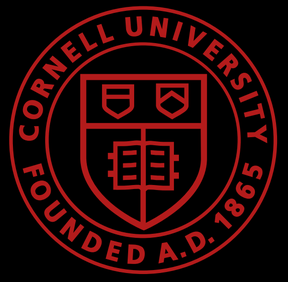

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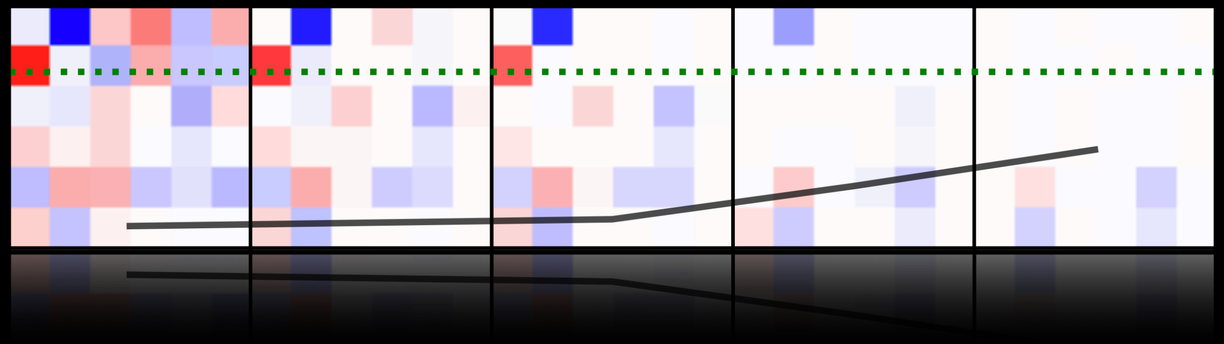
Preprint: bit.ly/lcl-paper
Code: bit.ly/lcl-code
Data: bit.ly/lcl-data



Learning Interpretable Feature Context Effects in Discrete Choice

Kiran Tomlinson

@ Stanford SOAL

research with Austin R. Benson

1. Choices and context effects

Discrete choices are everywhere

amazon.com

Tervis Stanford Cardinal Logo Tumbler with Emblem and Black Lid 16oz Mug, Clear

★★★★★ 571

$16.99

✓prime FREE Delivery **Sun, Nov 22**

+4

Champion NCAA Men's Eco Powerblend Pocketless 1/4 Zip

★★★★☆ 18

$53.00

FREE Delivery for Prime members

Champion NCAA Men's Shirt Short Sleeve Officially Licensed Team Color Tee

★★★★½ 1,486

$18.80

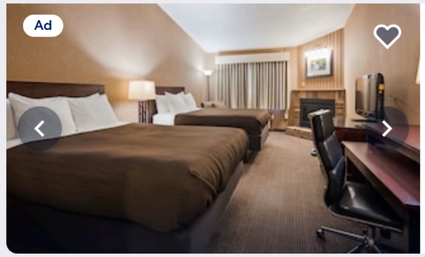
✓prime

Top of the World NCAA Mens Adjustable Vintage Team Icon hat

★★★★★ 7

$19.99

✓prime



Best Western University Inn

Ithaca

Expedia

Black Friday / Cyber Monday Deals Now

Free Shuttle Transportation, Grab & Go Breakfast, WiFi & Parking. Pet friendly, Outdoor Pool, Fitness Center. Sanitizing Daily

Breakfast included

3.9/5 Good (999 reviews)

$63

per night

$71 total

Includes taxes & fees

Quality Inn Ithaca - University Area

Ithaca

Black Friday / Cyber Monday Deals Now

Complimentary Breakfast. Free Airport Shuttle, WiFi & parking. Close to Ithaca College & Cornell University. Pets welcome.

Breakfast included

3.6/5 Good (694 reviews)

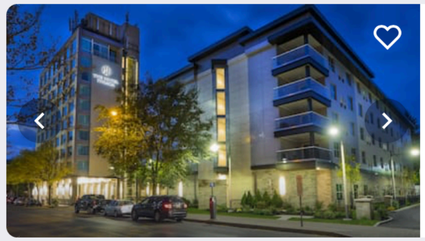
Member Price available

$59

per night

$66 total

Includes taxes & fees



Hotel Ithaca

Ithaca

Member Price available

$94

per night

$106 total

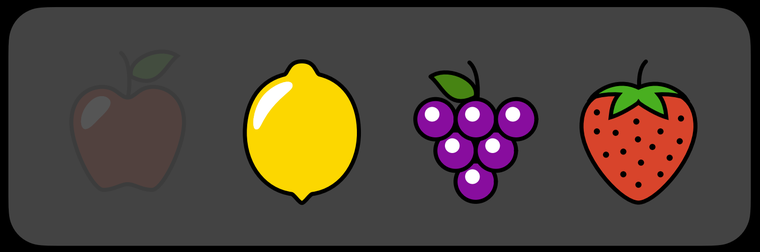
Includes taxes & fees

4.0/5 Very Good (842 reviews)

“The fundamental problem of discrete choice”

“The fundamental problem of discrete choice”

choice set



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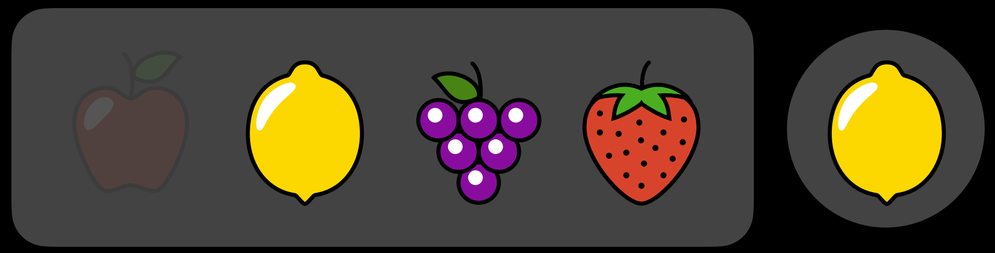

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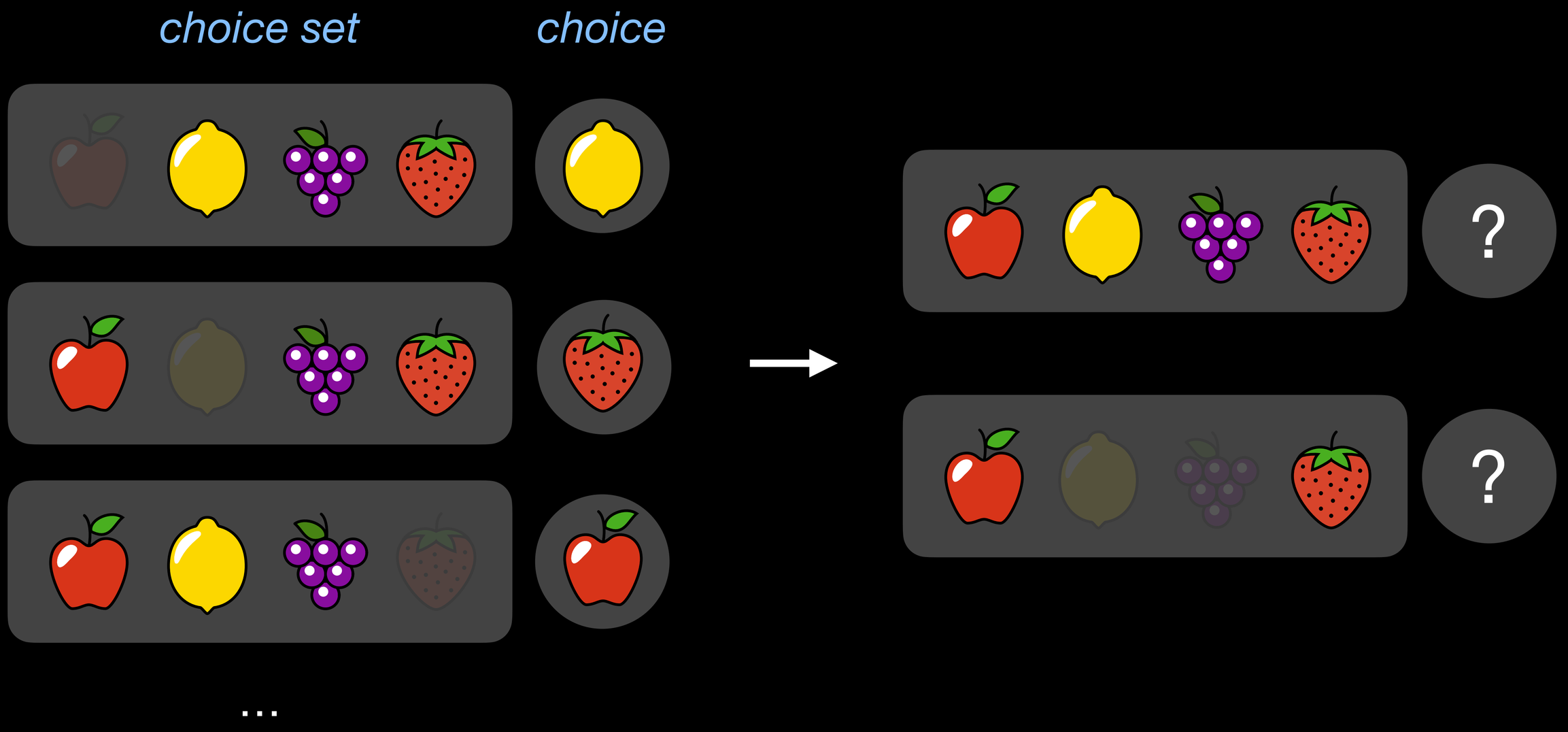

The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

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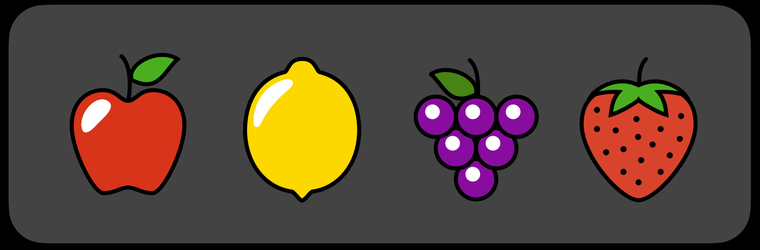

u_i	1	-1	0	2
$\Pr(i \mid C)$	.24	.03	.09	.64

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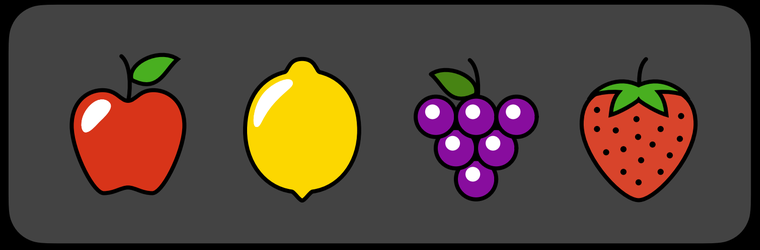

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Econometric derivation:

- Draw random utilities $u_i + \epsilon_i$ for each item (ϵ_i i.i.d. Gumbel)
- Rational agent picks $\text{argmax}_{i \in C}\, u_i + \epsilon_i$

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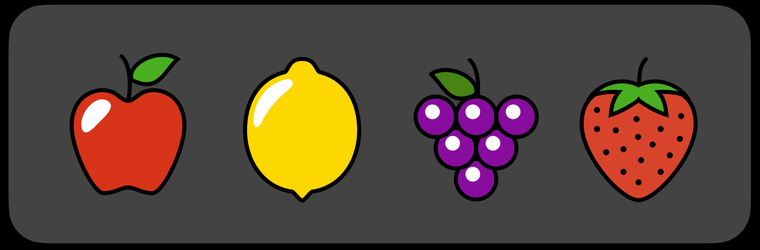

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→ MNL

Unique choice model satisfying
independence of irrelevant alternatives (IIA):

(Luce, *Individual Choice Behavior* 1959)

$$\frac{\Pr(i \mid C)}{\Pr(j \mid C)} = \frac{\Pr(i \mid C')}{\Pr(j \mid C')}$$

Learning an MNL from choice data

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Likelihood of utilities u given dataset $\mathscr{D}$: $\mathscr{L}(u;\mathscr{D}) = \prod_{(i,C)\in\mathscr{D}} \frac{\exp(u_i)}{\sum_{j\in C}\exp(u_j)}$

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→ gradient descent to learn u
(SGD, Adam, …)

Problem for MNL: *context effects*

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The choice set influences preferences.

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Compromise
(Simonson, 1989)

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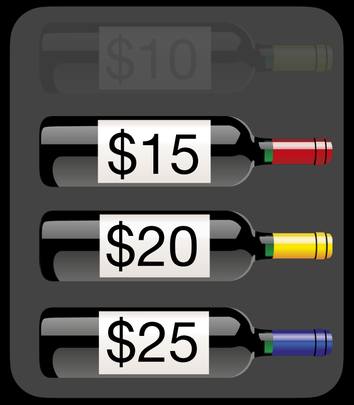

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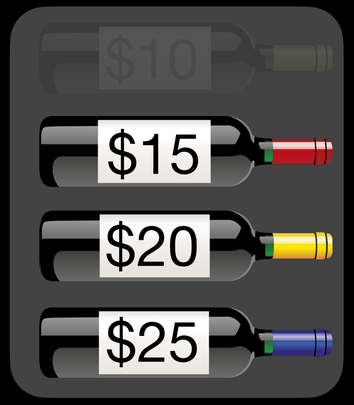

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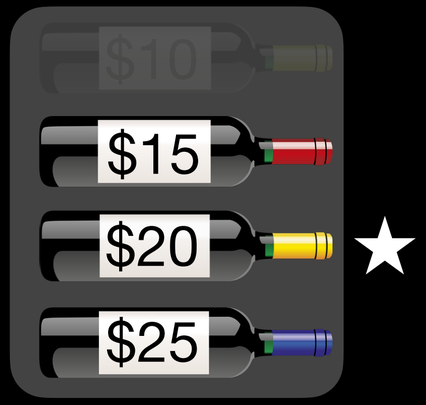

Asymmetric dominance

(Huber et al., 1982)

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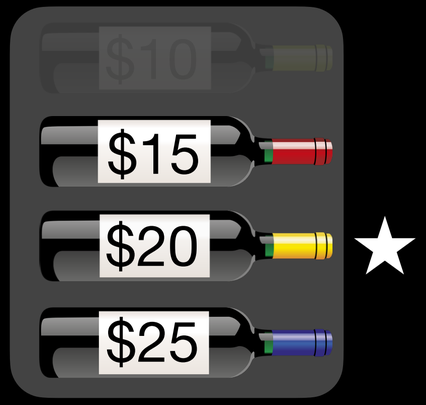

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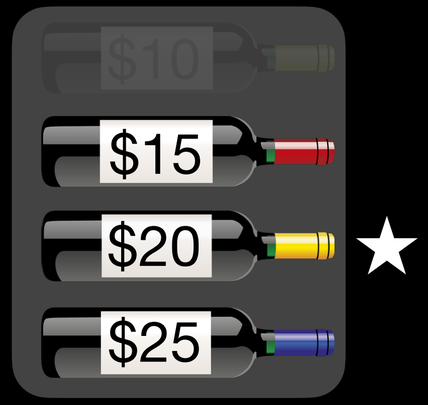

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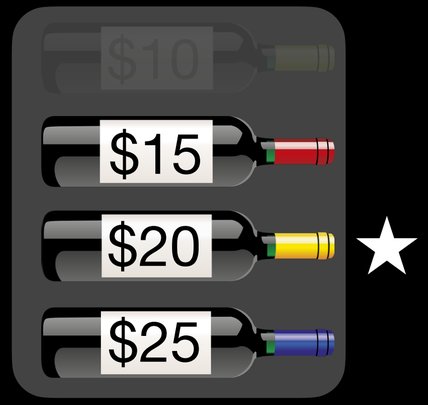

