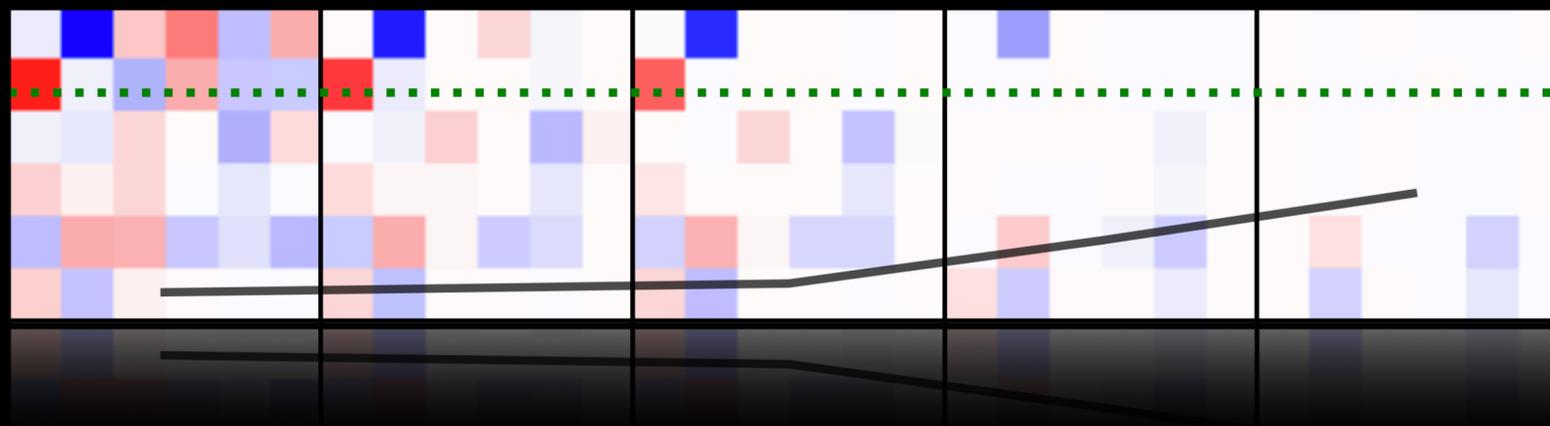


Code: bit.ly/lcl-code
Data: bit.ly/lcl-data
Slides: bit.ly/lcl-kdd-slides



Learning Interpretable Feature Context Effects in Discrete Choice

Kiran Tomlinson
PhD Student, Cornell CS



research with Austin R. Benson

Choices and context effects

Discrete choices are everywhere



amazon.com

Amazon's Choice

KDD Chocolate Flavored Milk 180ML (18 PACK)
6 Fl Oz (Pack of 18)
★★★★☆ ~ 57
\$27⁹⁹ (\$0.26/Fl Oz)
Save \$2.00 with coupon
✓prime FREE Delivery Thu, Jun 24

KDD Banana Flavored Milk 180ML (18 PACK)
6.33 Fl Oz (Pack of 18)
★★★★☆ ~ 31
\$27⁹⁹ (\$0.26/Fl Oz)
Save \$2.00 with coupon
✓prime FREE Delivery Thu, Jun 24

KDD Original Milk 180ML (18 PACK)
★★★★★ ~ 2
\$27⁹⁹ (\$4.60/Ounce)
✓prime FREE Delivery Thu, Jun 24



Best Western University Inn
Ithaca

Black Friday / Cyber Monday Deals Now
Free Shuttle Transportation, Grab & Go Breakfast, WiFi & Parking. Pet friendly, Outdoor Pool, Fitness Center. Sanitizing Daily

Breakfast included

3.9/5 Good (999 reviews)



\$63
per night
\$71 total
Includes taxes & fees



Quality Inn Ithaca - University Area
Ithaca

Black Friday / Cyber Monday Deals Now
Complimentary Breakfast. Free Airport Shuttle, WiFi & parking. Close to Ithaca College & Cornell University. Pets welcome.

Breakfast included

3.6/5 Good (694 reviews)

Member Price available

\$59
per night
\$66 total
Includes taxes & fees



Hotel Ithaca
Ithaca

4.0/5 Very Good (842 reviews)

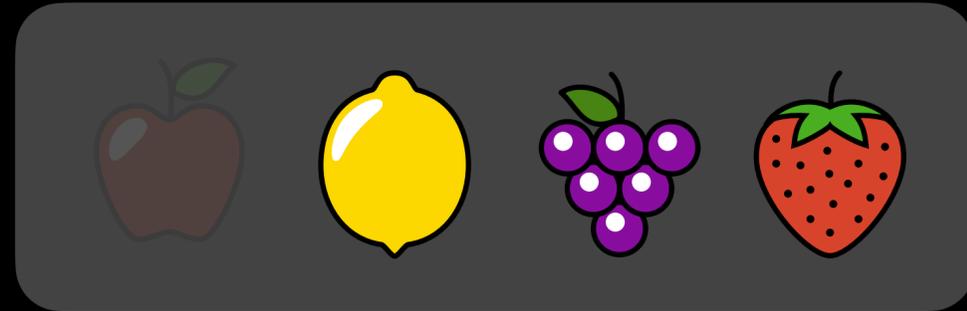
Member Price available

\$94
per night
\$106 total
Includes taxes & fees

“The fundamental problem of discrete choice”

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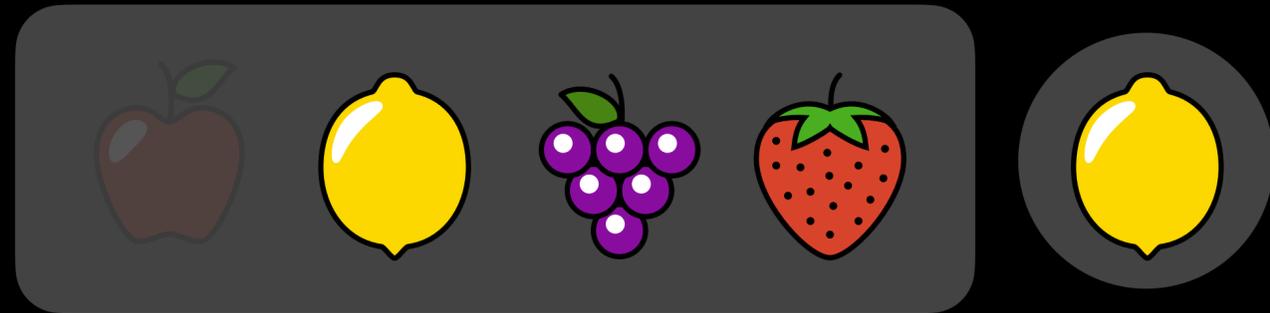
choice set



“The fundamental problem of discrete choice”

choice set

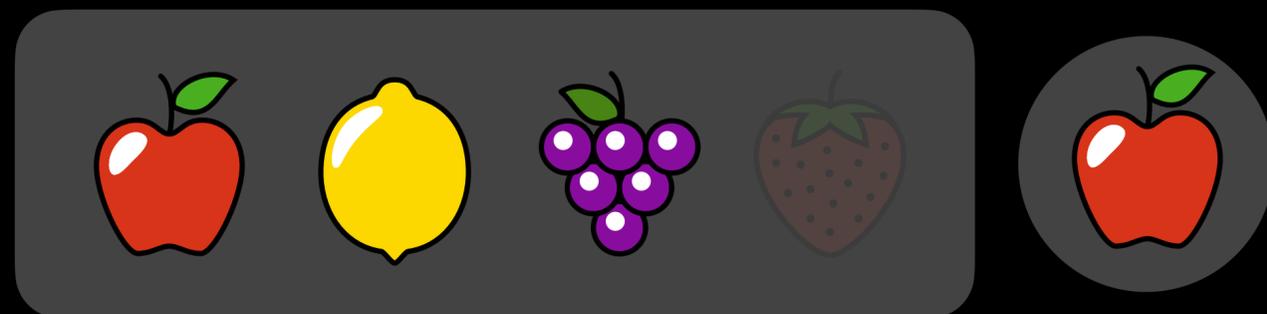
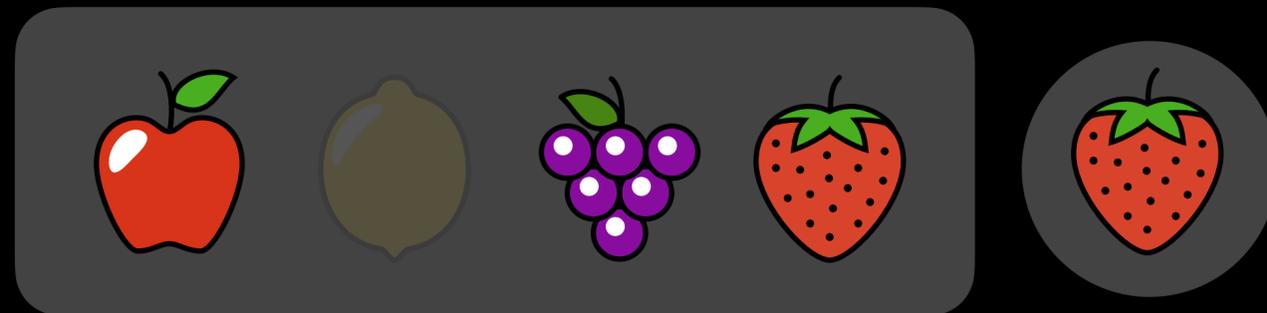
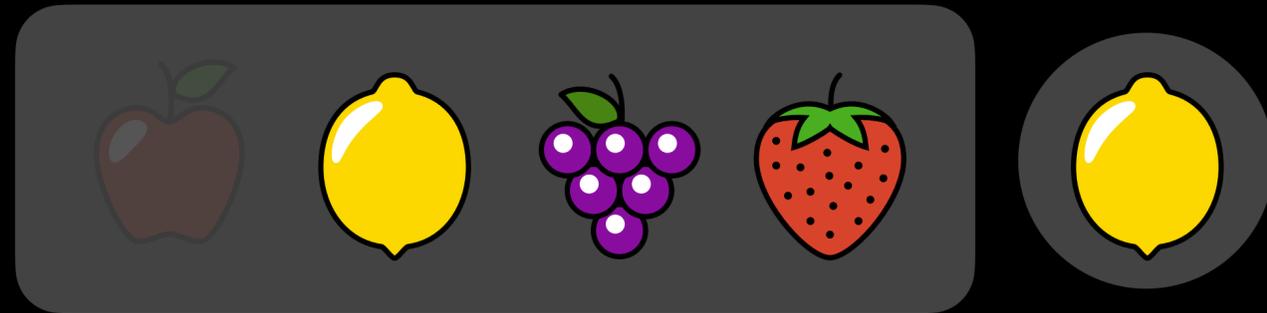
choice



“The fundamental problem of discrete choice”

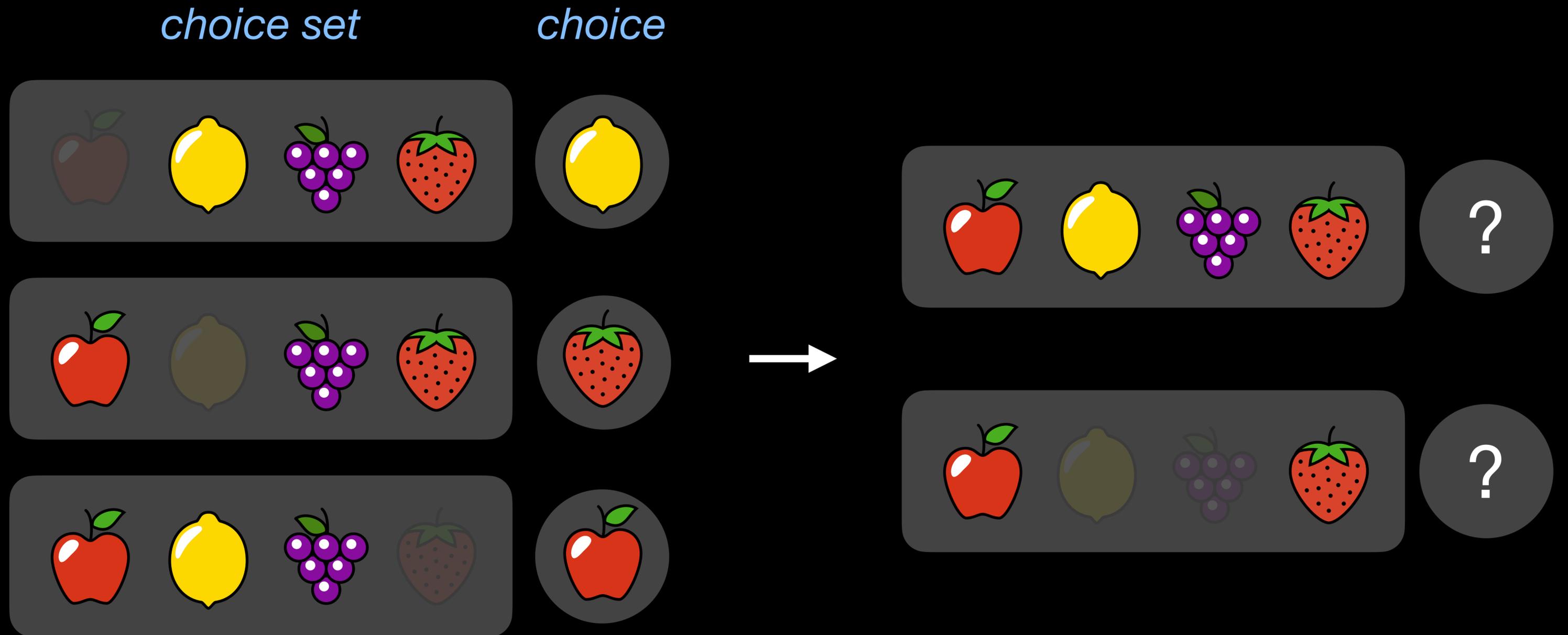
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...

“The fundamental problem of discrete choice”



The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

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Assume *item* i has *utility* u_i

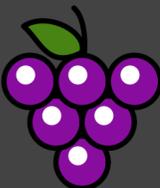
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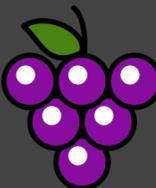
C				
u_i	1	-1	0	2
$\Pr(i C)$.24	.03	.09	.64

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Unique choice model satisfying
independence of irrelevant alternatives (IIA):

(Luce, *Individual Choice Behavior* 1959)

$$\frac{\Pr(i | C)}{\Pr(j | C)} = \frac{\Pr(i | C')}{\Pr(j | C')}$$

Problem for MNL: *context effects*

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The choice set influences preferences.

