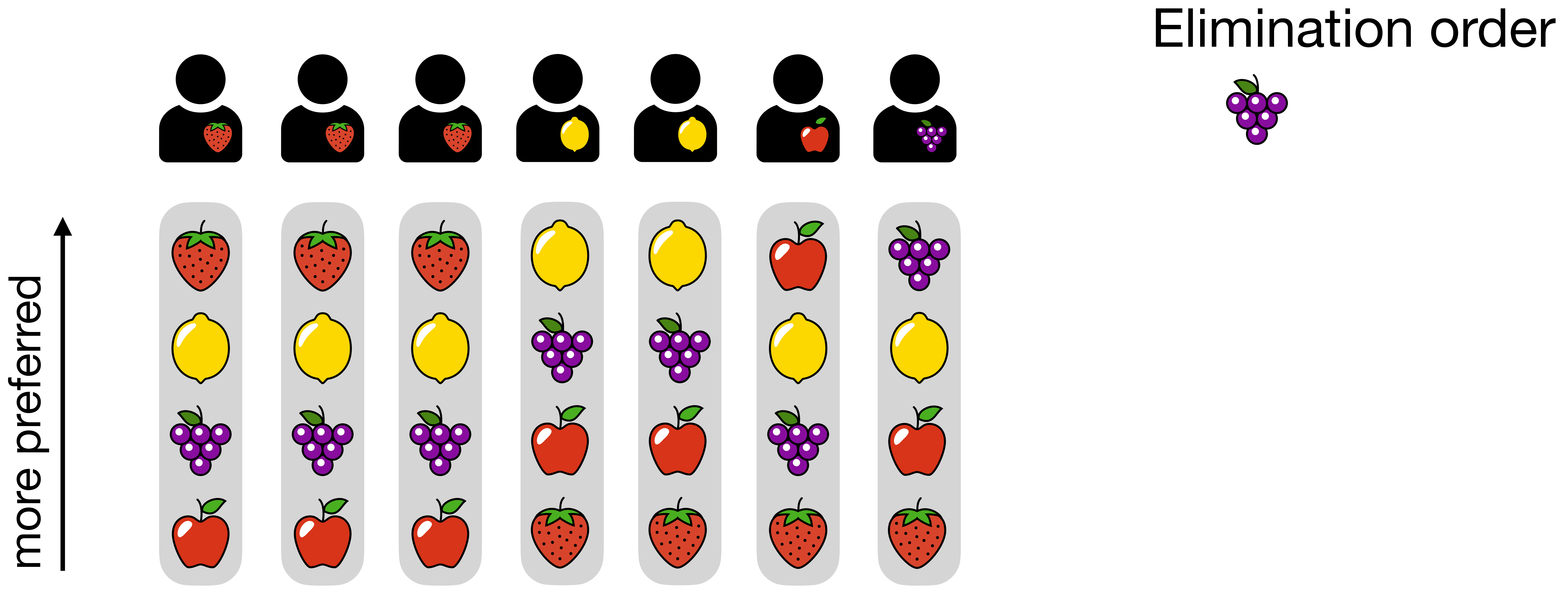

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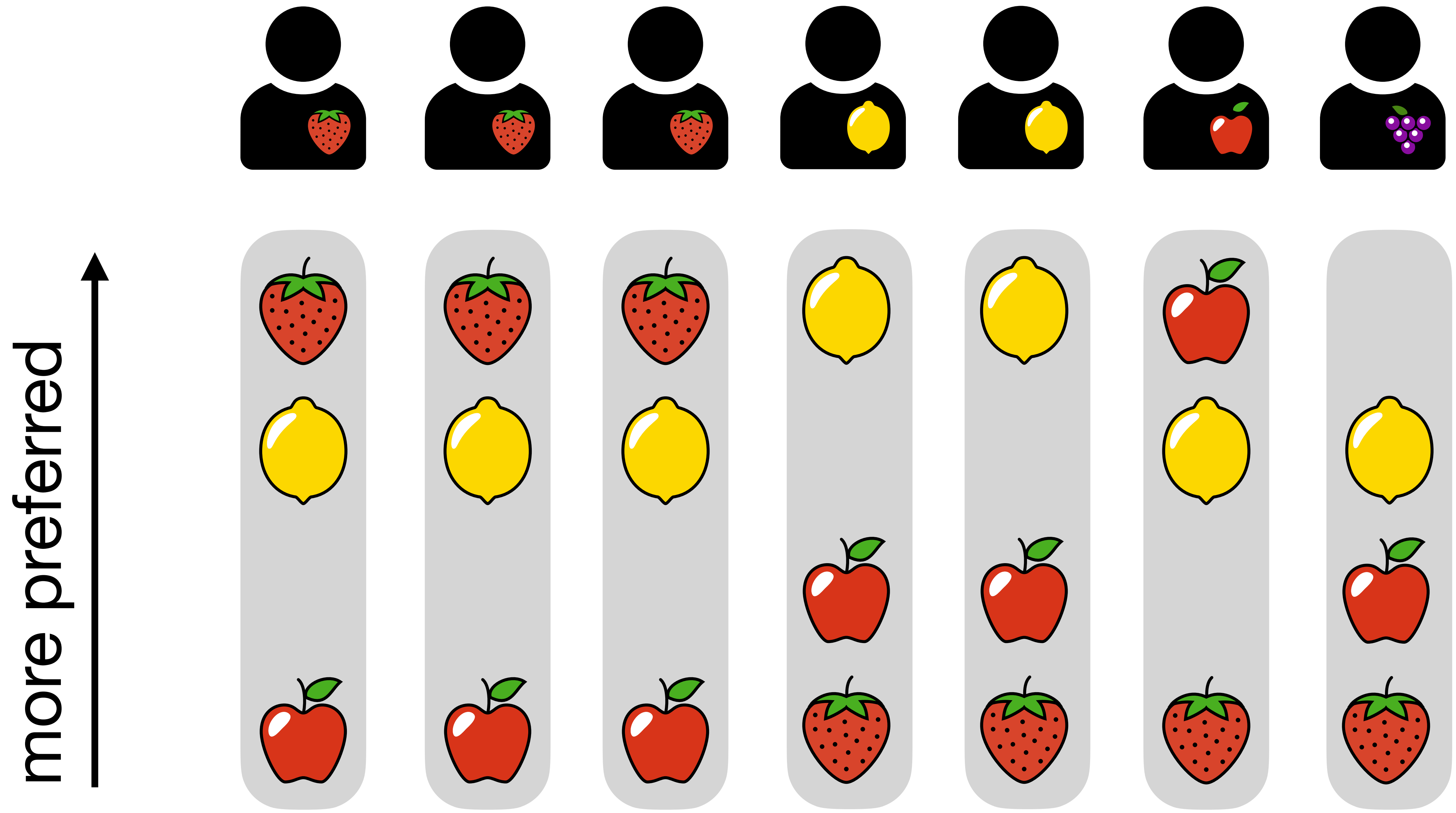

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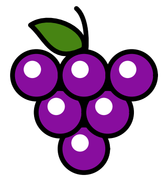

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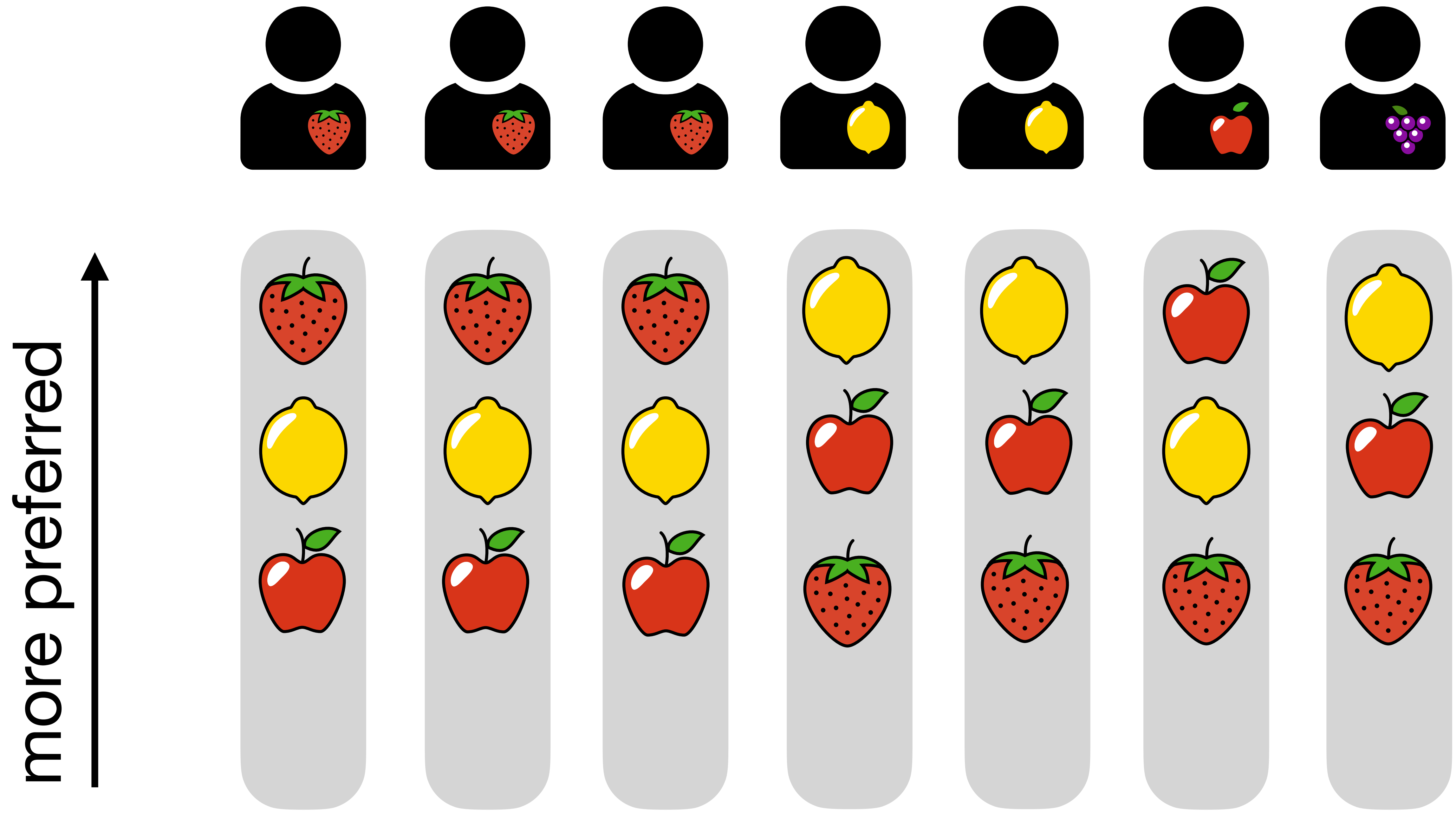

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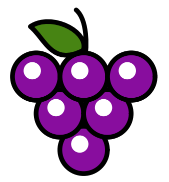

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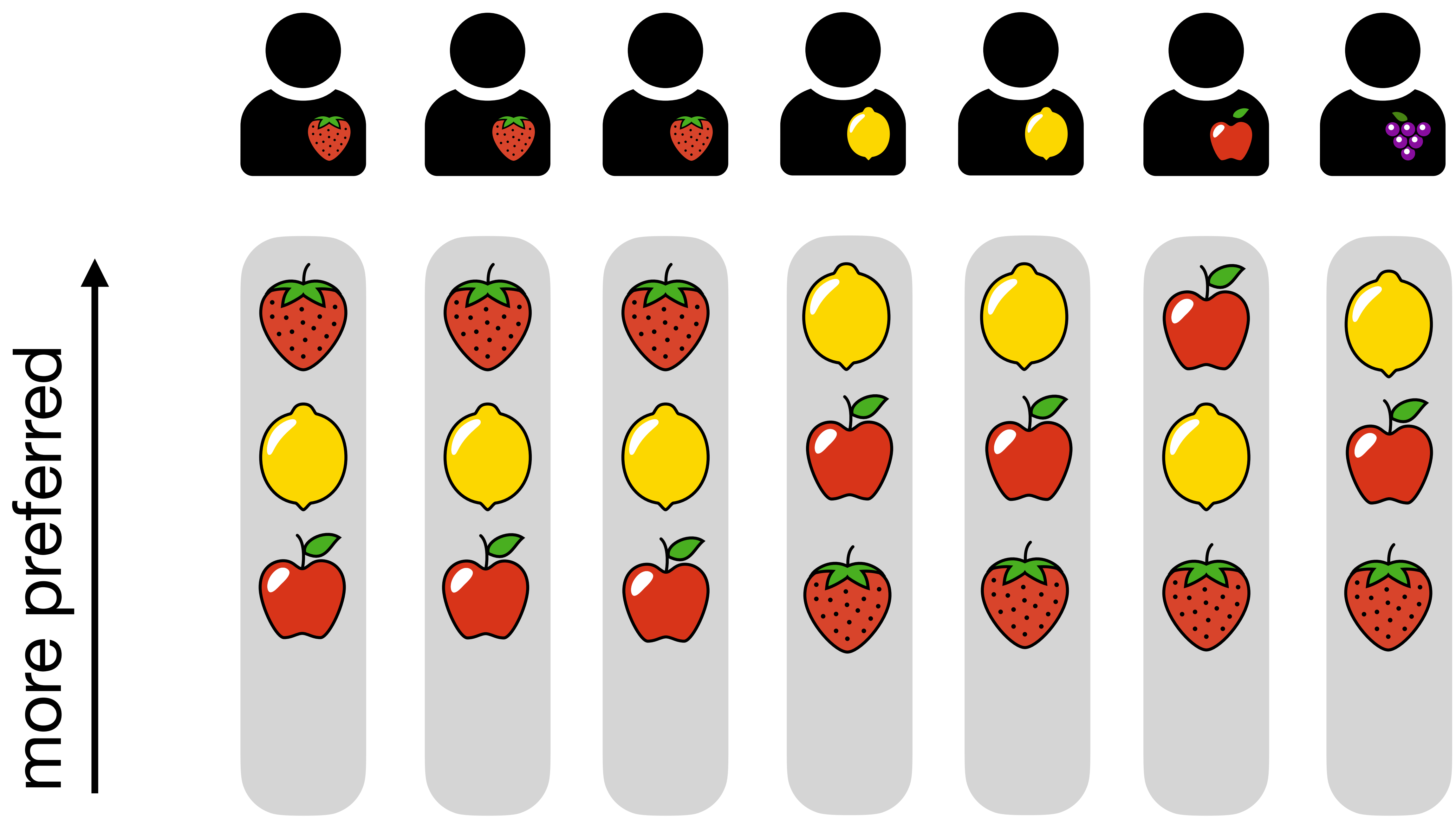

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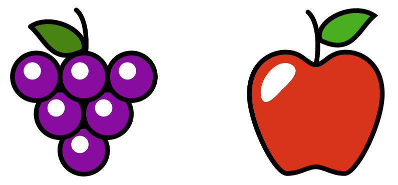

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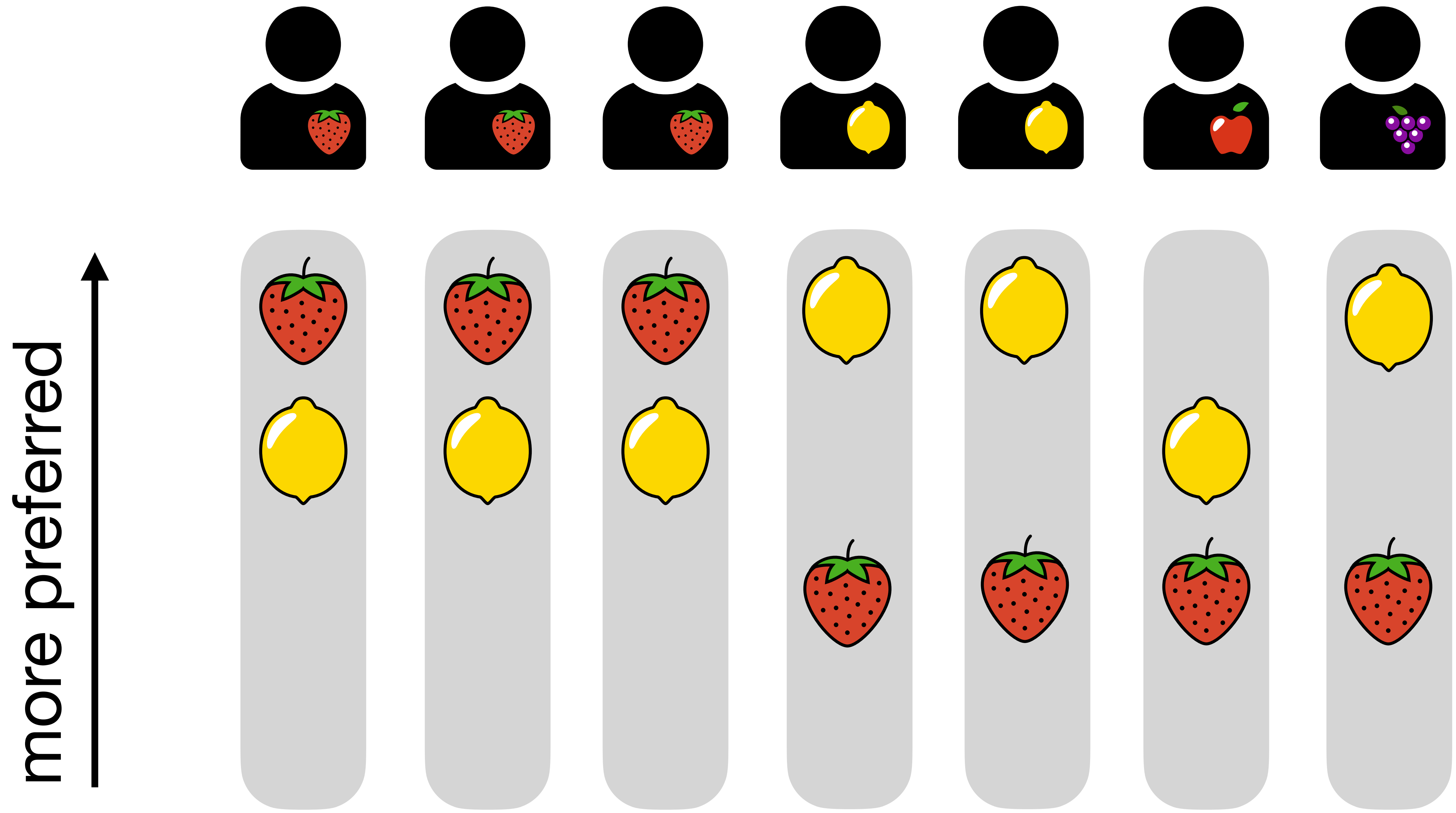

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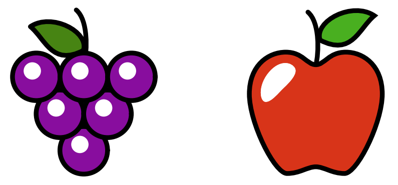

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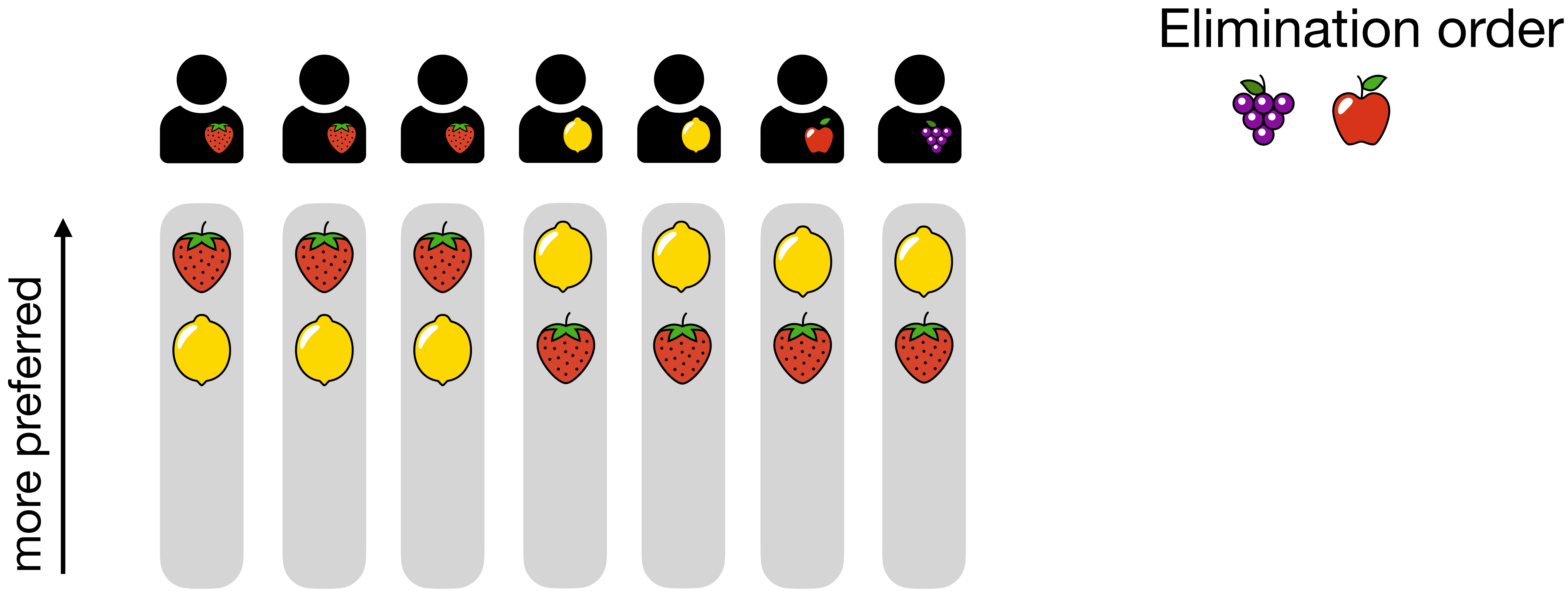

