The Moderating Effect of Instant Runoff Voting

AAA‘24

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How do we elect a winner given the preferences of voters?
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Plurality voting
choose the candidate with the most first-place votes
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Winner!
**Instant runoff voting (IRV)**

repeatedly eliminate candidate w/ fewest first-place votes

**Elimination order**
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![Elimination order](image)
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Elimination order

more preferred
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Elimination order

![Fruits](image-url)
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a.k.a. ranked-choice voting (+ AV, STV, Hare, …)
Who uses IRV?
Following a big year, more states push ranked-choice voting

Lawmakers in 14 states have already introduced 27 bills proposing ranked-choice voting models, according to an NBC News review.

WSJ | OPINION

Ranked-Choice Voting Was a Bad Choice

Arlington County, Va., halts a system that left many voters confused.

By The Editorial Board
July 25, 2023 at 6:44 pm ET

The New York Times | OPINION

Can Ranked-Choice Voting Cure American Politics?

Alaskans know the truth about this confusing, coercive voting system.

By Kimberley A. Strassel
Oct. 27, 2022 at 6:14 pm ET

Fox News | OPINION

Supreme Court shoots down GOP attempt to stop ranked-choice voting in Maine

The system allows voters to rank candidates in order of preference on the ballot.

By Paul Steinhauser
July 7, 2021 at 12:10 pm ET
Common debate: does IRV benefit moderates?

[Under IRV,] civility is substantially improved. Needing to reach out to more voters leads candidates to reduce personal attacks and govern more inclusively.

Howard Dean. How to move beyond the two-party system. NY Times, 10/8/2016

The ranked-choice system [...] is biased towards extreme candidates and away from moderate ones.

Nathan Atkinson and Scott Ganz. The flaw in ranked-choice voting: rewarding extremists. The Hill, 10/30/2022
Common debate: does IRV benefit moderates?

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The ranked-choice system [...] is biased towards extreme candidates and away from moderate ones.


**case studies**
(Fraenkel & Grofman, *Public Choice* 2004)
(Mitchell, *Electoral Studies* 2014)
(Reilly, *Nationalism and Ethnic Politics* 2018)

**simulation**
(Chamberlin and Cohen, *APSR* 1978)
(Merrill, *AJPS* 1984)
(McGann, Grofman, & Koetzle, *Public Choice* 2002)

**some limited theory**
(Grofman & Feld, *Electoral Studies* 2004)
(Dellis, Gauthier-Belzile, & Oak, *JITE* 2017)

**case studies**
(Horowitz, *Comparative Political Studies* 2006)
(Horowitz, *Public Choice* 2007)
Does IRV *provably* favor moderates compared to plurality?
1-Euclidean preference model
1-Euclidean preference model

- [0, 1]: left-right ideology

![Diagram of 1-Euclidean preference model with candidates A, B, C, D at positions 0, 2, 4, 6, 8, 1 on the x-axis representing left-right ideology. The diagram shows the distribution of preferences with regions shaded to indicate voter preferences.](image)
1-Euclidean preference model

- [0, 1]: left-right ideology
- Candidates are at points
1-Euclidean preference model

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- Symmetric distribution of voters
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- Voters prefer candidates in order of distance
1-Euclidean preference model

- [0, 1]: left-right ideology
- Candidates are at points
- Moderate = close to 0.5
- Symmetric distribution of voters
- Voters prefer candidates in order of distance

Three example voter distributions in one dimension. Candidates A, B, C, D are positioned at 0.2, 0.3, 0.4, and 0.85. The black line shows the density function of the distribution. Regions are colored according to the most preferred candidate of voters in that region. As an example, the preference ordering of a voter at 0.5 is C, B, A, D.
1-Euclidean preference model

- \([0, 1]\): left-right ideology
- Candidates are at points
- Moderate = close to 0.5
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- Voters prefer candidates in order of distance

C is the plurality and IRV winner
1-Euclidean preference model

- [0, 1]: left-right ideology
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C is the plurality and IRV winner

D is the plurality winner, A is the IRV winner
Formalizing a moderating effect
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**Definition**
A voting system has a **combinatorial moderating effect** if there is an interval $I \subset [0,1]$ s.t. a candidate from $I$ always wins (when present).

We call $I$ an **exclusion zone** of the voting system.
Formalizing a moderating effect

Definition
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We call \( I \) an **exclusion zone** of the voting system.

\[\downarrow \text{implies}\]

Definition
A voting system has a **probabilistic moderating effect** if
\[\Pr(\text{winner is in } I) \to 1\] as the number of candidates \( k \to \infty\).
Starting simple: uniform voters

That is, IRV exhibits an exclusion zone in the middle of voter randomly chosen according to how many voters are between them and each candidate. That is, Tomlinson et al. voters have 1-Euclidean preferences but rank candidates according to how many voters are between them and each candidate. That is, to provide another perspective on the uniform voter assumption, consider the following preference proof.