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Compromise

(Simonson, 1989)

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 Jon Ossoff ✓	Dem.	2,374,519	47.9



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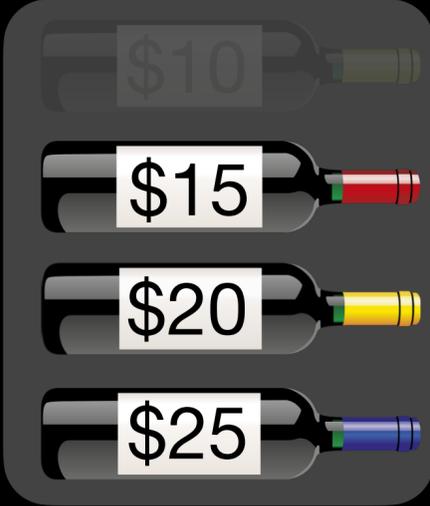
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Item features and the LCL

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Use item features:



genre: drama,
in_top_10: True,
has_new_episodes: True,
producer: Netflix



genre: comedy,
in_top_10: False,
has_new_episodes: False,
producer: NBC



genre: drama,
in_top_10: True,
has_new_episodes: False,
producer: Netflix



genre: reality,
in_top_10: True,
has_new_episodes: False,
producer: Banijay

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Preference coefficient θ_k is easy to interpret: importance of the k^{th} feature

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$\rightarrow u_{i,C} = (\theta + Ax_C)^T x_i$ ($x_C = \frac{1}{|C|} \sum_{j \in C} x_j$ is the *mean feature vector*)

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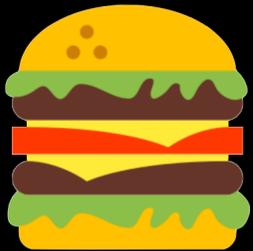
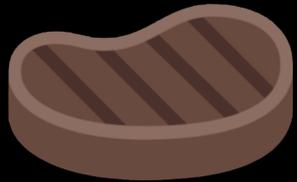
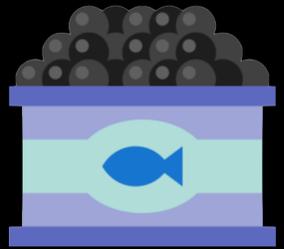
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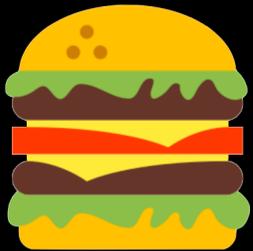
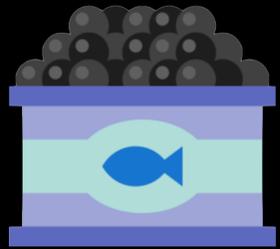
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LCL example: restaurant selection

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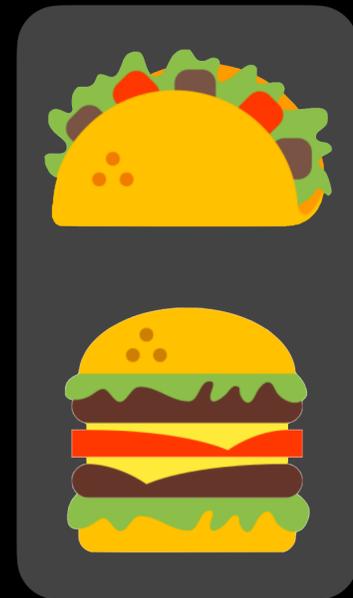
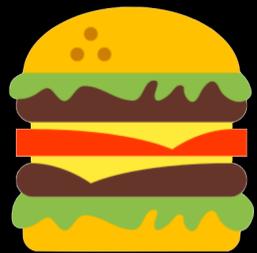
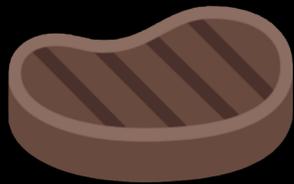
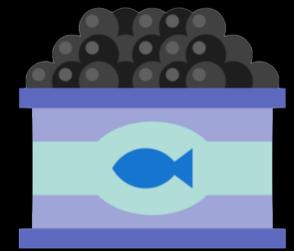
LCL example: restaurant selection



item features:

- price
- service speed
- wine selection

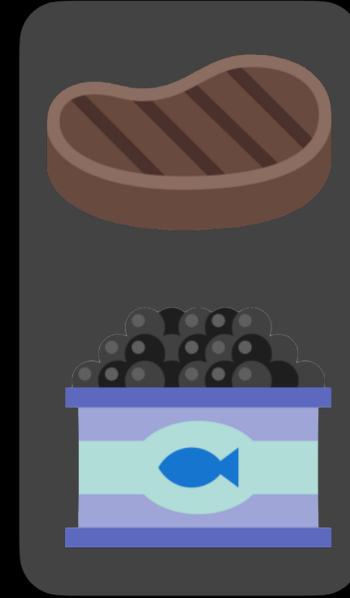
LCL example: restaurant selection



C_1



C_2

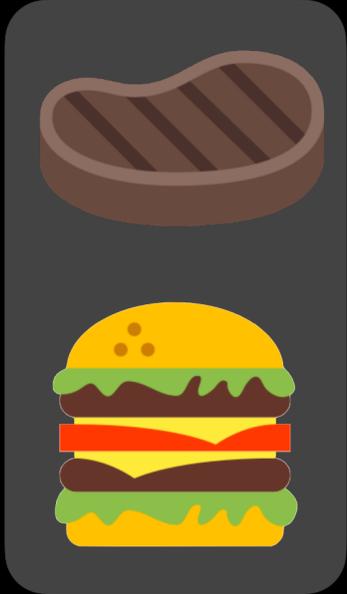
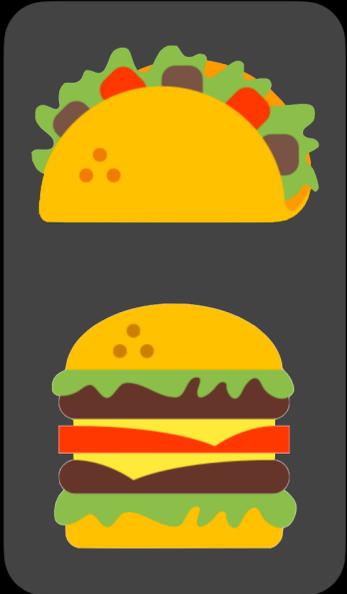
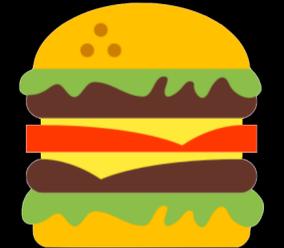
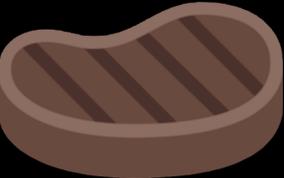
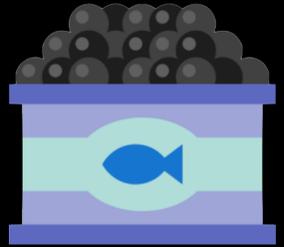


C_3

item features:

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LCL example: restaurant selection



C_1

C_2

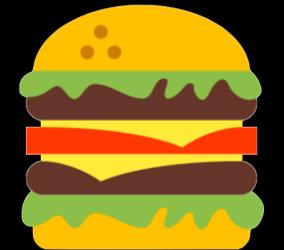
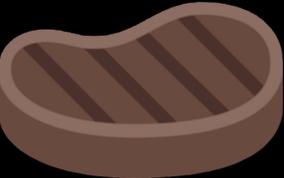
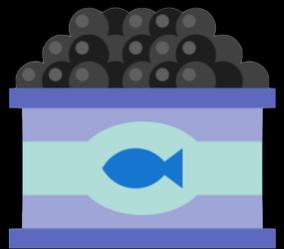
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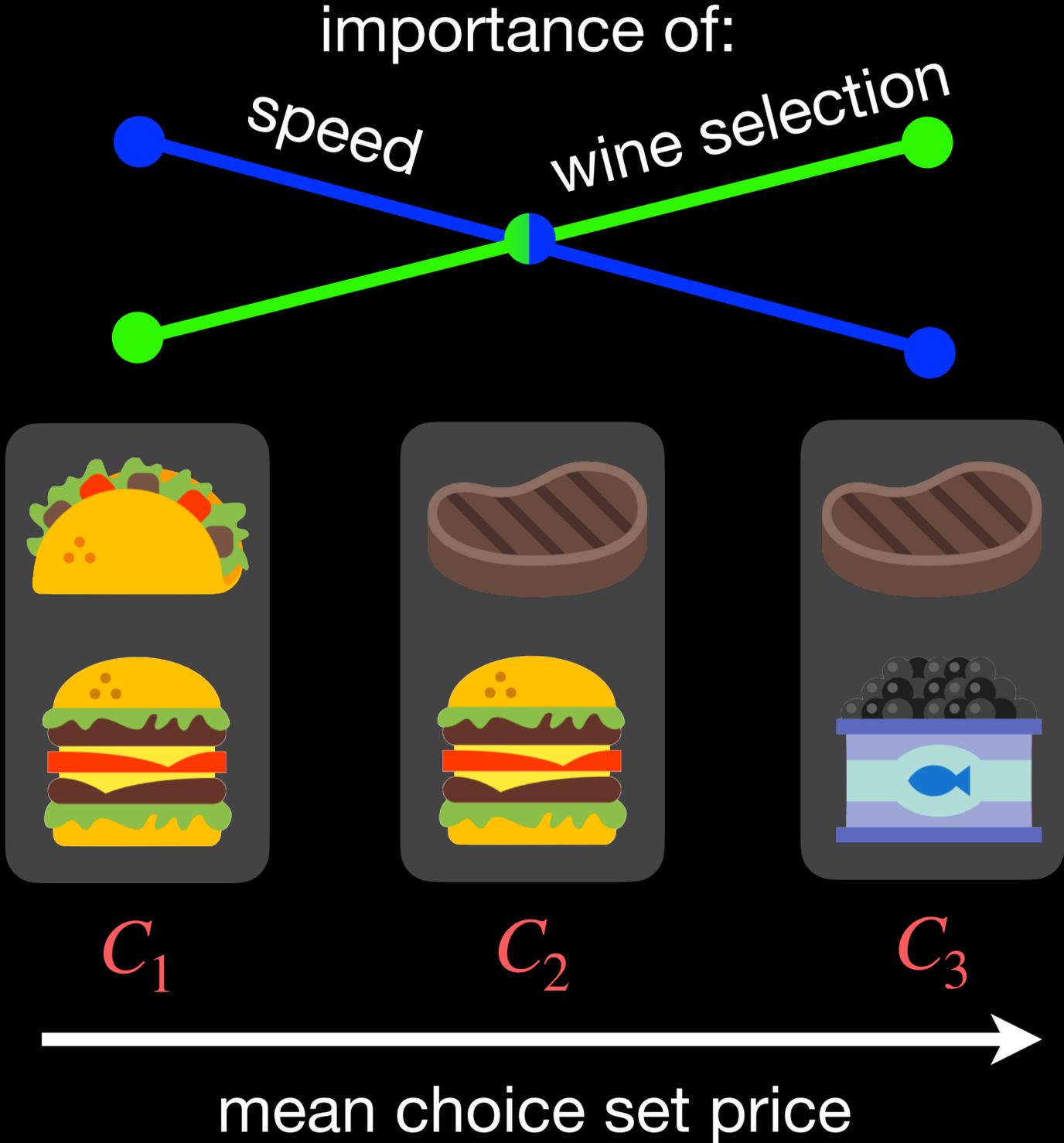
mean choice set price

- item features:
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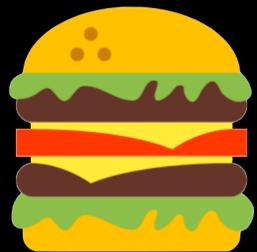
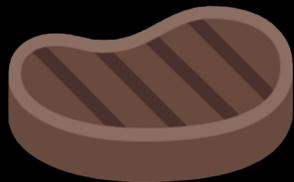
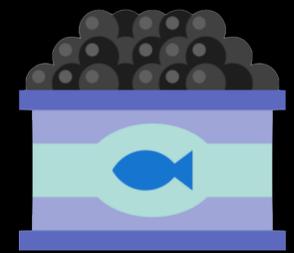
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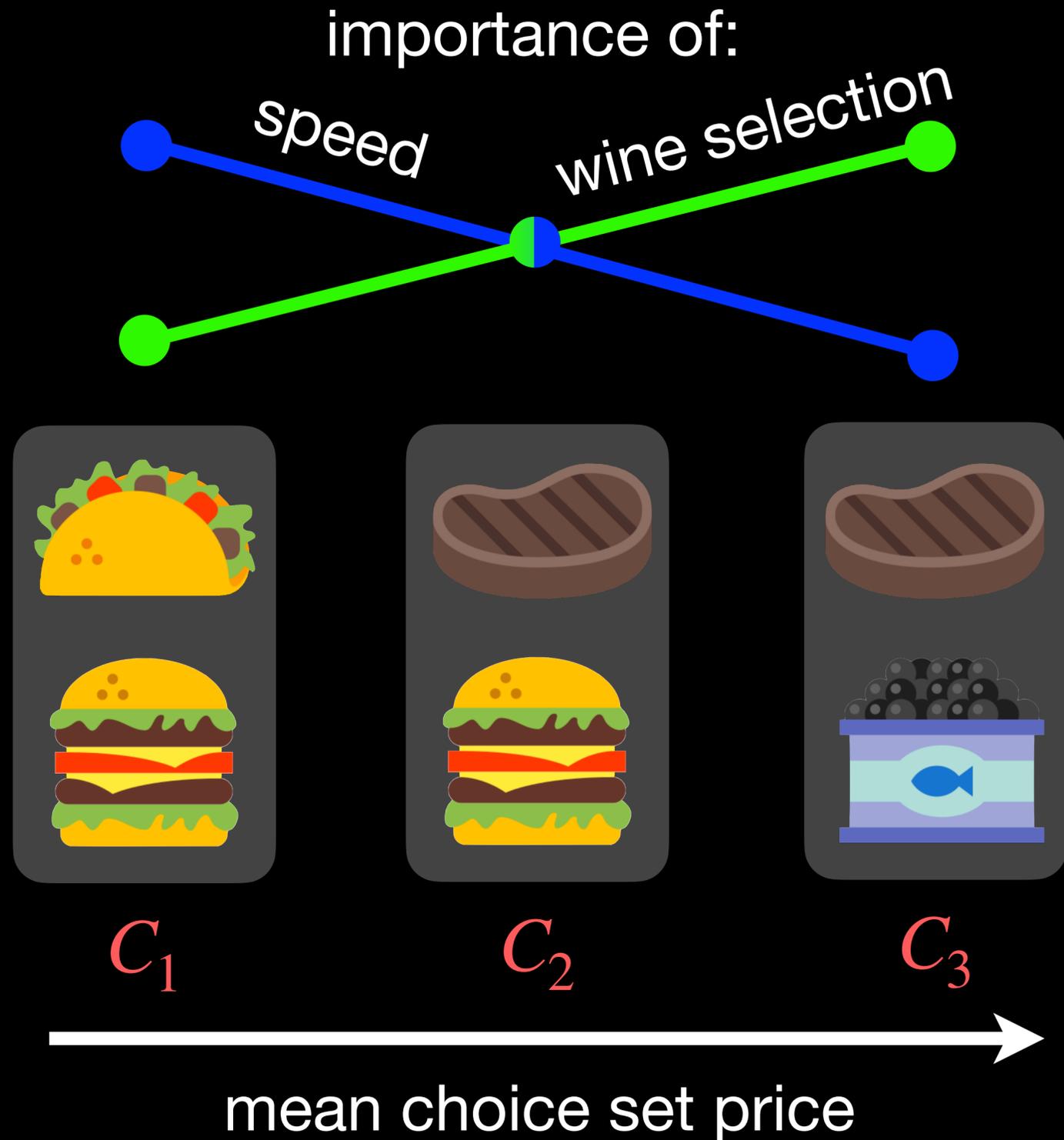