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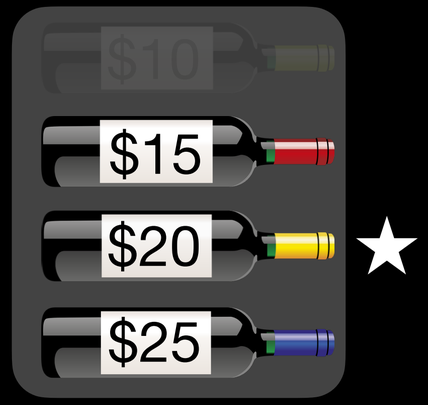

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David Perdue* ✔	Rep.	2,462,617	**49.7%**
Jon Ossoff ✔	Dem.	2,374,519	**47.9**

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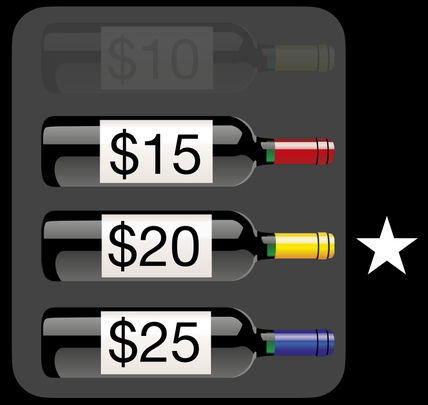

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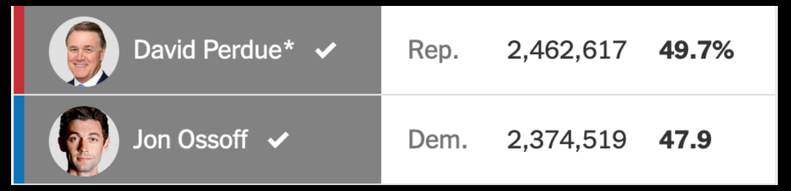

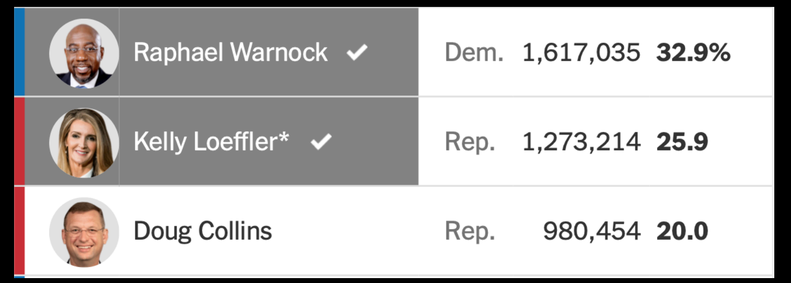

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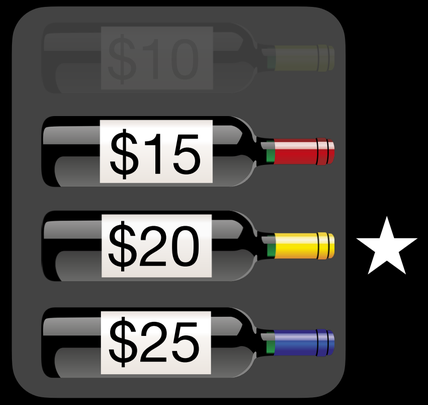

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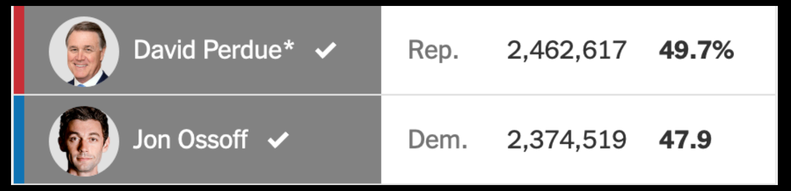

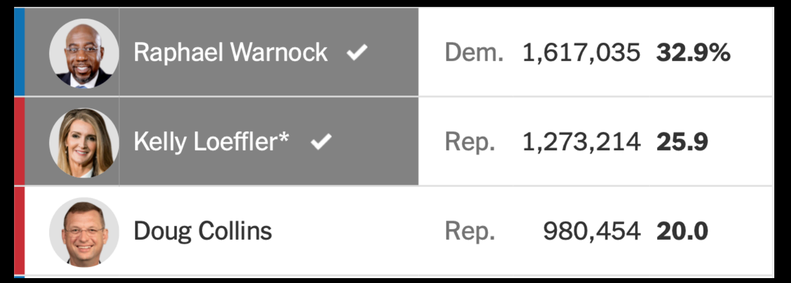

IIA violations:

$$\frac{\Pr(i \mid C)}{\Pr(j \mid C)} \neq \frac{\Pr(i \mid C')}{\Pr(j \mid C')}$$

Natural context effect model: *CDM*

(Seshadri, Peysakhovich, & Ugander, ICML 2019)

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Item j exerts *pull* u_{ij} on item i, item utility is sum of pulls:

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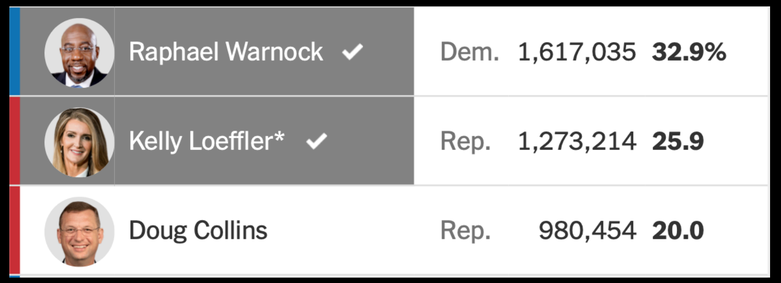

$$u_{\text{Loeffler, Collins}} < 0$$
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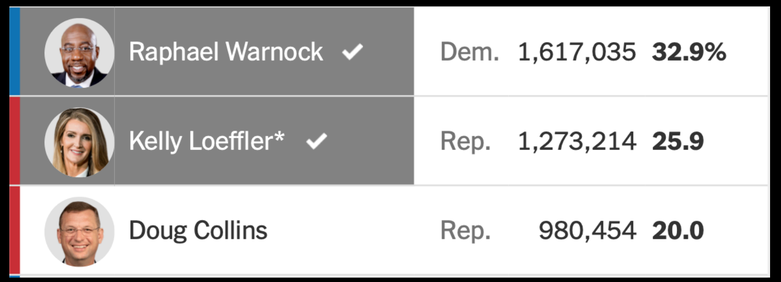

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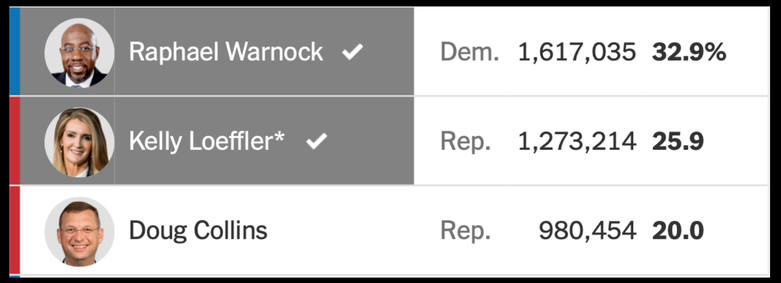

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Assumes no higher-order effects
(i.e., context effects decompose additively into effects of items)

2. Item features and the LCL

Choice models with *item features*

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So far, models have per-item parameters

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→ can’t generalize to new items not in training set

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Use item features:

genre: drama,
in_top_10: True,
has_new_episodes: True,
producer: Netflix

genre: comedy,
in_top_10: False,
has_new_episodes: False,
producer: NBC

genre: drama,
in_top_10: True,
has_new_episodes: False,
producer: Netflix

genre: reality,
in_top_10: True,
has_new_episodes: False,
producer: Banijay

MNL with item features: *conditional logit*

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Feature vector $x_i \in \mathbb{R}^d$ for each item i

Preference vector $\theta \in \mathbb{R}^d$

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Conditional logit:

$$\Pr(i \mid C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

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Conditional logit:

$$\Pr(i \mid C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

Preference coefficient θ_k is easy to interpret: importance of the k^{th} feature

Incorporating *feature context effects* into conditional logit

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Conditional logit utility: $u_i = \theta^T x_i$

Incorporating *feature context effects* into conditional logit

Conditional logit utility: $u_i = \theta^T x_i$ → *Contextual utility*: $u_{i,C} = [\theta + F(C)]^T x_i$

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Simplifying assumptions on $F(C)$:

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1. *Additivity*: $F(C) \propto \sum_{j \in C} f(x_j)$ for some function f

Incorporating *feature context effects* into conditional logit

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Simplifying assumptions on $F(C)$:

1. *Additivity*: $F(C) \propto \sum_{j \in C} f(x_j)$ for some function f

2. *Dilution*: $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$

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Simplifying assumptions on $F(C)$:

1. *Additivity*: $F(C) \propto \sum_{j \in C} f(x_j)$ for some function f
2. *Dilution*: $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$
3. *Linearity*: $f(x_j) = Ax_j$ for some matrix $A \in \mathbb{R}^{d \times d}$

Incorporating *feature context effects* into conditional logit

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Simplifying assumptions on $F(C)$:

1. *Additivity*: $F(C) \propto \sum_{j \in C} f(x_j)$ for some function f
2. *Dilution*: $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$
3. *Linearity*: $f(x_j) = Ax_j$ for some matrix $A \in \mathbb{R}^{d \times d}$

→ $u_{i,C} = (\theta + Ax_C)^T x_i$ ($x_C = \frac{1}{|C|} \sum_{j \in C} x_j$ is the *mean feature vector*)

The *Linear Context Logit (LCL)*

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$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

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LCL example: restaurant selection

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Icons by icons8.com

LCL example: restaurant selection

item features:
- price
- service speed
- wine selection

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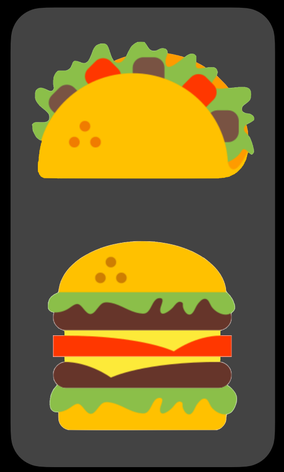
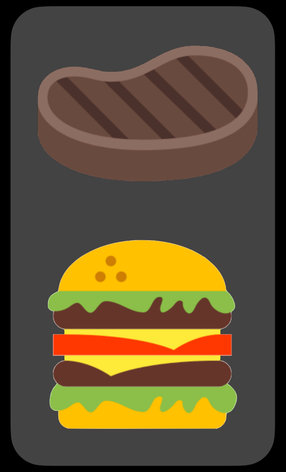
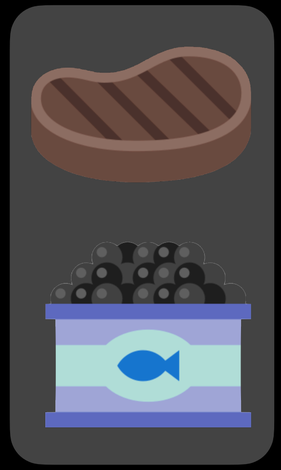


C_1



C_2



C_3

Icons by icons8.com

LCL example: restaurant selection

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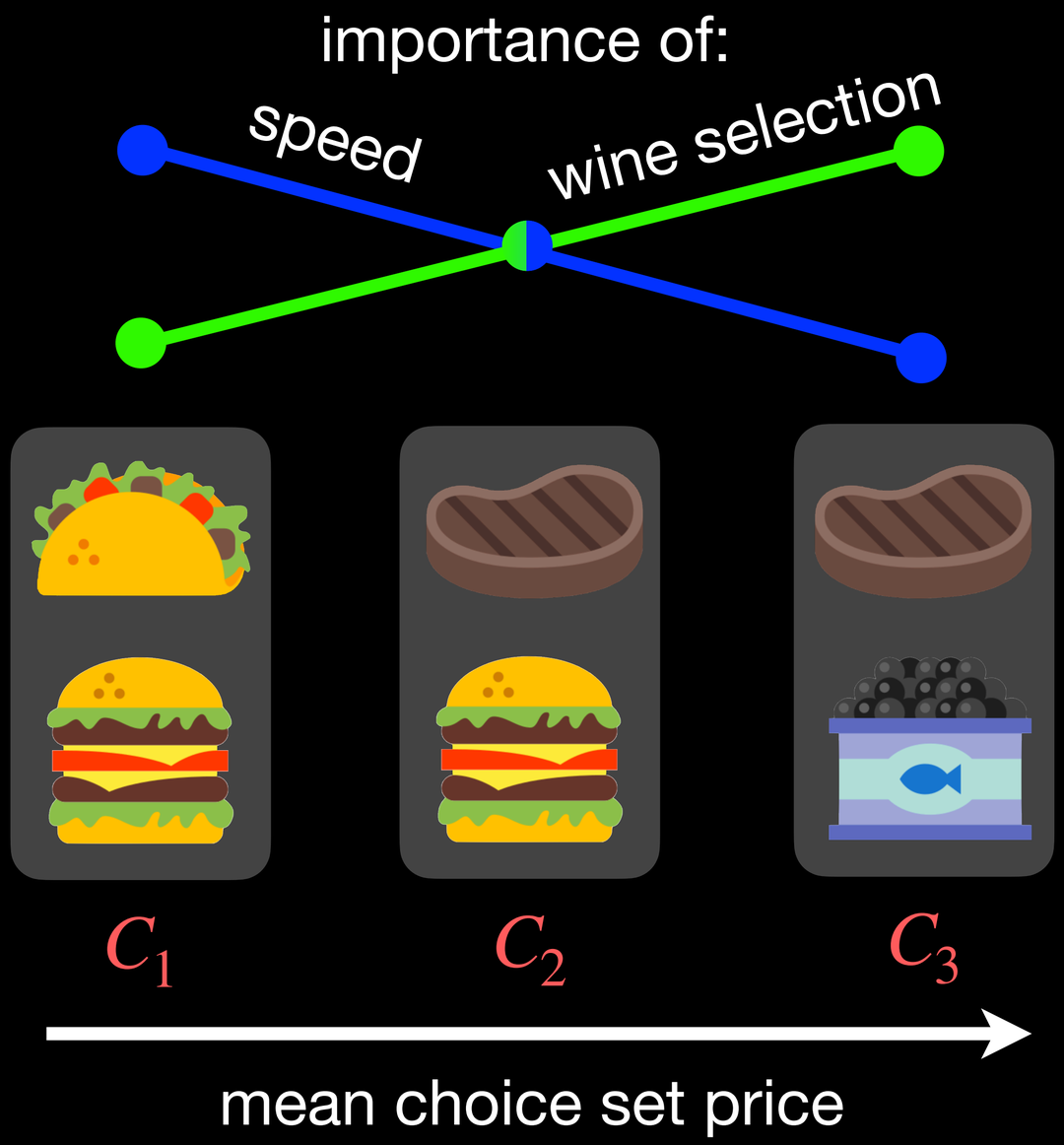