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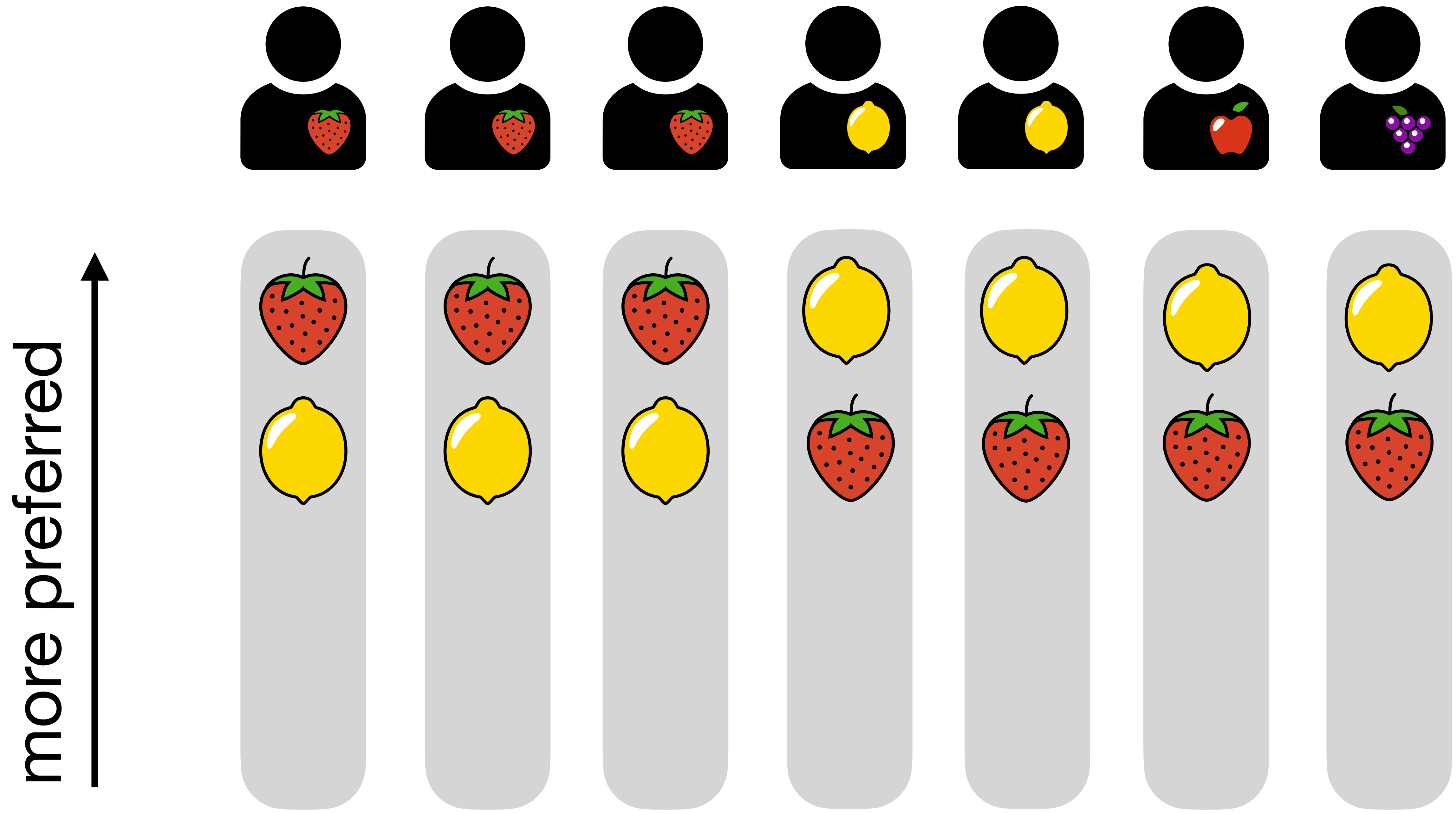

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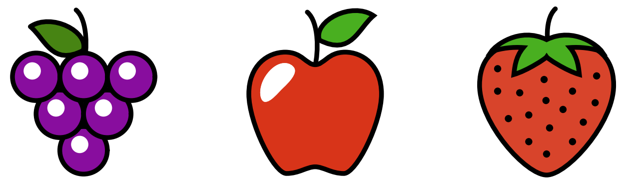

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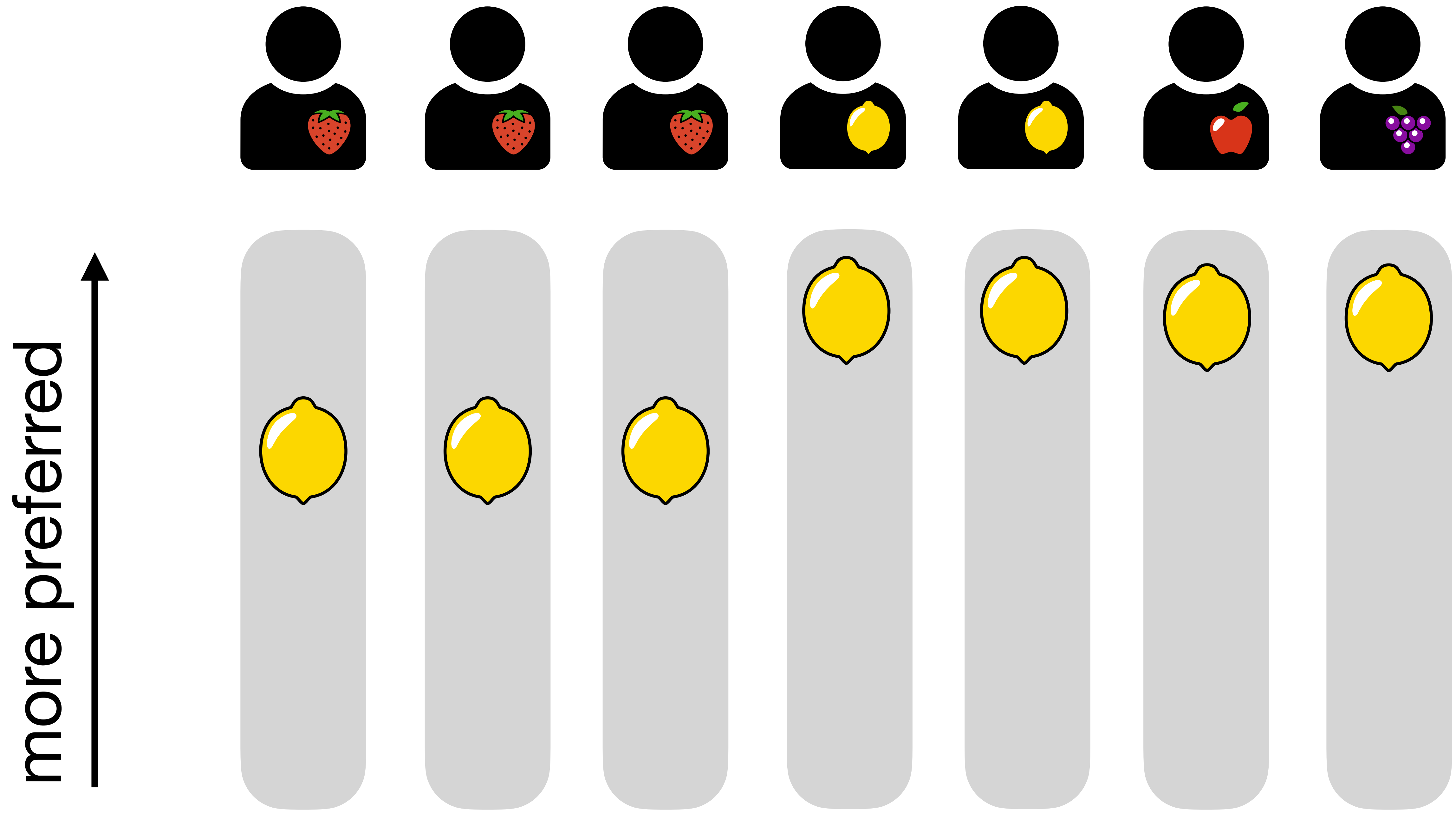

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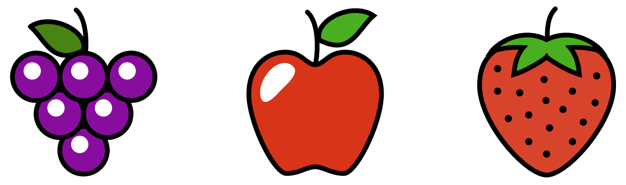

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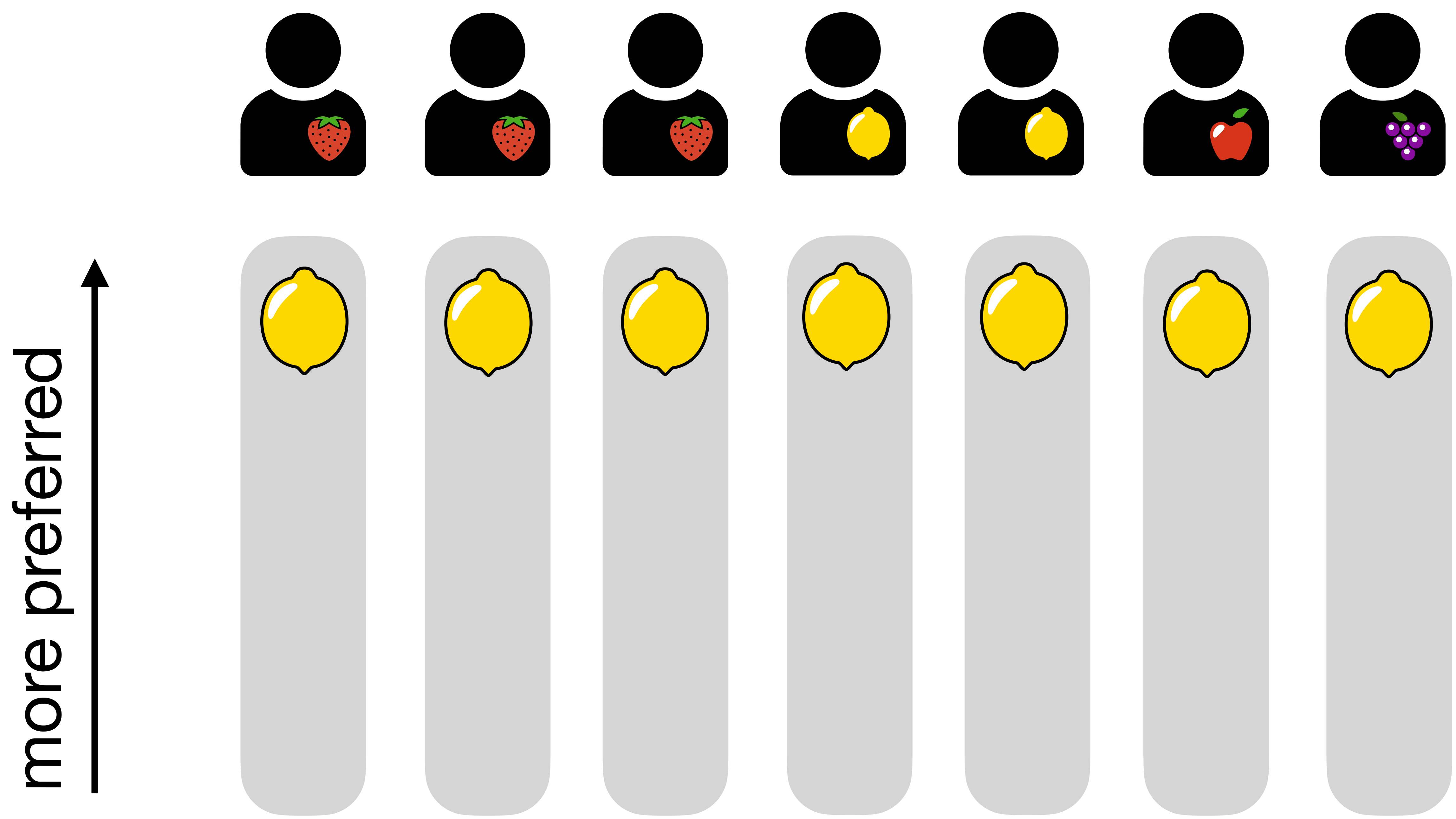

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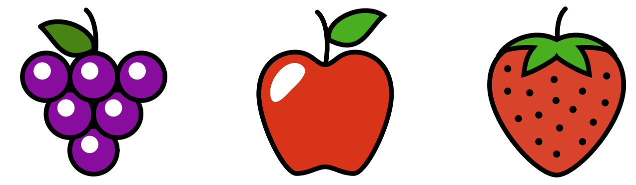

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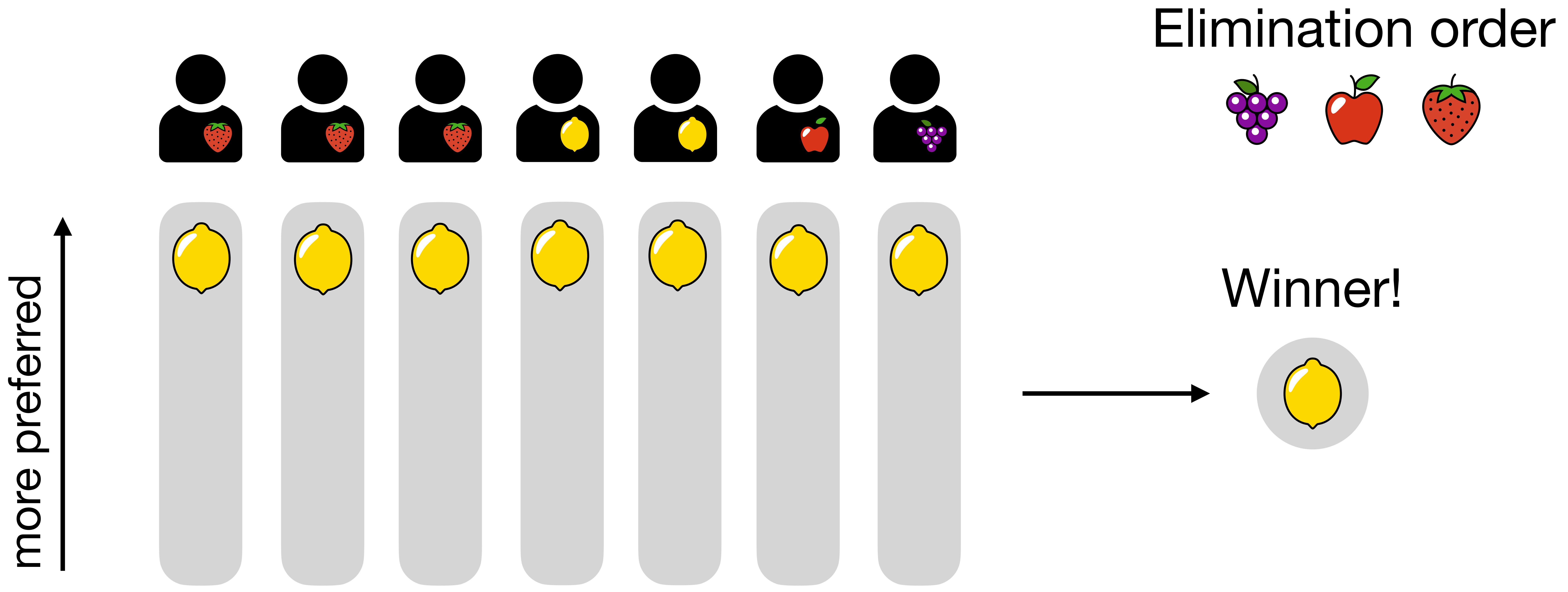