LCL example: restaurant selection



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$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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model is *identifiable* from dataset \mathcal{D} if no two parameter values result in the same probability distribution

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($\mathcal{C}_{\mathcal{D}}$: unique choice sets in \mathcal{D} , \otimes : Kronecker product)

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intuition: need varied choice sets containing varied items

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- see paper for details

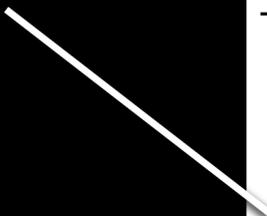
Results on choice data

Choice datasets

Dataset	Choices	Features	Largest Choice Set
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

Choice datasets

favorite sushi types



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Choice datasets

favorite sushi types

hotel bookings

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LCL improves model fit

whole-dataset negative log-likelihood (lower = better)

	CL	LCL	Mixed logit	DLCL
DISTRICT	3313	3130	3258	3206
DISTRICT-SMART	3426	3278*	3351	3303 [†]
EXPEDIA	839505	837649*	839055	837569[†]
SUSHI	9821	9773*	9793	9764
CAR-A	1702	1694	1696	1692
CAR-B	1305	1295	1297	1284
CAR-ALT	7393	6733*	7301	7011 [†]

*significant likelihood ratio test vs MNL ($p < 0.001$)

[†]significant likelihood ratio test vs mixed logit ($p < 0.001$)

LCL can improve out-of-sample prediction performance

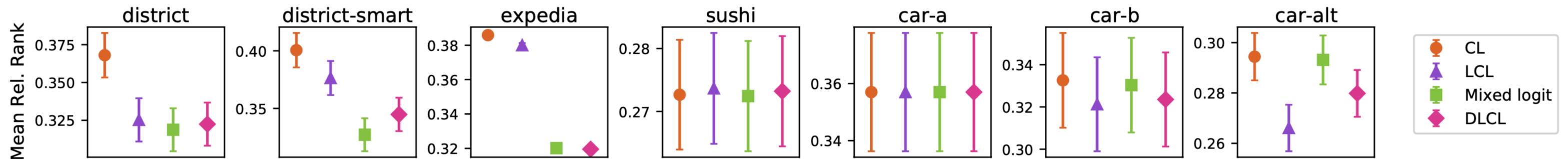


Figure 2: Mean relative rank of predictions on held-out test data (lower is better). Error bars show standard error of the mean.

LCL can test individual effects for significance

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Compute std. errs. (and z-scores) for each parameter estimate using MLE *asymptotic normality*

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Table 4: Five largest context effects in SUSHI.

Effect (q on p)	A_{pq} (std. err.)	p -value
<i>popularity</i> on <i>popularity</i>	-0.28 (0.15)	0.066
<i>availability</i> on <i>is maki</i>	0.24 (0.14)	0.087
<i>oiliness</i> on <i>oiliness</i>	-0.20 (0.08)	0.0089
<i>popularity</i> on <i>availability</i>	0.19 (0.14)	0.16
<i>availability</i> on <i>oiliness</i>	-0.18 (0.10)	0.064

LCL can test individual effects for significance

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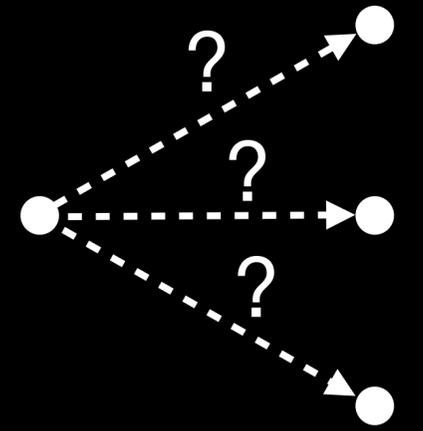
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Table 5: Five largest context effects in EXPEDIA.

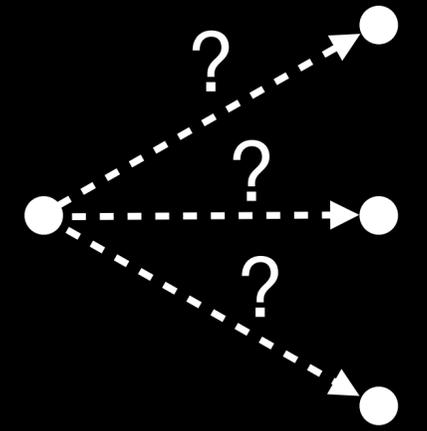
Effect (q on p)	A_{pq} (std. err.)	p -value
<i>location score</i> on <i>price</i>	-0.47 (0.05)	$< 10^{-16}$
<i>on promotion</i> on <i>price</i>	0.27 (0.03)	$< 10^{-16}$
<i>review score</i> on <i>price</i>	-0.19 (0.03)	1.4×10^{-9}
<i>star rating</i> on <i>price</i>	0.15 (0.04)	6.7×10^{-5}
<i>price</i> on <i>star rating</i>	0.10 (0.00)	$< 10^{-16}$

Social network application

What factors drive edge formation?

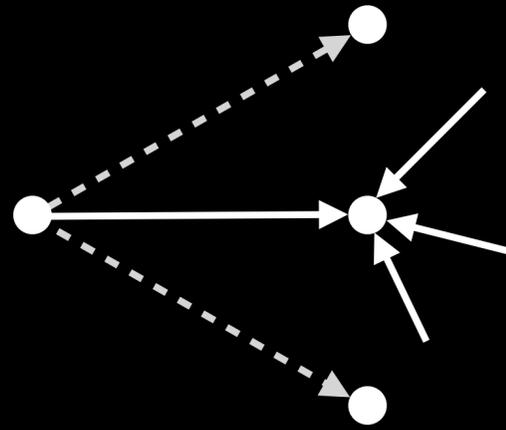


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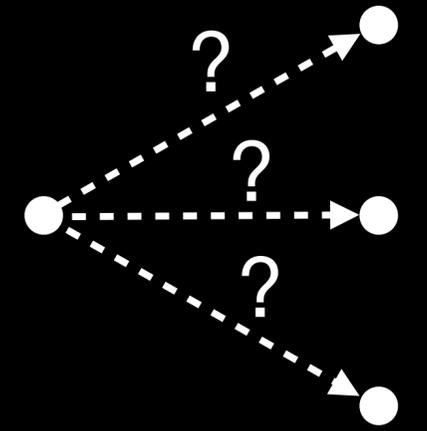


Preferential attachment

(Barabási & Albert, *Science* 1999)

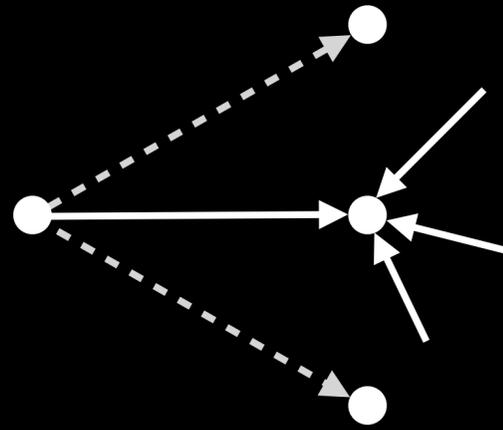


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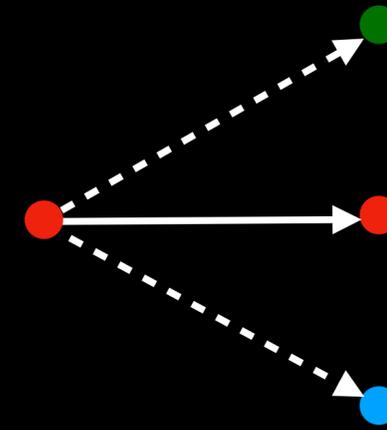
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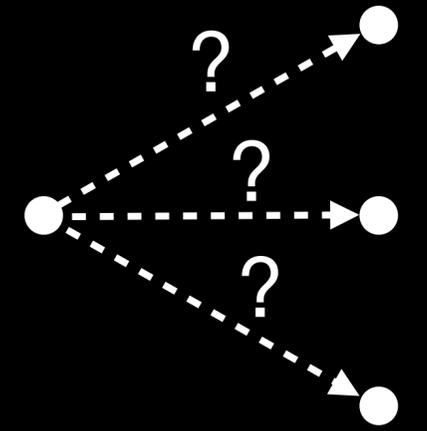


Homophily

(McPherson et al., *Annual Review of Sociology* 2001)
(Papadopoulos et al., *Nature* 2012)

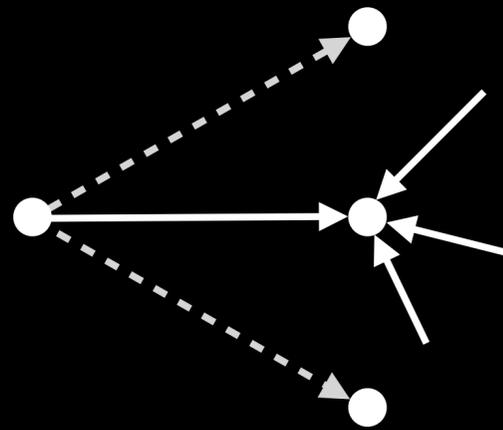


What factors drive edge formation?



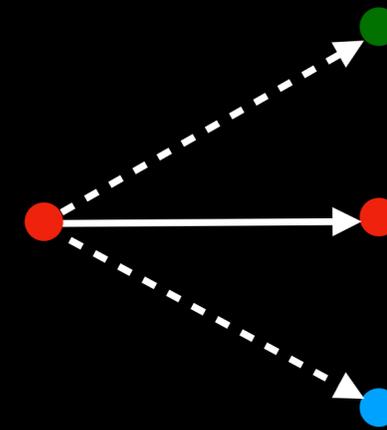
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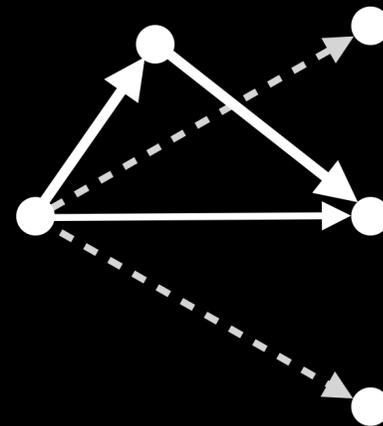
Homophily

(McPherson et al., *Annual Review of Sociology* 2001)
(Papadopoulos et al., *Nature* 2012)



Triadic closure

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)
(Jin et al., *Physical Review E* 2001)



“Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

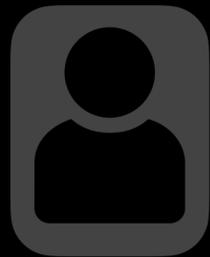
(Gupta & Porter, *arXiv* 2020)

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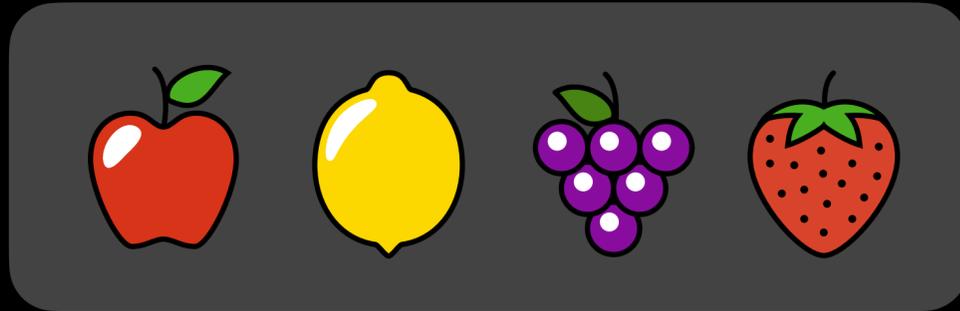
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so far:



chooser



choice set

“Choosing to grow a graph”

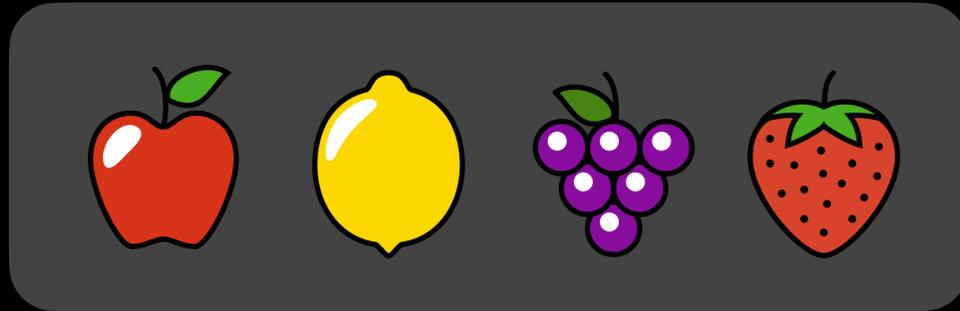
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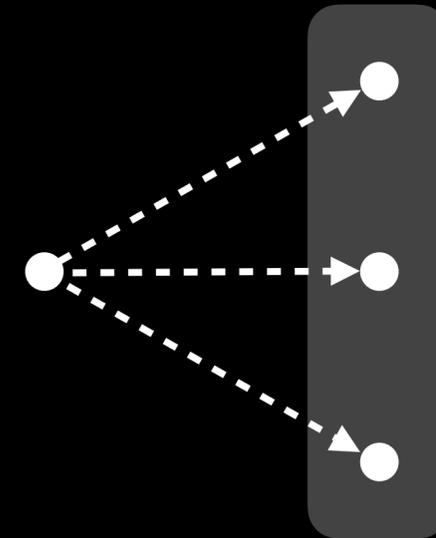
chooser



choice set

in network growth:

chooser



choice set

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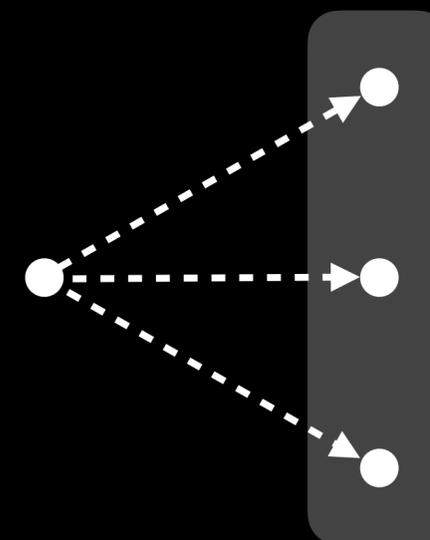
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choice set

in network growth:

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choice set

Key usage

Timestamped edges

→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data

“Choosing to grow a graph”