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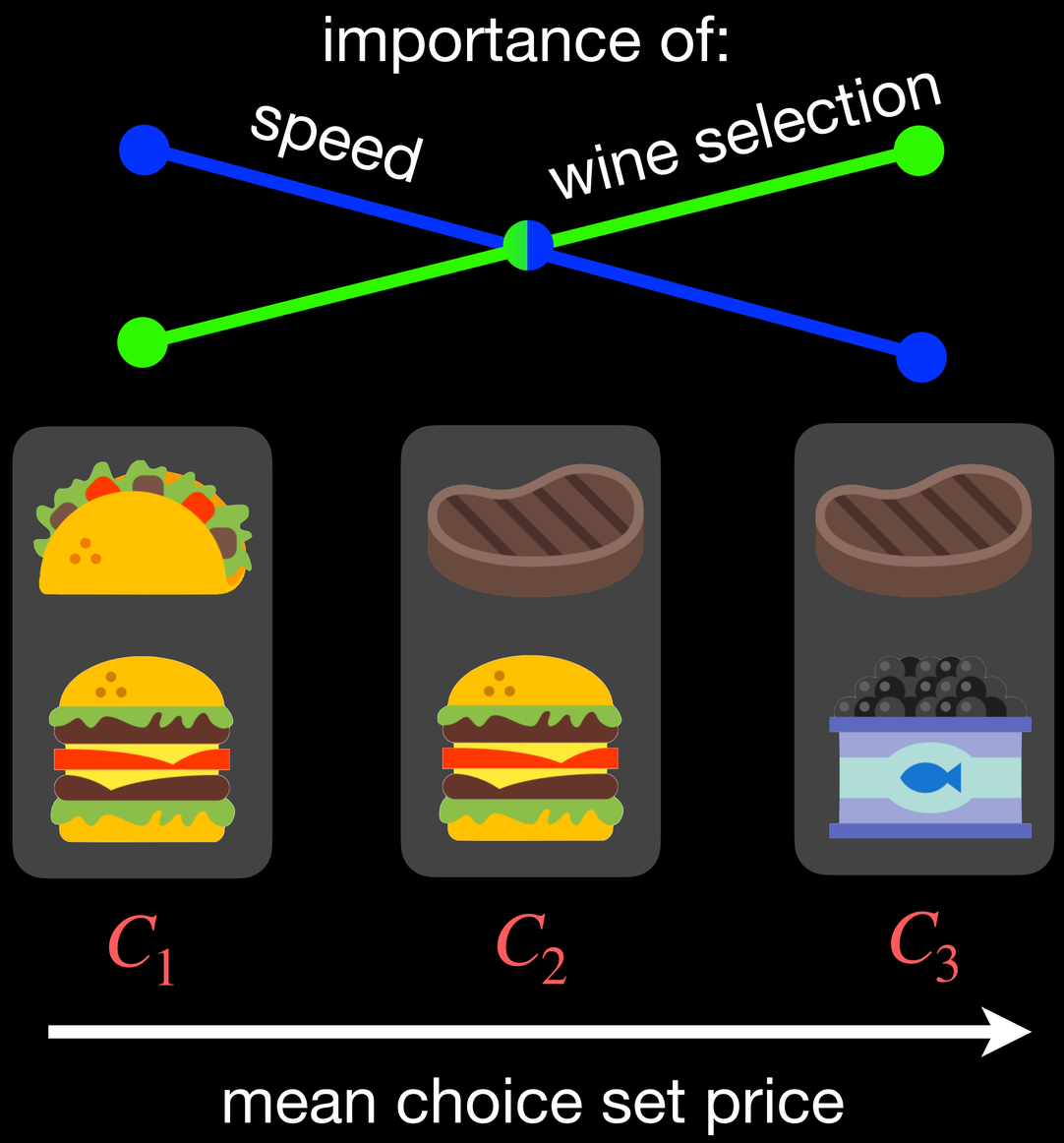
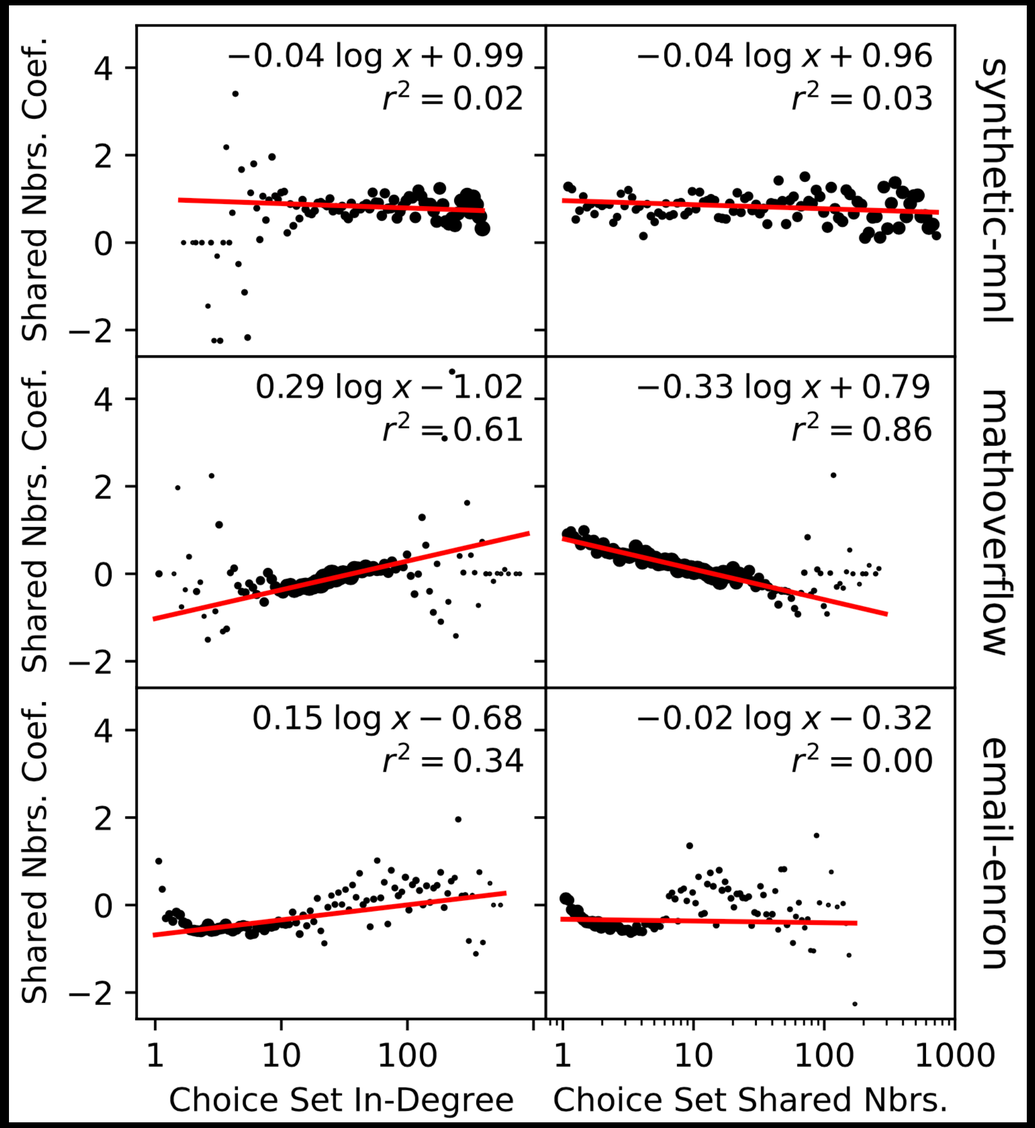


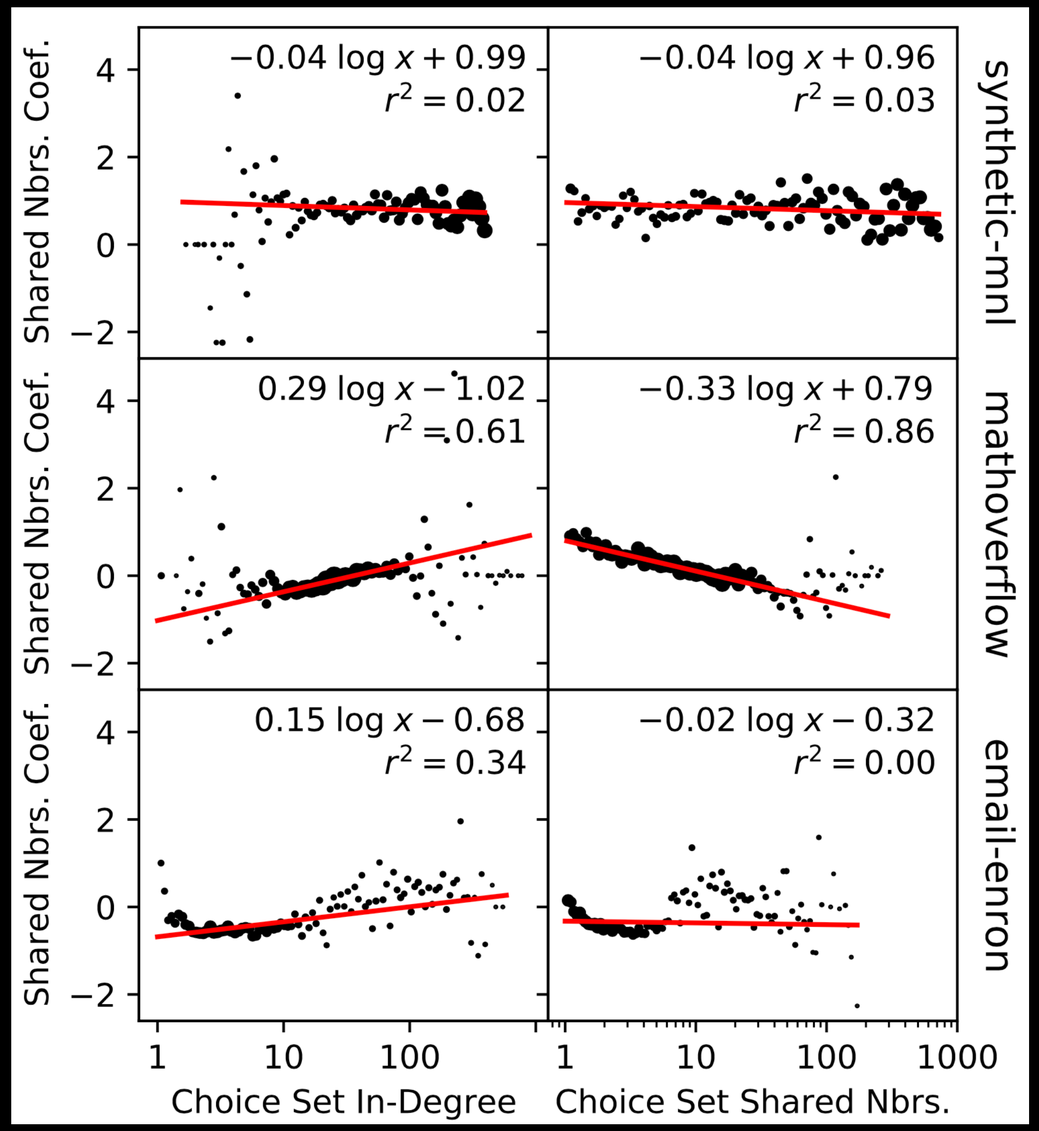
$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Linear feature context effects appear in real data



3. Decomposed LCL

Motivation: the problem of intercepts



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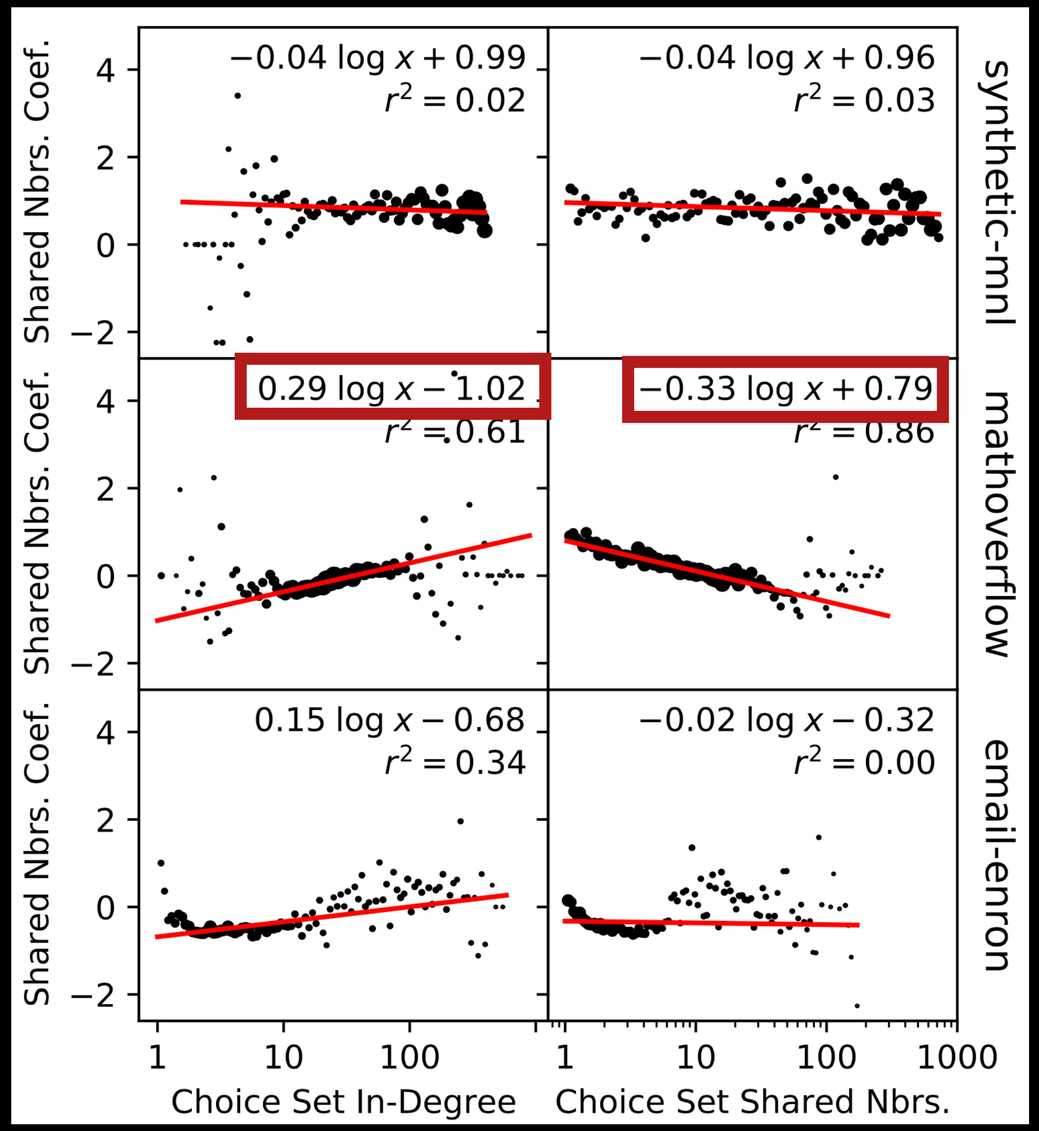