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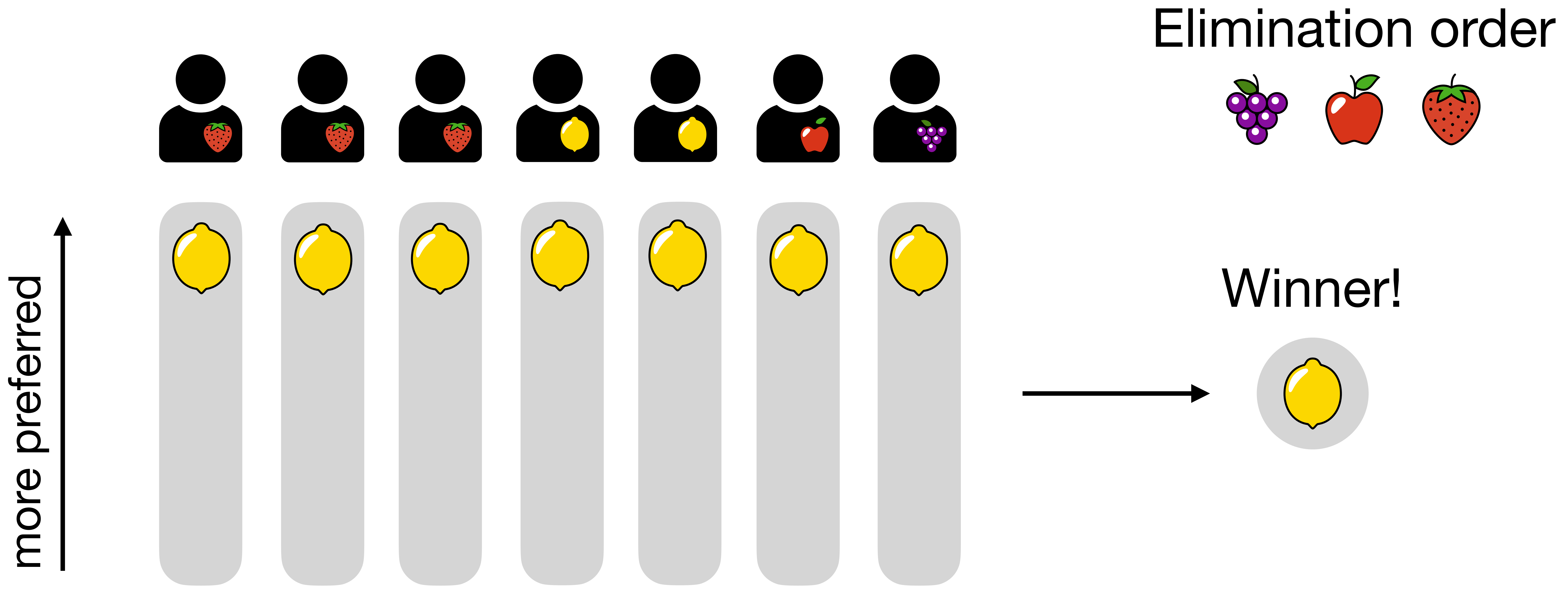
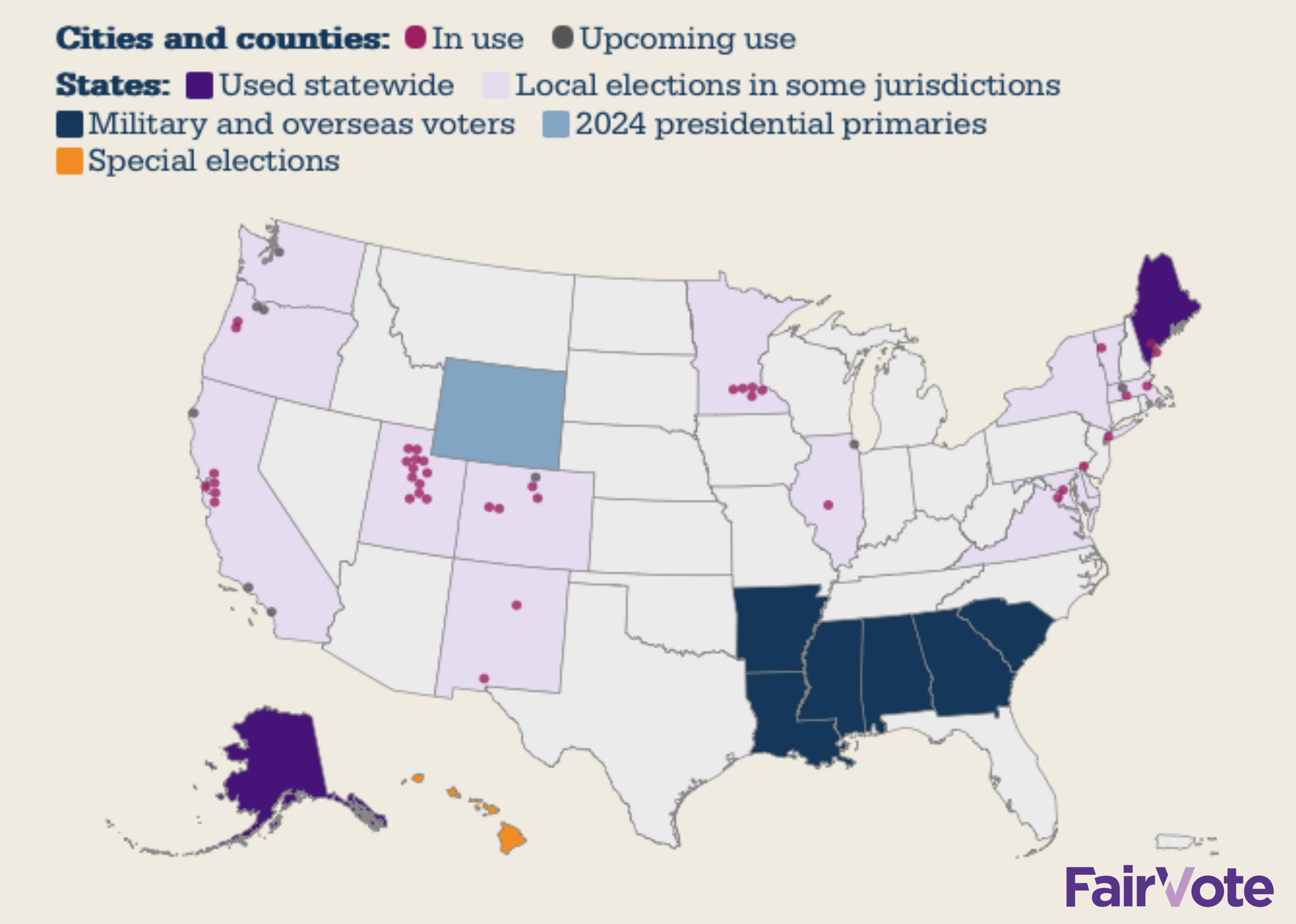
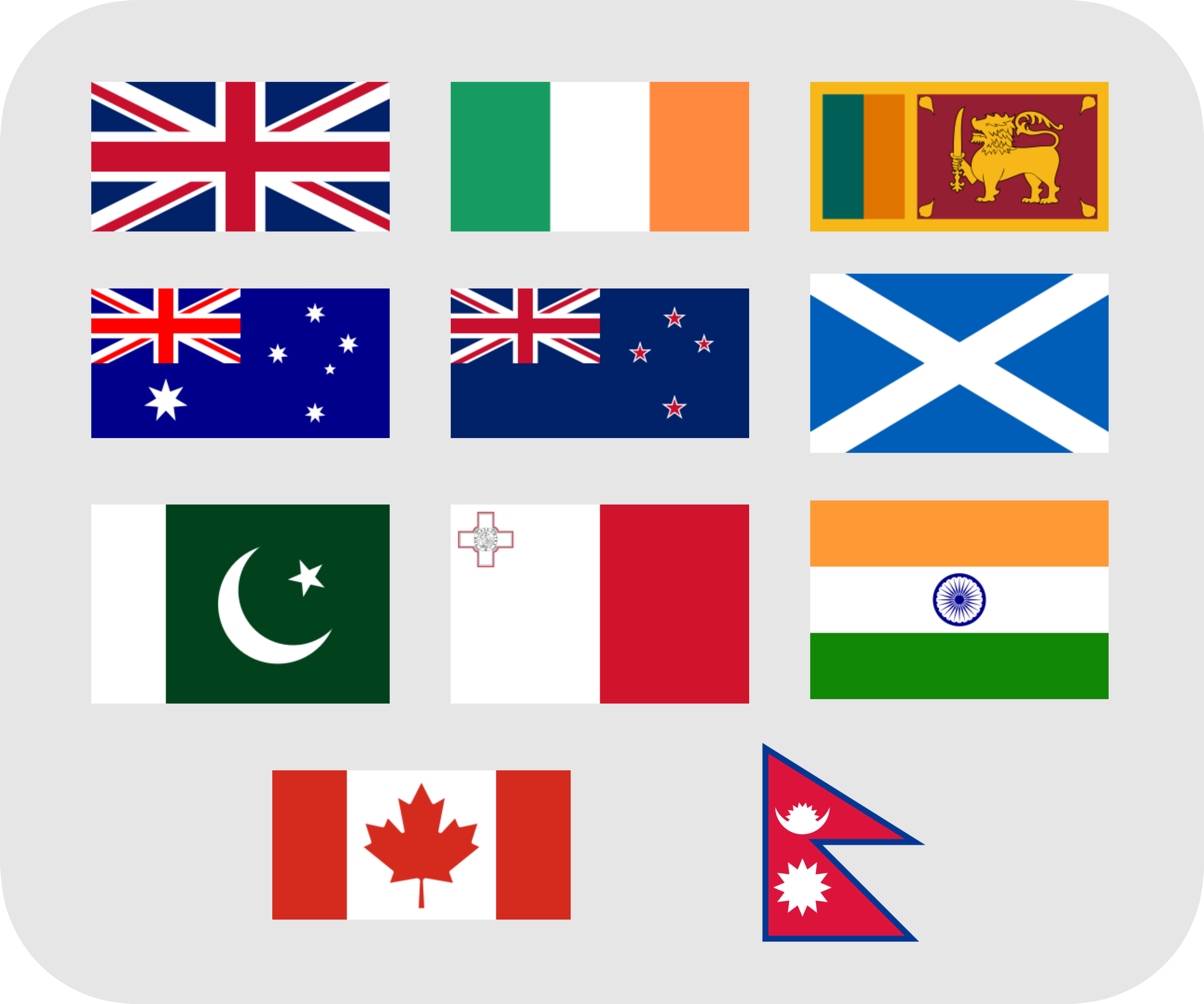


a.k.a. ranked-choice voting (+ AV, STV, Hare, …)

Who uses IRV?


Cities and counties: In use Upcoming use
States: Used statewide Local elections in some jurisdictions
Military and overseas voters 2024 presidential primaries
Special elections
FairVote



NBC NEWS

Following a big year, more states push ranked-choice voting

Lawmakers in 14 states have already introduced 27 bills proposing ranked-choice voting models, according to an NBC News review.

Jan. 16, 2023, 7:00 AM EST

By Adam Edelman

Ranked choice voting is being touted as a cure-all for U.S. deep partisan divides

DECEMBER 3, 2023 · 5:54 PM ET

HEARD ON ALL THINGS CONSIDERED

npr

The New York Times

OPINION

Can Ranked-Choice Voting Cure American Politics?

June 24, 2021

By Spencer Bokat-Lindell

Supreme Court shoots down GOP attempt to stop ranked-choice voting in Maine

FOX NEWS channel

The system allows voters to rank candidates in order of preference on the ballot

By Paul Steinhauser · Fox News

WSJ OPINION

Ranked-Choice Voting Was a Bad Choice

Arlington County, Va., halts a system that left many voters confused.

By The Editorial Board Follow

July 25, 2023 at 6:44 pm ET

OPINION | POTOMAC WATCH

The 'Ranked Choice' Scam

Alaskans know the truth about this confusing, coercive voting system.

By Kimberley A. Strassel Follow

Oct. 27, 2022 at 6:14 pm ET

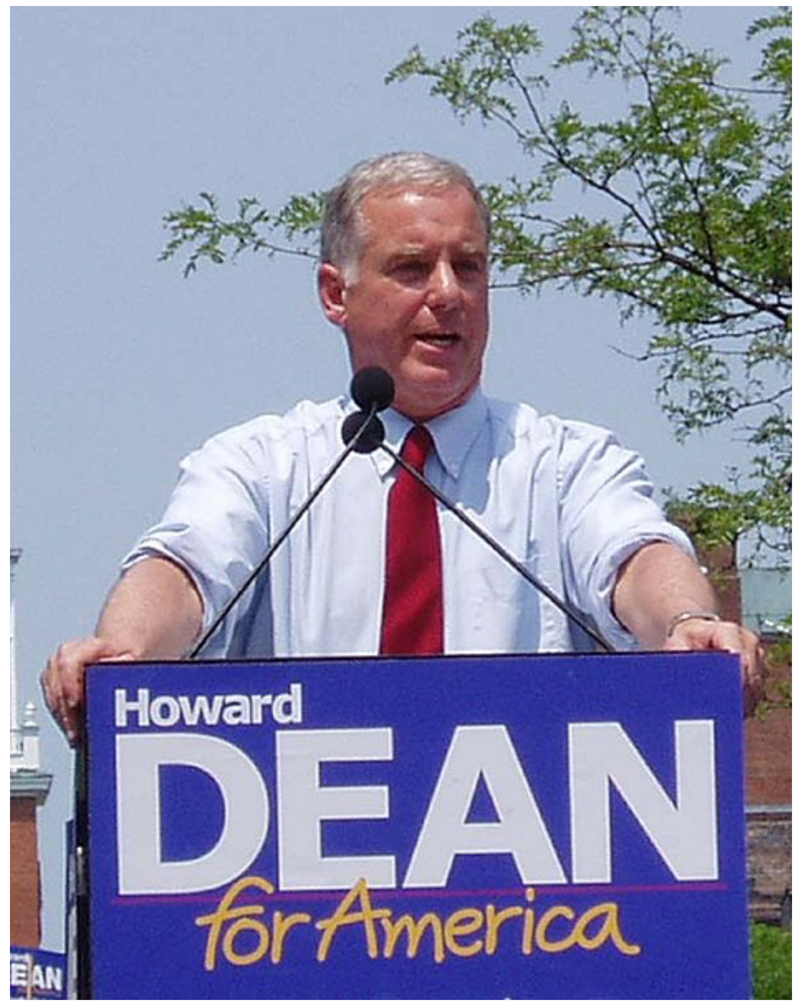
Ranked-Choice Voting Is Bad for Everyone

It appeals to progressives because it allows them to vote twice—once for show and once for real.

By Harvey Mansfield

July 7, 2021 at 12:10 pm ET

Common debate: does IRV benefit moderates?



[Under IRV,] civility is substantially improved. Needing to reach out to more voters leads candidates to reduce personal attacks and govern more inclusively.

Howard Dean. How to move beyond the two-party system. *NY Times*, 10/8/2016

The ranked-choice system […] is biased towards extreme candidates and away from moderate ones.

Nathan Atkinson and Scott Ganz. The flaw in ranked-choice voting: rewarding extremists. *The Hill*, 10/30/2022

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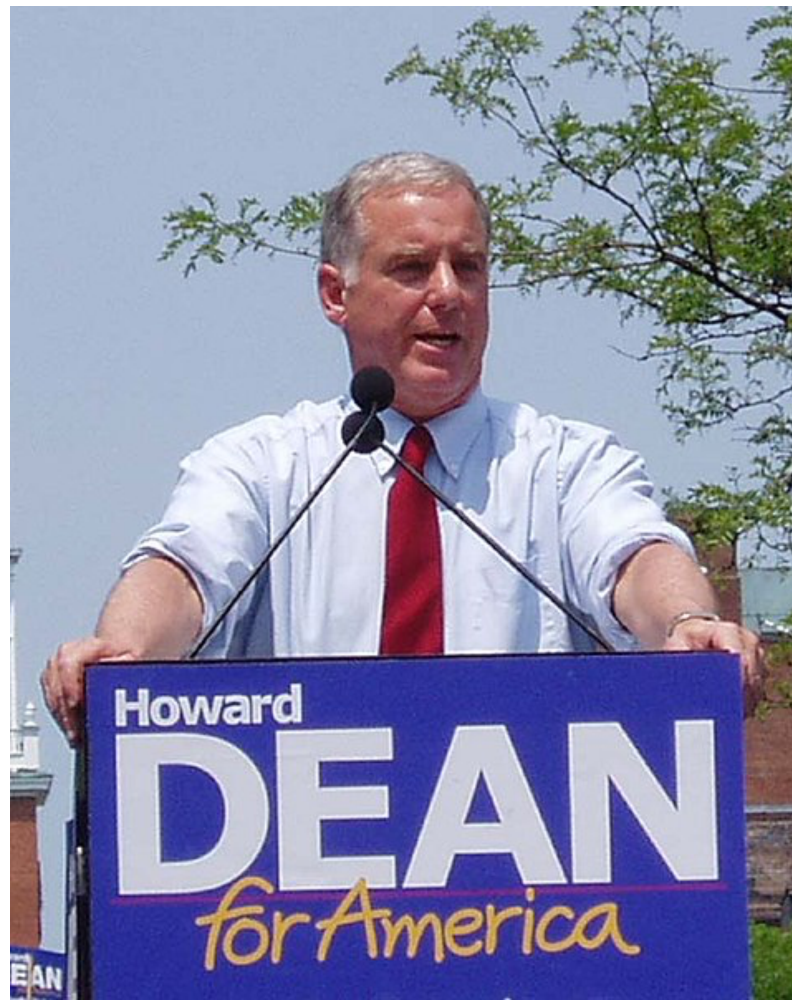

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case studies

(Fraenkel & Grofman, *Public Choice* 2004)
(Mitchell, *Electoral Studies* 2014)
(Reilly, *Nationalism and Ethnic Politics* 2018)

simulation

(Chamberlin and Cohen, *APSR* 1978)
(Merrill, *AJPS* 1984)
(McGann, Grofman, & Koetzle, *Public Choice* 2002)

some limited theory

(Grofman & Feld, *Electoral Studies* 2004)
(Dellis, Gauthier-Belzile, & Oak, *JITE* 2017)

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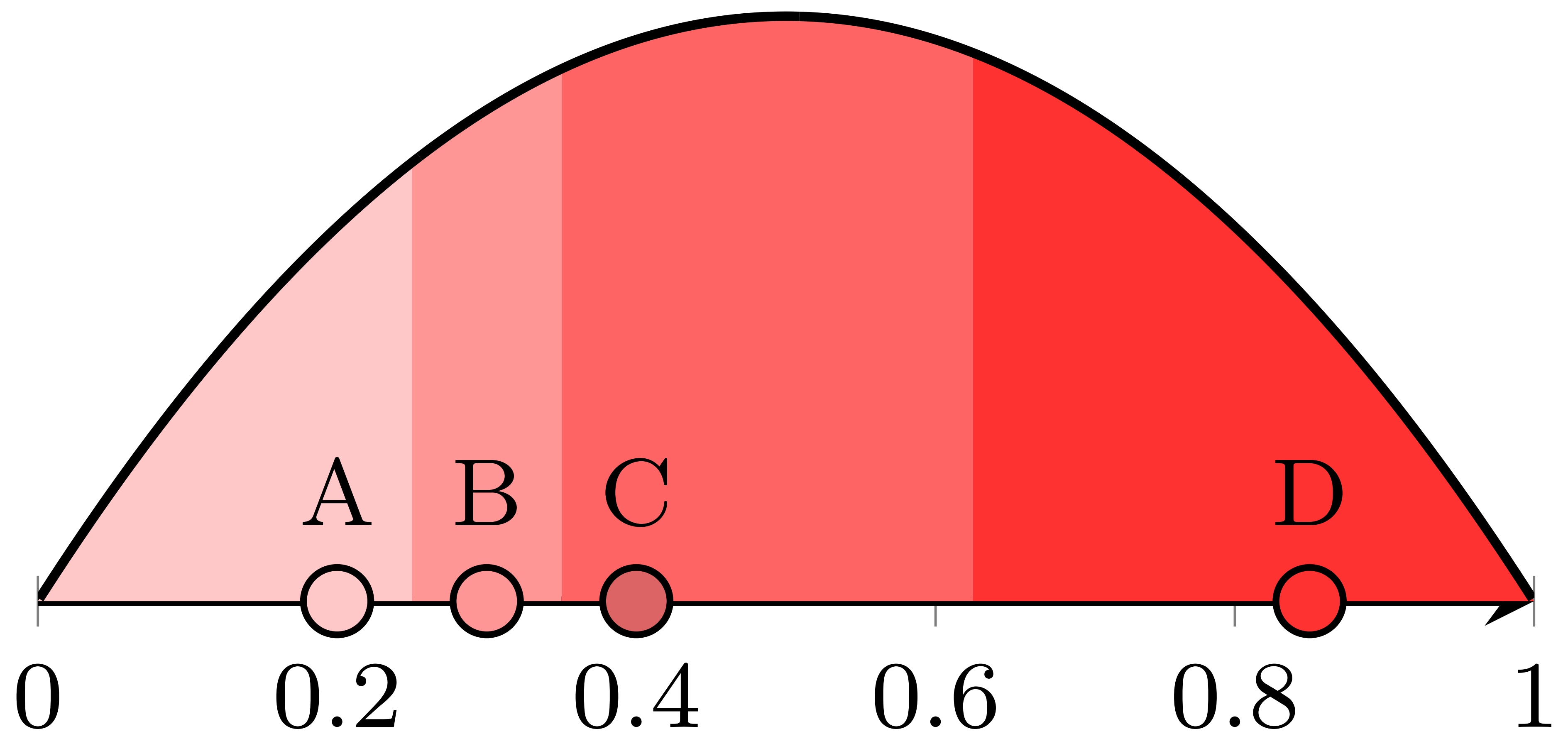
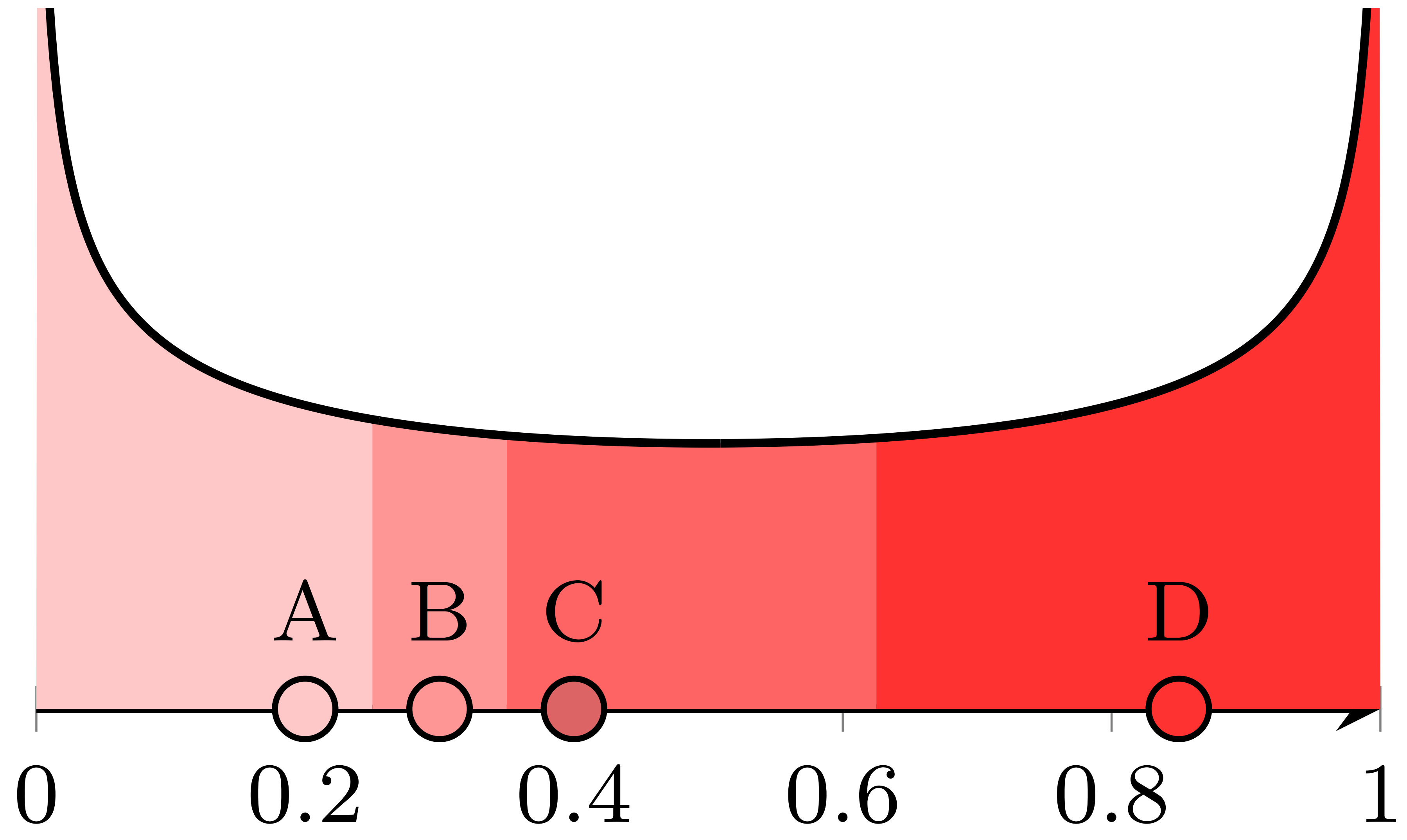
(Horowitz, *Public Choice* 2007)

Does IRV *provably* favor moderates compared to plurality?