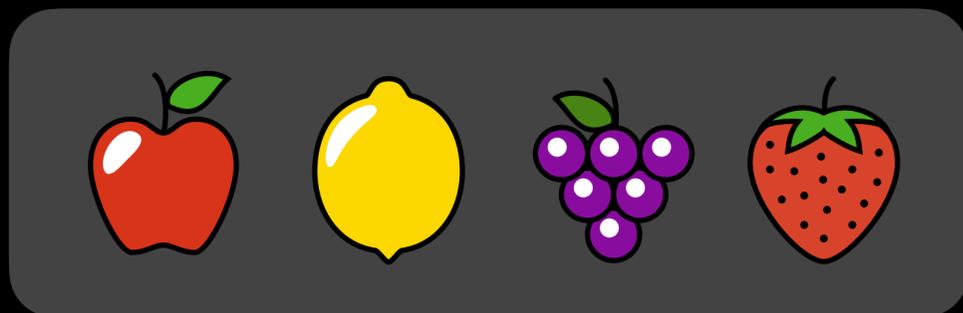
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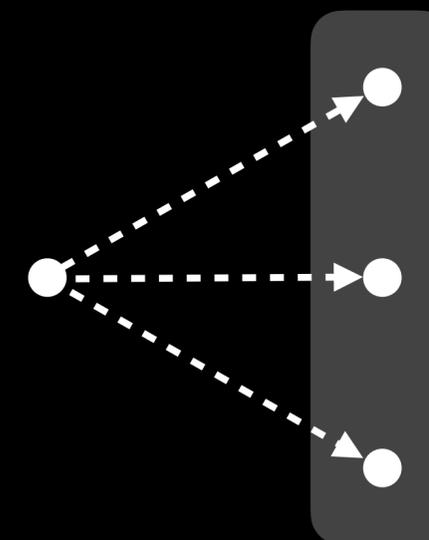
chooser



choice set

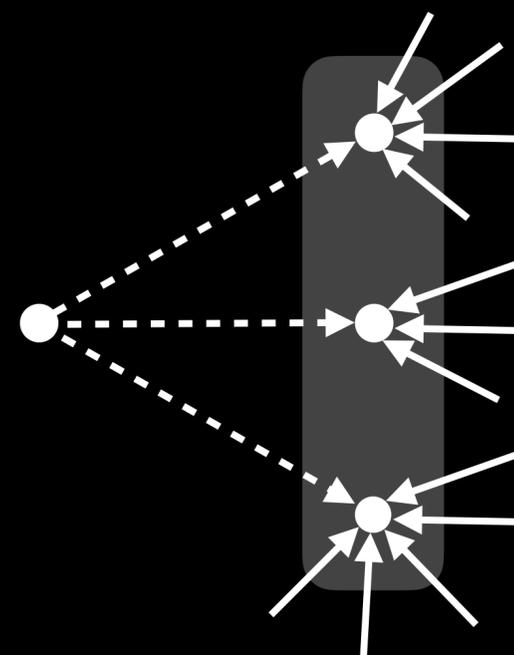
in network growth:

chooser

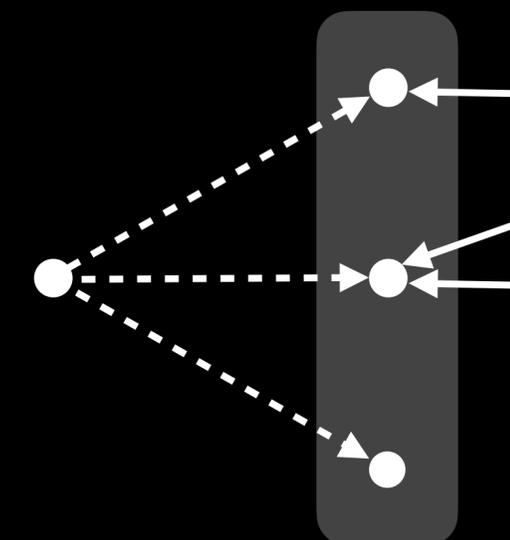


choice set

feature context effects:



vs.



Key usage

Timestamped edges

→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data

Choosing to close triangles

- Triadic closure* offers small choice sets
- tractable inference
- varied choice sets

Choosing to close triangles

Triadic closure offers small choice sets
→ tractable inference
→ varied choice sets

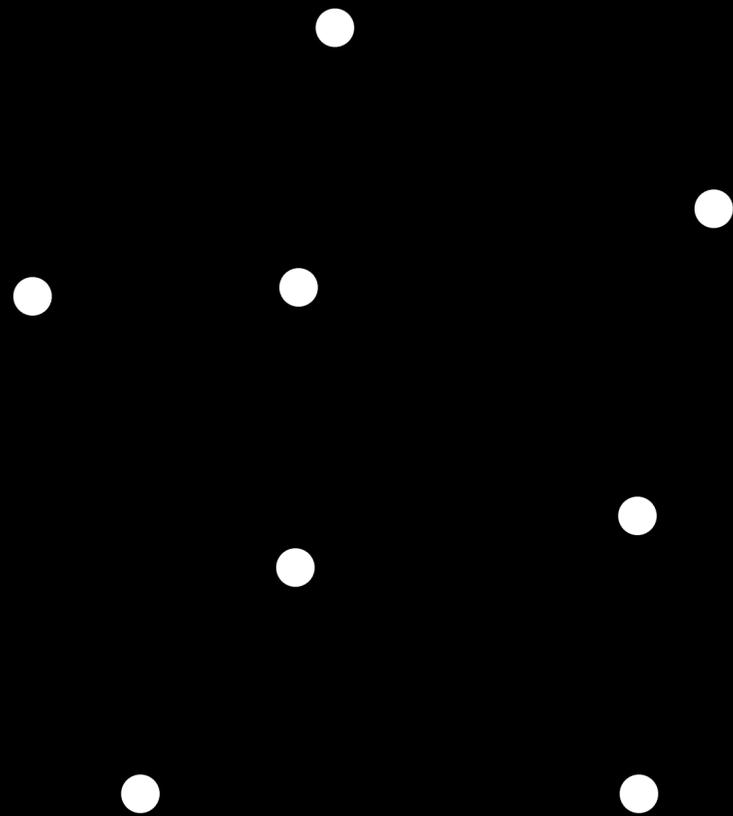
Our data
Timestamped edges
(including repeats)

Choosing to close triangles

Triadic closure offers small choice sets
→ tractable inference
→ varied choice sets

Our data

Timestamped edges
(including repeats)

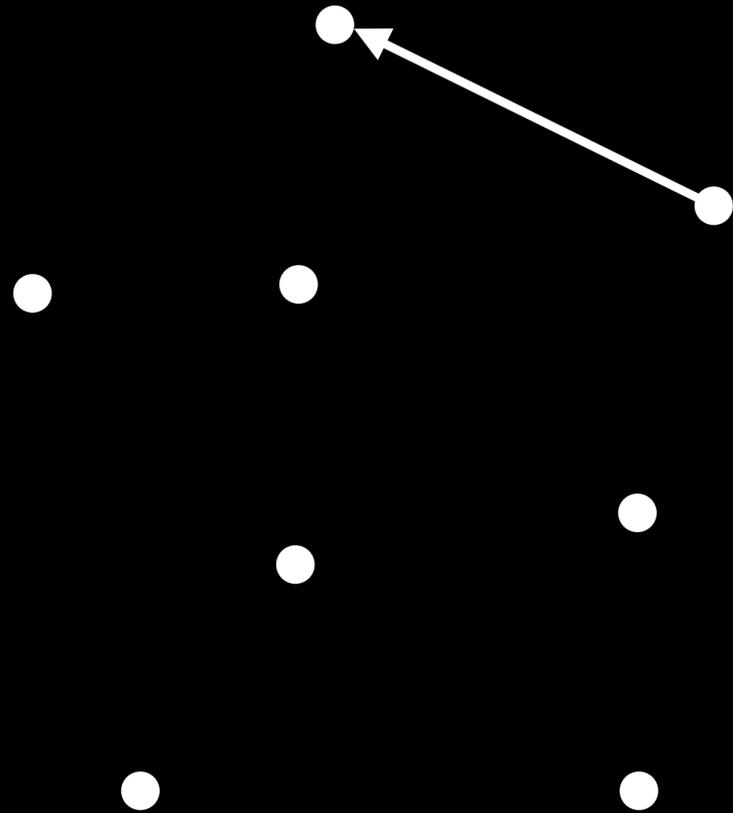


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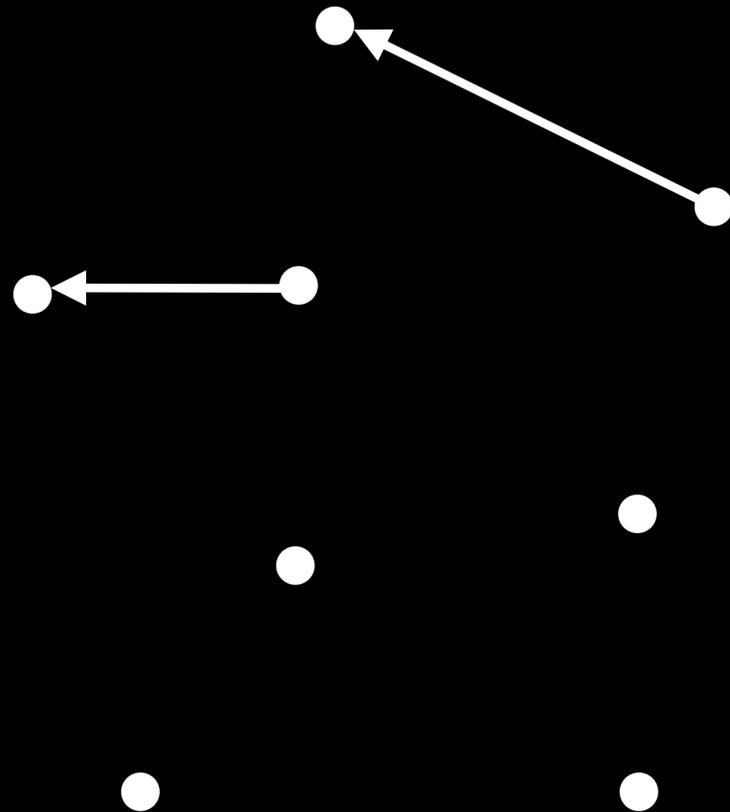


Choosing to close triangles

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Our data

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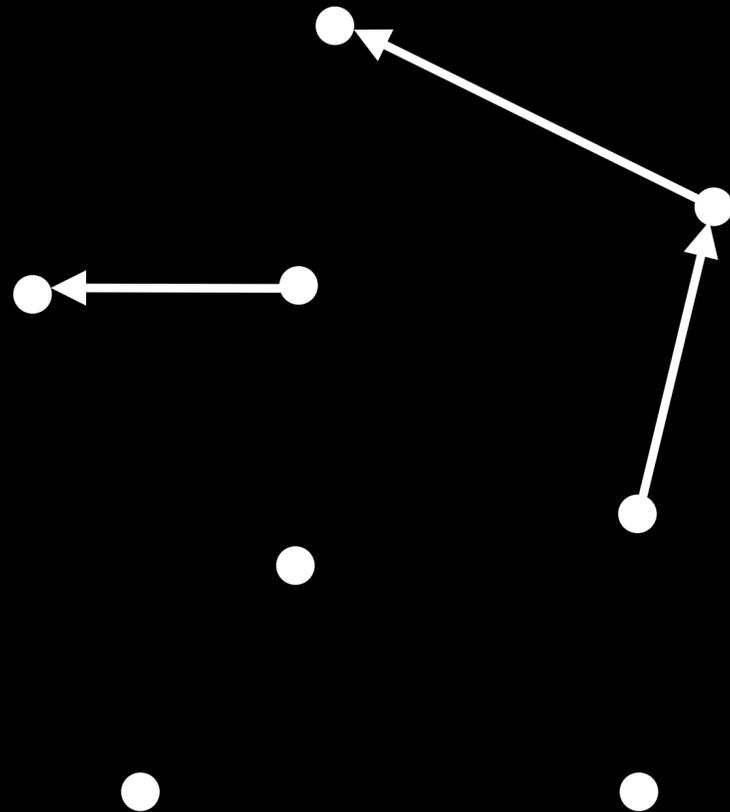


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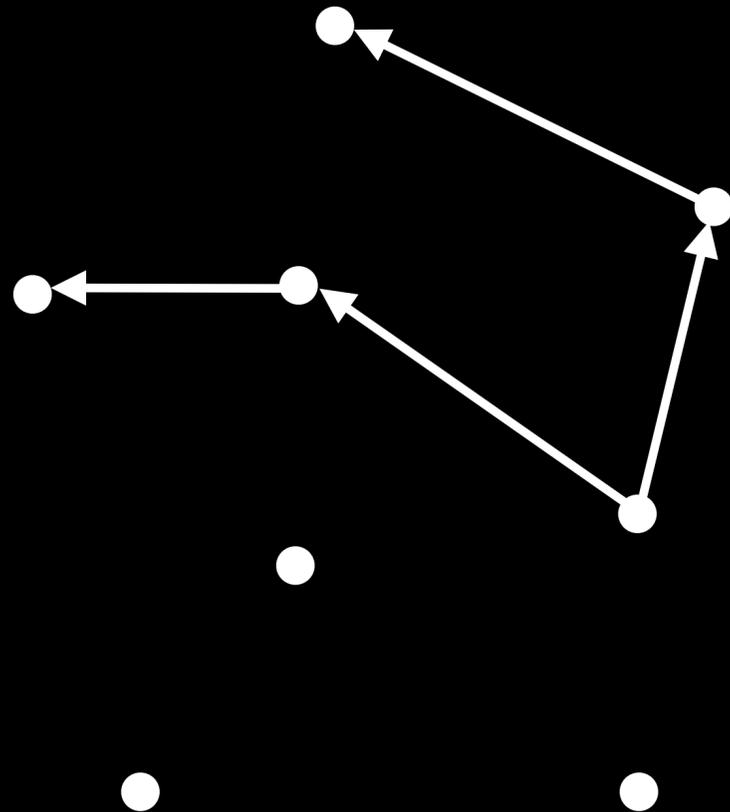