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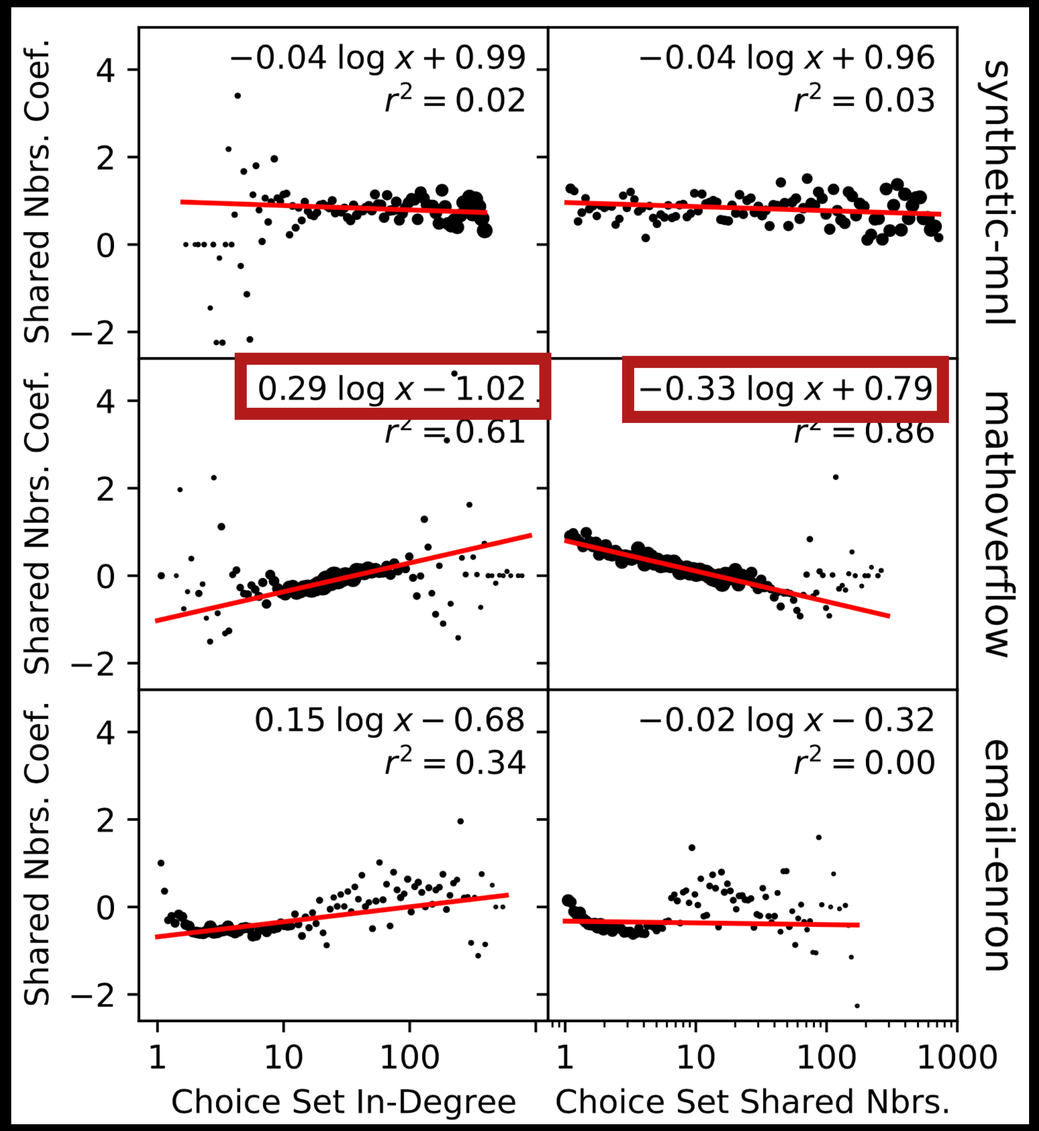

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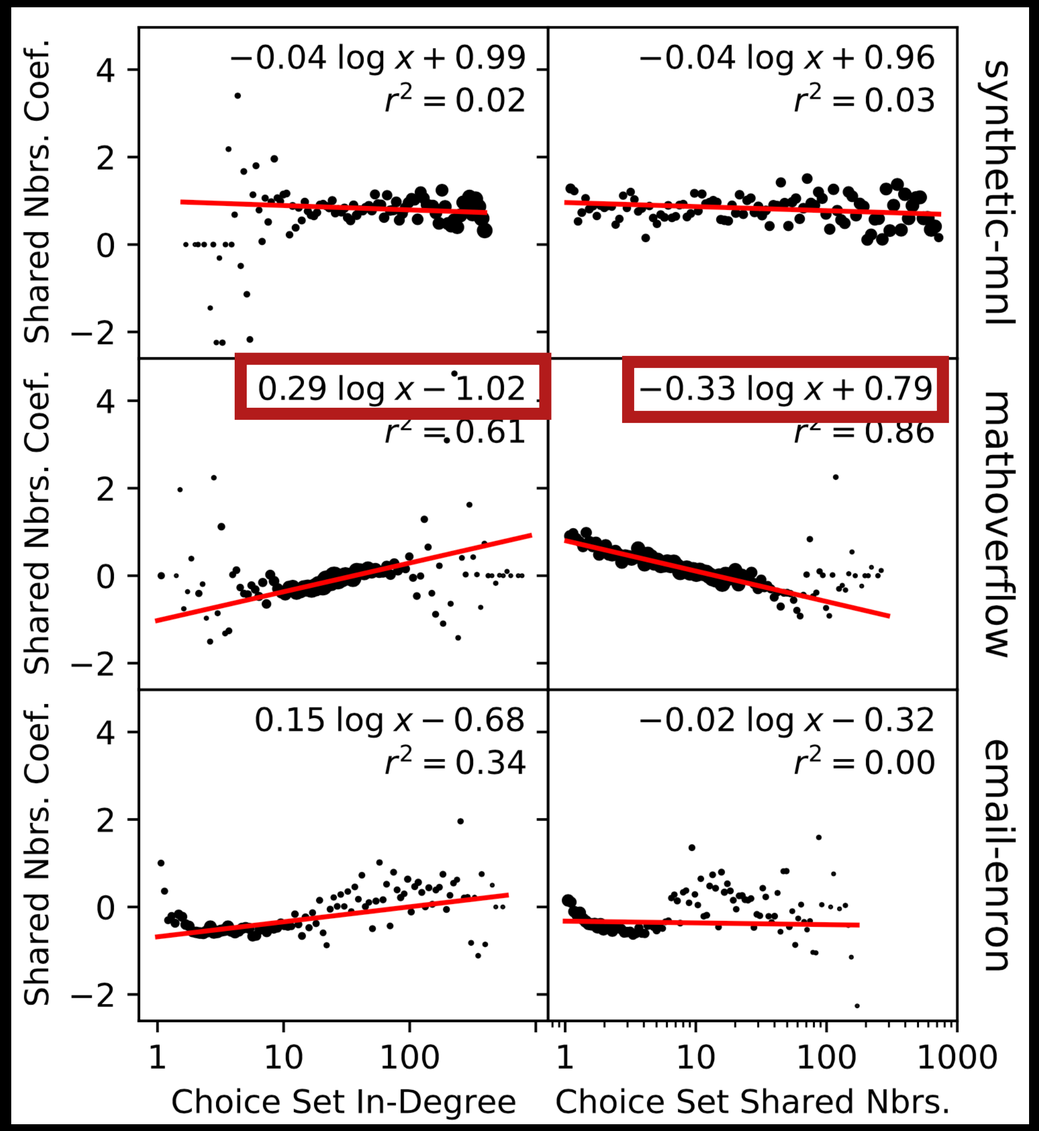

what happens with mean vector zero?

d intercepts d^2 slopes

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

One option: *Decomposed Linear Context Logit (DLCL)*

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(matrix subscript = column)

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→ combine sub-models in mixture (w/ proportions π)

$$\Pr(i \mid C) = \sum_{k=1}^{d} \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

4. Identifiability and estimation

LCL identifiability, fully characterized

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model is *identifiable* from dataset $\mathscr{D}$ if no two parameter values result in the same probability distribution

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Theorem 1. A d-feature linear context logit is identifiable from a dataset $\mathcal{D}$ if and only if

$$\operatorname{span}\left\{\begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C\right\} = \mathbb{R}^{d^2+d}. \tag{6}$$

($\mathscr{C}_{\mathscr{D}}$: unique choice sets in $\mathscr{D}$, $\otimes$: Kronecker product)

LCL identifiability, fully characterized

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($\mathscr{C}_{\mathscr{D}}$: unique choice sets in $\mathscr{D}$, $\otimes$: Kronecker product)

intuition: need varied choice sets containing varied items

LCL identifiability, more intuition

LCL identifiability, more intuition

necessary:

PROPOSITION 2. *No d-feature linear context logit is identifiable from a dataset $\mathcal{D}$ if it does not include a set of $d + 1$ choice sets with affinely independent mean feature vectors.*

LCL identifiability, more intuition

necessary:

Proposition 2. *No d-feature linear context logit is identifiable from a dataset $\mathcal{D}$ if it does not include a set of $d+1$ choice sets with affinely independent mean feature vectors.*

sufficient:

Proposition 3. *If a dataset contains $d+1$ distinct choice sets $C_0, \ldots, C_d$ such that*

i. *the set of mean feature vectors $\{x_{C_0}, \ldots, x_{C_d}\}$ is affinely independent (the necessary condition from Proposition 2) and*
ii. *in each choice set C_i, there is some set of $d+1$ items with affinely independent features,*

then we can uniquely identify a d-feature LCL.

DLCL identifiability…. for another day

Mixture models are rough

see:

Chierichetti, F., Kumar, R., & Tomkins, A. (2018). Learning a mixture of two multinomial logits. In *International Conference on Machine Learning* (pp. 961-969).

Zhao, Z., Piech, P., & Xia, L. (2016). Learning mixtures of Plackett-Luce models. In *International Conference on Machine Learning* (pp. 2906-2914).

Grün, B., & Leisch, F. (2008). Identifiability of finite mixtures of multinomial logit models with varying and fixed effects. *Journal of Classification*, *25*(2), 225-247.

LCL and DLCL Estimation

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negative log-likelihood convex, so use gradient descent

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$$-\ell(\theta, A; \mathscr{D}) = \sum_{(i,C)\in\mathscr{D}} -\left(\theta + Ax_C\right)^T x_i + \log \sum_{j\in C} \exp\left(\left[\theta + Ax_C\right]^T x_j\right)$$

LCL and DLCL Estimation

LCL

negative log-likelihood convex, so use gradient descent

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DLCL

log-likelihood not convex…
but, *expectation-maximation (EM)* algorithm performs well

Table 9: Adam vs. EM algorithm for DLCL

Dataset	Adam NLL	EM NLL
DISTRICT	3206	**3041**
DISTRICT-SMART	3303	**3144**
EXPEDIA	837569	**805055**
SUSHI	9764	**9709**
CAR-A	1692	**1684**
CAR-B	1284	**1246**
CAR-ALT	7011	**6369**

5. Results on choice data

Choice datasets

Dataset	Choices	Features	Largest Choice Set
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

Choice datasets

favorite sushi types
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Choice datasets

favorite sushi types
(random choice sets in survey)

hotel bookings
(choice sets = search results)

car purchasing survey
(handcrafted choice sets)

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LCL improves model fit

whole-dataset negative log-likelihood (lower = better)

	MNL	LCL	Mixed logit	DLCL
DISTRICT	3313	**3130**	3258	3206
DISTRICT-SMART	3426	**3278***	3351	3303†
EXPEDIA	839505	837649*	839055	**837569**†
SUSHI	9821	9773*	9793	**9764**
CAR-A	1702	1694	1696	**1692**
CAR-B	1305	1295	1297	**1284**
CAR-ALT	7393	**6733***	7301	7011†

*significant likelihood ratio test vs MNL ($p < 0.001$)

†significant likelihood ratio test vs mixed logit ($p < 0.001$)

LCL can improve out-of-sample prediction performance

mean (std. dev) relative rank (0 = perfect prediction, 1 = always wrong)

	MNL	LCL	Mixed logit	DLCL
DISTRICT	.3680 (.4823)	.3253 (.4685)	**.3188** (.4660)	.3225 (.4674)
DISTRICT-SMART	.4006 (.4900)	.3764 (.4845)	**.3271** (.4692)	.3448 (.4753)
EXPEDIA	.3859 (.2954)	.3800* (.2945)	.3201 (.2825)	**.3195**† (.2823)
SUSHI	.2727 (.2751)	.2737 (.2781)	**.2724** (.2765)	.2732 (.2765)
CAR-A	**.3570** (.4791)	**.3570** (.4791)	**.3570** (.4791)	**.3570** (.4791)
CAR-B	.3326 (.4711)	**.3213** (.4670)	.3303 (.4703)	.3235 (.4678)
CAR-ALT	.2944 (.2875)	**.2661*** (.2819)	.2931 (.2966)	.2798 (.2837)

*significant Wilcoxon test vs MNL ($p < 0.001$)

LCL can test individual effects for significance

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constrain all but one entry of A to 0

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Table 5: Five largest context effects in SUSHI.

Effect (q on p)	A_{pq}	$\overline{A}_{pq}$	**LRT**	p-value
popularity on *popularity*	−0.28	−0.11	3.0	0.081
availability on *is maki*	0.24	0.04	0.39	0.53
oiliness on *oiliness*	−0.20	−0.26	23	1.5×10^{-6}
popularity on *availability*	0.19	0.09	2.3	0.13
availability on *oiliness*	−0.18	−0.04	0.74	0.39

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Table 6: Five largest context effects in EXPEDIA.

Effect (q on p)	A_{pq}	$\overline{A}_{pq}$	LRT	p-value
location score on *price*	−0.47	−0.13	10	0.002
on promotion on *price*	0.27	0.13	17	3.5×10^{-5}
review score on *price*	−0.19	−0.13	29	8.6×10^{-8}
star rating on *price*	0.15	0.20	65	6.0×10^{-16}
price on *star rating*	0.10	0.03	4.0	0.046

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Effect (q on p)	A_{pq}	$\overline{A}_{pq}$	LRT	p-value
truck on *truck*	1.06	0.83	239	$< 10^{-16}$
van on *van*	0.94	0.97	309	$< 10^{-16}$
suv on *station wagon*	0.89	0.98	−0.21	1.0
station wagon on *station wagon*	0.88	0.93	153	$< 10^{-16}$
sports car on *station wagon*	0.86	0.96	−0.21	1.0

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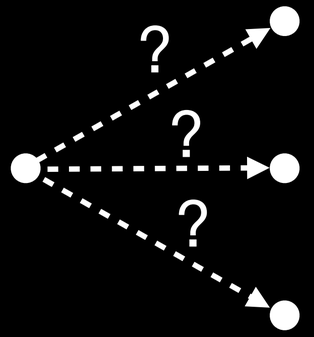
6. Social network application

What factors drive edge formation?



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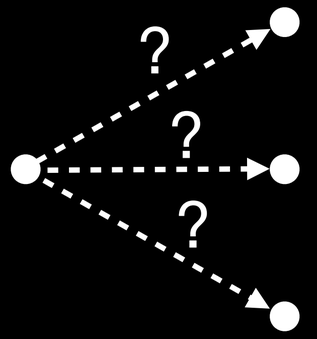
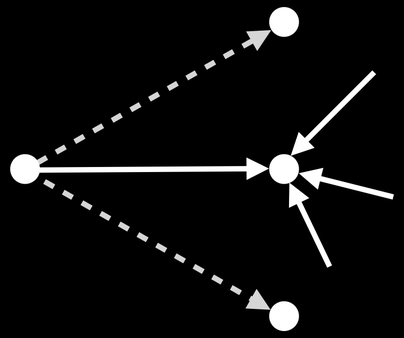


Preferential attachment
(Barabási & Albert, *Science* 1999)



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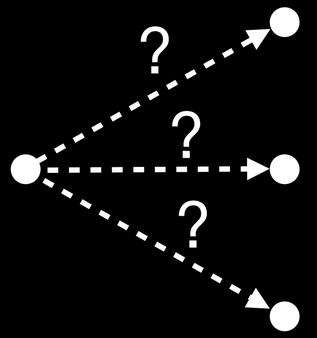

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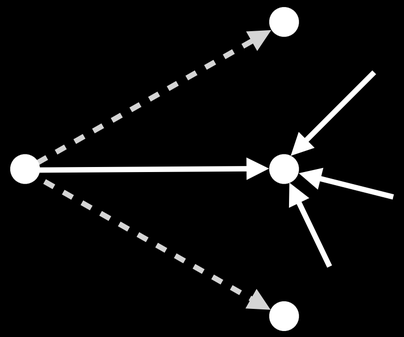

Homophily

(McPherson et al., *Annual Review of Sociology* 2001)
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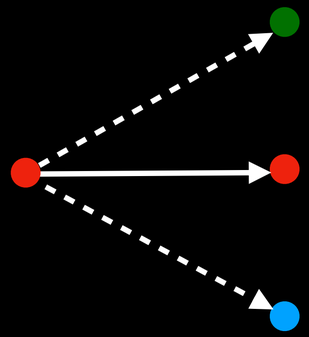

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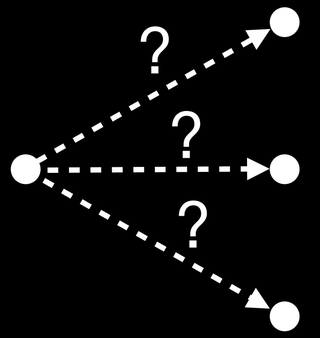