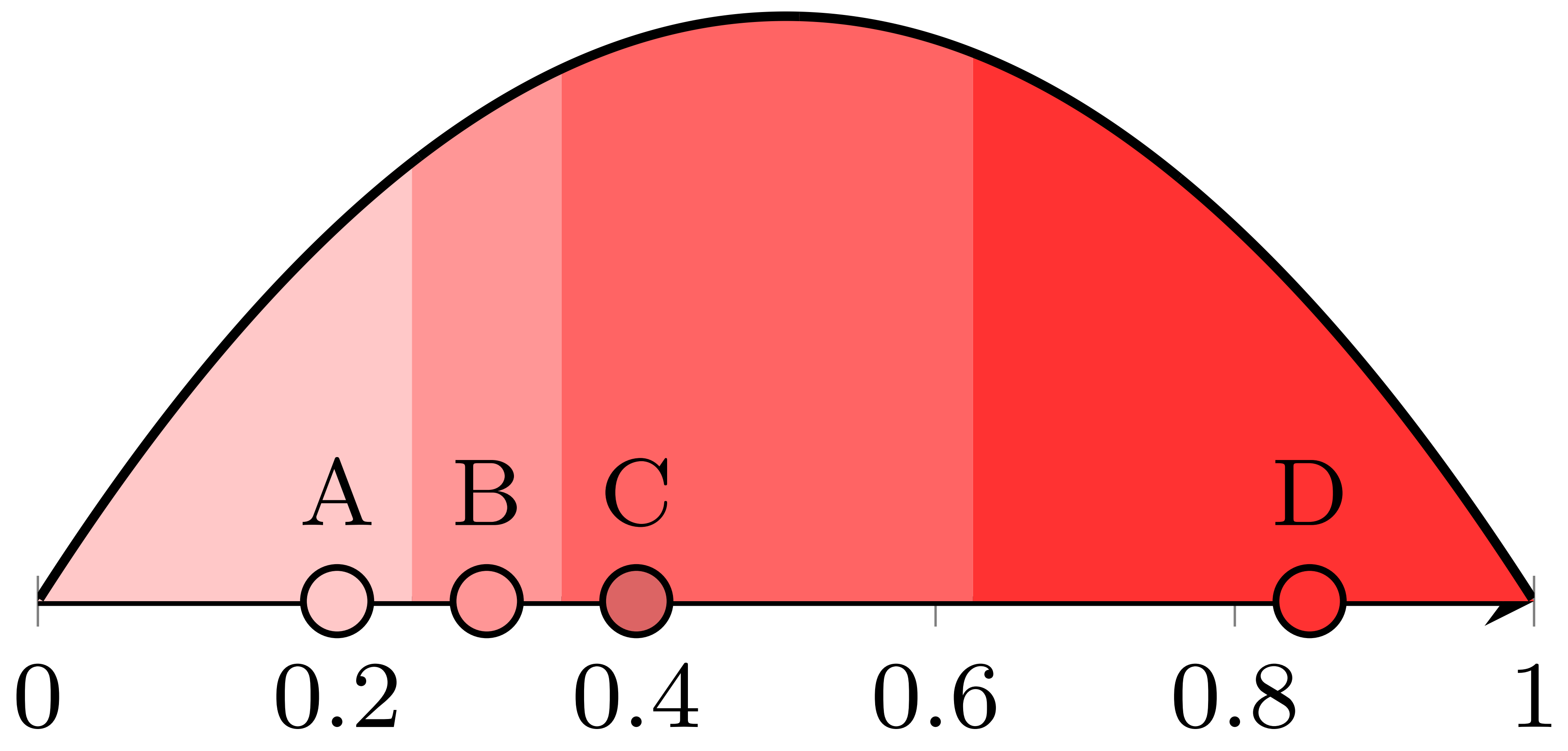
1-Euclidean preference model

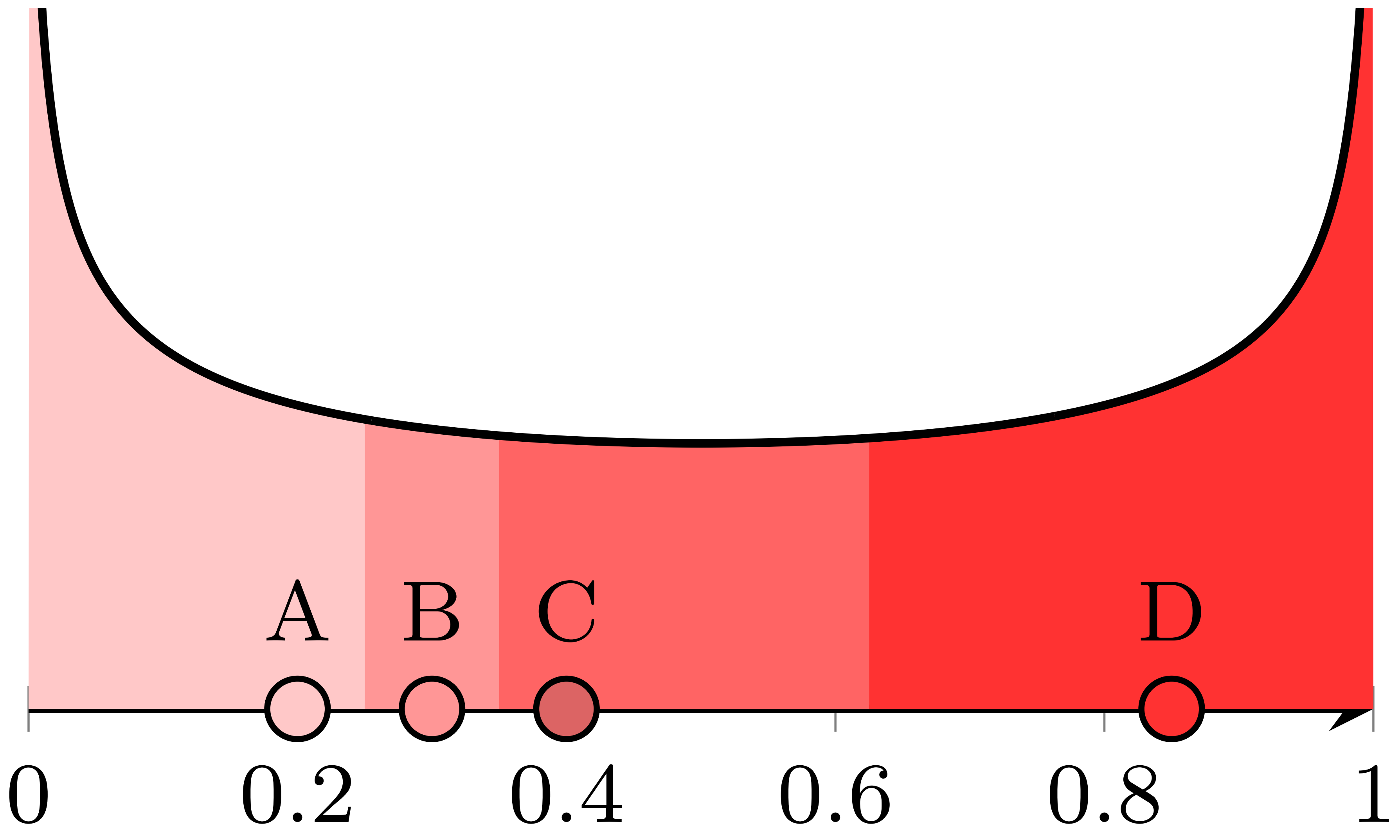




1-Euclidean preference model

- [0, 1]: left-right ideology





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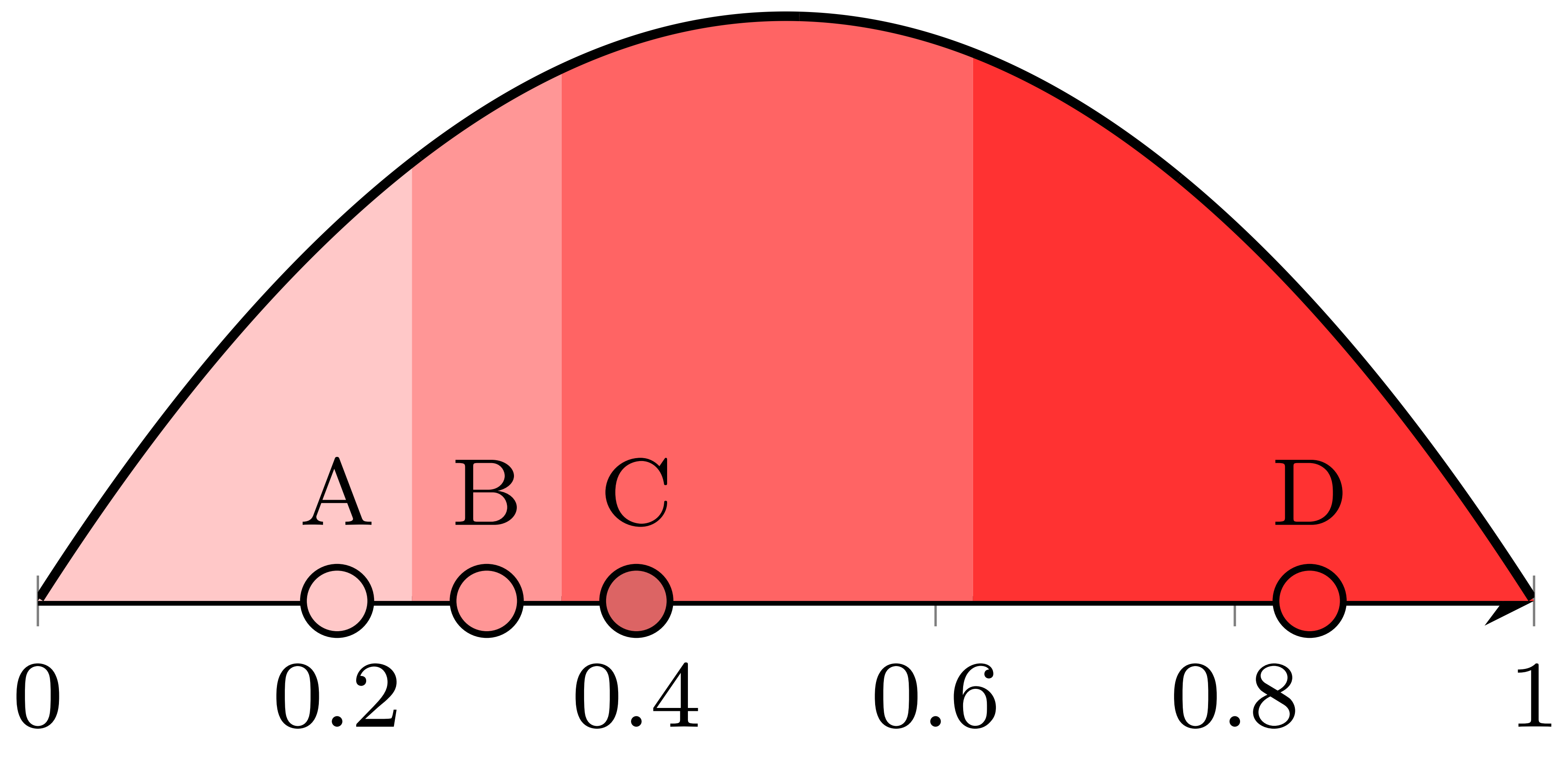
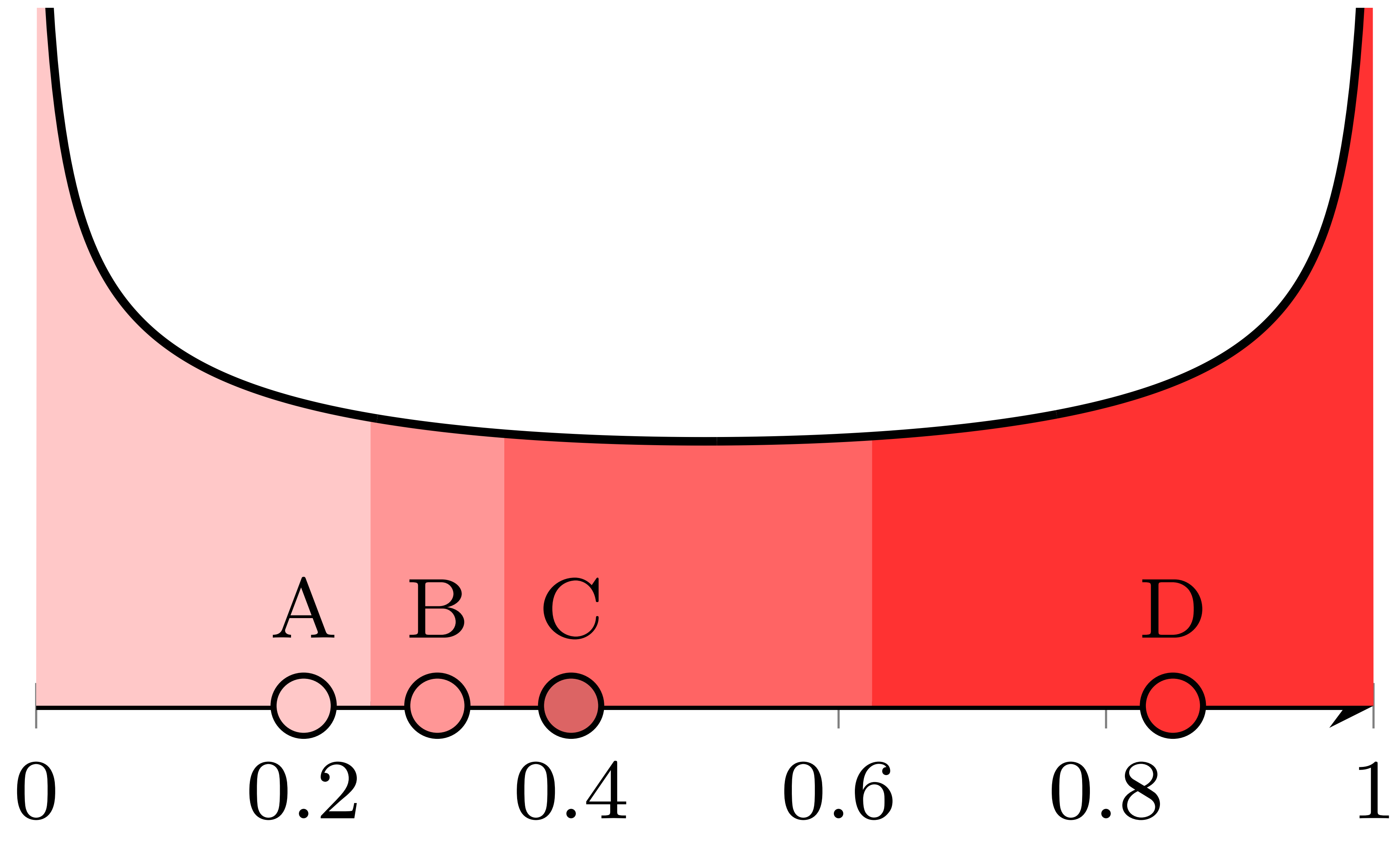
- [0, 1]: left-right ideology
- Candidates are at points





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- [0, 1]: left-right ideology
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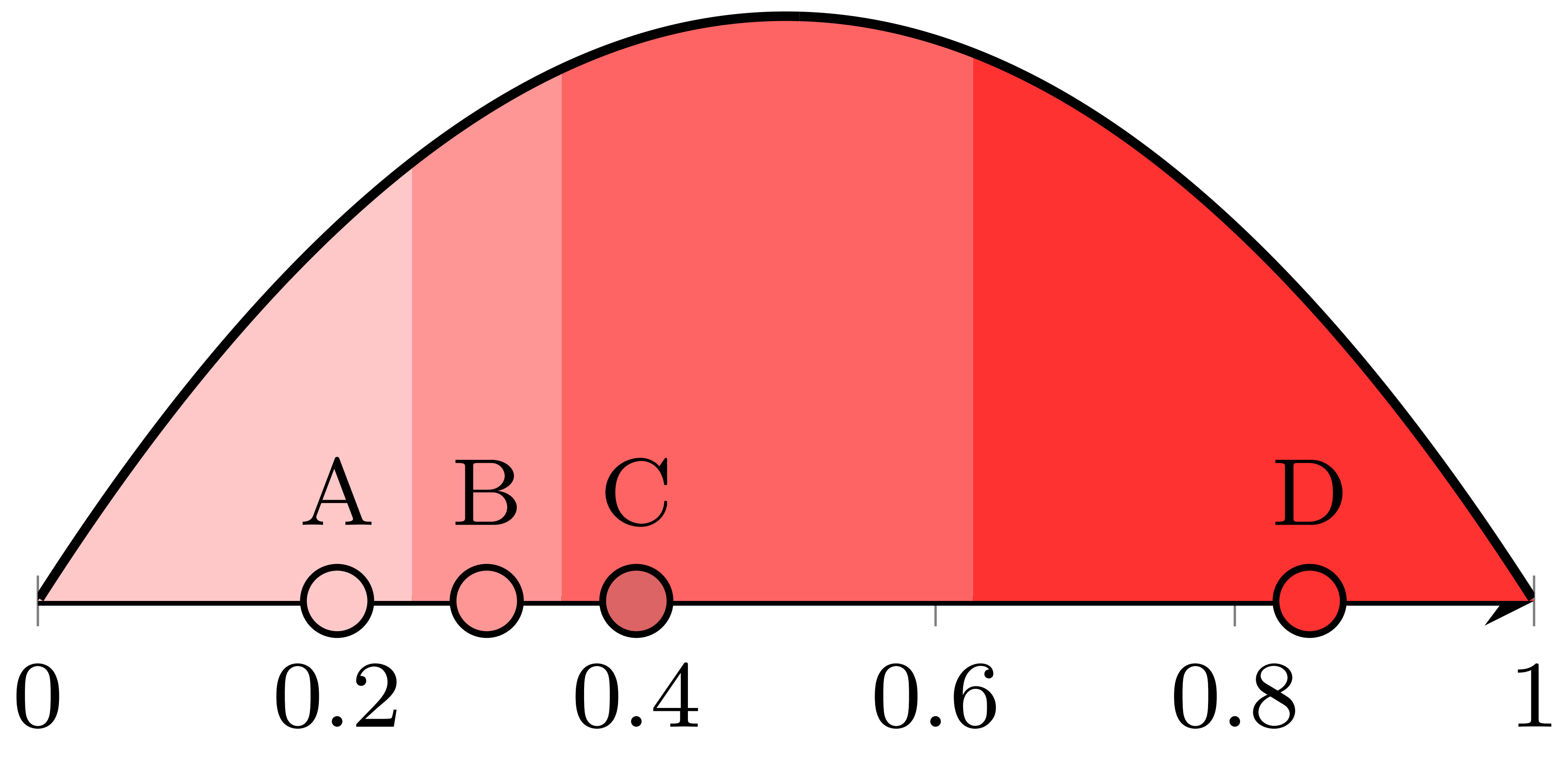
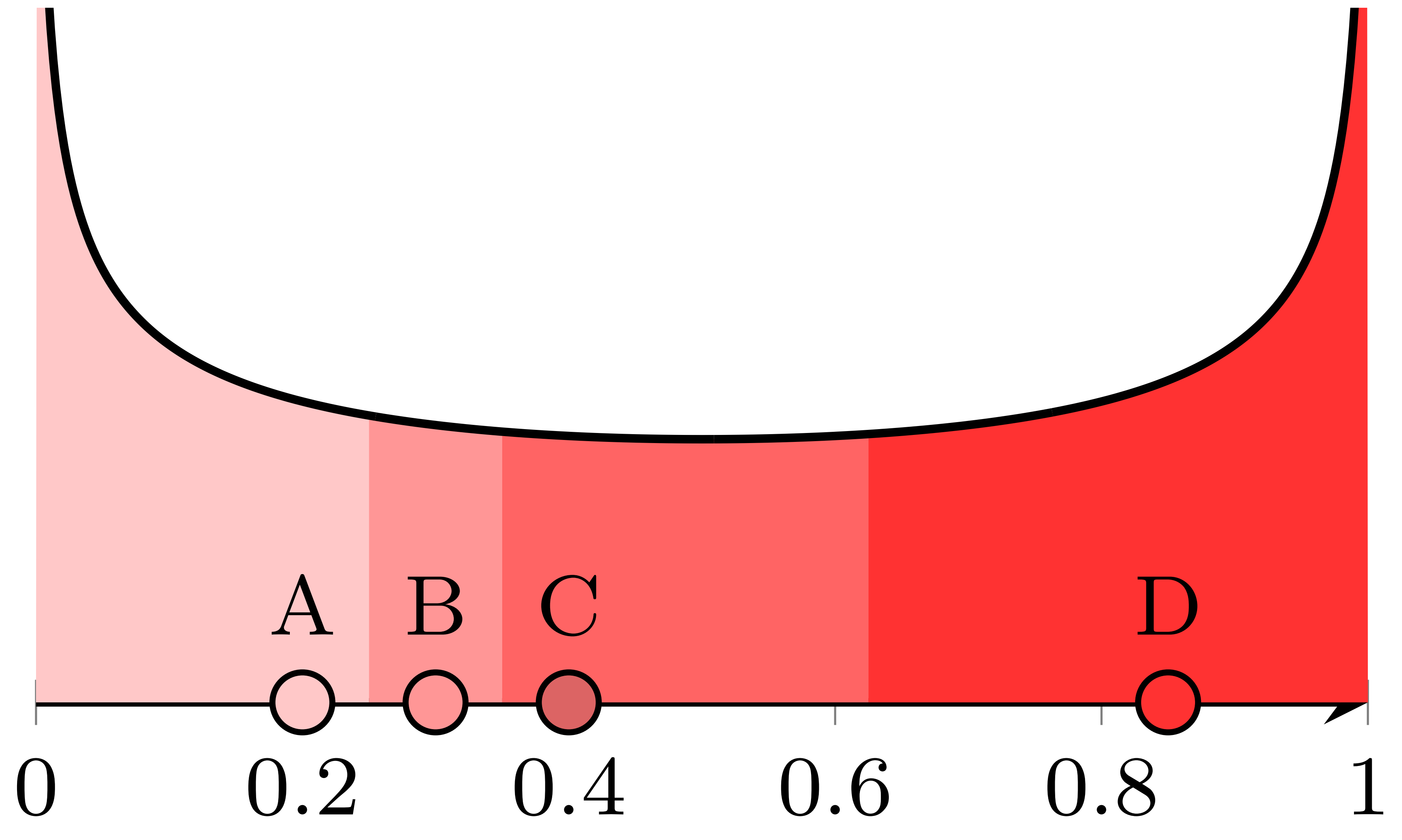
- Symmetric distribution of voters