Choosing to close triangles

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Our data

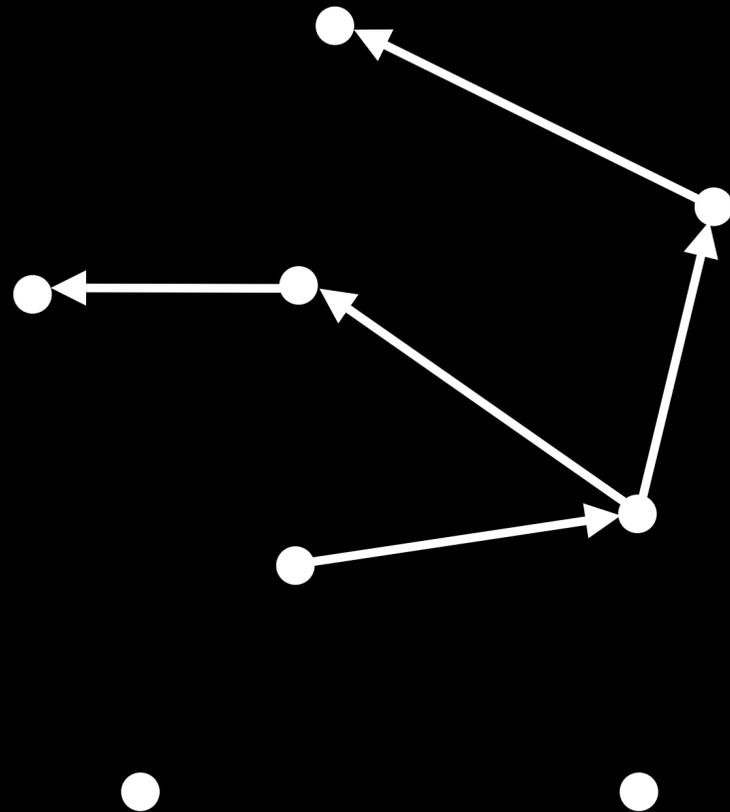
Timestamped edges
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Timestamped edges
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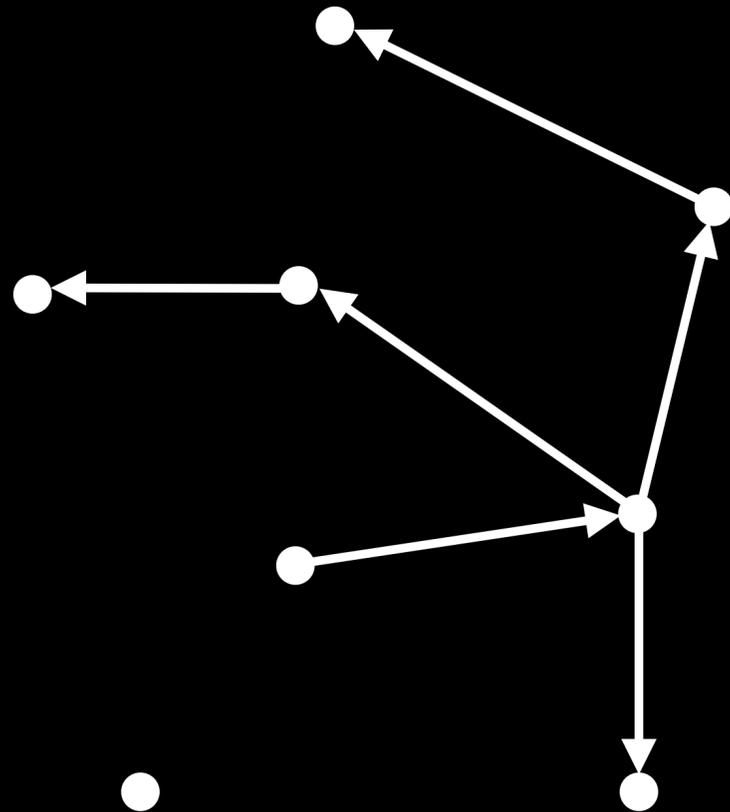


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Timestamped edges
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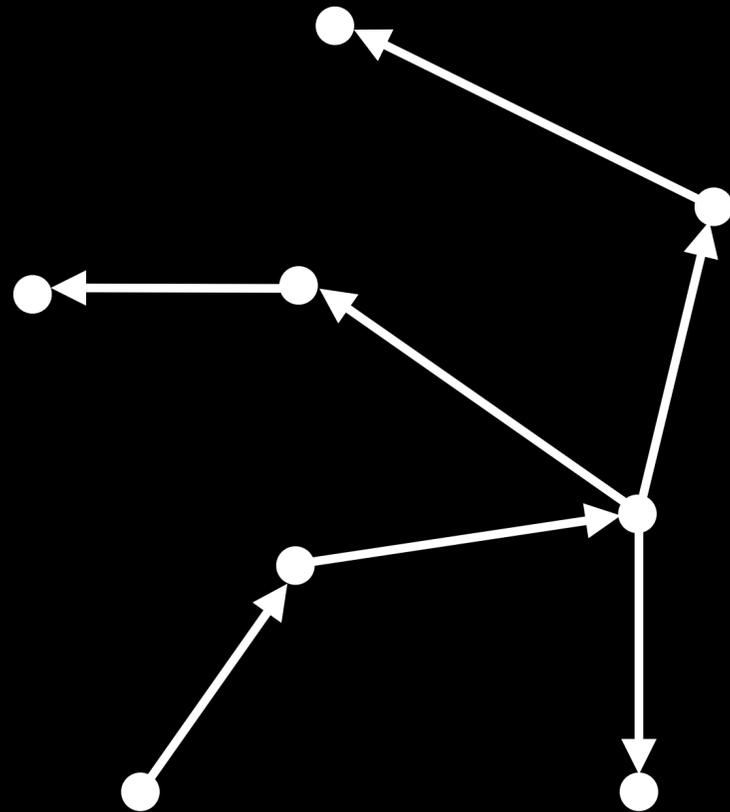


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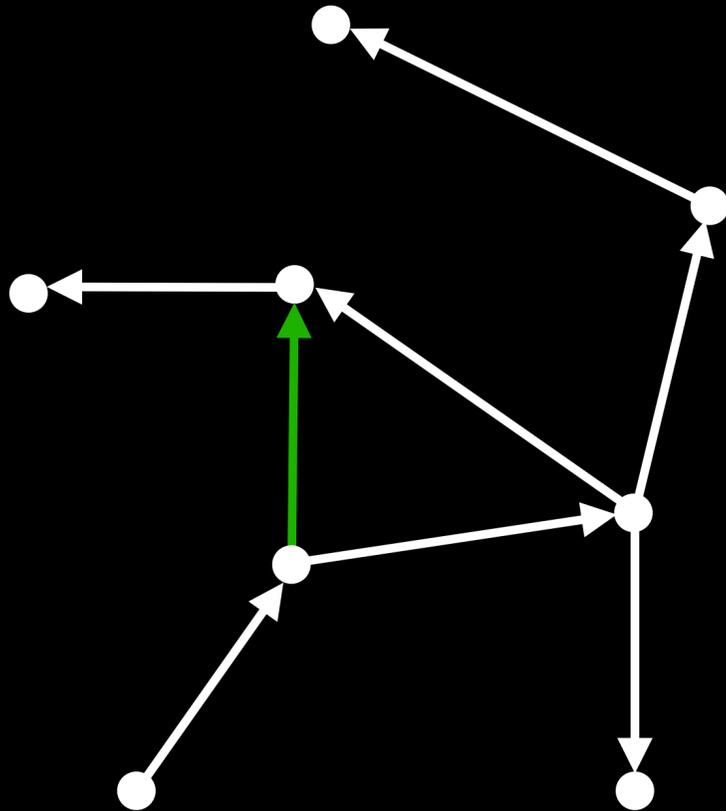


Choosing to close triangles

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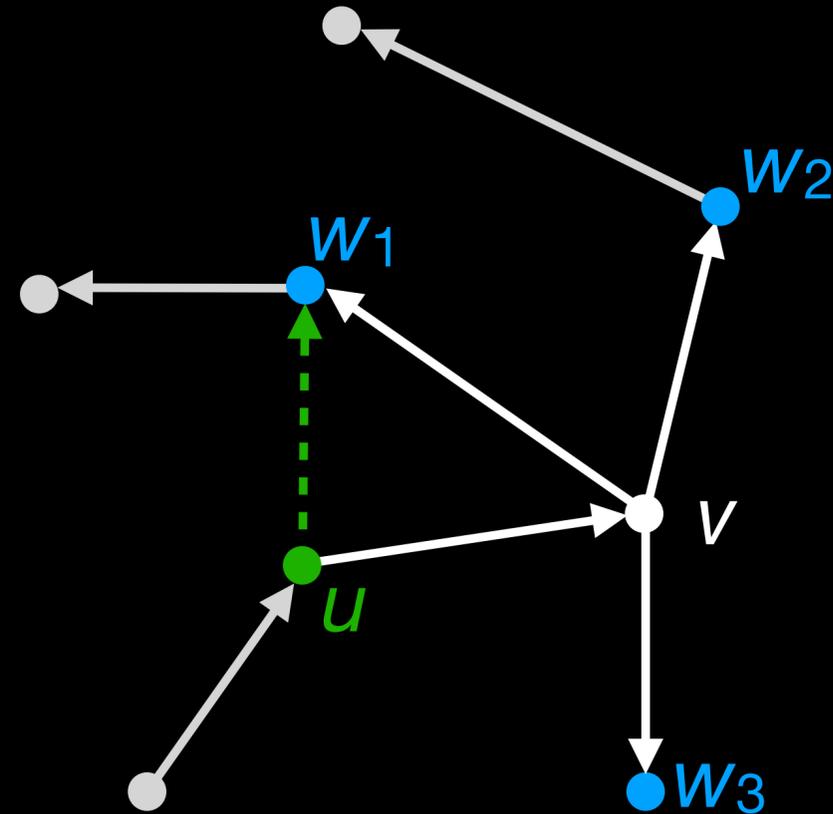
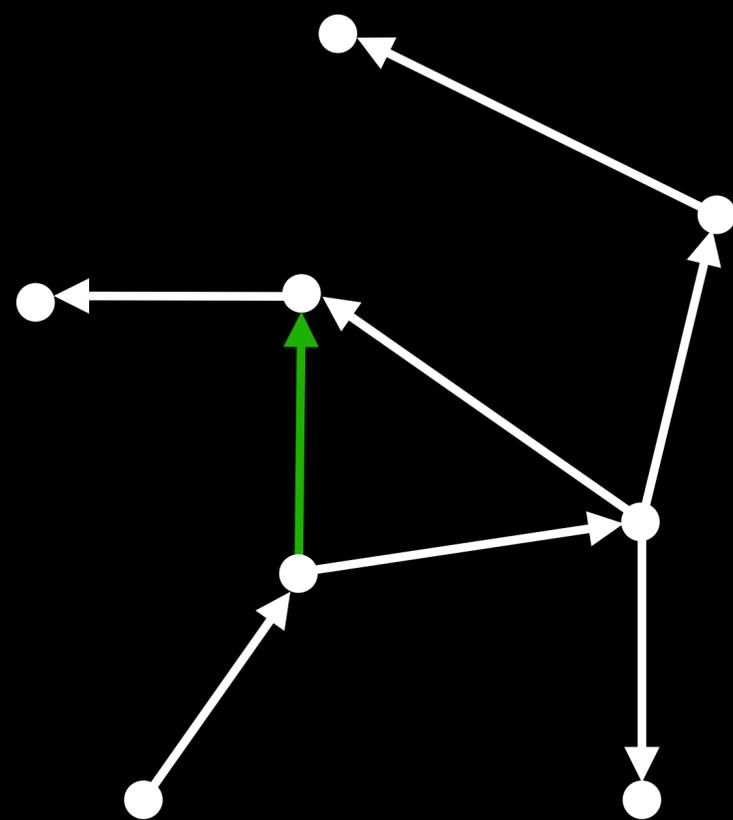
Our data

Timestamped edges
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Choosing to close triangles

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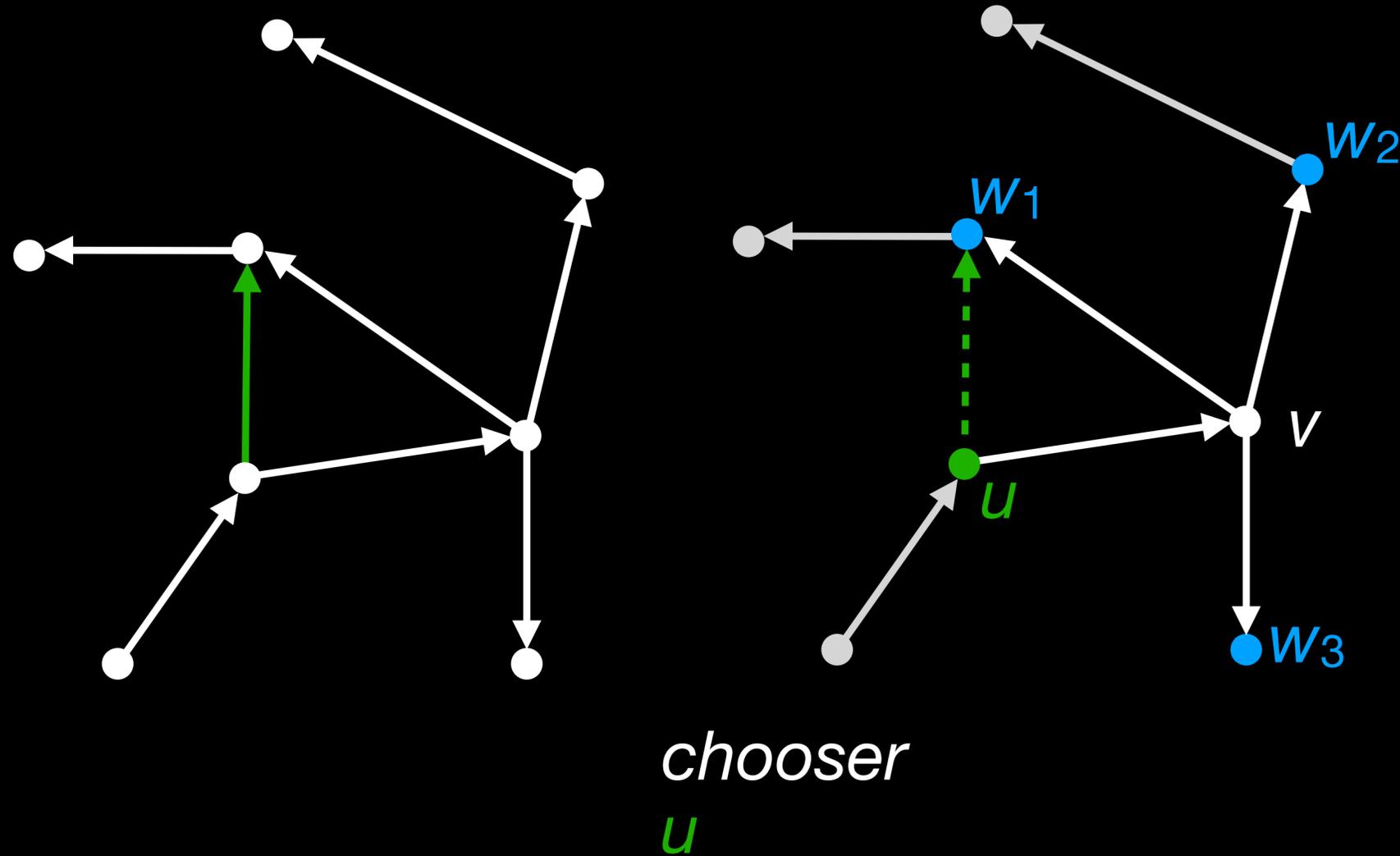
Our data

Timestamped edges
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Choosing to close triangles