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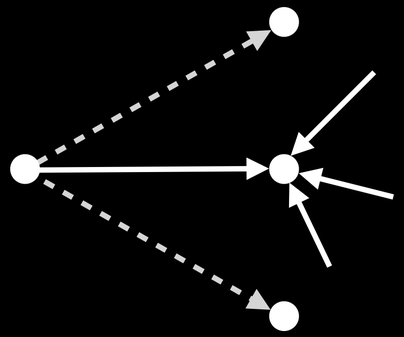

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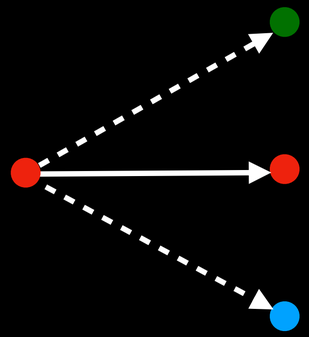
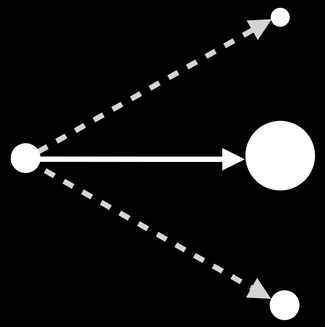


Fitness

(Bianconi & Barabási, *Europhysics Letters* 2001)
(Caldarelli et al., *Physical Review Letters* 2002)



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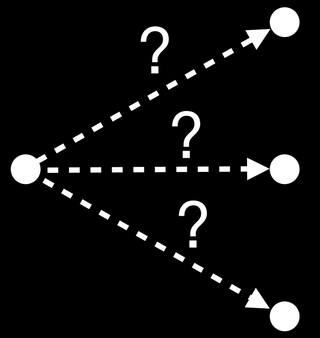

Preferential attachment

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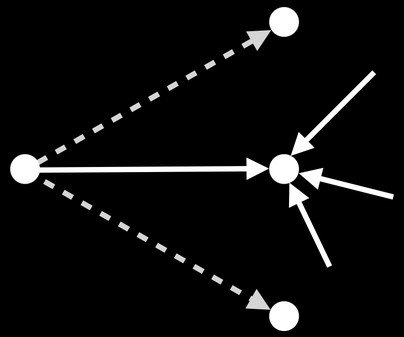

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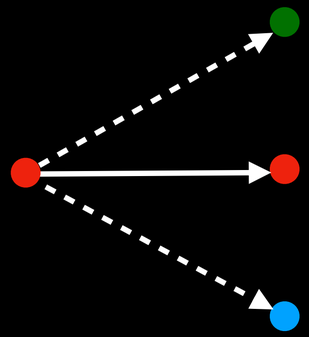

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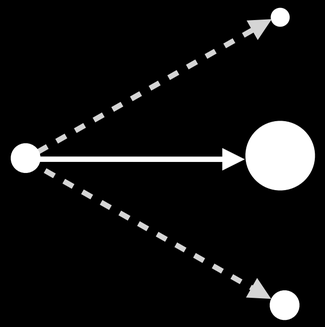
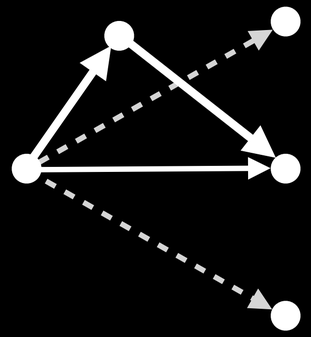


Triadic closure

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)
(Jin et al., *Physical Review E* 2001)



“Choosing to grow a graph”

(Overgoor et al., *SINM* ’19 & *WWW* ’19)

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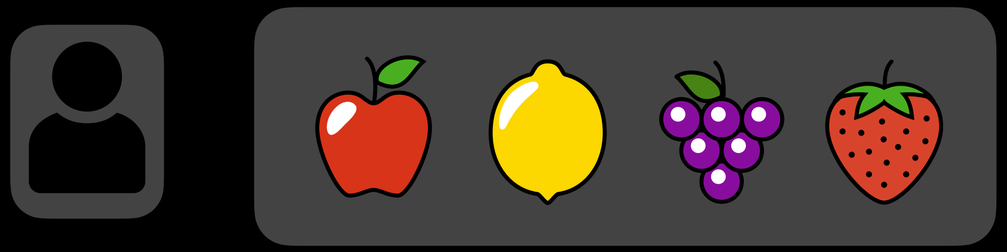
so far:



chooser *choice set*

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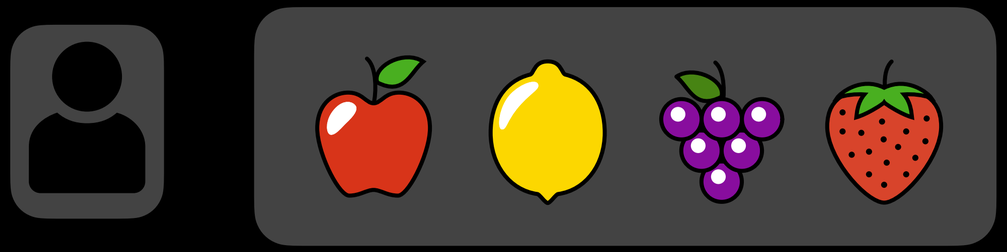
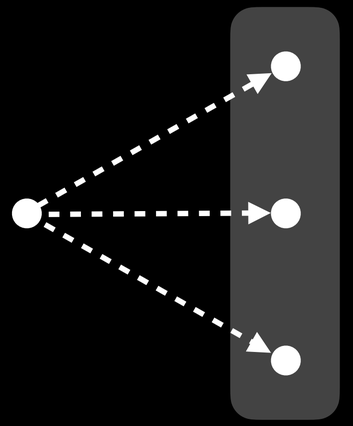
so far:



chooser *choice set*

in network growth:



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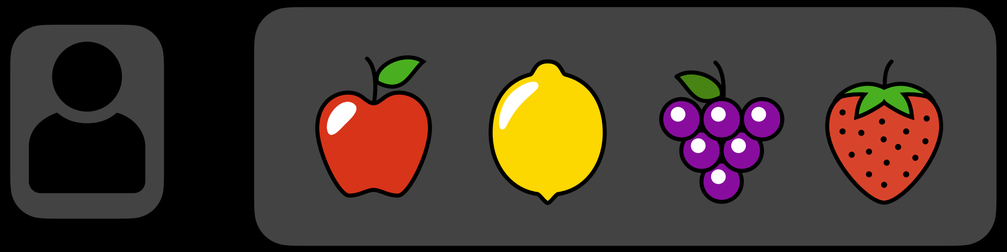

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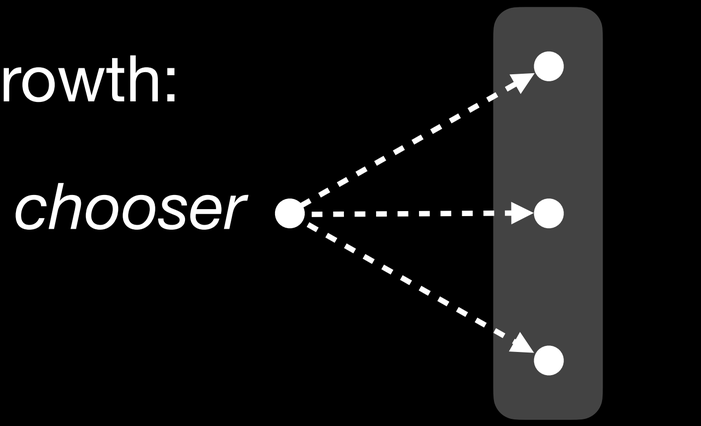

choice set

Key usage

Timestamped edges
→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data

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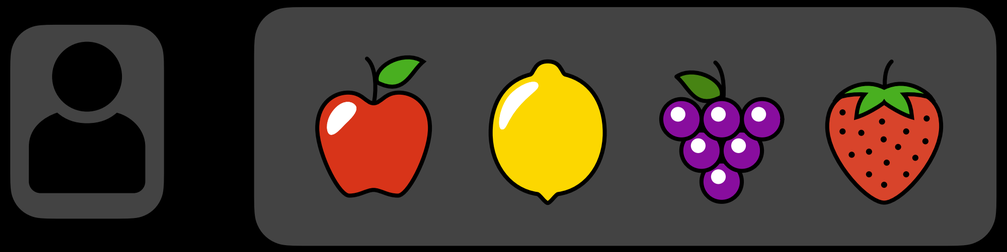
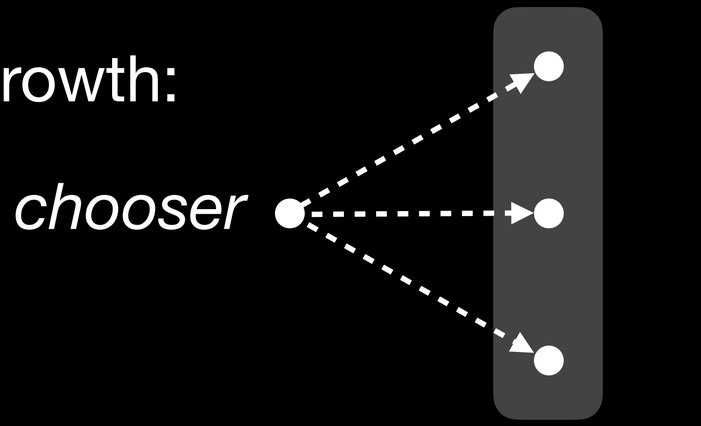


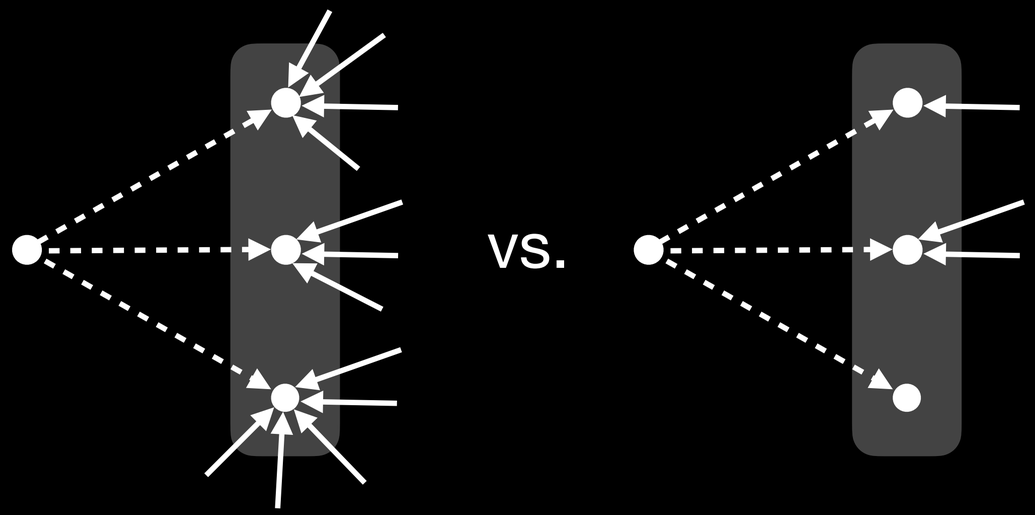
chooser *choice set*

in network growth:



choice set

feature context effects:



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Choosing to close triangles

Triadic closure offers small choice sets
→ tractable inference
→ varied choice sets

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Our data
Timestamped edges
(including repeats)

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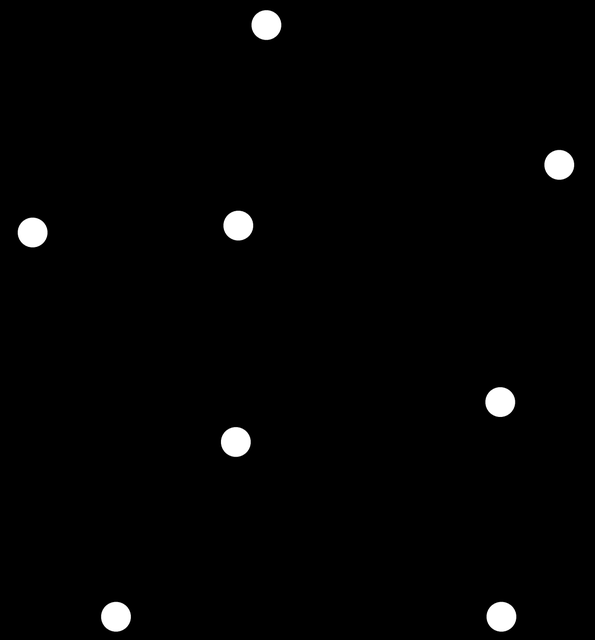

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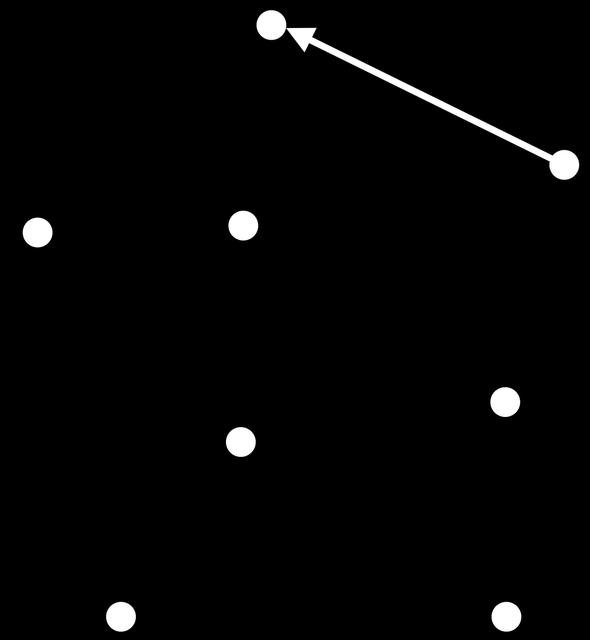

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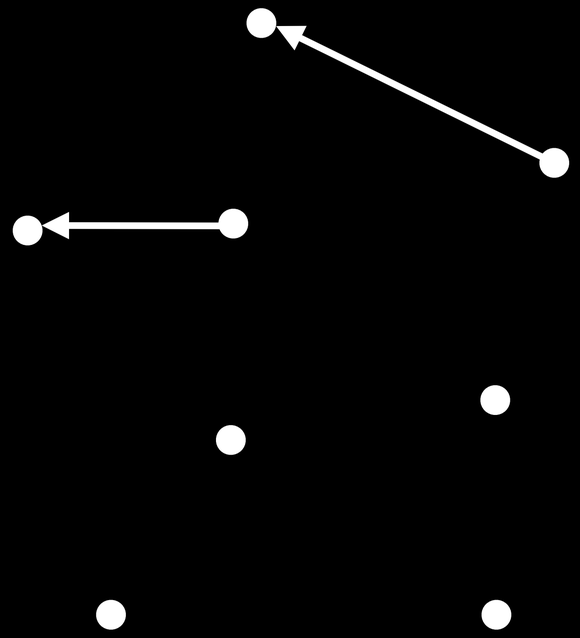

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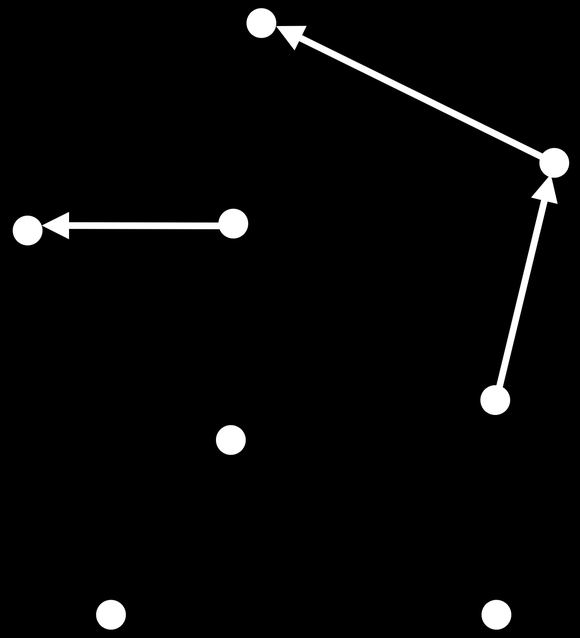