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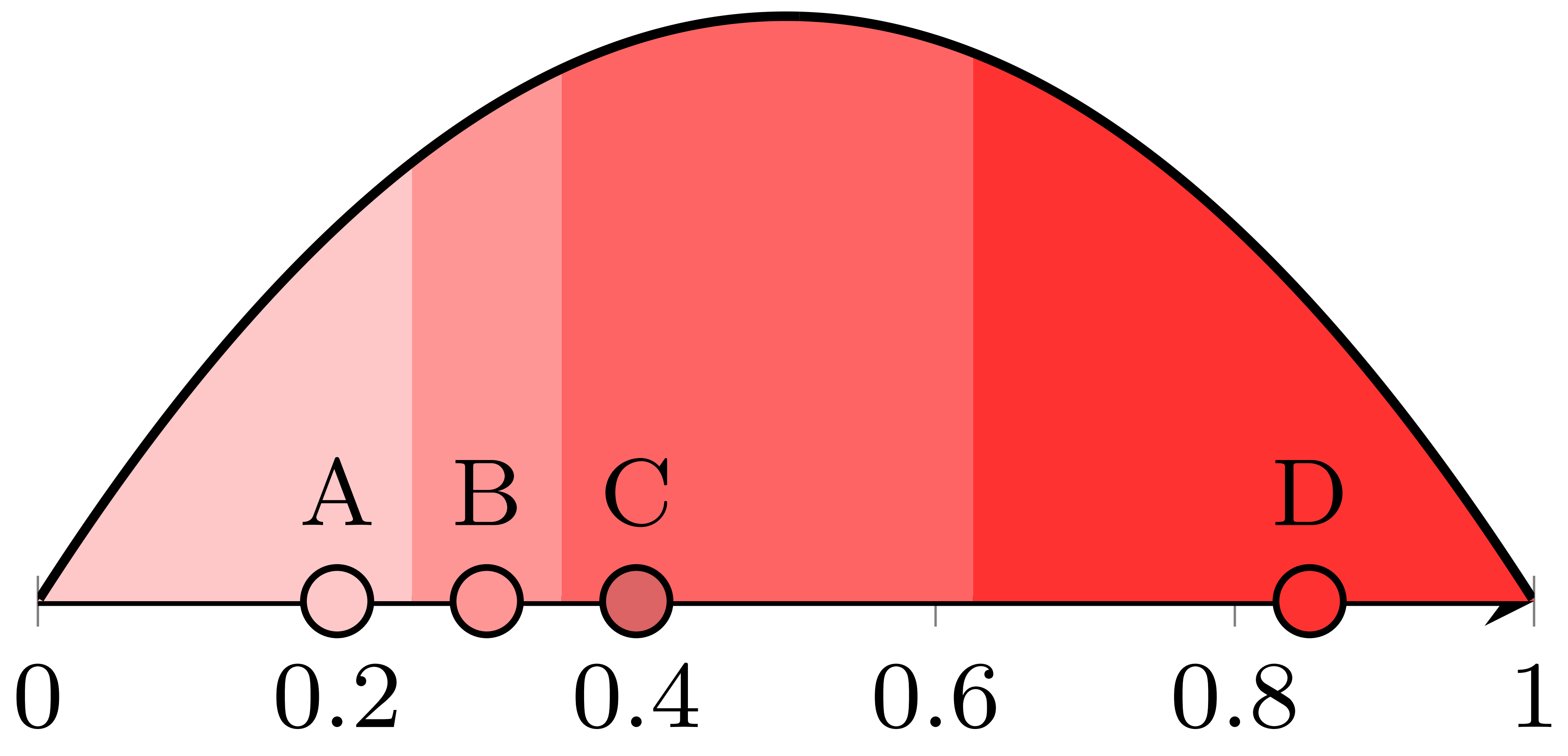
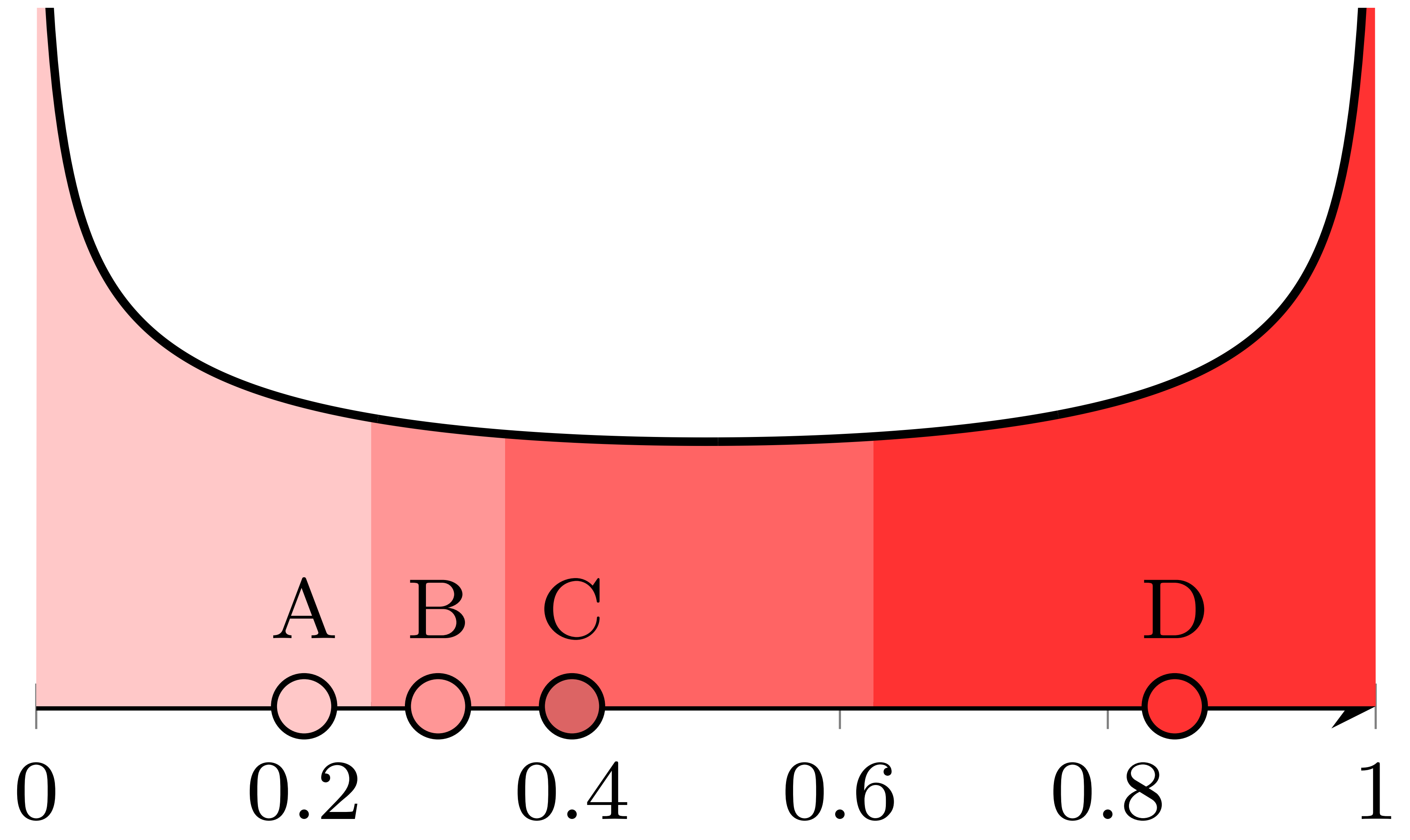
- Voters prefer candidates in order of distance





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- Moderate = close to 0.5
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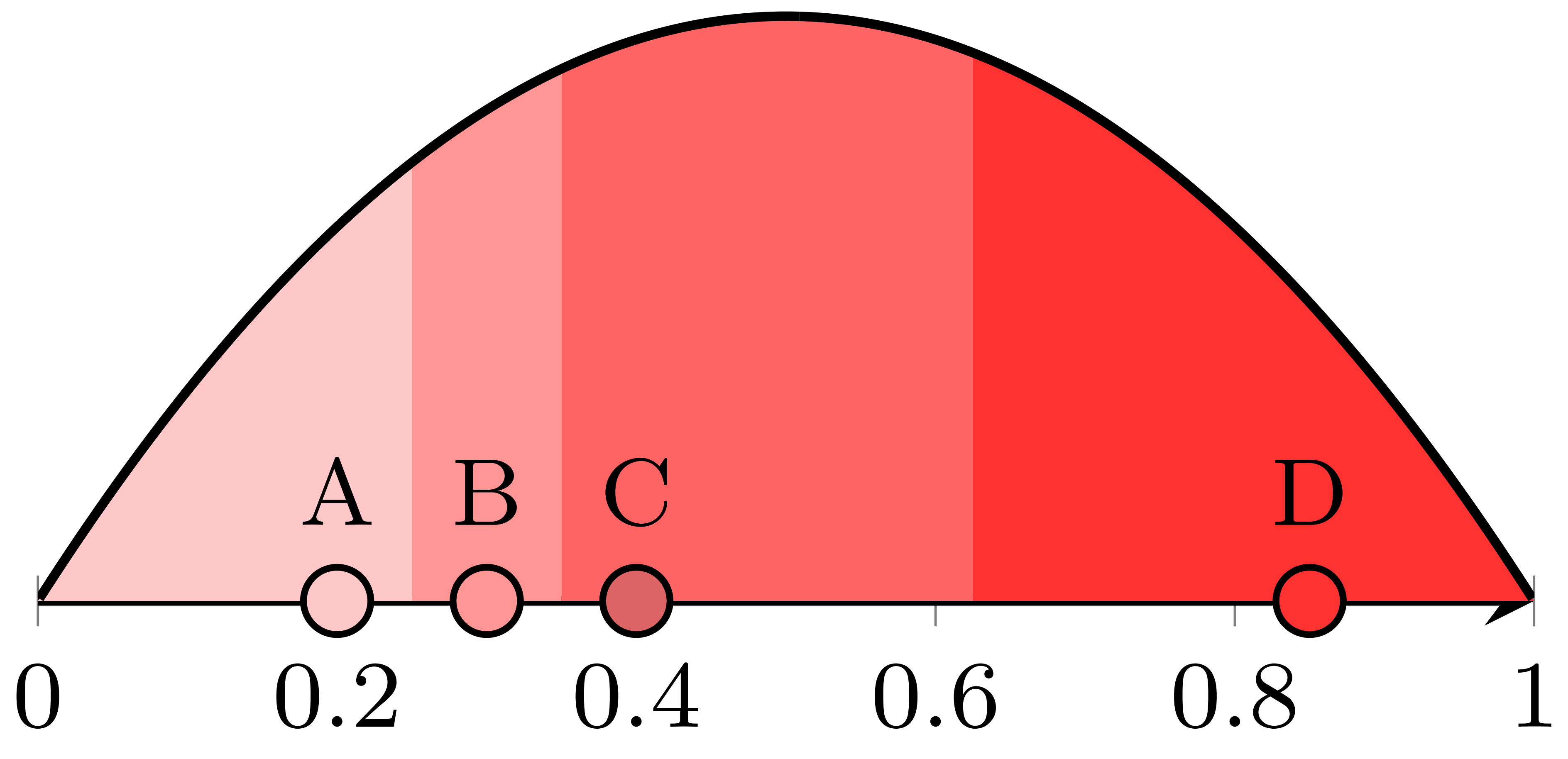

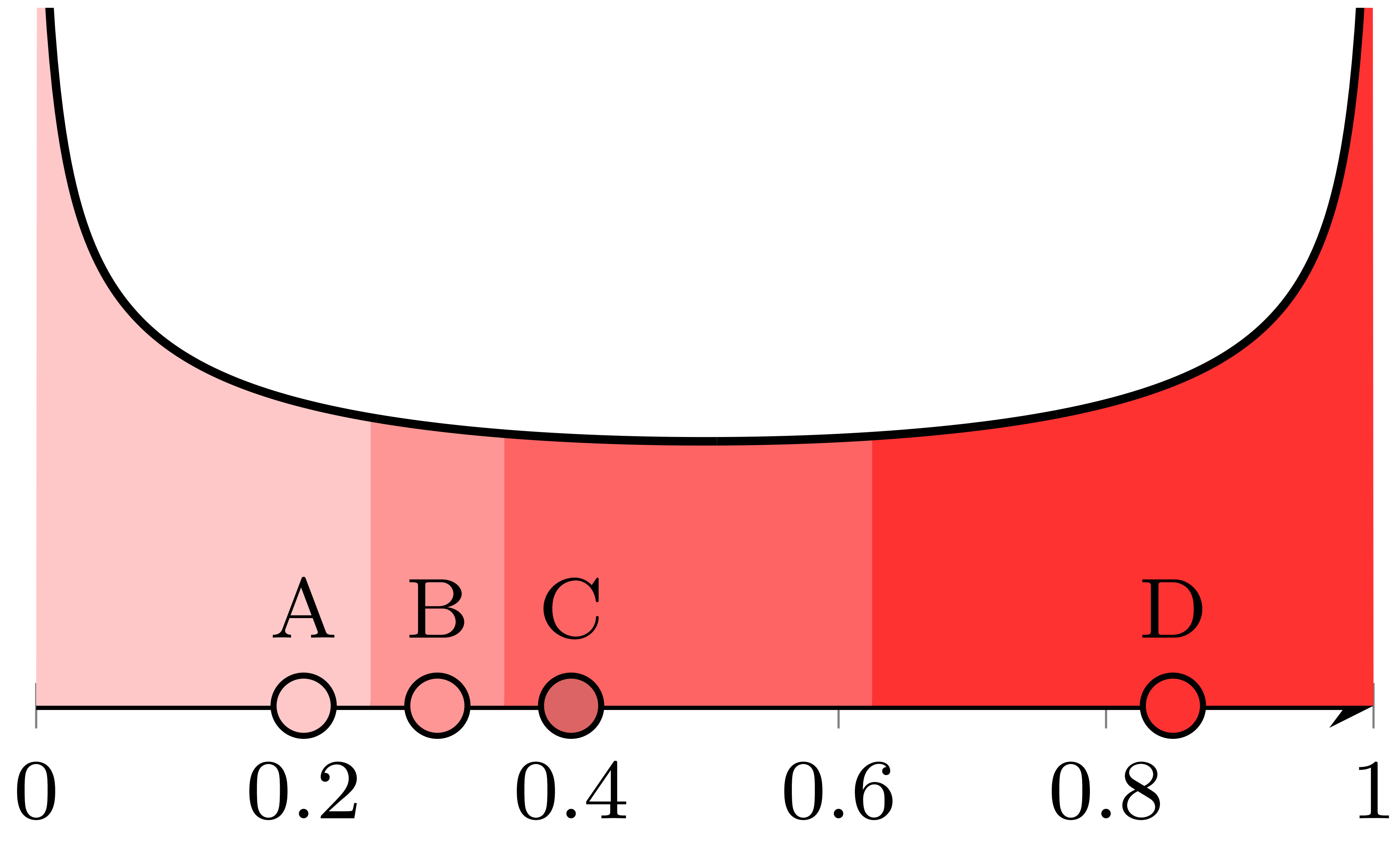

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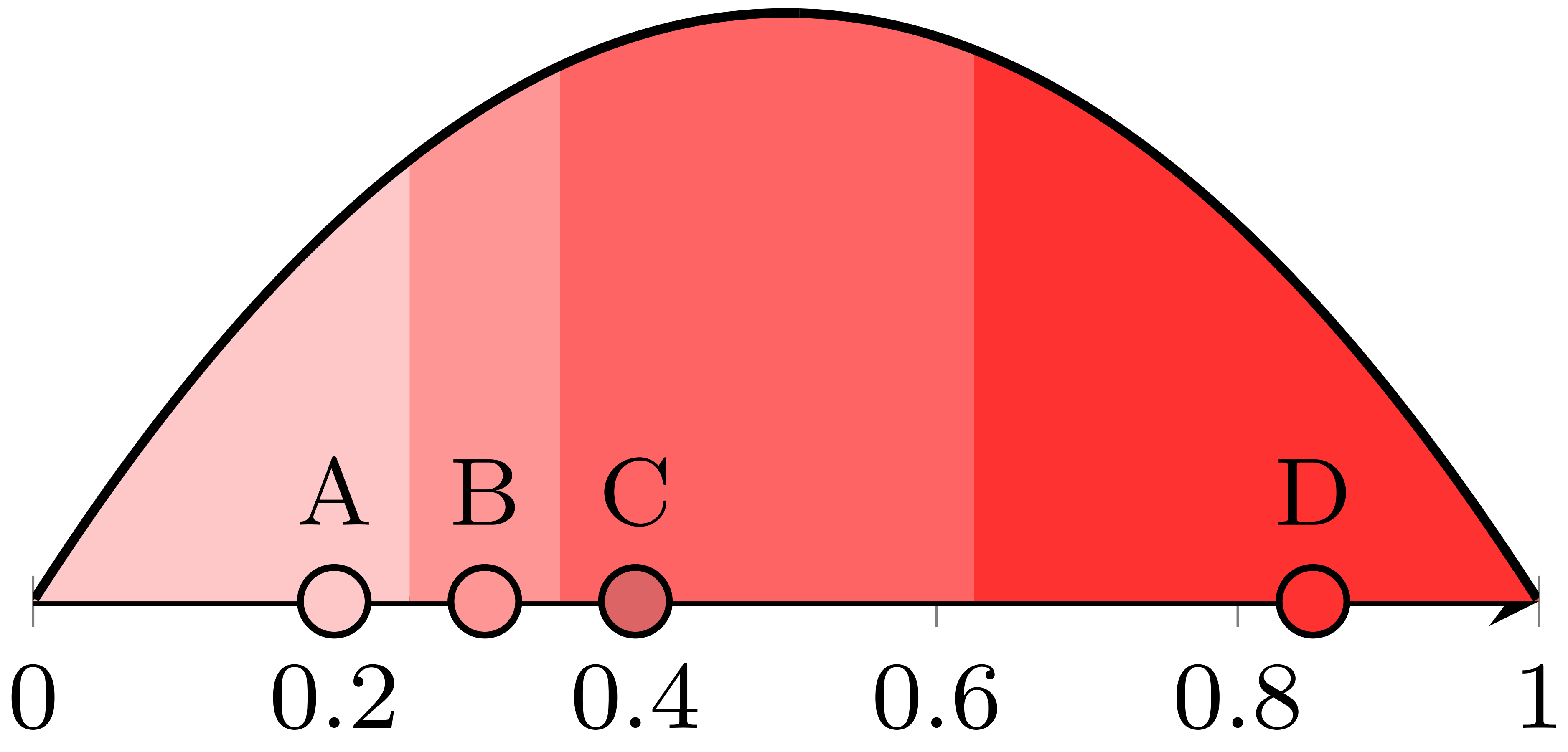
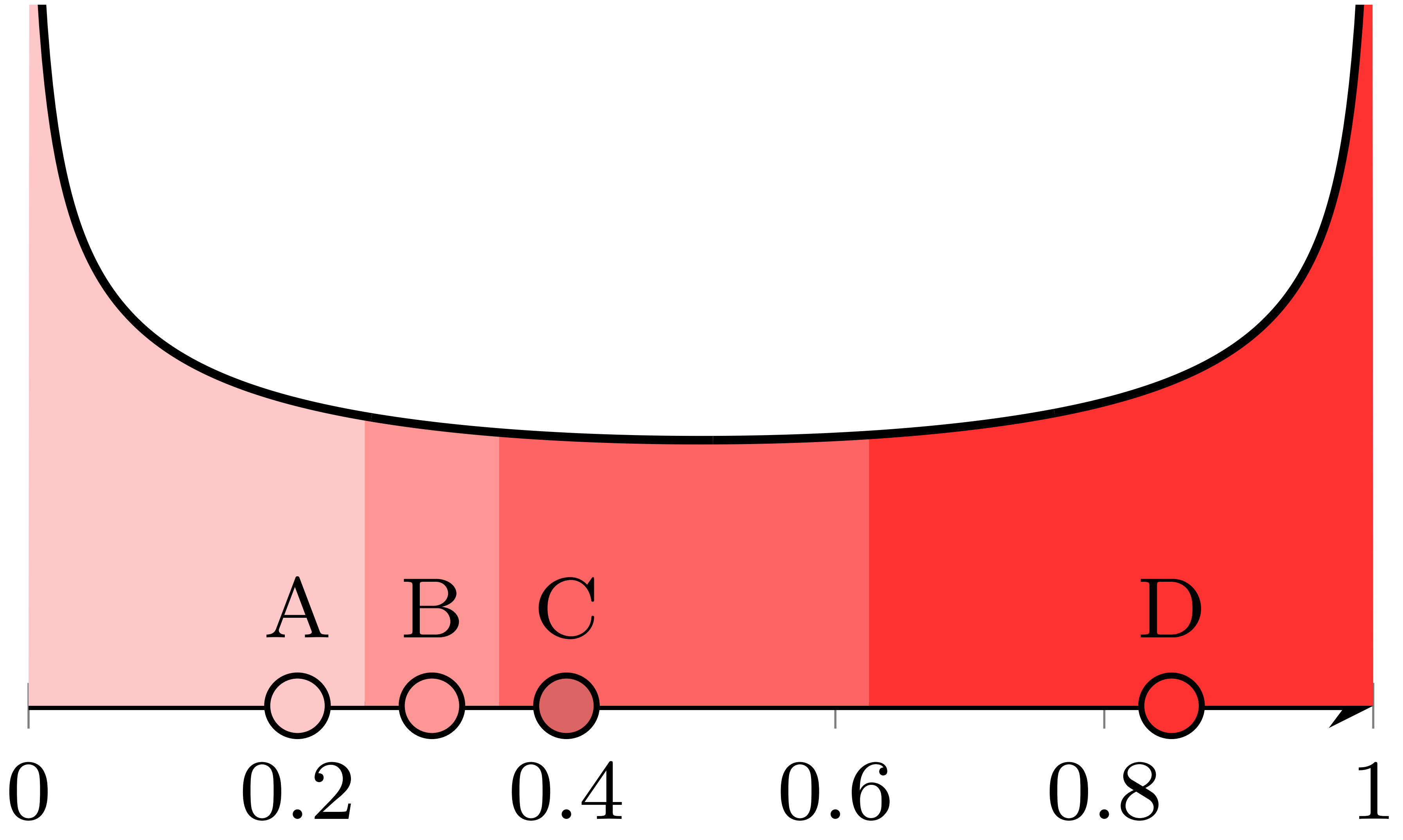