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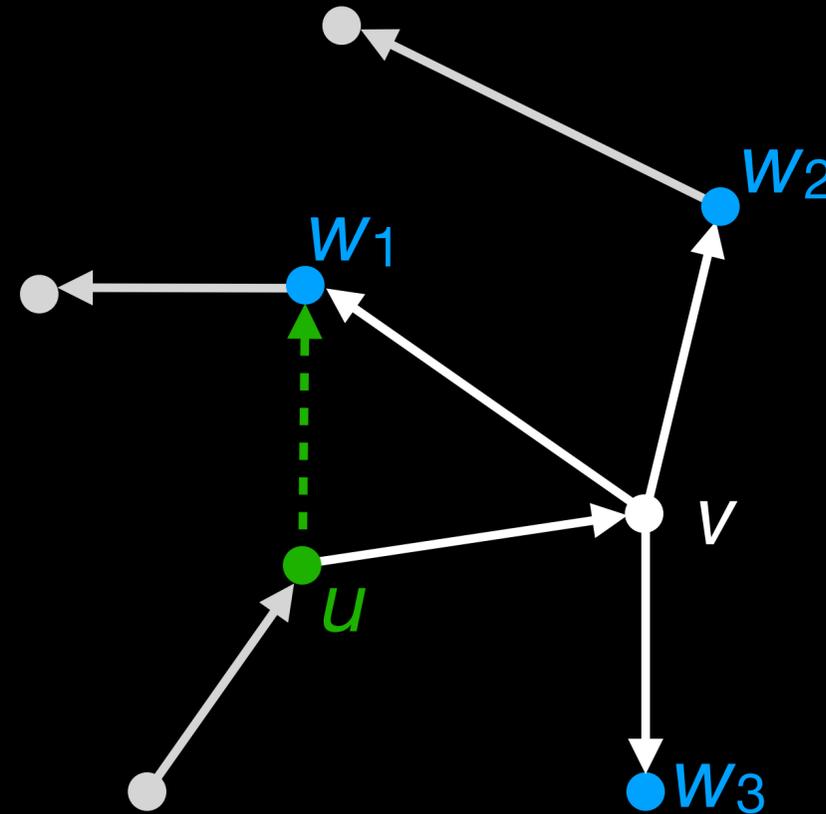
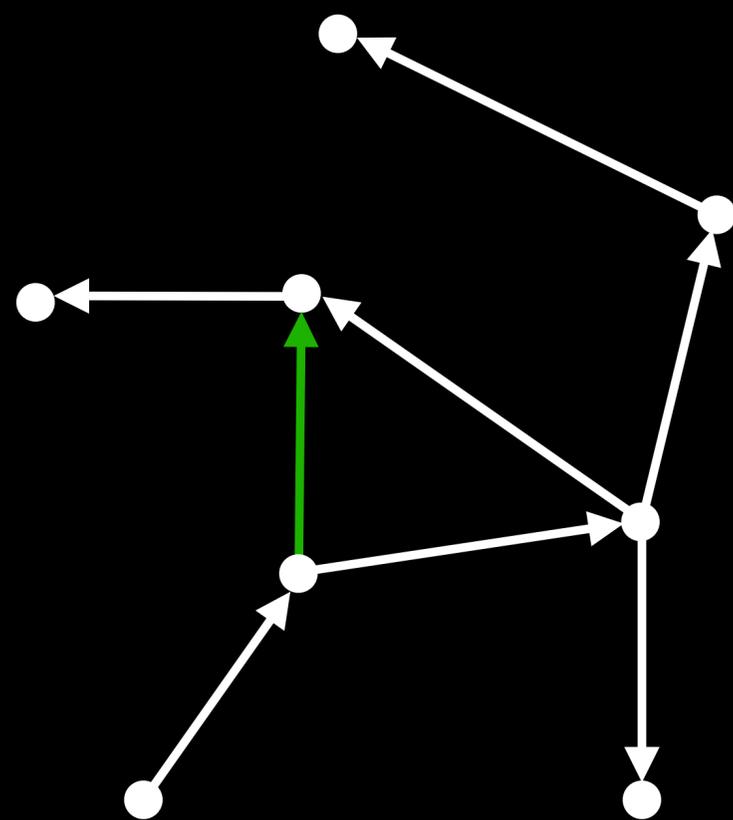
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Timestamped edges
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Our data
Timestamped edges
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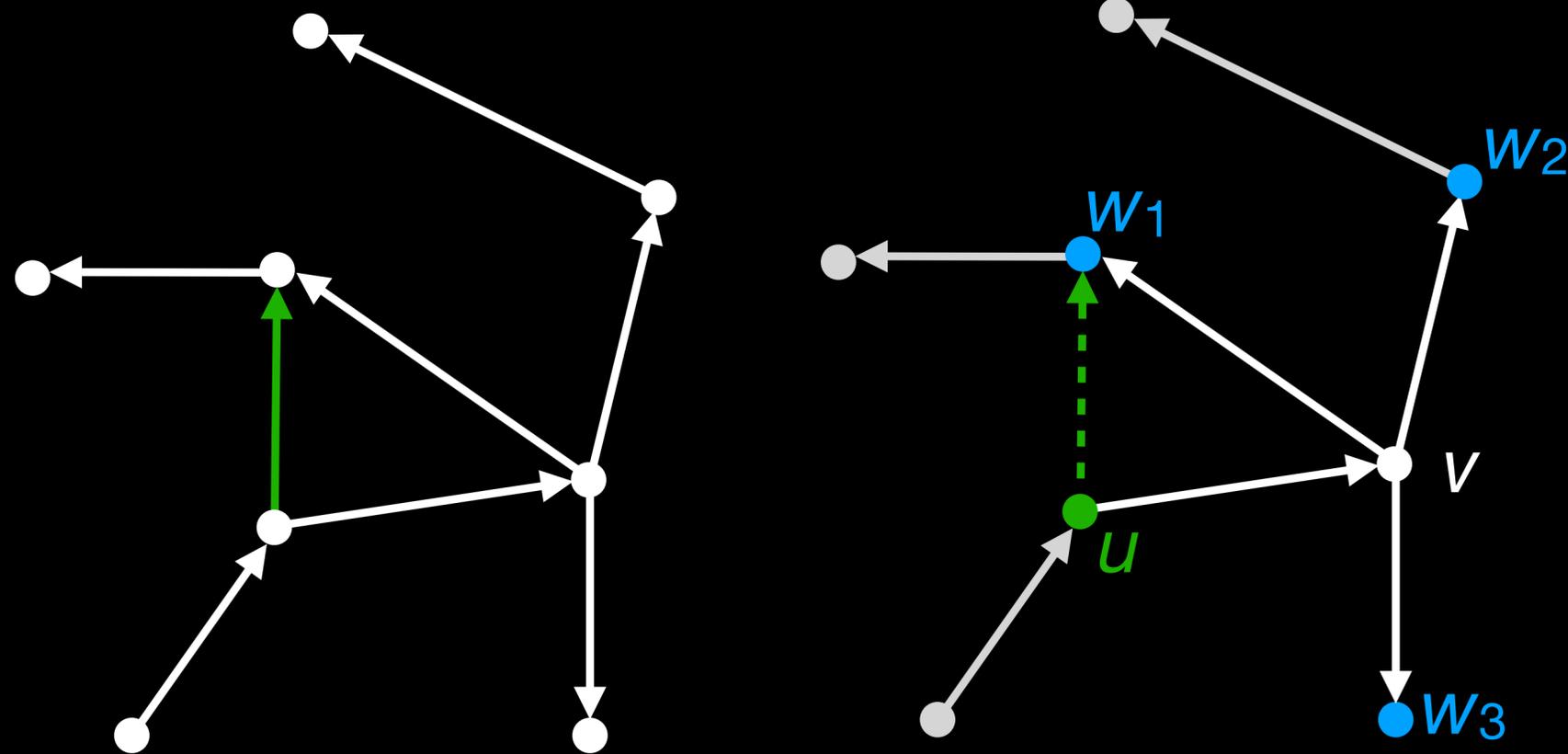
<i>chooser</i>	<i>choice set</i>	<i>choice</i>
u	$\{W_1, W_2, W_3\}$	W_1

Choosing to close triangles

Triadic closure offers small choice sets

→ tractable inference

→ varied choice sets



chooser

u

choice set

$\{w_1, w_2, w_3\}$

choice

w₁

Our data

Timestamped edges
(including repeats)

Node features

1. in-degree of w
2. # shared neighbors of u, w
3. weight of edge $w \rightarrow u$
4. time since last edge into w
5. time since last edge out of w
6. time since last $w \rightarrow u$ edge