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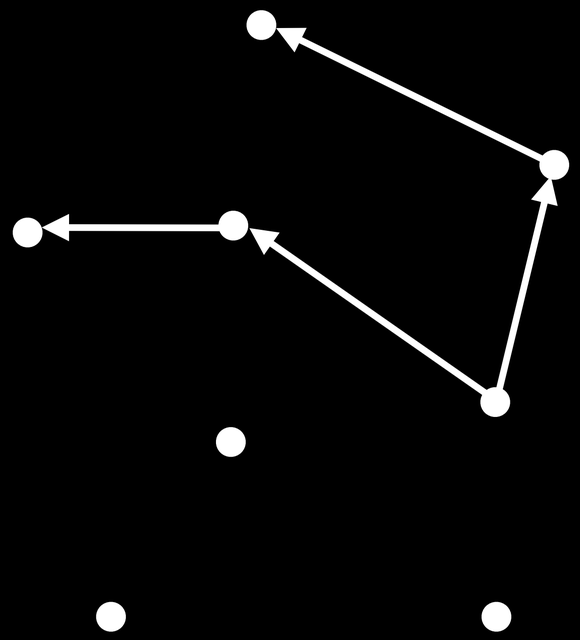

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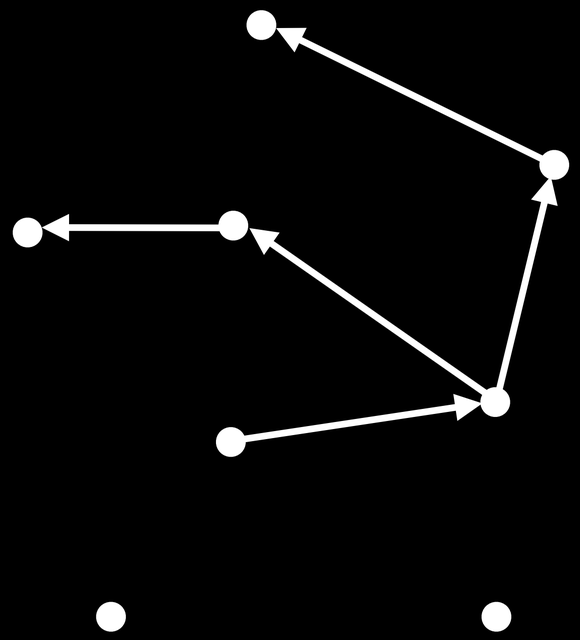

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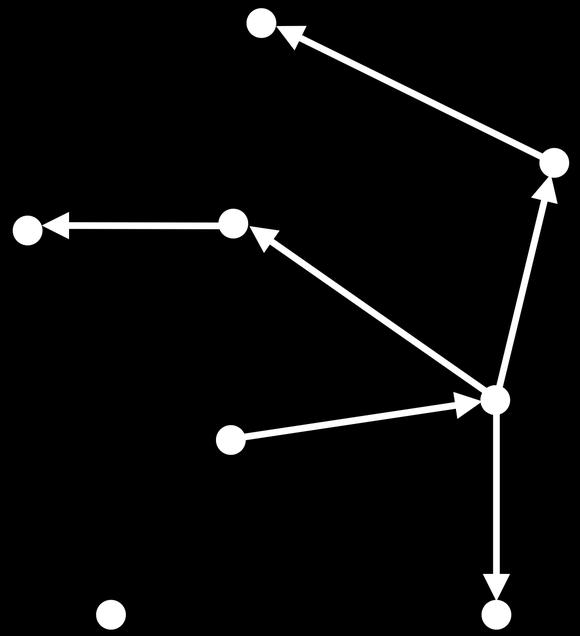

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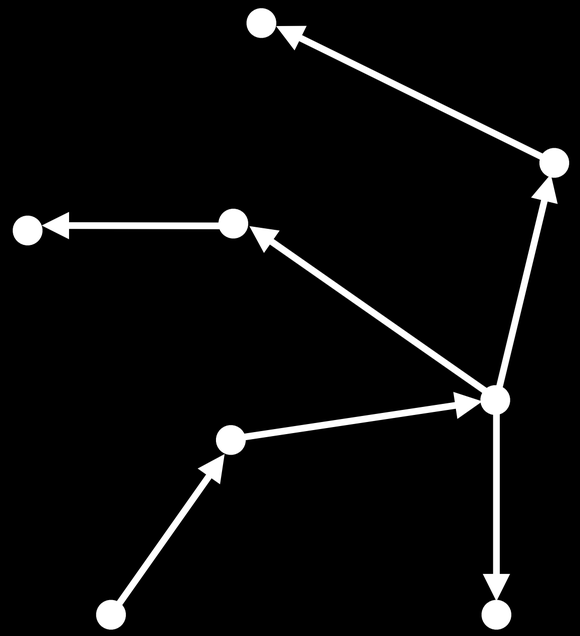

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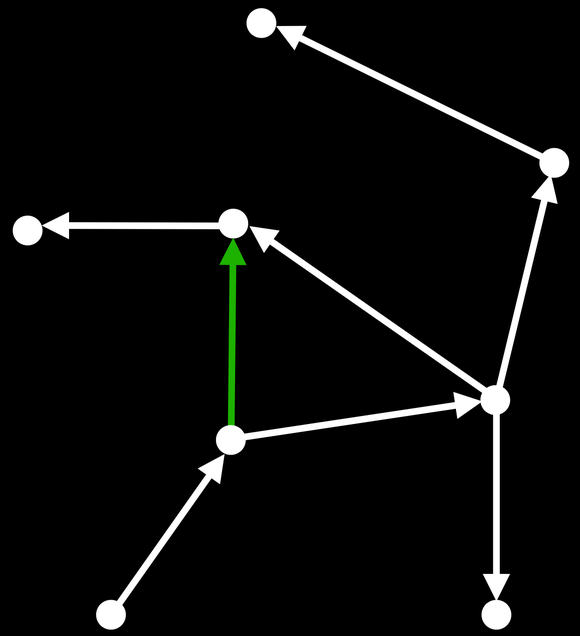

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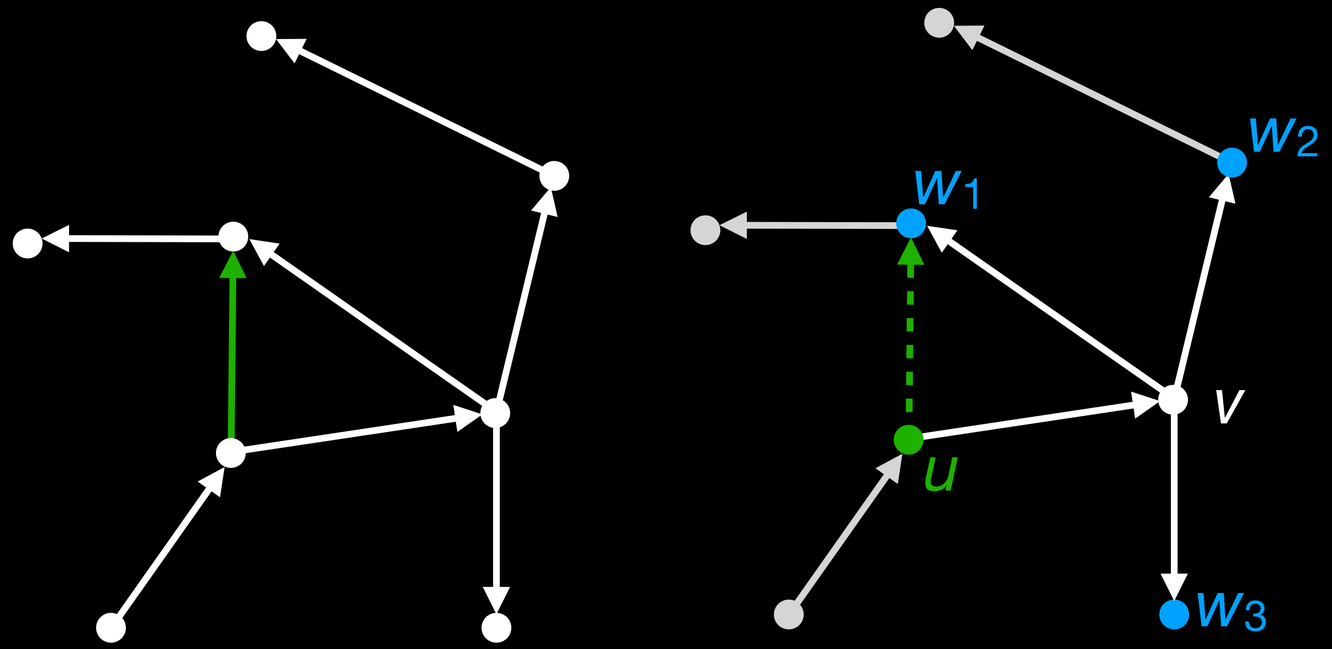

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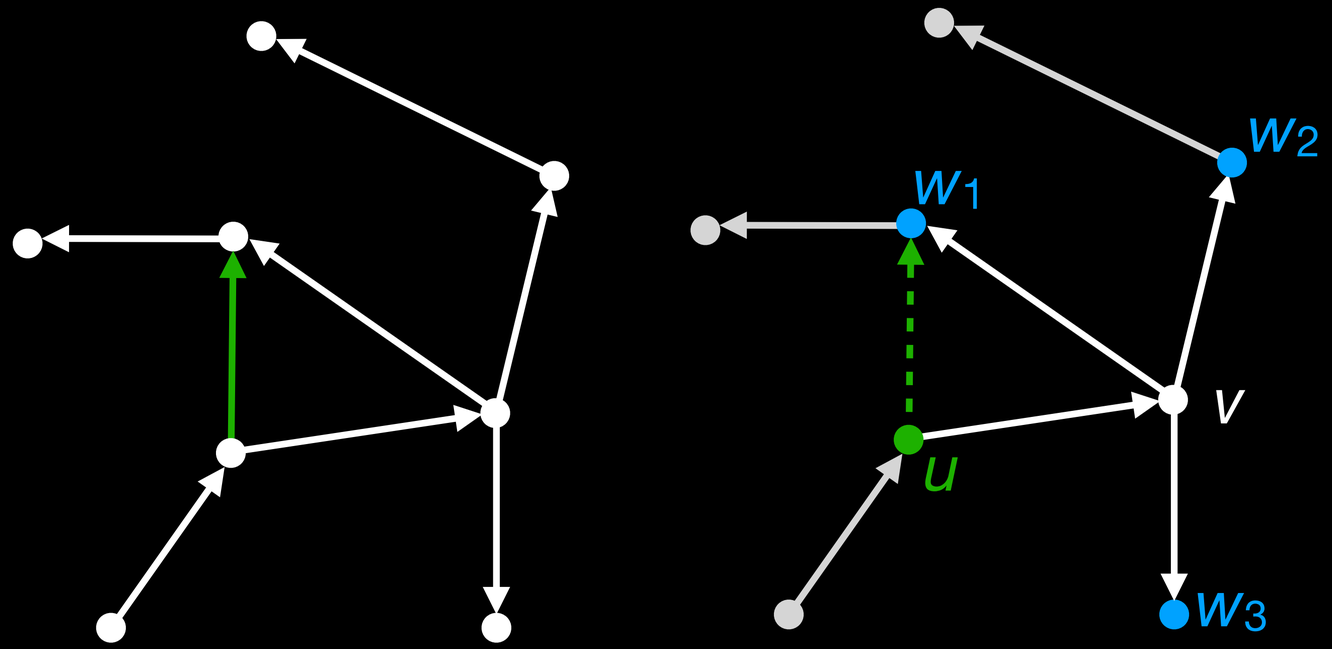

chooser
u

Our data
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(including repeats)

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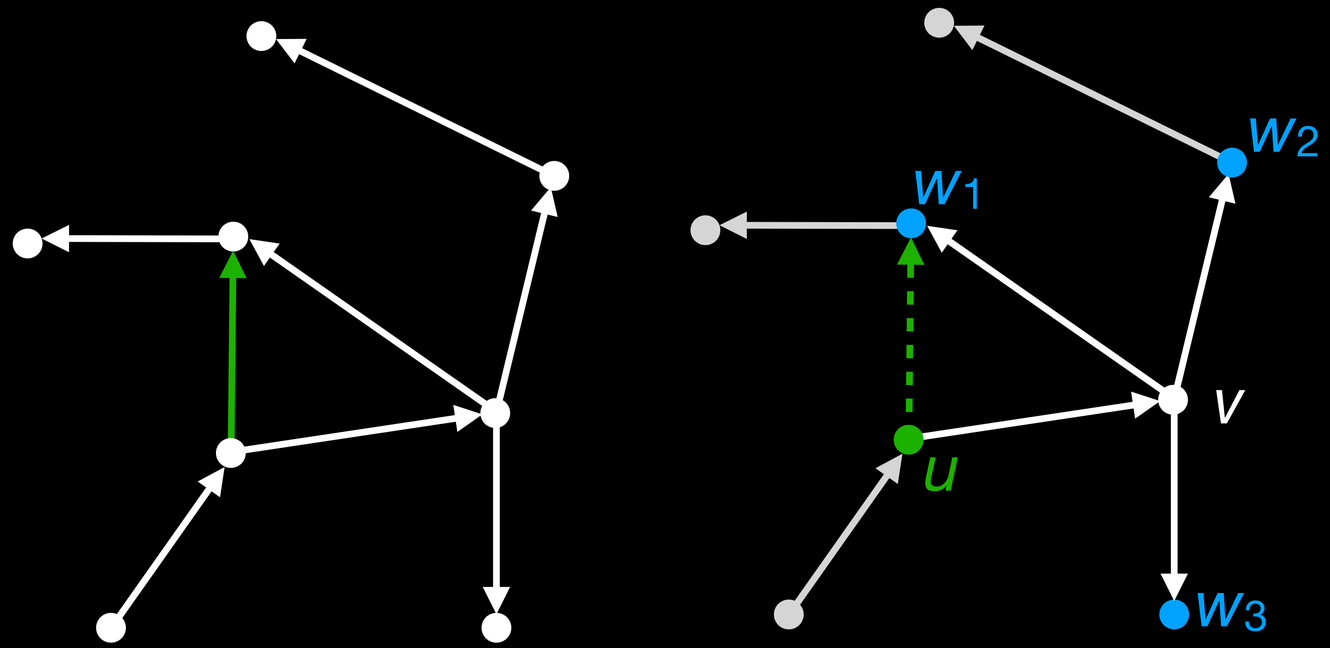

chooser u

choice set $\{w_1, w_2, w_3\}$

Our data
Timestamped edges
(including repeats)