C is the plurality and IRV winner



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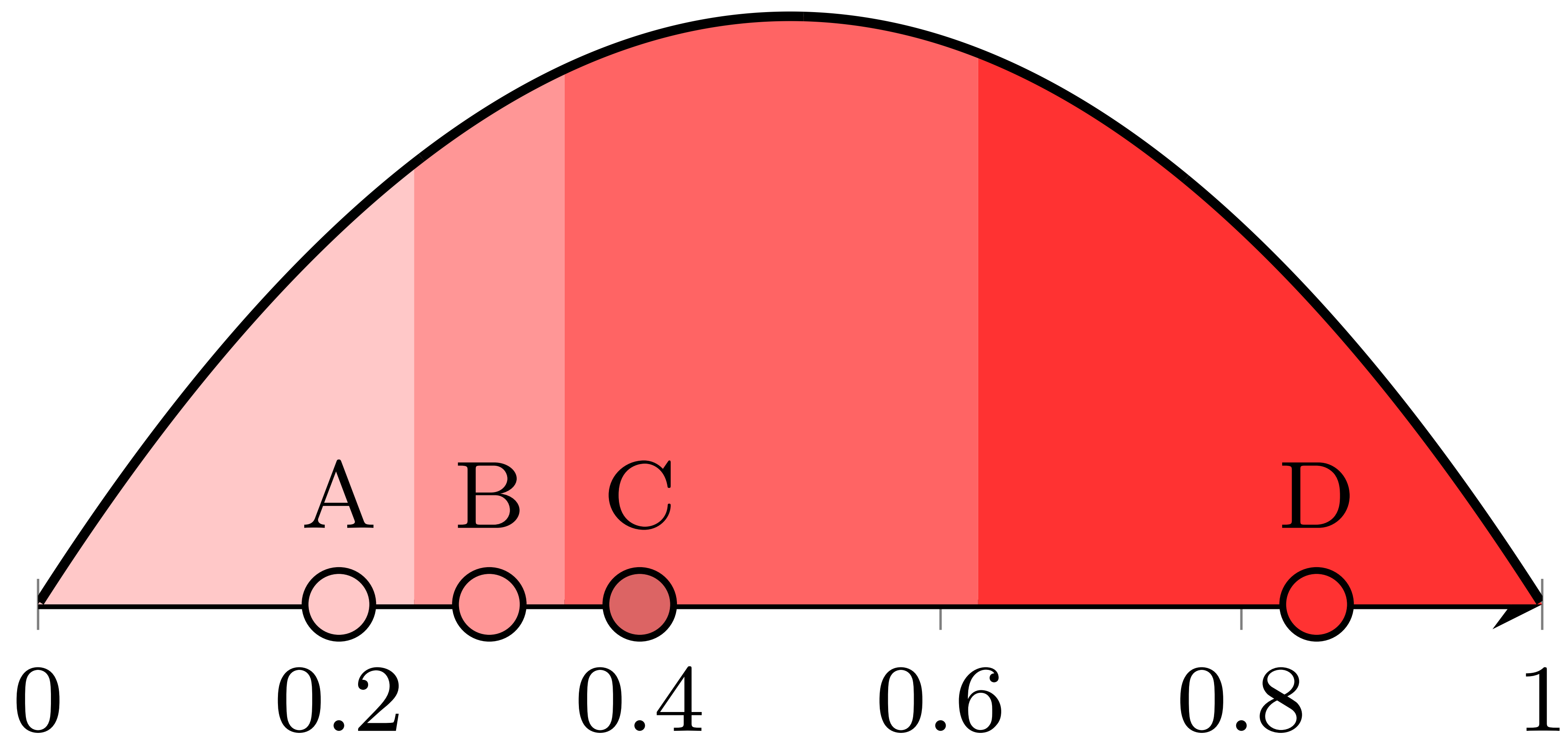

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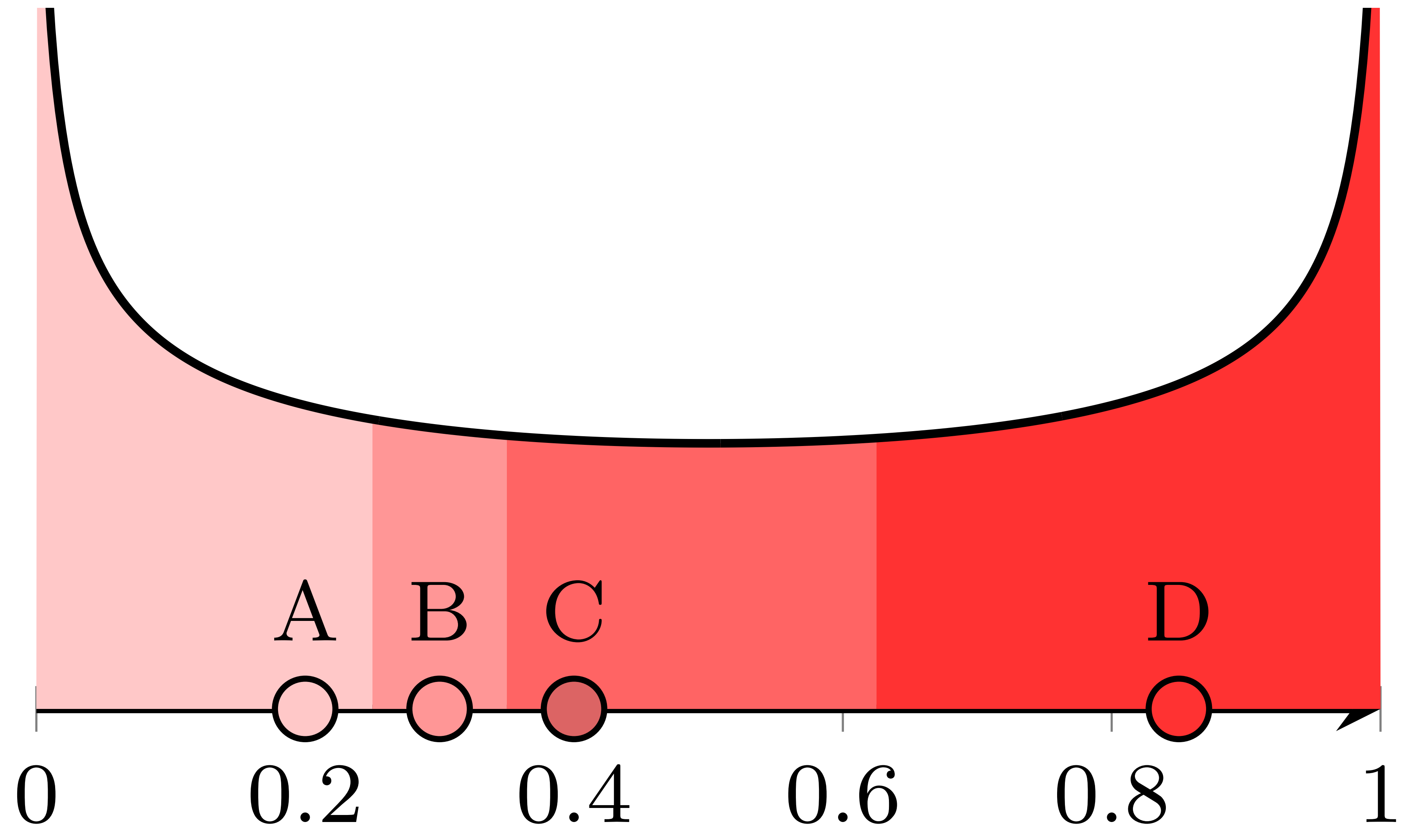


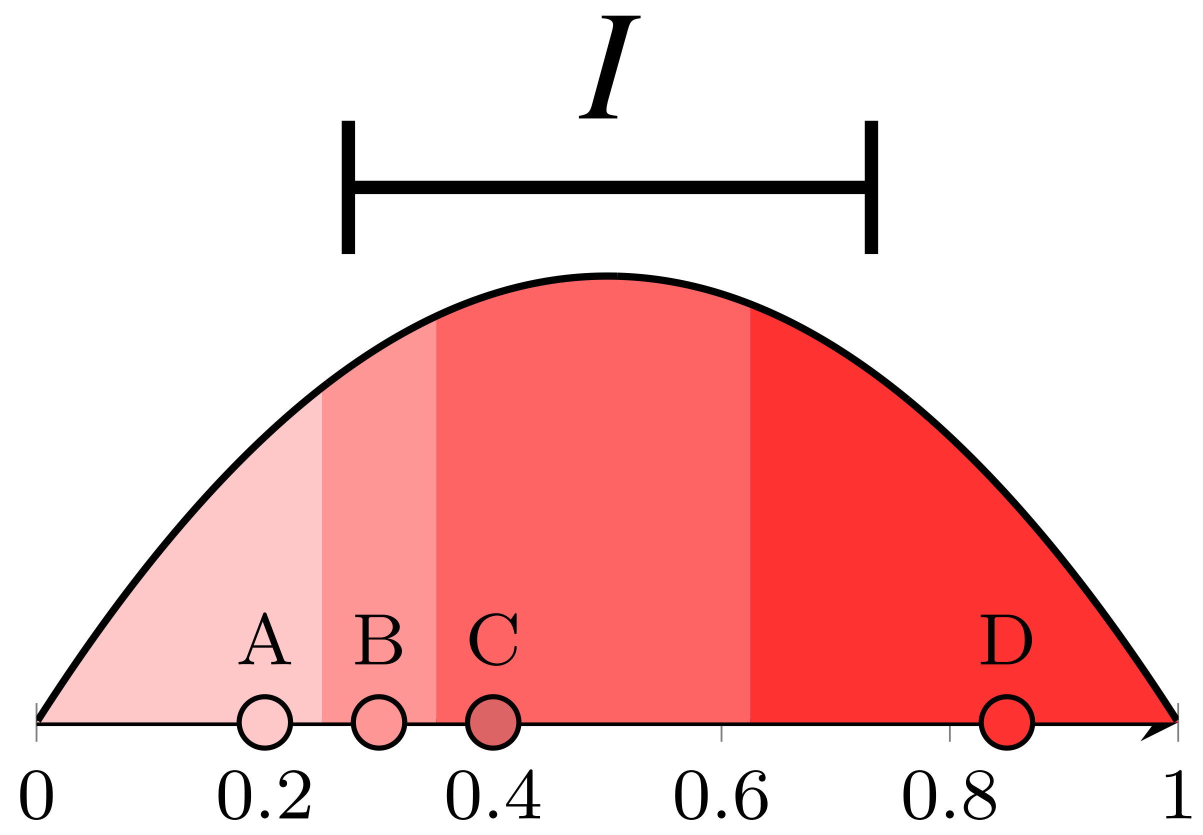
D is the plurality winner, A is the IRV winner

Formalizing a moderating effect

Formalizing a moderating effect

Definition
A voting system has a **combinatorial moderating effect** if there is an interval $I \subset [0,1]$ s.t. a candidate from I always wins (when present).

We call I an **exclusion zone** of the voting system.



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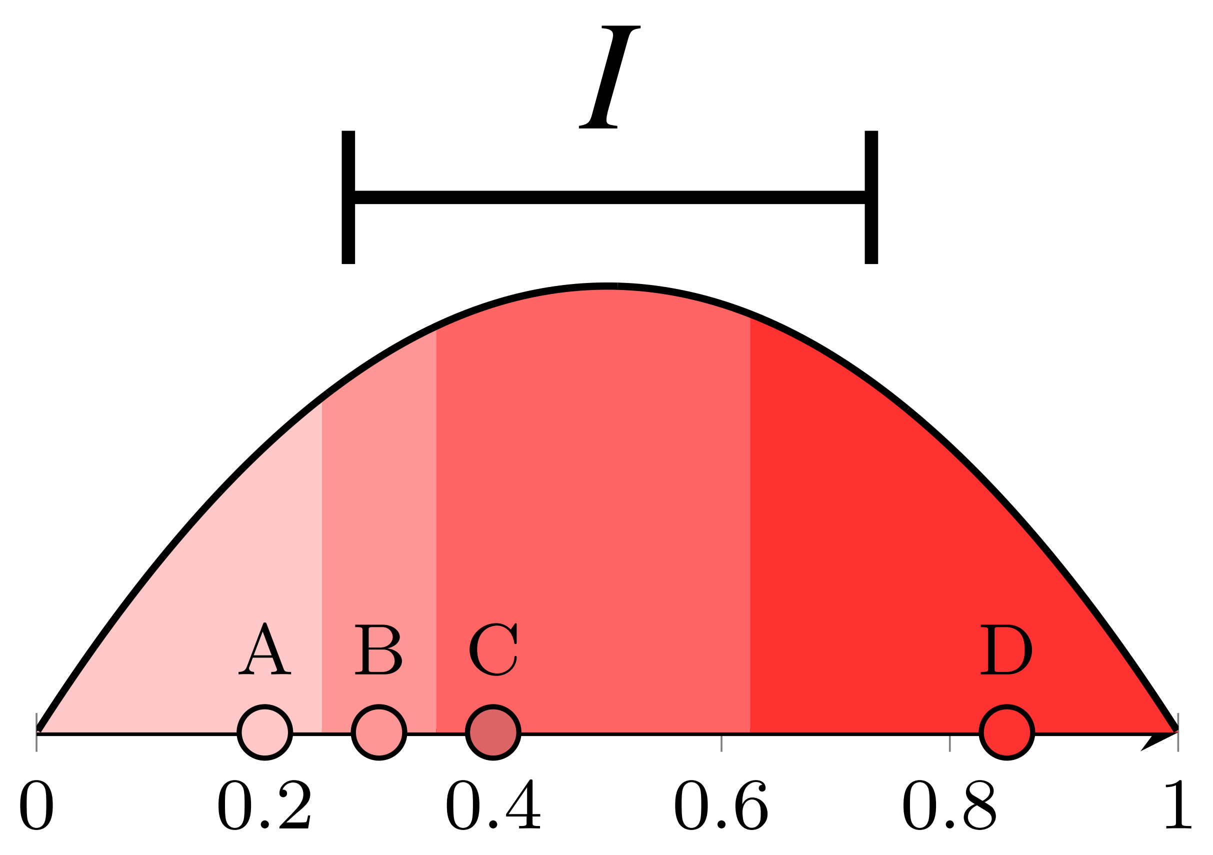
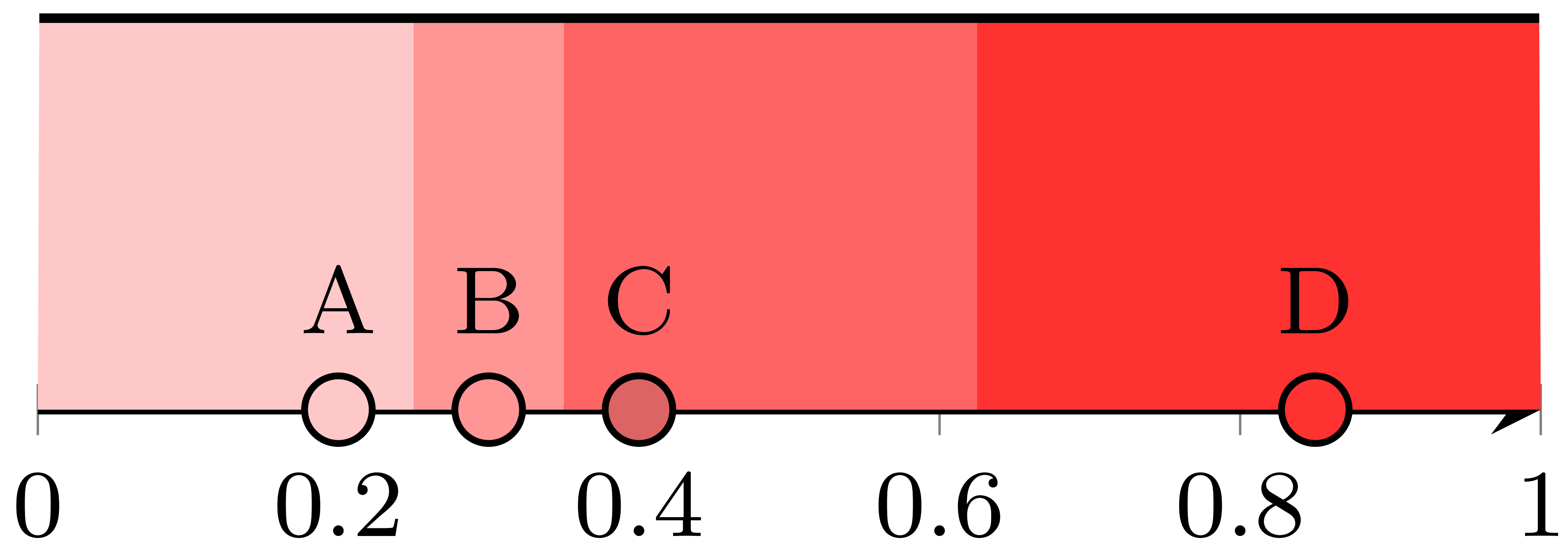


↓ implies

Definition
A voting system has a **probabilistic moderating effect** if $\Pr(\text{winner is in } I) \to 1$ as the number of candidates $k \to \infty$.

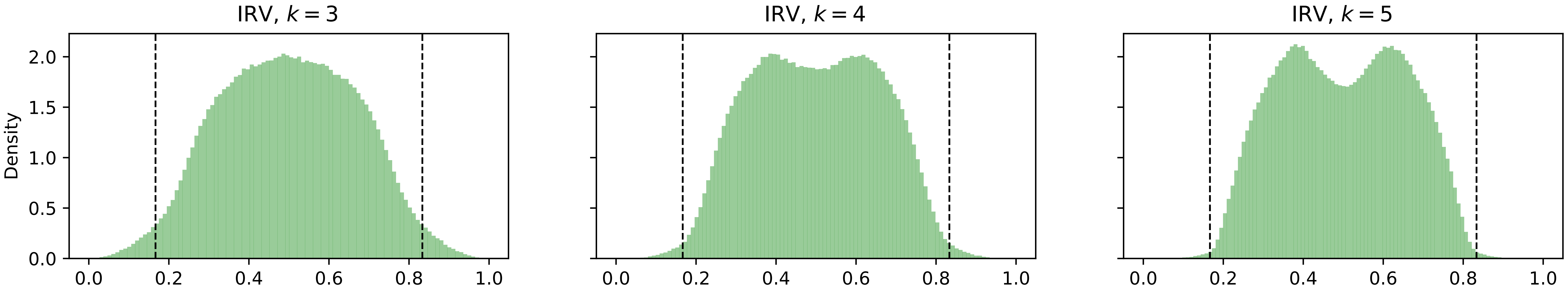
Starting simple: uniform voters



IRV has a moderating effect!

Theorem 1 (Combinatorial moderation for IRV)
For any $k \geq 3$, [1/6, 5/6] is an exclusion zone of IRV with uniform voters.

No smaller interval has this property.