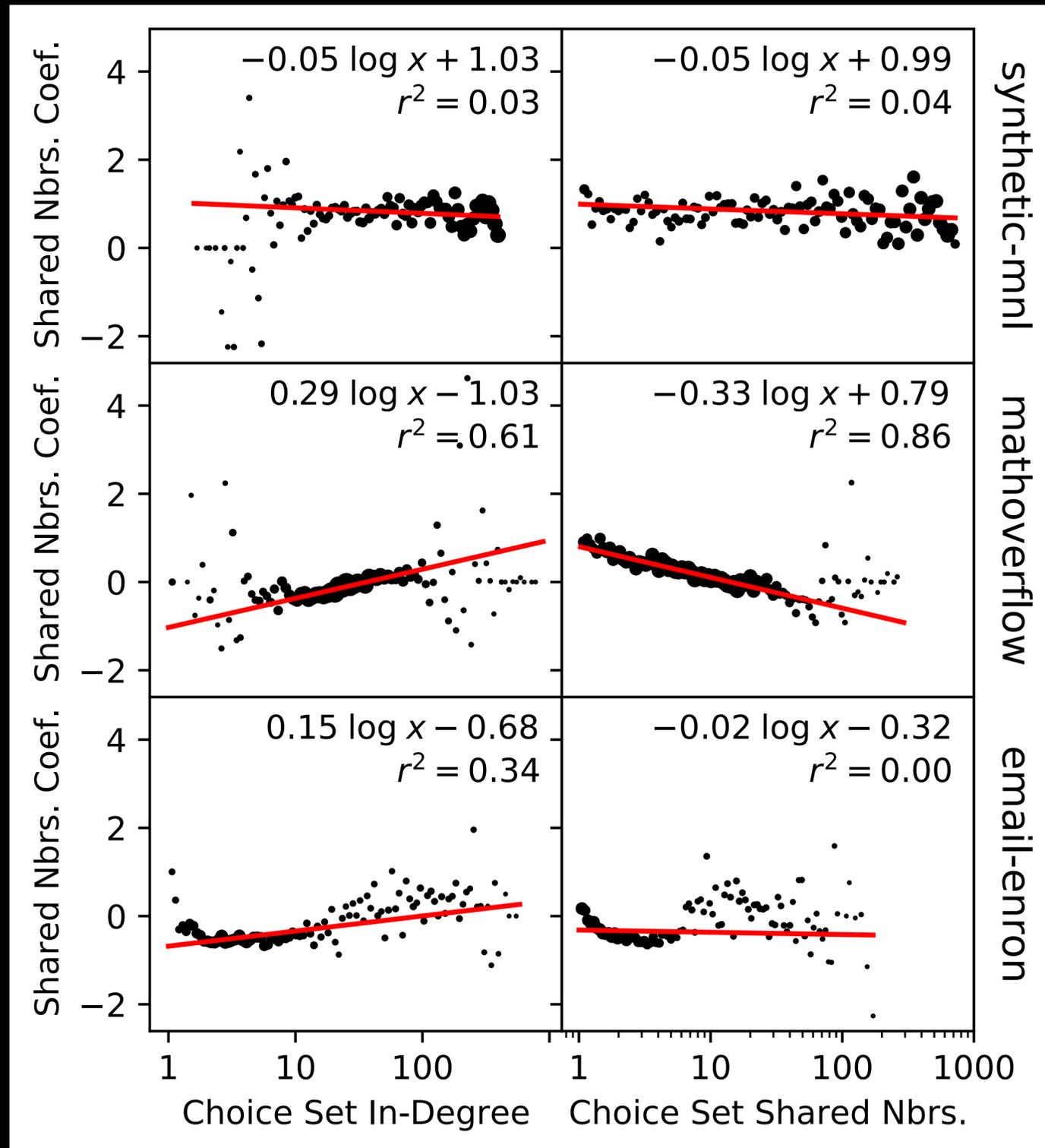
Context matters in triadic closure

Context matters in triadic closure

Datasets

email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

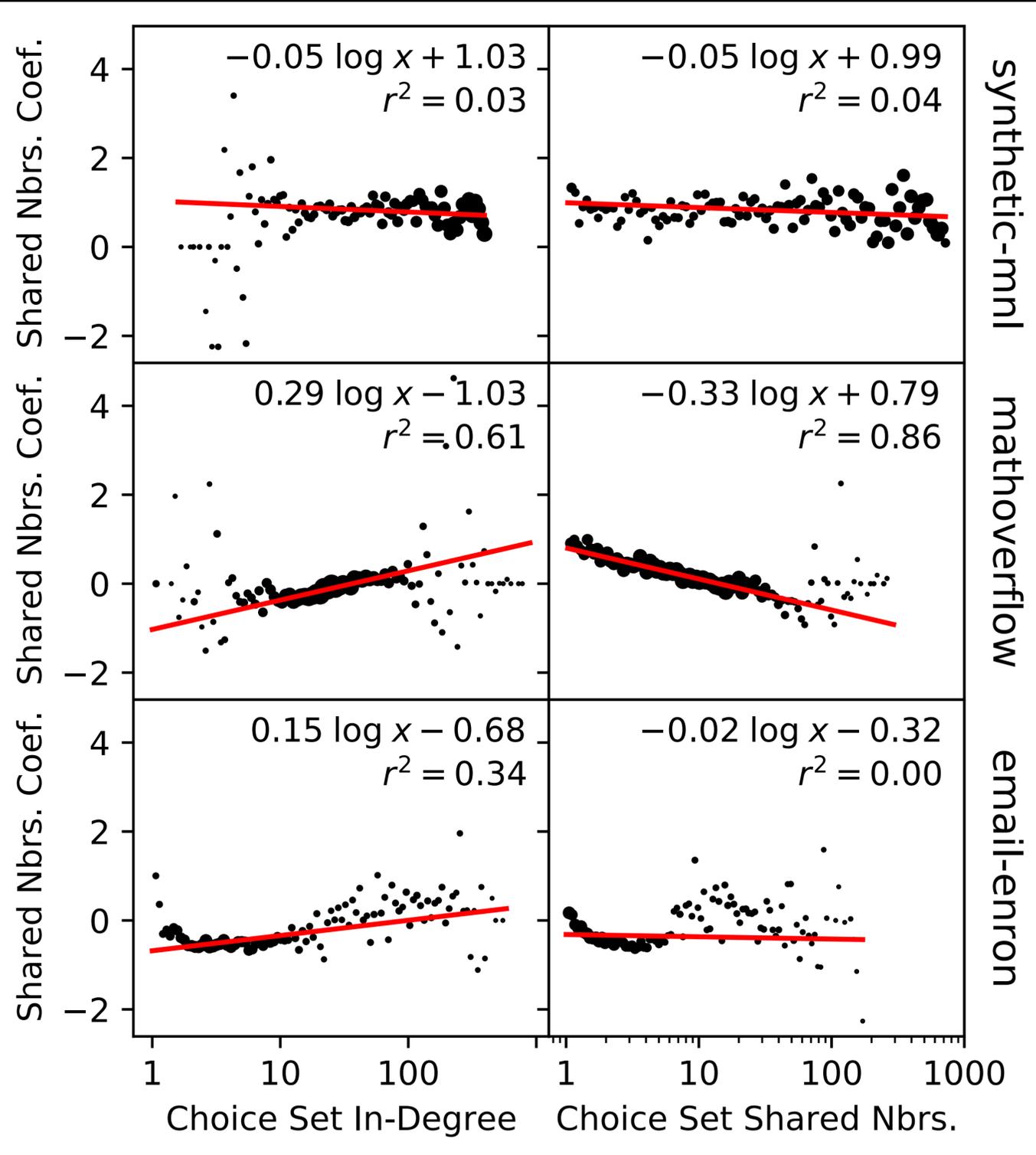
Context matters in triadic closure



Datasets
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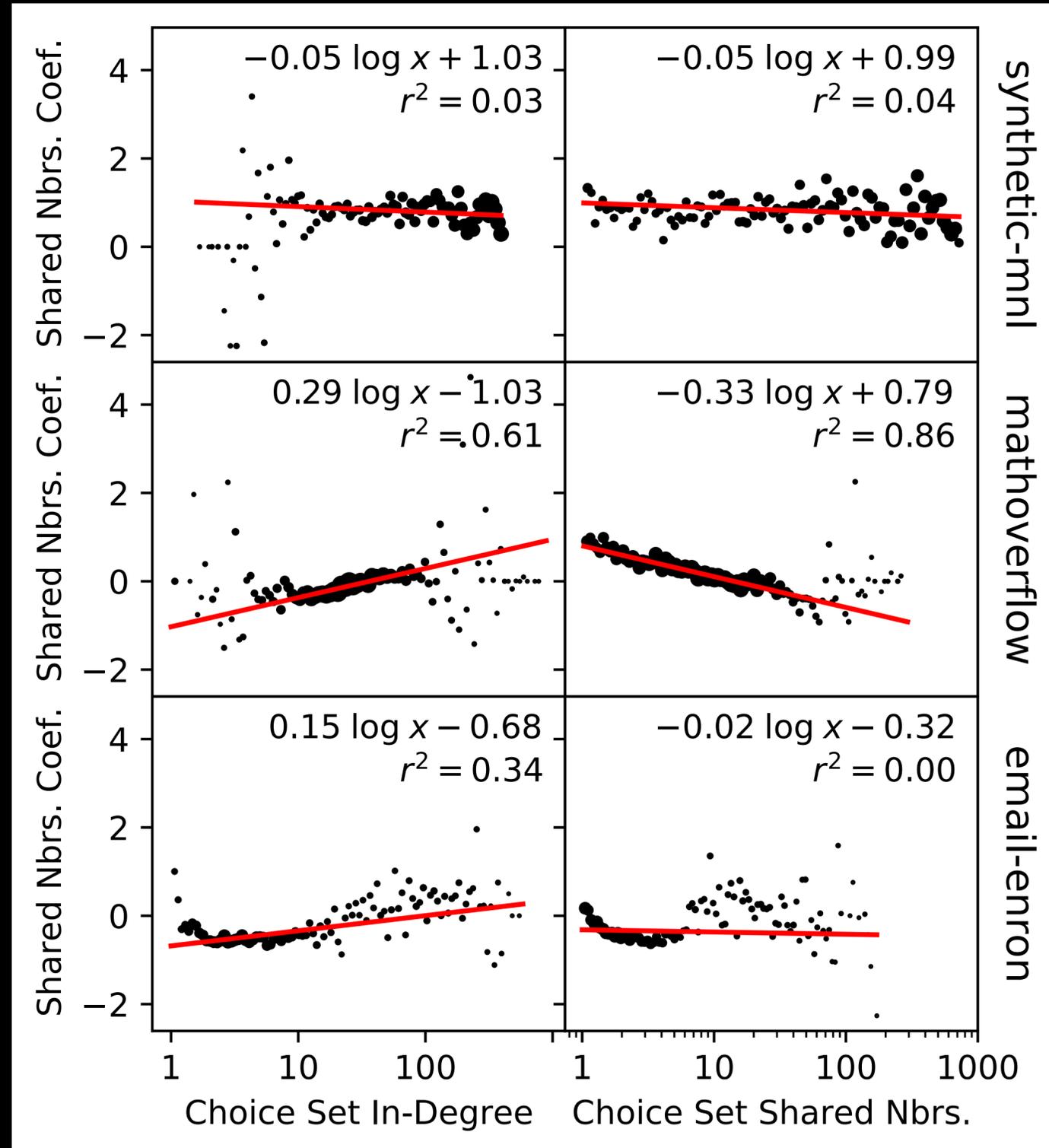
Context matters in triadic closure

Synthetic data,
no context effects



- Datasets**
- email-enron
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Context matters in triadic closure

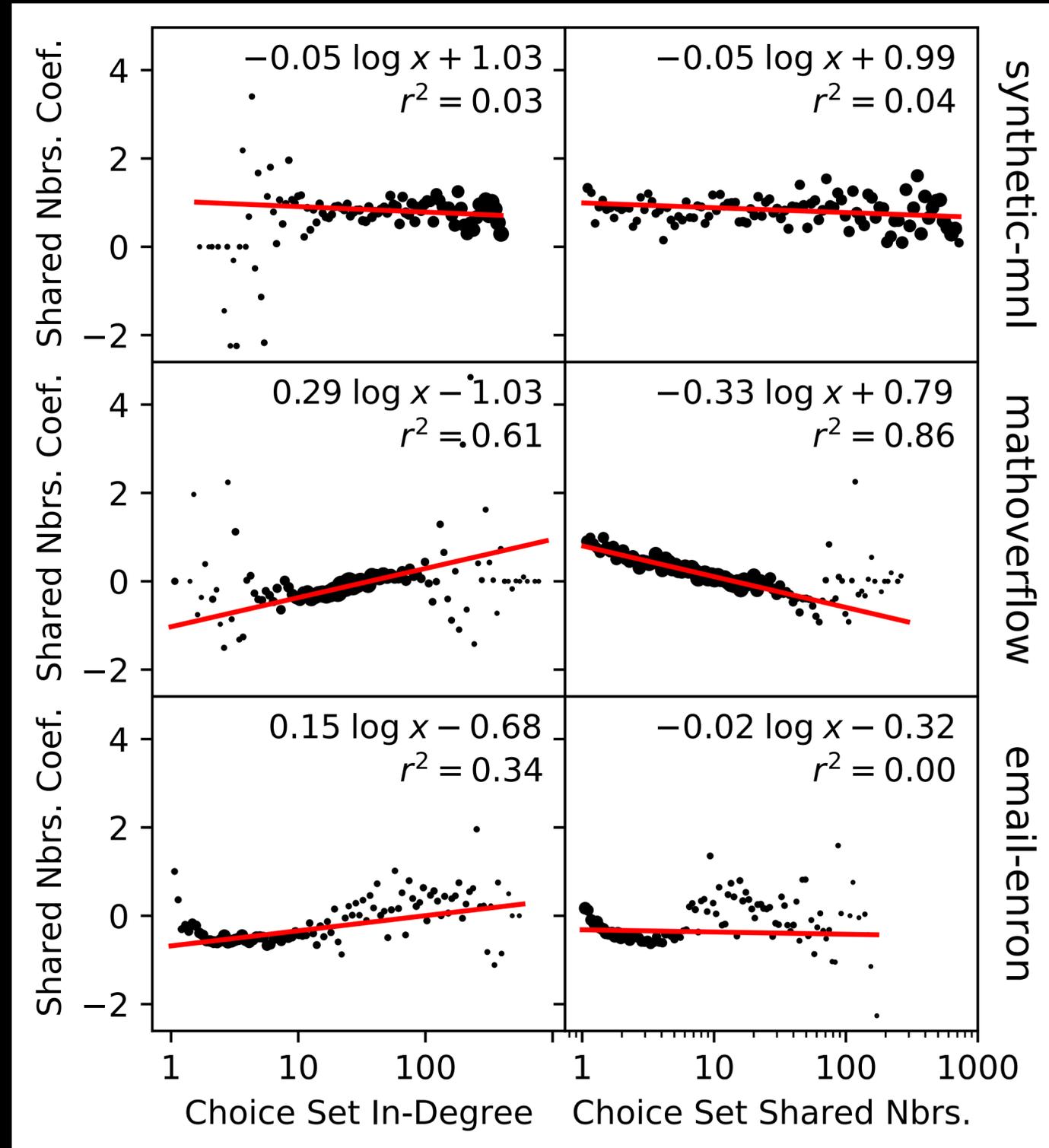


Synthetic data,
no context effects

Commenting network,
linear context effects

Datasets
email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

Context matters in triadic closure



Synthetic data,
no context effects

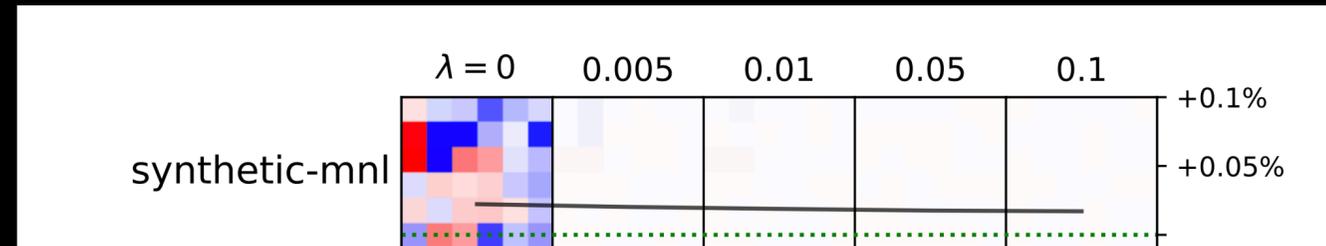
Commenting network,
linear context effects

Email network,
nonlinear context effects?

Datasets
 email-enron
 email-eu
 email-w3c
 wiki-talk
 reddit-hyperlink
 bitcoin-alpha
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 mathoverflow
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 sms-b
 sms-c

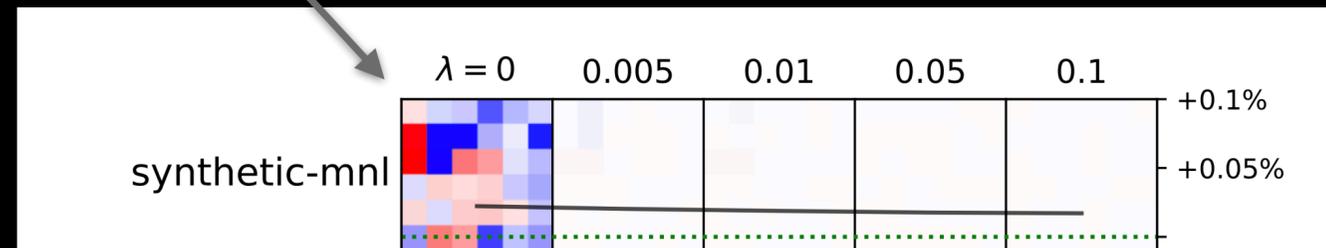
LCL reveals interpretable feature context effects

LCL reveals interpretable feature context effects



LCL reveals interpretable feature context effects

context effect matrix A
red: +, blue: -, white: 0
(column acts on row)



Node features

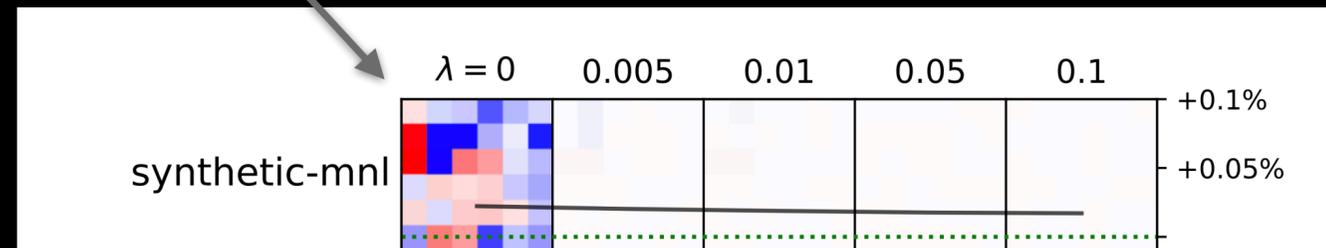
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

LCL reveals interpretable feature context effects

context effect matrix A
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increasing L_1 regularization on A



Node features

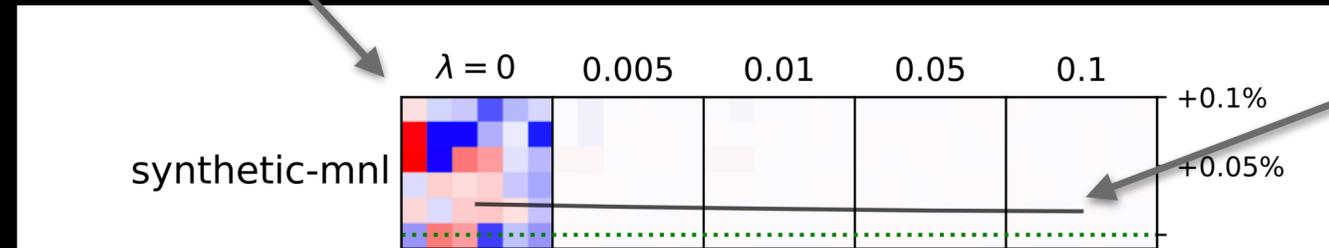
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NLL (lower = better fit)

Node features

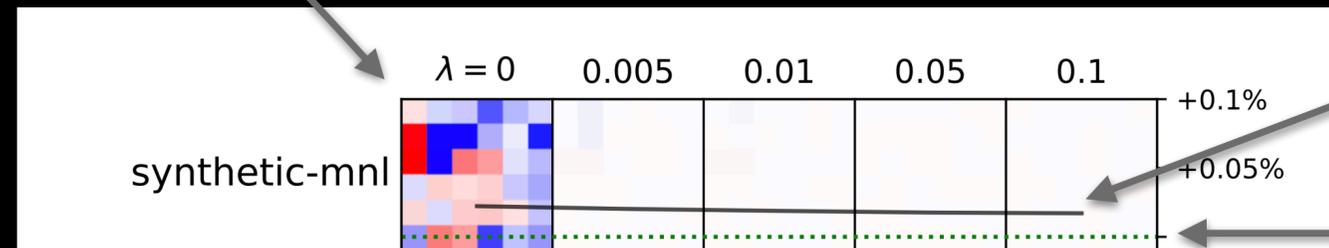
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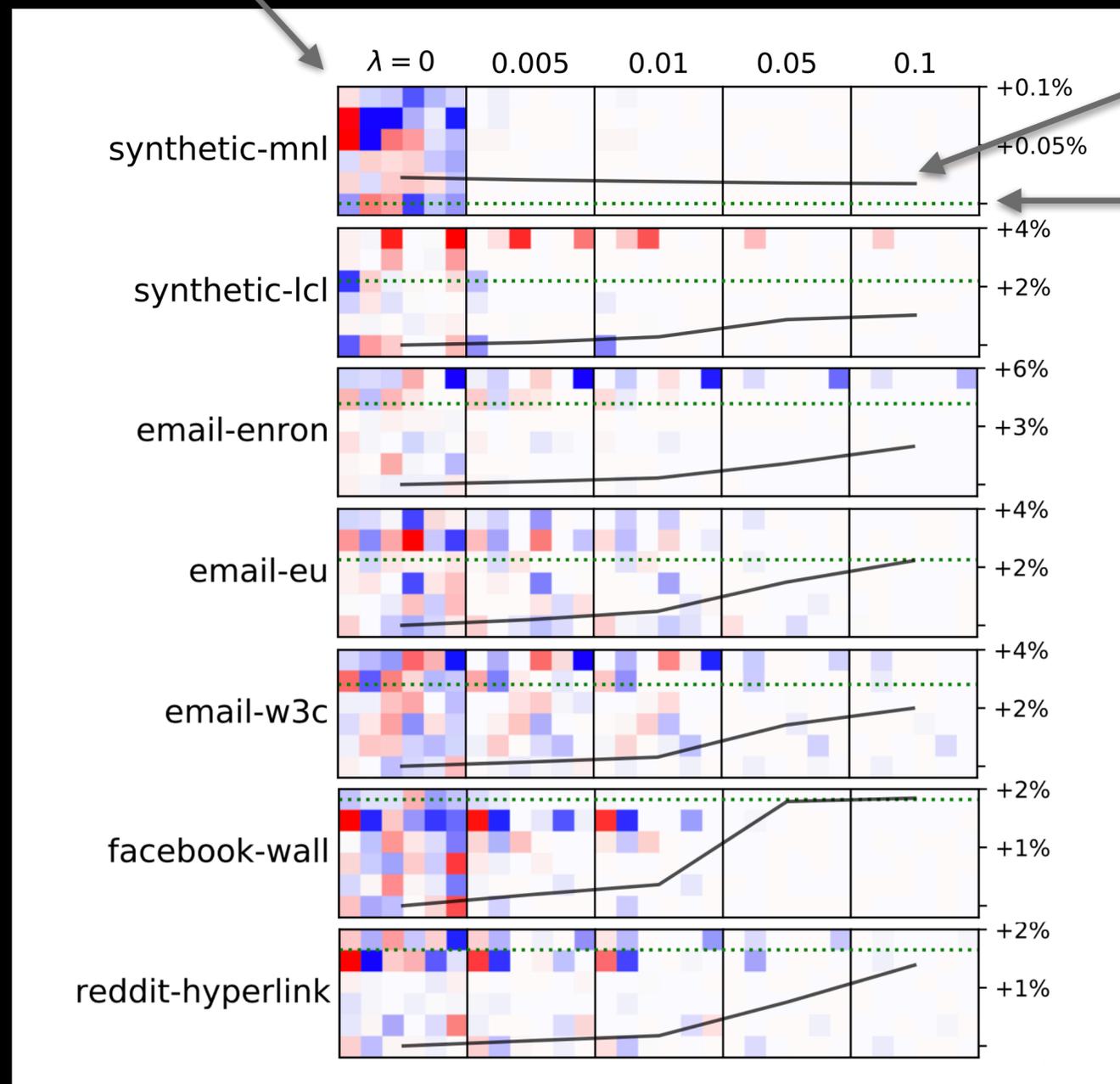
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NLL (lower = better fit)