Choosing to close triangles

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Our data
Timestamped edges
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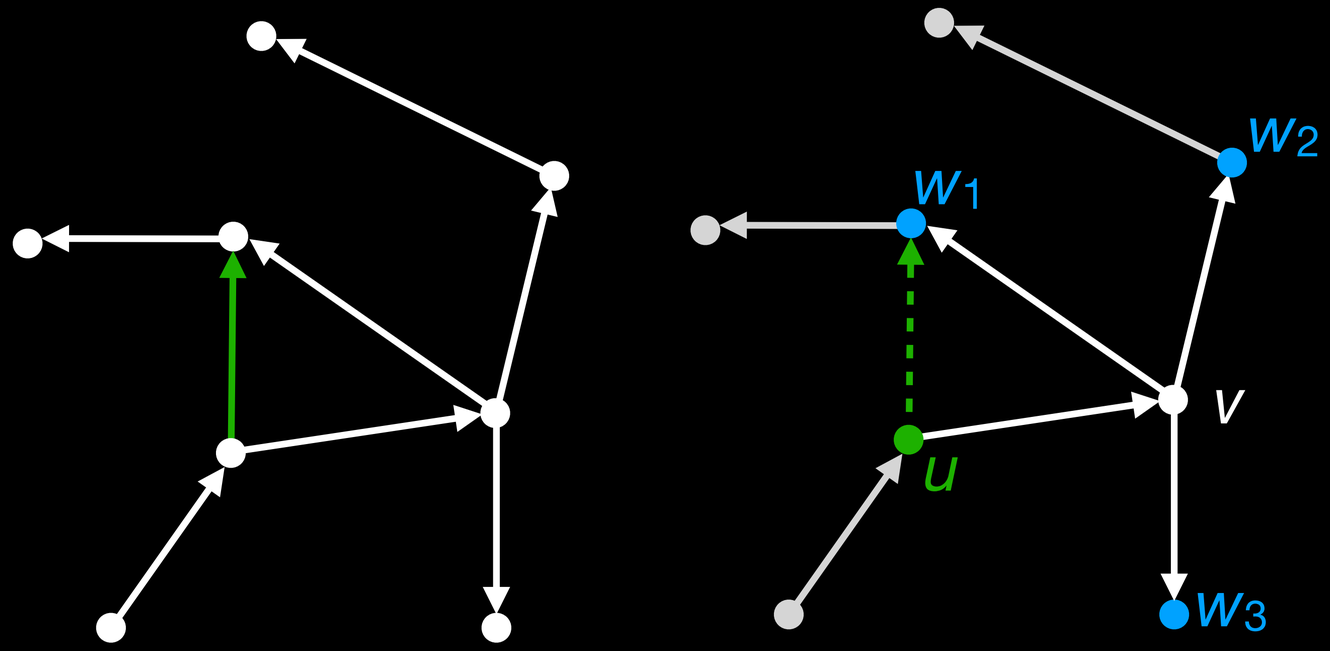

chooser	*choice set*	*choice*
u	$\{w_1, w_2, w_3\}$	w_1

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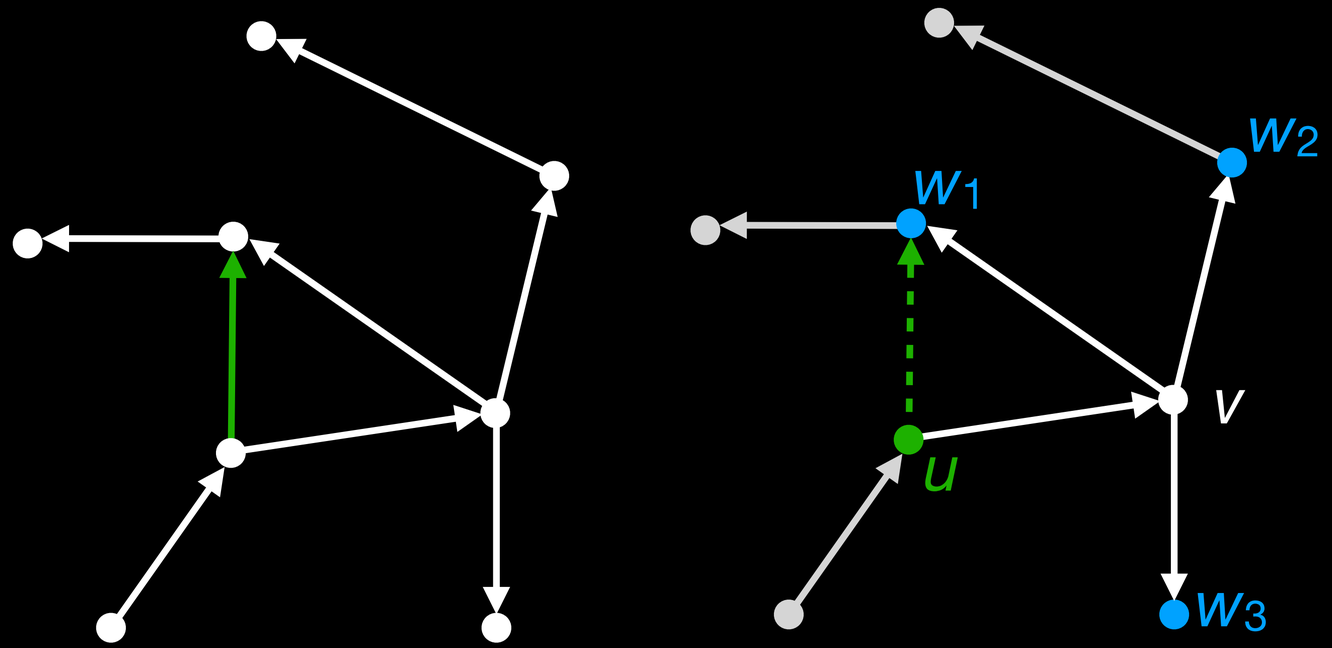


chooser	*choice set*	*choice*
u	$\{w_1, w_2, w_3\}$	w_1

Our data
Timestamped edges
(including repeats)

Node features

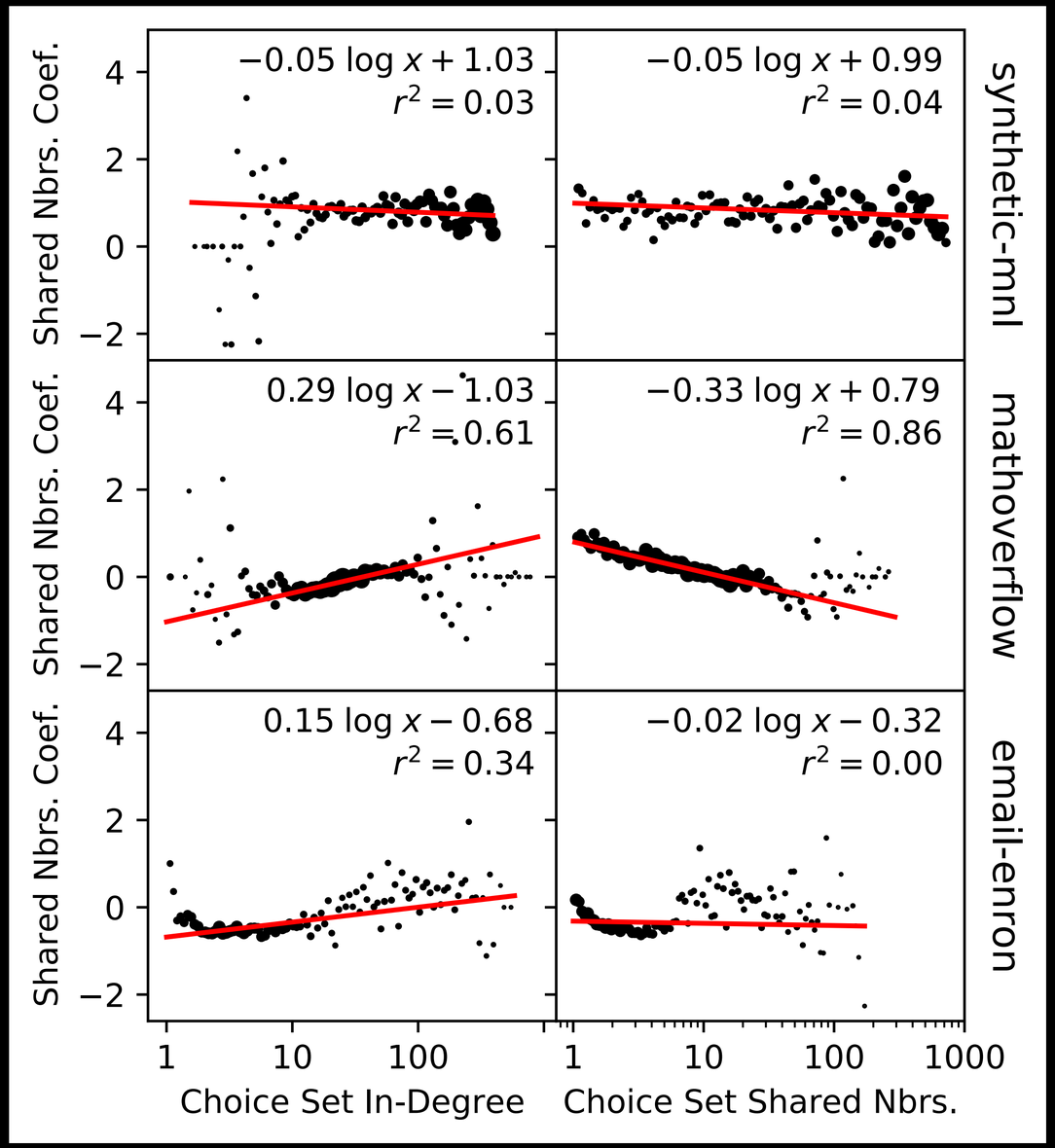
1. in-degree of w
2. # shared neighbors of u, w
3. weight of edge $w \rightarrow u$
4. time since last edge into w
5. time since last edge out of w
6. time since last $w \rightarrow u$ edge

Context matters in triadic closure

Context matters in triadic closure

Datasets
email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

Context matters in triadic closure



Datasets
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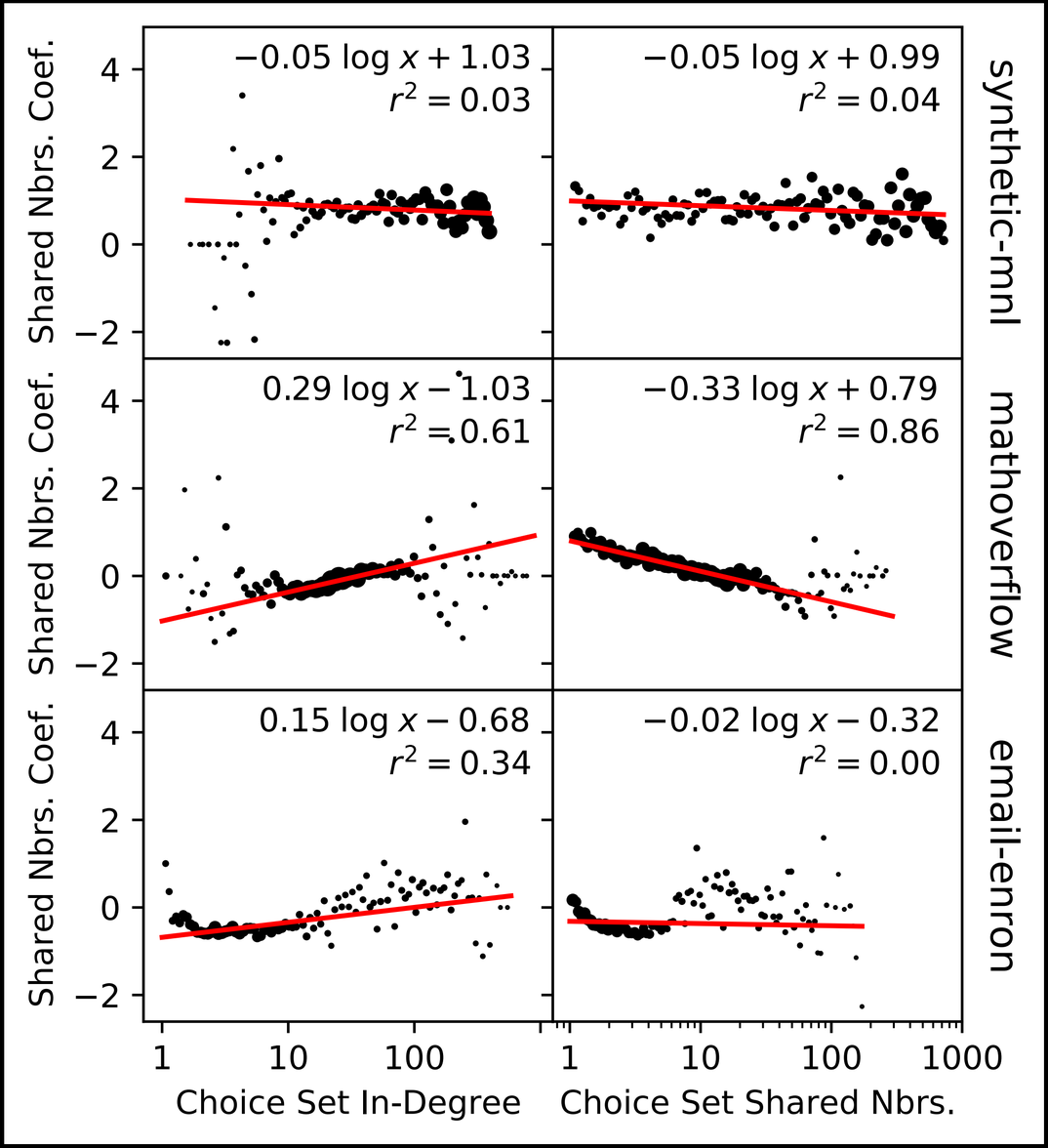
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

Context matters in triadic closure



Synthetic data,
no context effects

Datasets
email-enron
email-eu
email-w3c
wiki-talk
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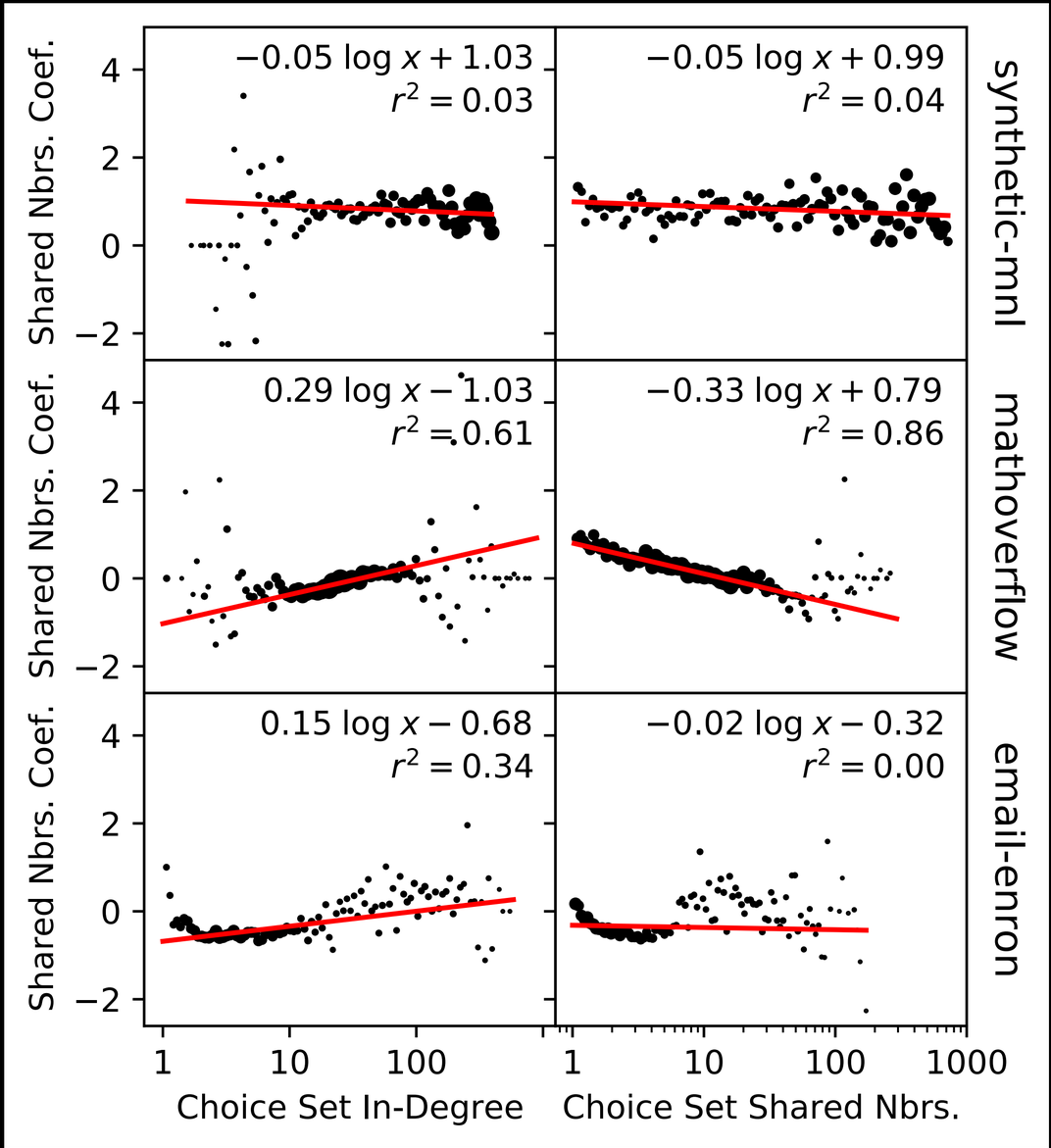
Context matters in triadic closure