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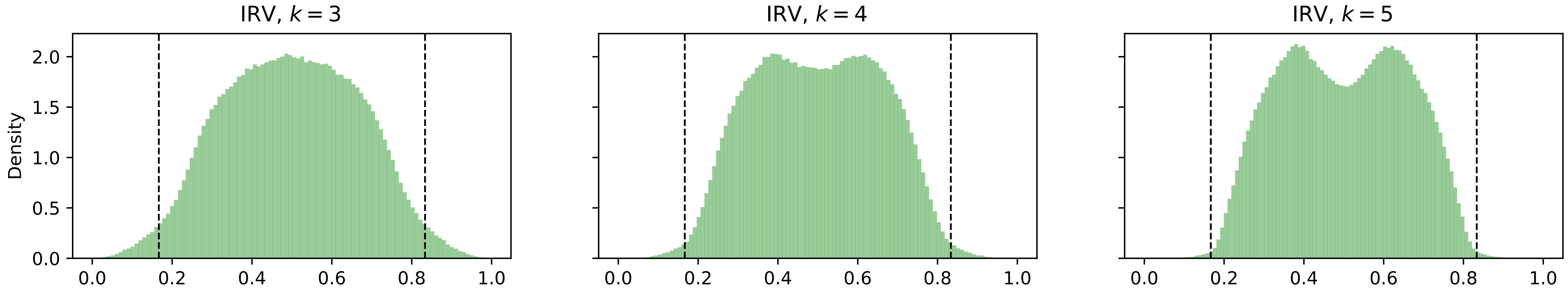
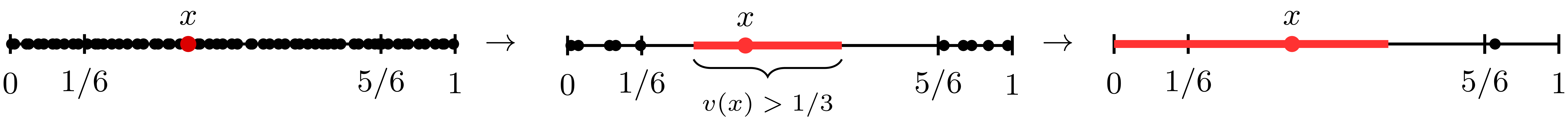
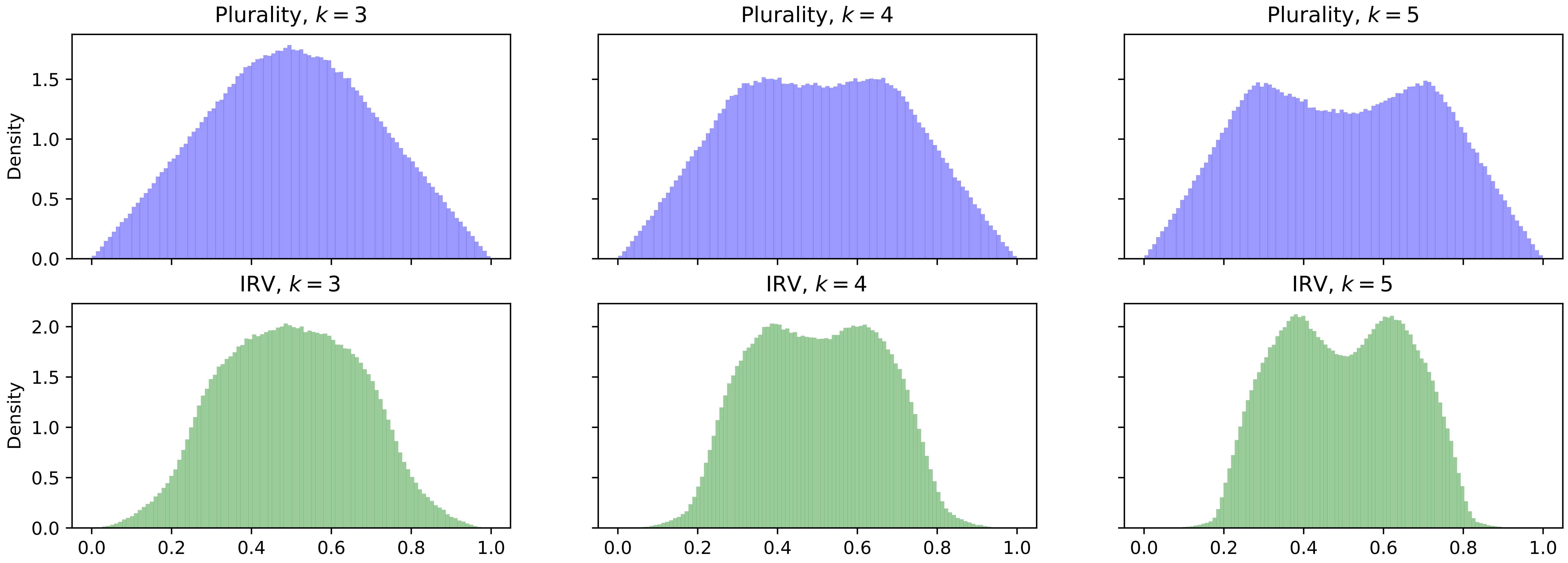




Plurality allows extreme winners



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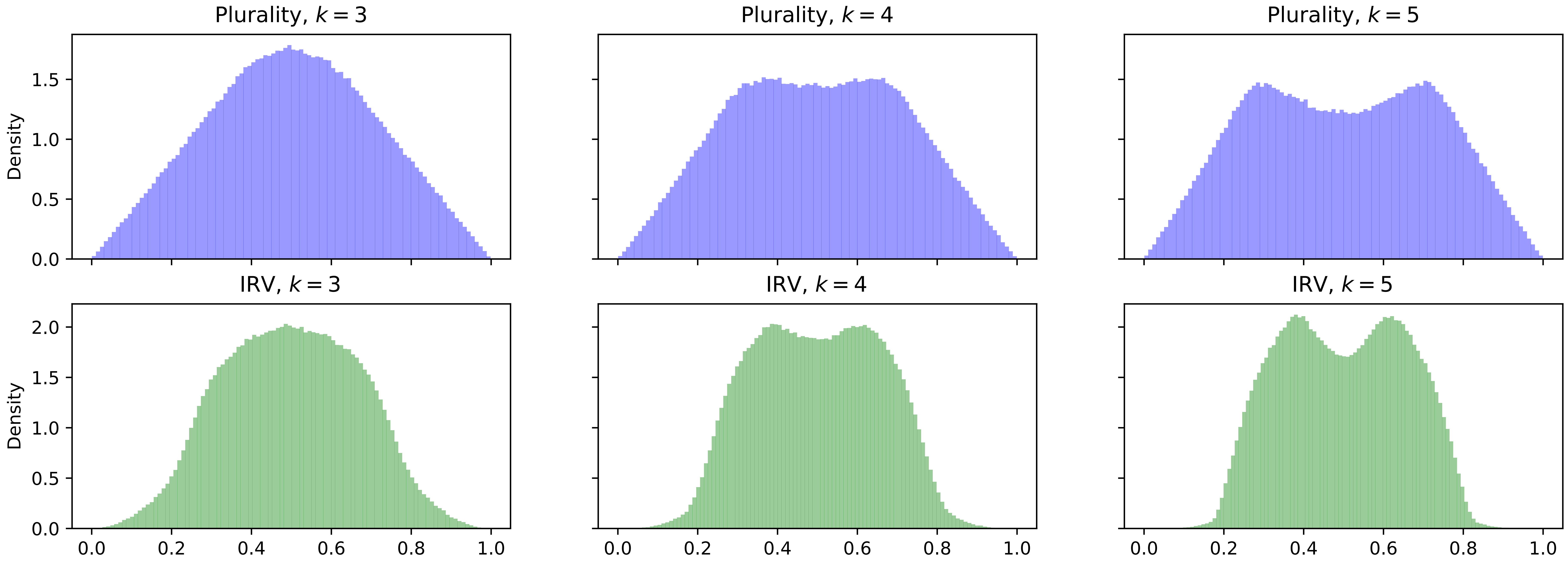
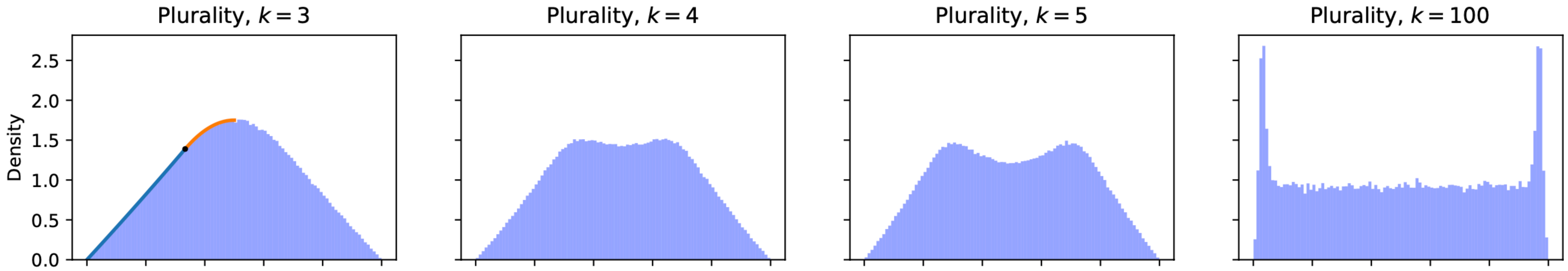


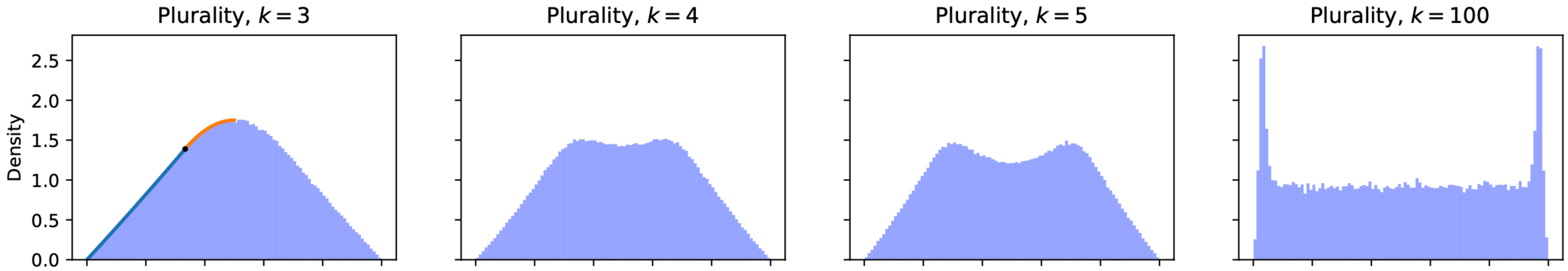
Theorem 2 (No combinatorial moderation for plurality, uniform voters)
Given any distinct candidate positions $x_1, \ldots, x_k$ (with $x_1 \notin \{0,1\}$), we can add more candidates to make x_1 the plurality winner.

No probabilistic moderation for plurality



Theorem 3 (No probabilistic moderation for plurality, uniform voters)
Let P_k be the position of the plurality winner with k candidates distributed uniformly. As $k \to \infty$, $P_k \to_d$ *Uniform*$(0,1)$.

No probabilistic moderation for plurality



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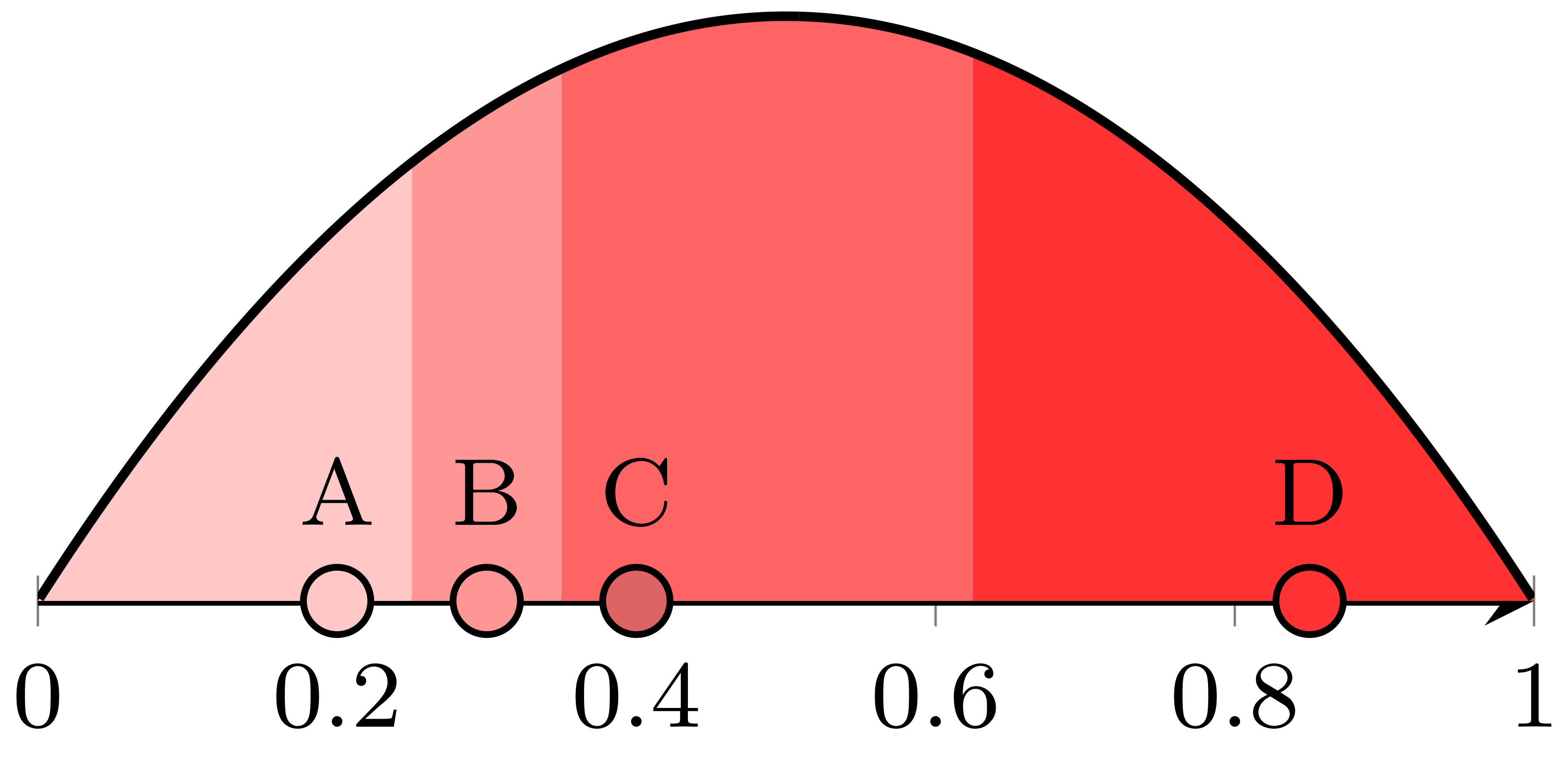
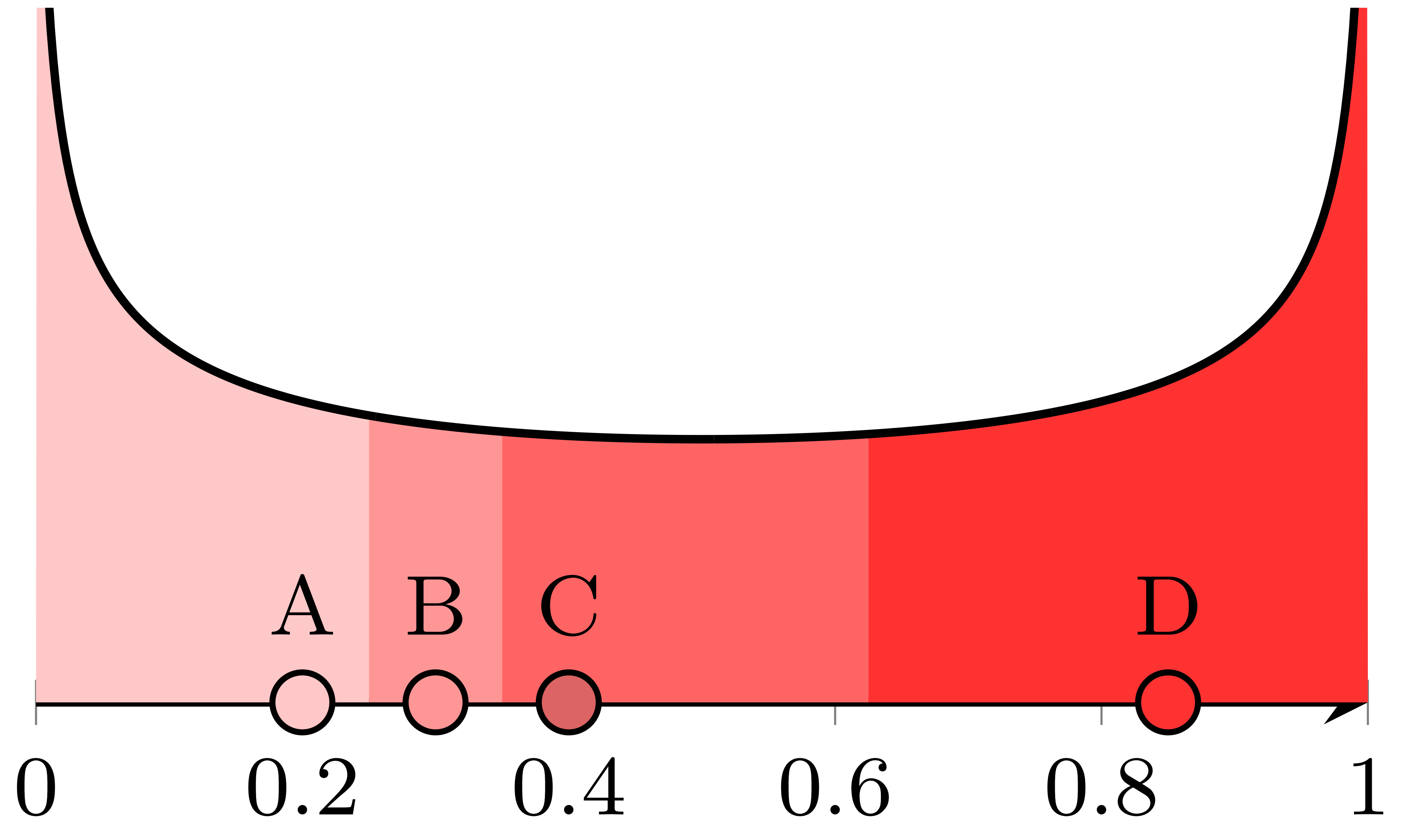
Proof idea:

Connection to stick-breaking processes to find winning vote share
+ circle-cutting argument

D. A. Darling. On a class of problems related to the random division of an interval. *The Annals of Mathematical Statistics*, 1953.
L. Holst. On the lengths of the pieces of a stick broken at random. *Journal of Applied Probability*, 1980.

What about non-uniform voters?





[1/6, 5/6] Theorem generalizes!

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Theorem 4 (Combinatorial moderation for IRV, general case)
Let the voter distribution be symmetric with CDF F and let $c \in (0,1/2)$.
If for all $x \in [c,1/2]$,

$$F\left(\frac{x+1-c}{2}\right) - F\left(\frac{c+x}{2}\right) > 1/3 \qquad (\star)$$

then $[c, 1-c]$ is an exclusion zone of IRV.

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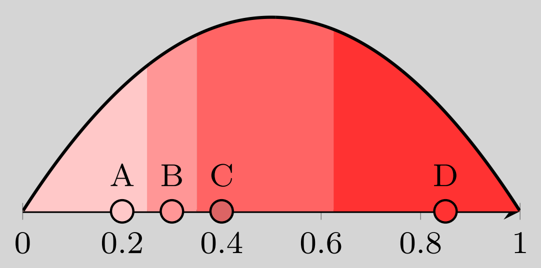
Theorem 5

exclusion zone:
$[F^{-1}(1/6),1-F^{-1}(1/6)]$



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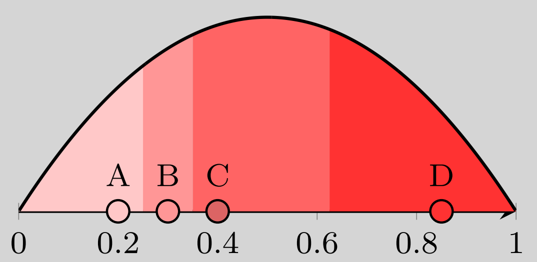

exclusion zone: centrist voters

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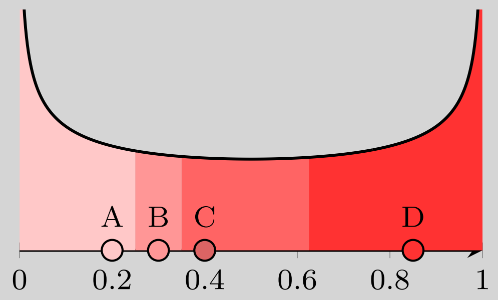
Theorem 6



exclusion zone: even with polarized voters!

$(F(1/4) < 1/3)$

$[2F^{-1}(1/3) - 1/2, 3/2 - 2F^{-1}(1/3)]$

If voters are too polarized, IRV can't elect moderates

Theorem 7 (hyper-polarized voters)
Suppose $F(1/4) > 1/3$. For any $c \geq 2F^{-1}(1/3)$, $[0,c] \cup [1-c,1]$ is an exclusion zone of IRV.

