Ballot Length in Instant Runoff Voting


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INSTANT RUNOFF VOTING (IRV)

- Voter submit (partial) rankings over k candidates
- Repeat until one candidate remains:
  - Eliminate candidate with fewest top rankings
  - Redistribute ballots
- How many candidates should voters be allowed to rank? This is the *ballot length*.
- **We study how ballot length affects IRV winners**

CONSTRUCTING ANY WINNER SEQUENCE

- Consider *consequential-tie-free* profiles (unique winner at every ballot length h)
- Label candidates 1, .., k in IRV elimination order
- Sequence of winners from h = 1, …, k - 1: *truncation winner sequence*
  - Feasible iff element-wise > 1, …, k - 1

Theorem. For all $k \geq 3$, given any truncation winner sequence, there is a consequential-tie-free profile with $2k^2 - 2k$ voters achieving that sequence.

- Explicit construction! See center example
- In the paper: constructions with other tie restrictions and with $\Theta(k)$ voter types

VOTER LOWER BOUNDS

Theorem. For all $k > 3$, a consequential-tie-free profile needs at least $2k^2 - 2k$ voters to have k - 1 different truncation winners.

- Construction is tight for k - 1 truncation winners!
- In the paper: lower bounds for other restrictions on ties

PREFERENCE RESTRICTIONS

Theorem. For $k \geq 5$, k - 1 truncation winners are impossible with single-peaked or single-crossing preferences.

- However, at least $\Theta(\sqrt{k})$ winners are possible with single-peaked preferences
- Open question: up to k - 2 winners?

FULL BALLOTS

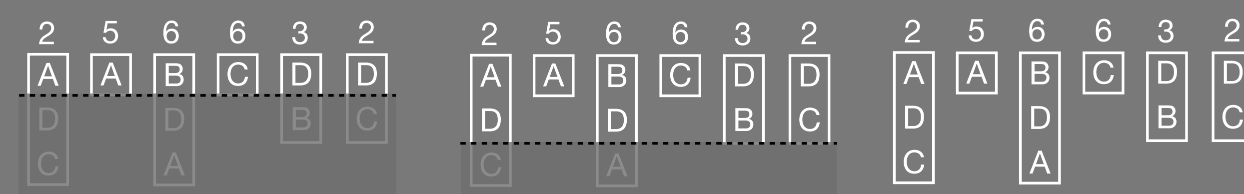
- Constructions so far use partial rankings; what if we require full ballots?
- Construction with full ballots with k / 2 winners
- Linear program found full-ballot k - 1 winner constructions up to k = 10

The number of candidates that voters are allowed to rank can have a huge effect on IRV election outcomes.

Given (almost) any length k - 1 sequence of k candidates, we can construct voter preferences so that the IRV winners at ballot lengths 1, …, k-1 follow the given sequence.

Example. k = 4 candidates, winner sequence ABC:

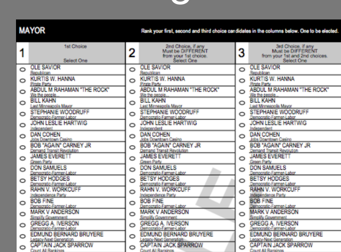
Ballot length 1 | Ballot length 2 | Ballot length 3



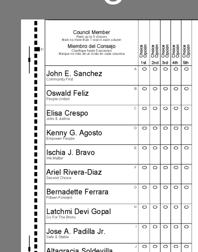
A wins | B wins | C wins

Our constructions use only $\Theta(k^2)$ voters to achieve any winner sequence, which is tight for k - 1 different winners.

Real-world IRV elections use various ballot lengths:

3


Minneapolis, MN

5

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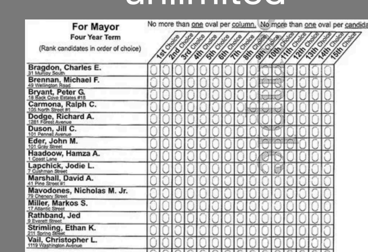


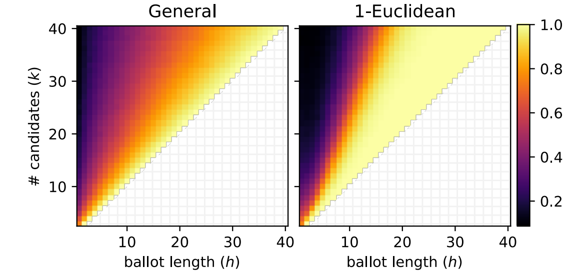
Portland, ME

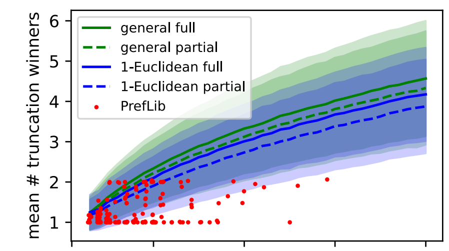
We truncate ballots in 168 real-world elections:
25% of them have multiple winners as ballot length varies.

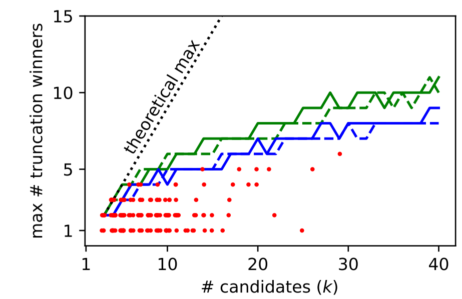
Paper: | Code + Data:

BALLOT LENGTH IN SIMULATION

- General profiles: 1000 uniform rankings
- 1-Euclidean profiles: uniform 1-dimensional voters
- Multiple truncation winners are common
- Extreme cases are rare (e.g., k - 1 winners)

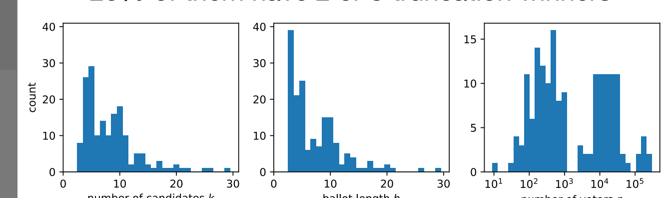




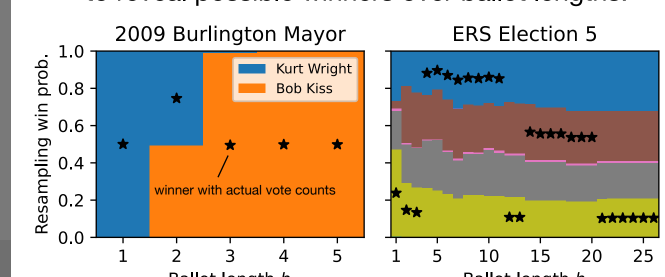


BALLOT LENGTH IN REAL-WORLD DATA

- 168 elections from PrefLib (1)
- 25% of them have 2 or 3 truncation winners



- We resample ballots w/o replacement 1k times to reveal possible winners over ballot lengths:



RELATED WORK

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(3) Ayadi, M.; Amor, N.; Lang, J.; and Peters, D. 2019. Single transferable vote: Incomplete knowledge and communication issues. In *AAMAS*.
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