Fig. 1. We can specify the IRV rule using these vote shares $\frac{x}{6}$, $\frac{5}{6}$, $\frac{1}{3}$ for candidate $C$, $\frac{2}{3}$ for candidate $A$, and are always uniformly distributed over moderates—regardless of the distribution of candidates.

Notice that the above argument still holds if we replace $\frac{1}{3}$ and $\frac{2}{3}$ with any $c$, $\frac{c}{6}$ for any $c$, $\frac{5}{6}$, and are always uniformly distributed over moderates; for instance, consider placing the most votes any candidate achieves a majority). In practice, voters submit a vote for candidate instead think of a continuum of voters who correspond to the uniform voters example (center), $D$ is the plurality winner and $C$ is the IRV winner. In the polarized voters example (right), $D$ is the plurality winner and $A$ is the IRV winner.
Theorem 1 (Combinatorial moderation for IRV)
For any $k \geq 3$, $[1/6, 5/6]$ is an exclusion zone of IRV with uniform voters.

No smaller interval has this property.
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**IRV has a moderating effect!**
Plurality allows extreme winners
**Theorem 2** (No combinatorial moderation for plurality, uniform voters)

Given any distinct candidate positions $x_1, \ldots, x_k$ (with $x_1 \not\in \{0,1\}$), we can add more candidates to make $x_1$ the plurality winner.
No probabilistic moderation for plurality

**Theorem 3** (No probabilistic moderation for plurality, uniform voters)
Let $P_k$ be the position of the plurality winner with $k$ candidates distributed uniformly. As $k \to \infty$, $P_k \to_d \text{Uniform}(0,1)$. 
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Proof idea:

Connection to stick-breaking processes to find winning vote share + circle-cutting argument


What about non-uniform voters?

[Diagram showing two voter distributions in one dimension. Candidates A, B, C, D are positioned at 0.2, 0.3, 0.4, and 0.85. The black line shows the density function of the distribution. See Figure 1. Three example voter distributions in one dimension. Candidates A, B, C, D are positioned at 0.2, 0.3, 0.4, and 0.85. The black line shows the density function of the distribution. See Figure 1.]
Theorem generalizes!
Theorem 4 (Combinatorial moderation for IRV, general case)
Let the voter distribution be symmetric with CDF $F$ and let $c \in (0, 1/2)$. If for all $x \in [c, 1/2]$,}

$$F\left(\frac{x + 1 - c}{2}\right) - F\left(\frac{c + x}{2}\right) > \frac{1}{3} \quad (\star)$$

then $[c, 1 - c]$ is an exclusion zone of IRV.

$(\star)$ intuitively: “the last moderate can’t be squeezed out”
[1/6, 5/6] Theorem generalizes!

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**Theorem 5**
exclusion zone: centrists voters

\([F^{-1}(1/6), 1 - F^{-1}(1/6)]\)
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Theorem 5
Exclusion zone: 
\[ [F^{-1}(1/6),1-F^{-1}(1/6)] \]

Theorem 6
Exclusion zone: 
\[ [2F^{-1}(1/3) - 1/2, 3/2 - 2F^{-1}(1/3)] \]
even with polarized voters! $(F(1/4) < 1/3)$
If voters are too polarized, IRV can’t elect moderates

**Theorem 7** (hyper-polarized voters)  
Suppose $F(1/4) > 1/3$. For any $c \geq 2F^{-1}(1/3)$,  
$[0,c] \cup [1 - c,1]$ is an exclusion zone of IRV.
If voters are too polarized, IRV can’t elect moderates

**Theorem 7 (hyper-polarized voters)**
Suppose $F(1/4) > 1/3$. For any $c \geq 2F^{-1}(1/3)$, $[0,c] \cup [1 - c,1]$ is an exclusion zone of IRV.

Beta-distributed voters
Theorem 8 (no combinatorial moderation for plurality)
As long as the voter distribution is continuous and positive over $(0,1)$, we can make an arbitrarily extreme candidate win by adding more candidates.
Plurality still elects arbitrarily extreme candidates

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As long as the voter distribution is continuous and positive over $(0,1)$, we can make an arbitrarily extreme candidate win by adding more candidates.

Open question: probabilistic moderation for plurality in general?
Moderation Takeaway:
IRV provably has a moderating effect in a way plurality doesn’t
Thank you!

Code:  
github.com/tomlinsonk/irv-moderation

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