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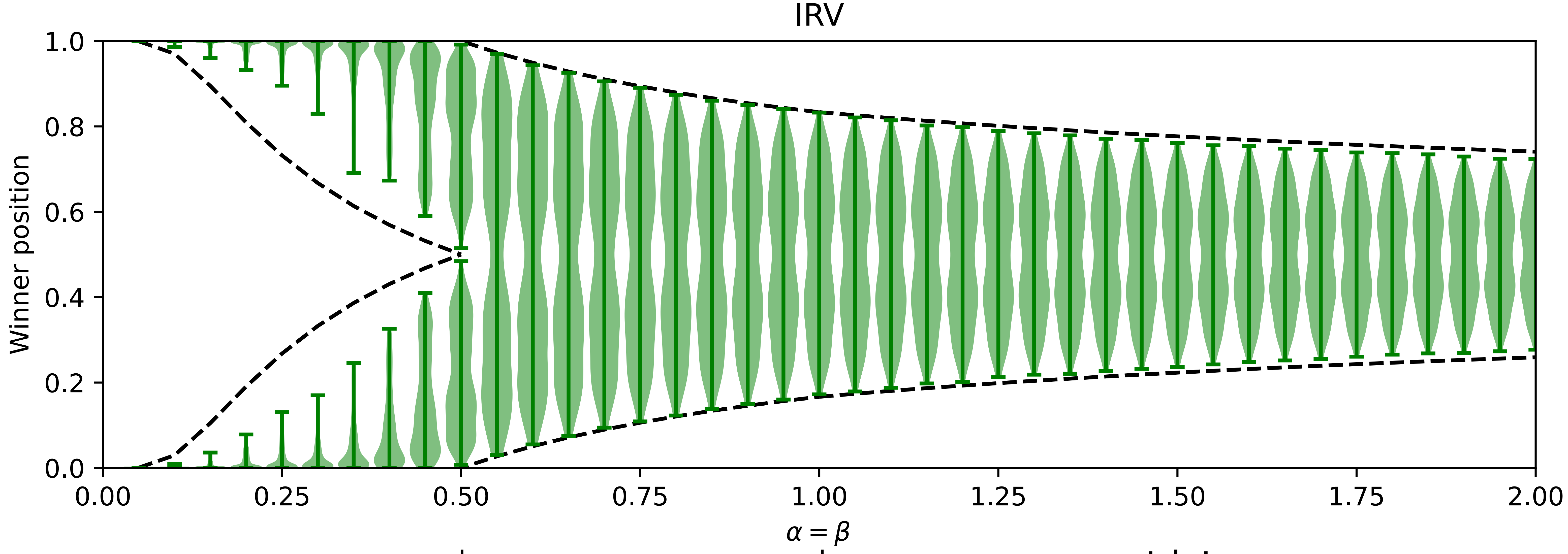
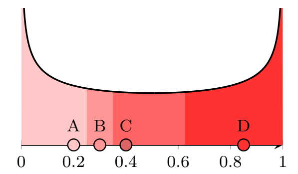
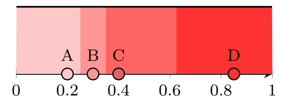


hyper-polarized (Theorem 7)

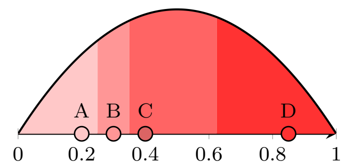
polarized (Theorem 6)



uniform (Theorem 1)

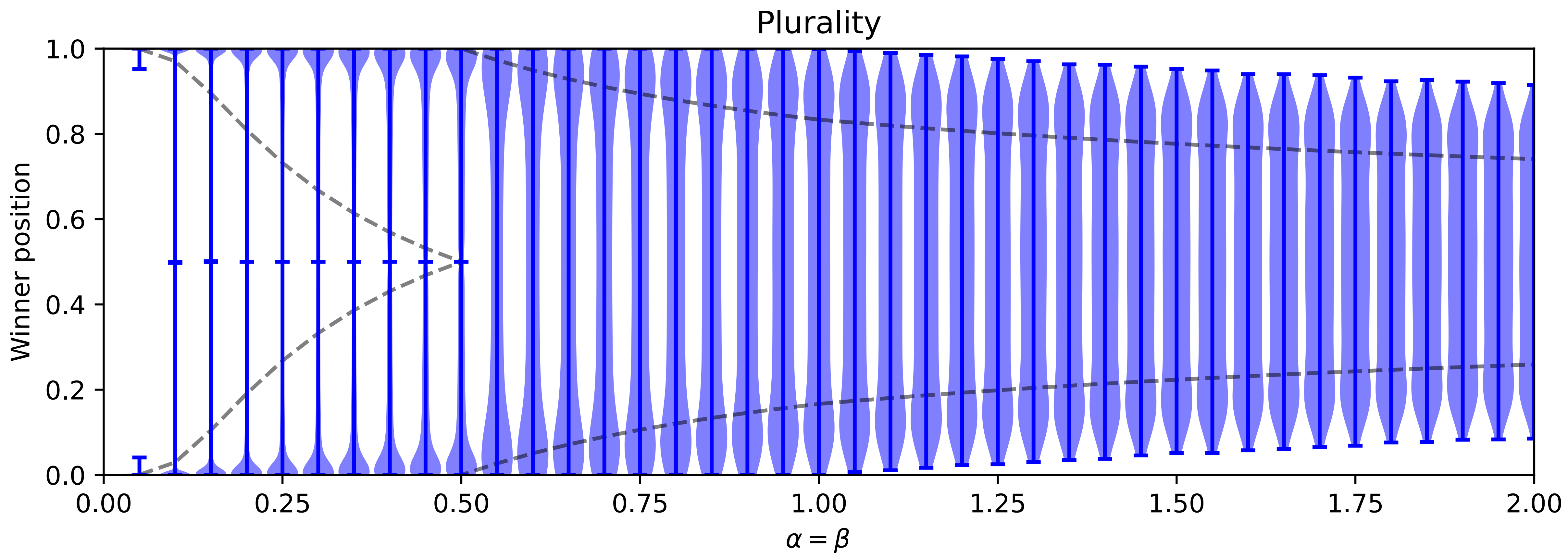


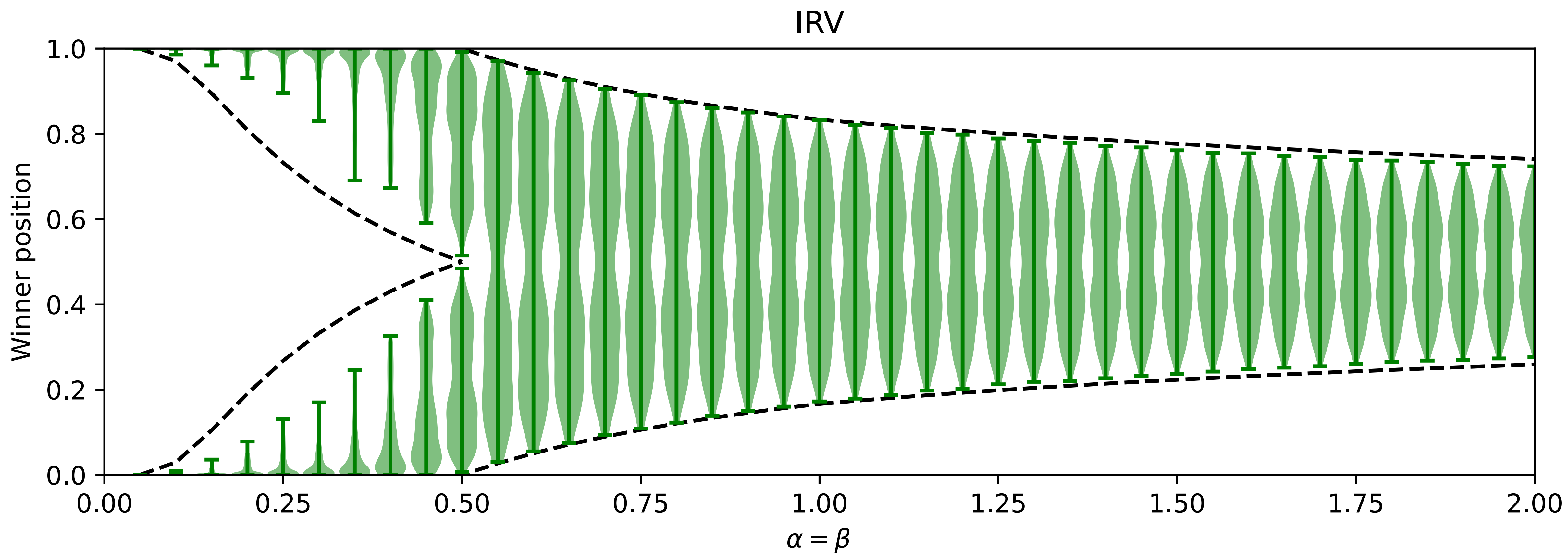
centrist (Theorem 5)



Beta-distributed voters

Plurality still elects arbitrarily extreme candidates





Theorem 8 (no combinatorial moderation for plurality)
As long as the voter distribution is continuous and positive over $(0,1)$, we can make an arbitrarily extreme candidate win by adding more candidates.

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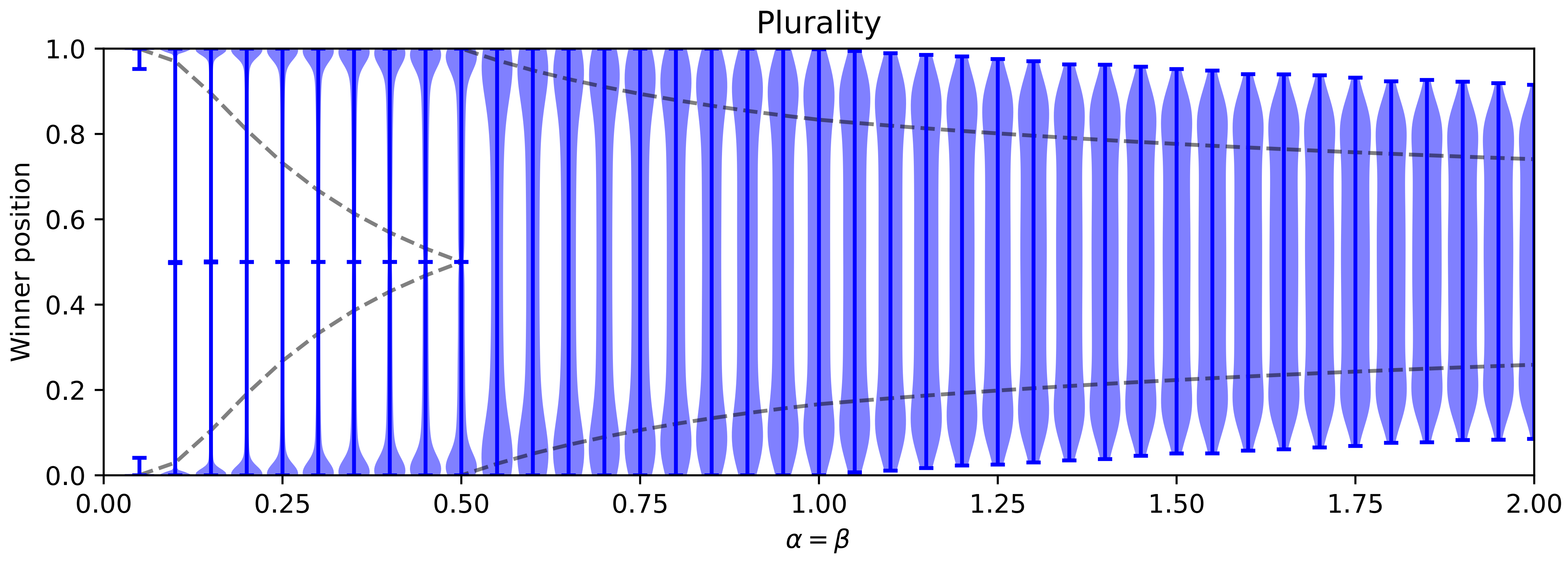

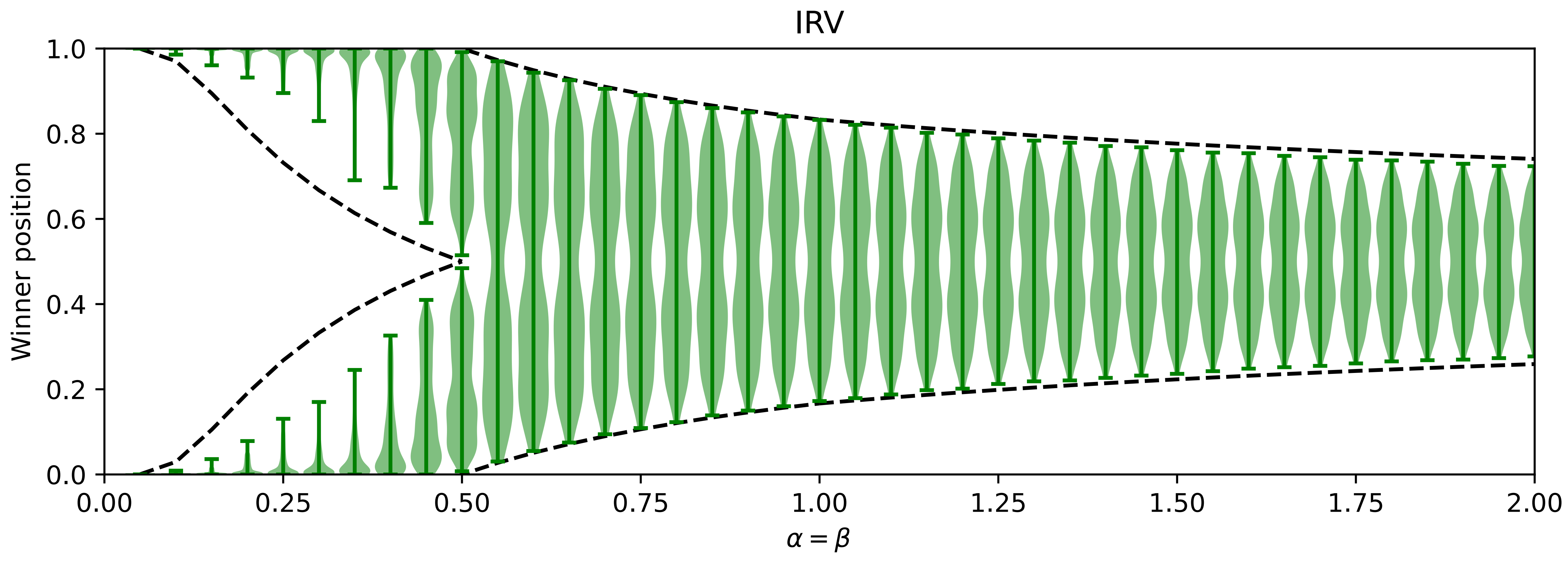

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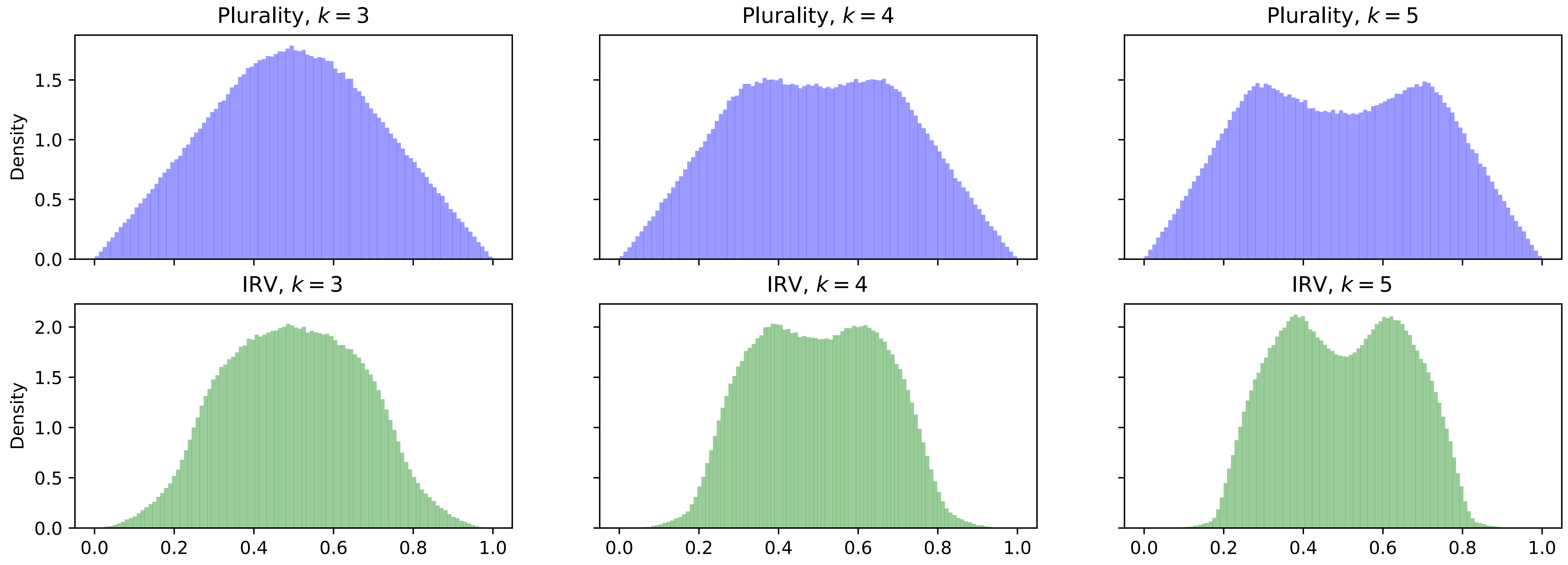
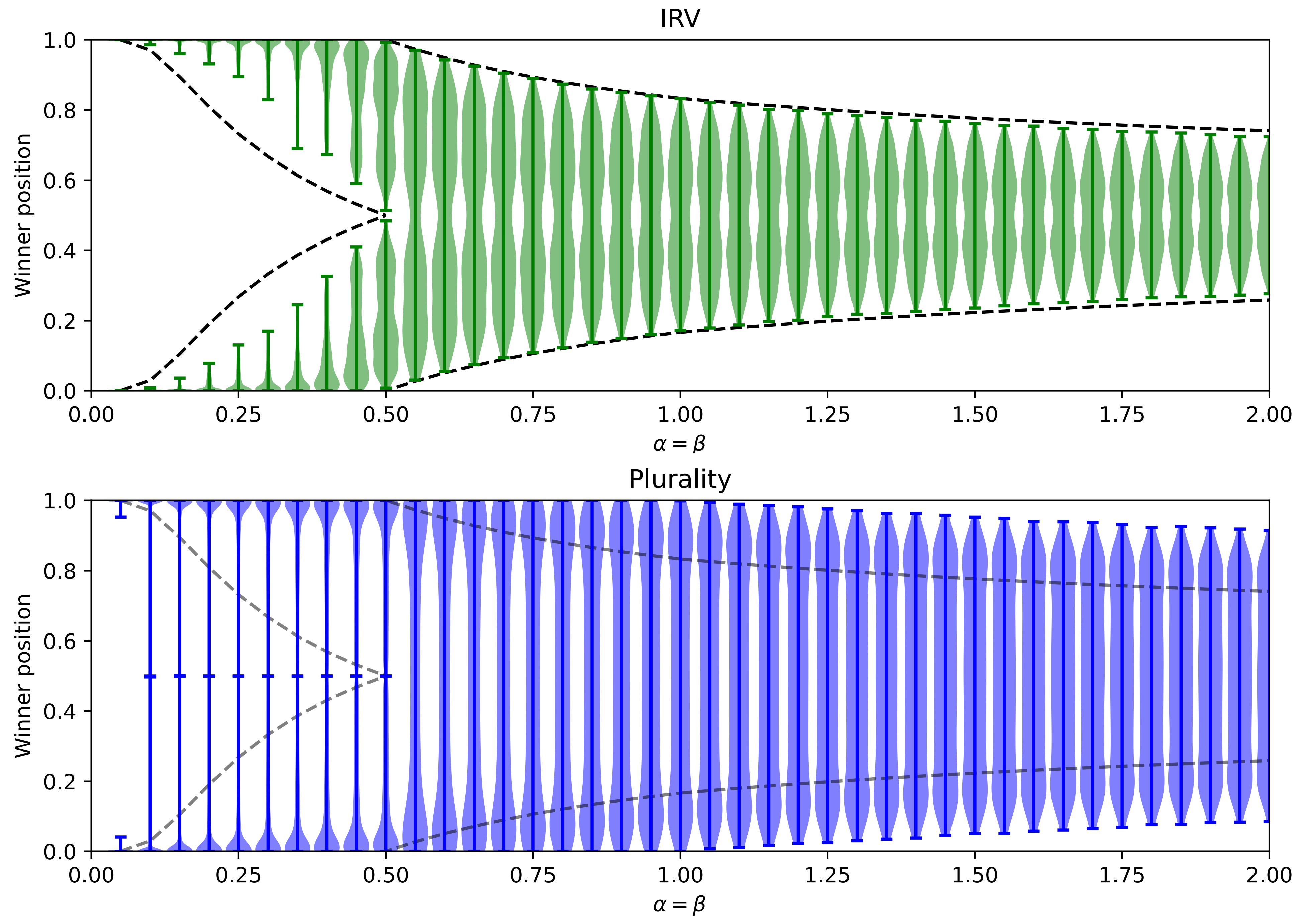
Open question: probabilistic moderation for plurality in general?

Moderation Takeaway: IRV provably has a moderating effect in a way plurality doesn't





Thank you!

Code:
github.com/tomlinsonk/irv-moderation

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