$p = 0.001$ LRT threshold

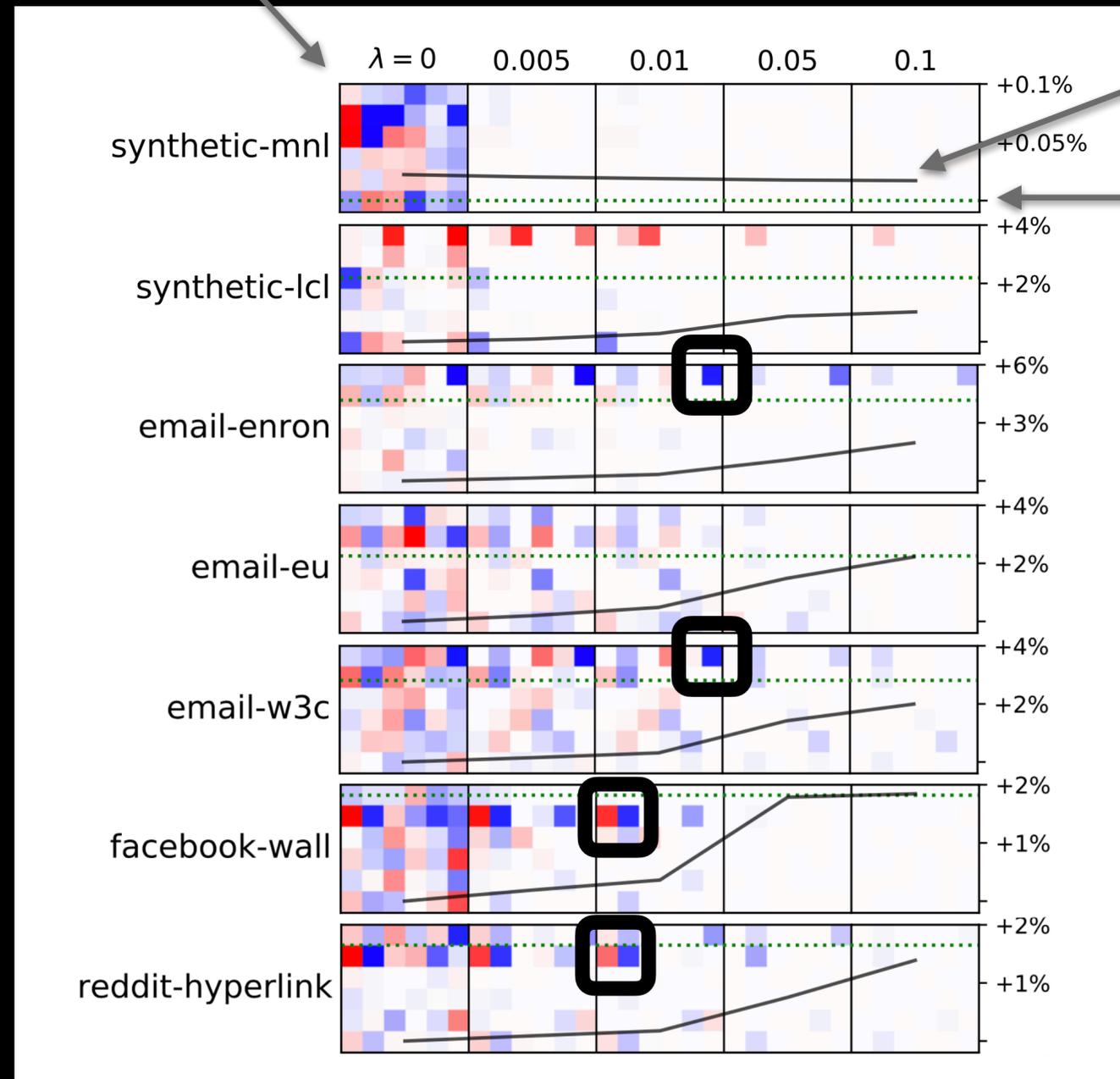
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increasing L_1 regularization on A

Node features (left-right, top-bottom)

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2. shared neighbors
3. reciprocal weight
4. send recency
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6. reciprocal recency



NLL (lower = better fit)

$p = 0.001$ LRT threshold

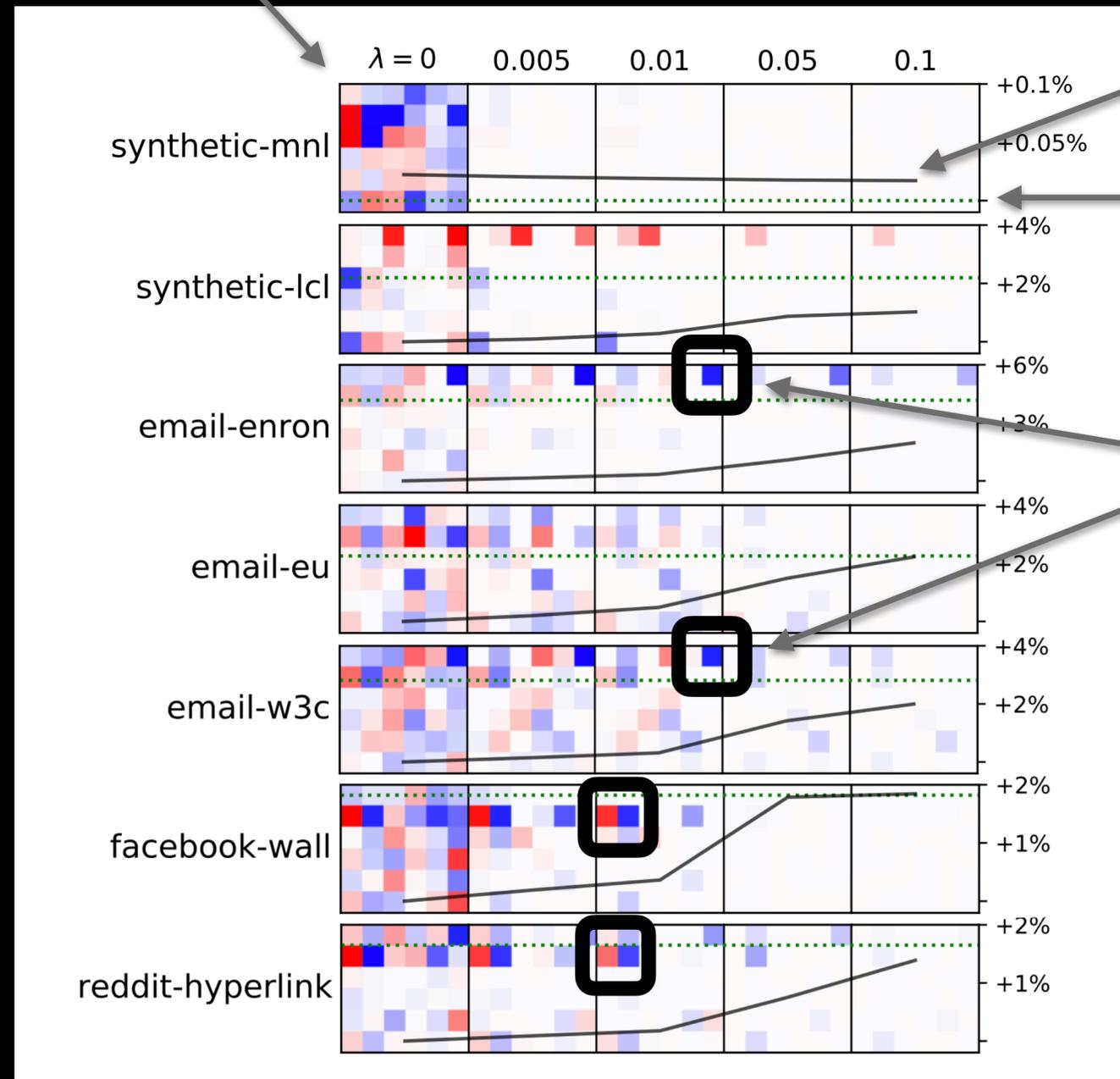
LCL reveals interpretable feature context effects

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 (column acts on row)

increasing L_1 regularization on A

Node features (left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency



NLL (lower = better fit)

$p = 0.001$ LRT threshold

“cluttered inbox”
 high choice set reciprocal recency
 → in-degree less important

LCL reveals interpretable feature context effects

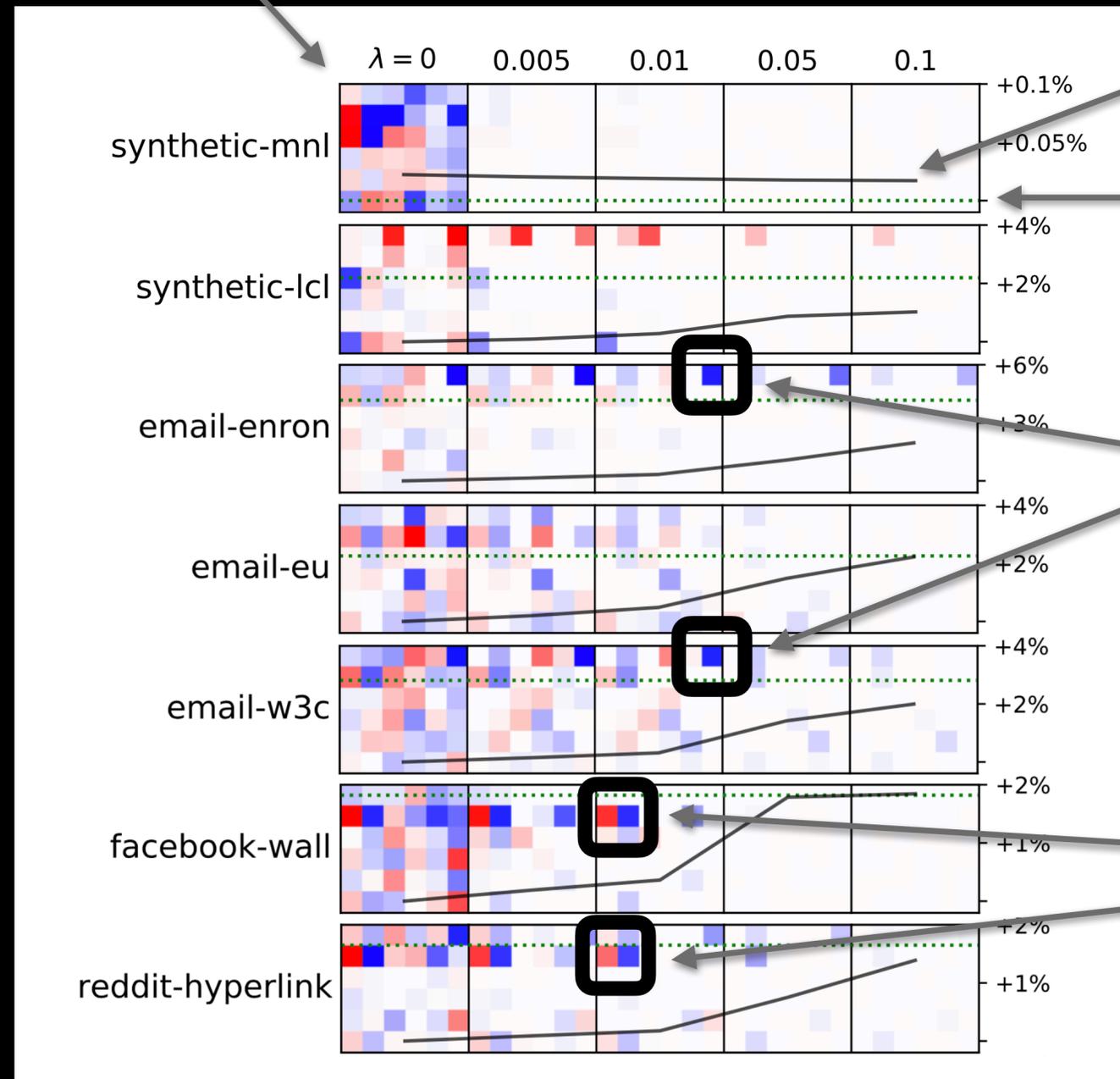
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6. reciprocal recency



NLL (lower = better fit)

$p = 0.001$ LRT threshold

“cluttered inbox”

high choice set reciprocal recency
 → in-degree less important

red: “cocktail party introduction”

high choice set in-degree
 → shared neighbors more important

blue: “familiarity saturation”

high choice set shared neighbors
 → shared neighbors less important

Concluding thoughts

Code: bit.ly/lcl-code
Data: bit.ly/lcl-data
Slides: bit.ly/lcl-kdd-slides

Key takeaways

Feature context effects extend item-level effects
LCL offers an interpretable and tractable way to reveal them

Future work

Non-linear context effects
Negative sampling
Discovering relational effects

Causal context effects?

See our other KDD '21 paper:
“Choice Set Confounding in Discrete Choice”

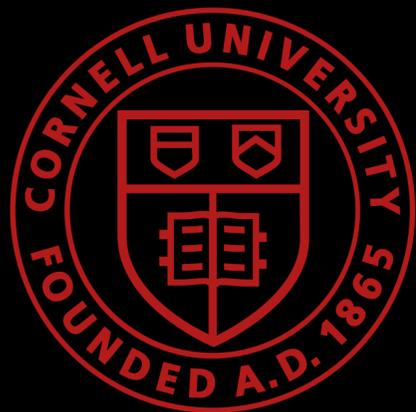
Submit to our NeurIPS '21 workshop!

bit.ly/WHMD2021

Thank you!

More questions or ideas?
Email me: kt@cs.cornell.edu

 @kiran_tomlinson



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