Synthetic data,
no context effects

Commenting network,
linear context effects

Datasets
email-enron
email-eu
email-w3c
wiki-talk
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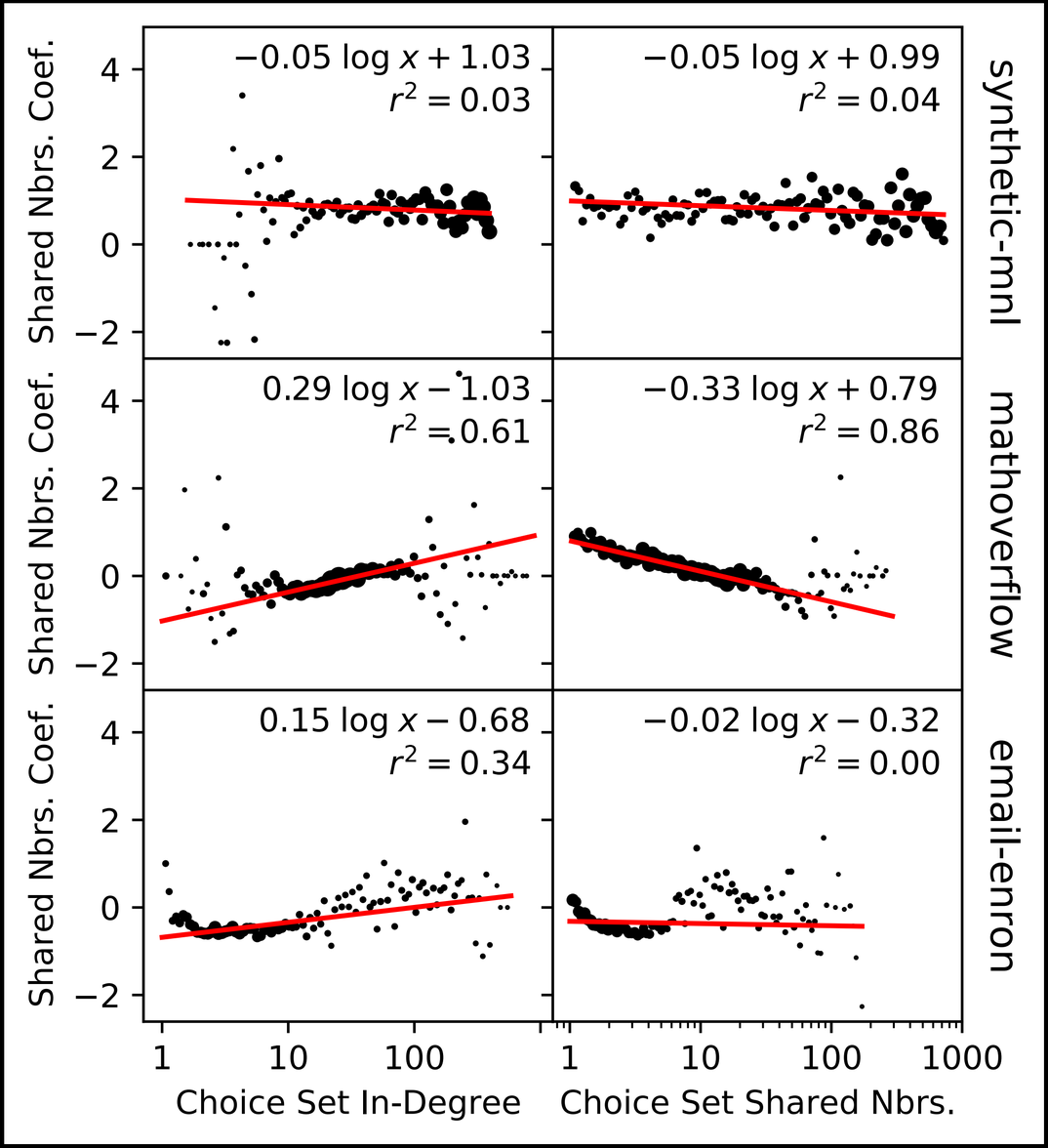
college-msg
facebook-wall
sms-a
sms-b
sms-c

Context matters in triadic closure



Synthetic data,
no context effects

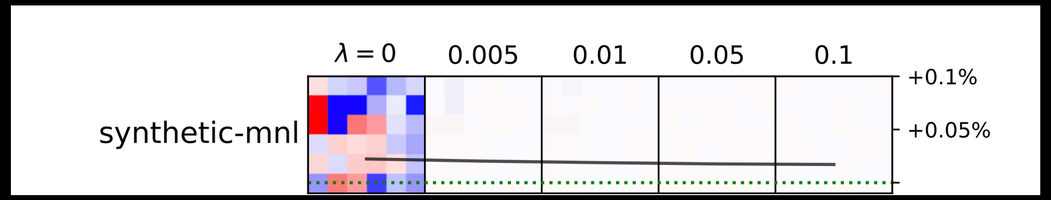
Commenting network,
linear context effects

Email network,
nonlinear context effects?

Datasets
email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

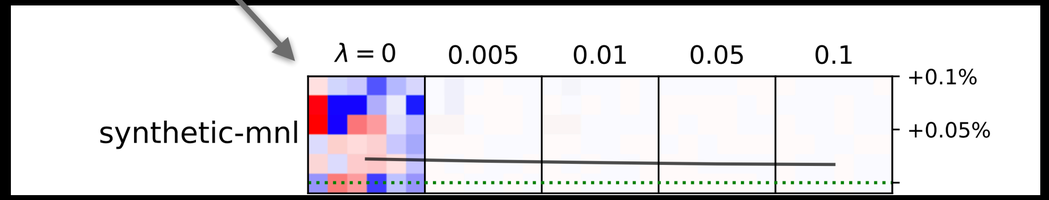
LCL reveals interpretable feature context effects

LCL reveals interpretable feature context effects



LCL reveals interpretable feature context effects

context effect matrix A
red: +, blue: -, white: 0
(column acts on row)



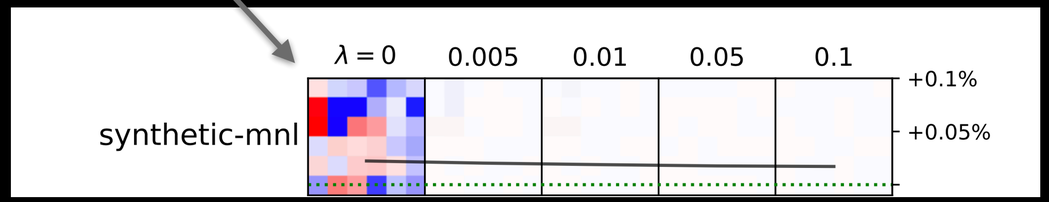
Node features
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

LCL reveals interpretable feature context effects

context effect matrix A
red: +, blue: -, white: 0
(column acts on row)

increasing L_1 regularization on A



Node features
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
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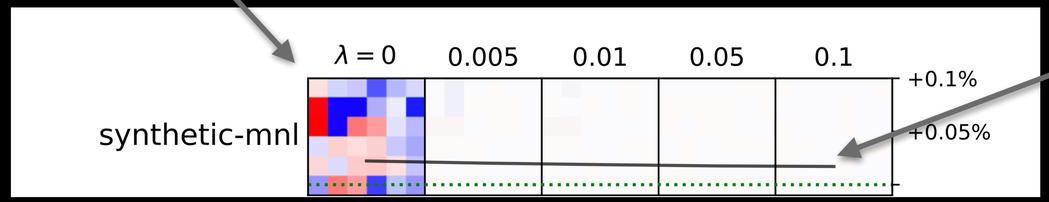

NLL (lower = better fit)

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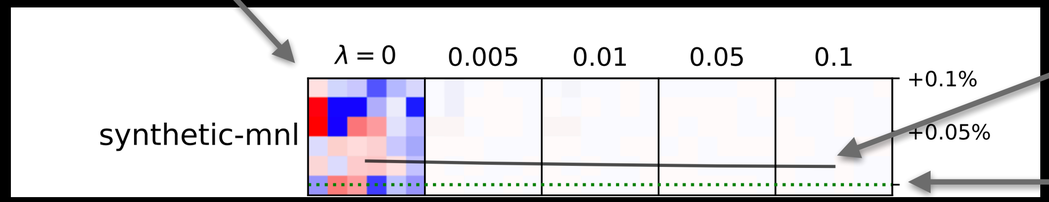

NLL (lower = better fit)

$p = 0.001$ LRT threshold

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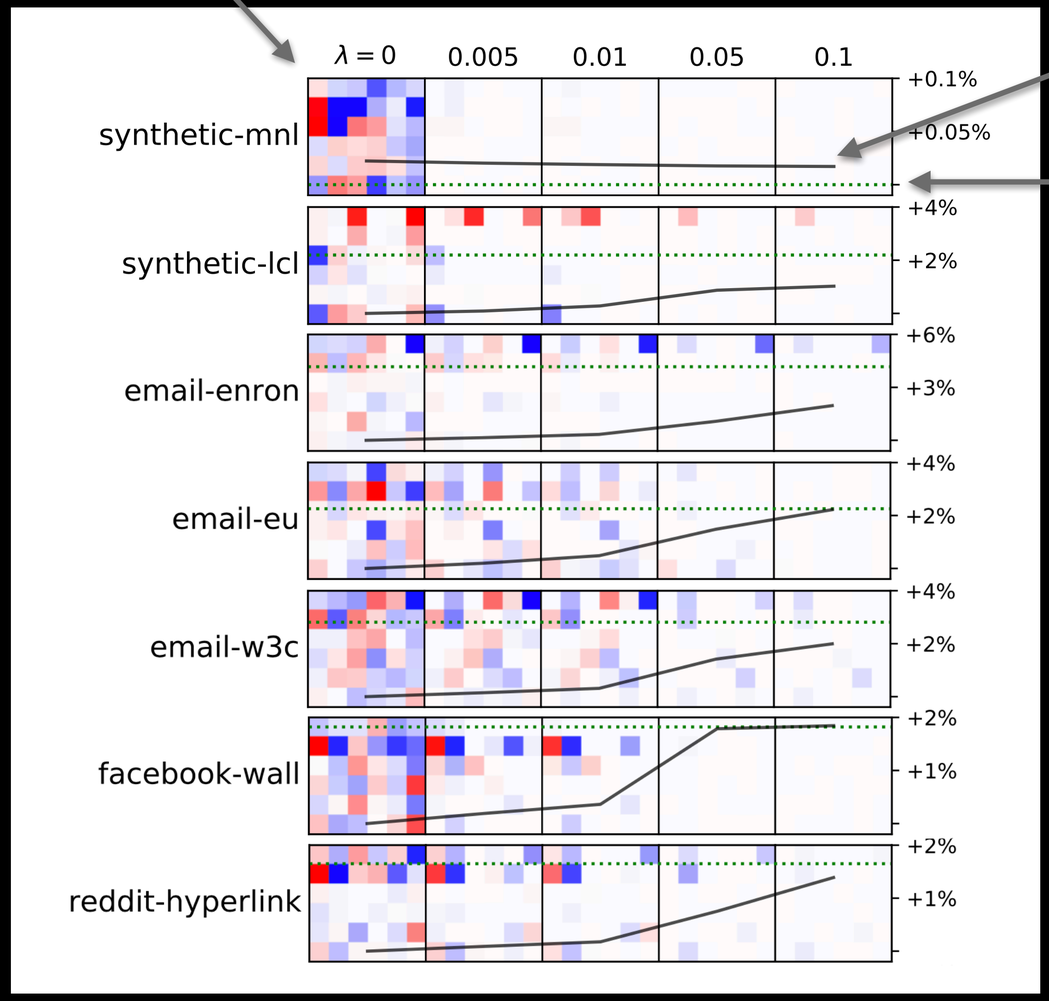

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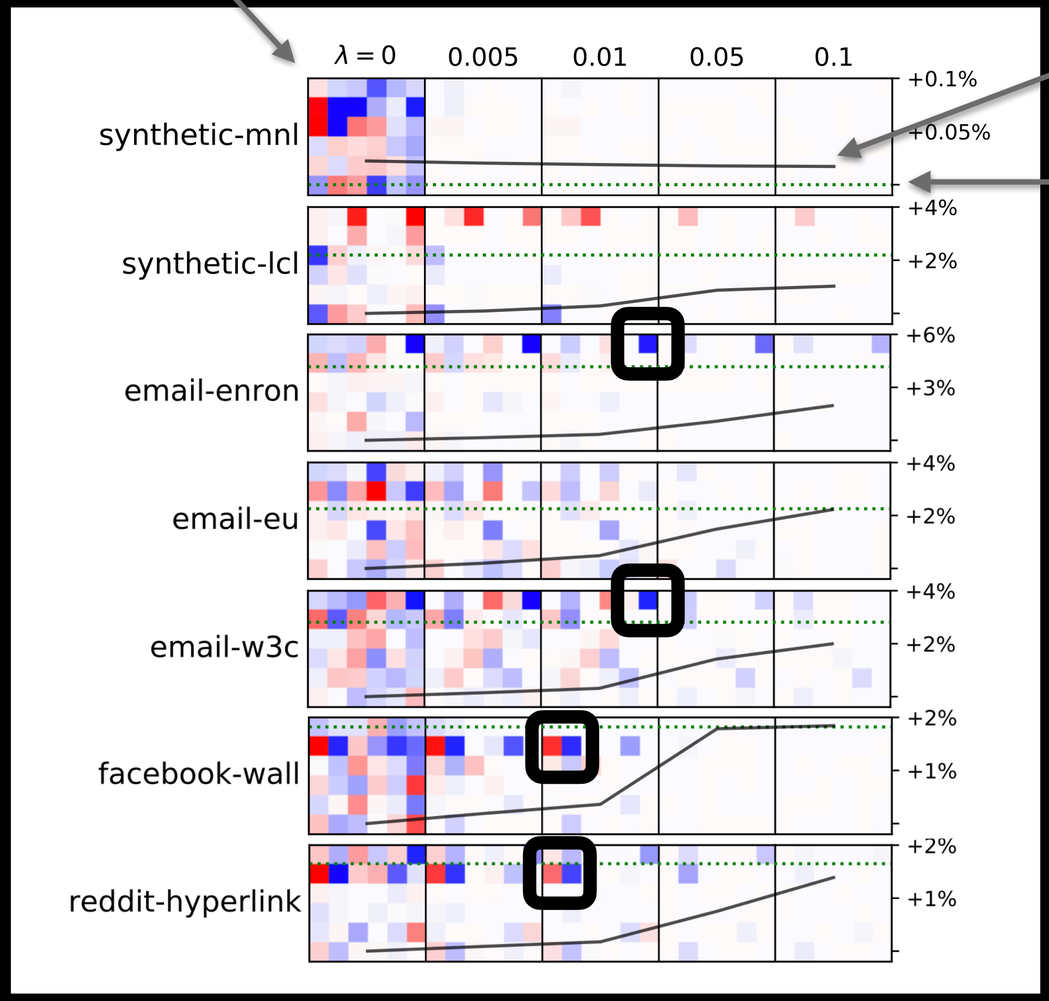

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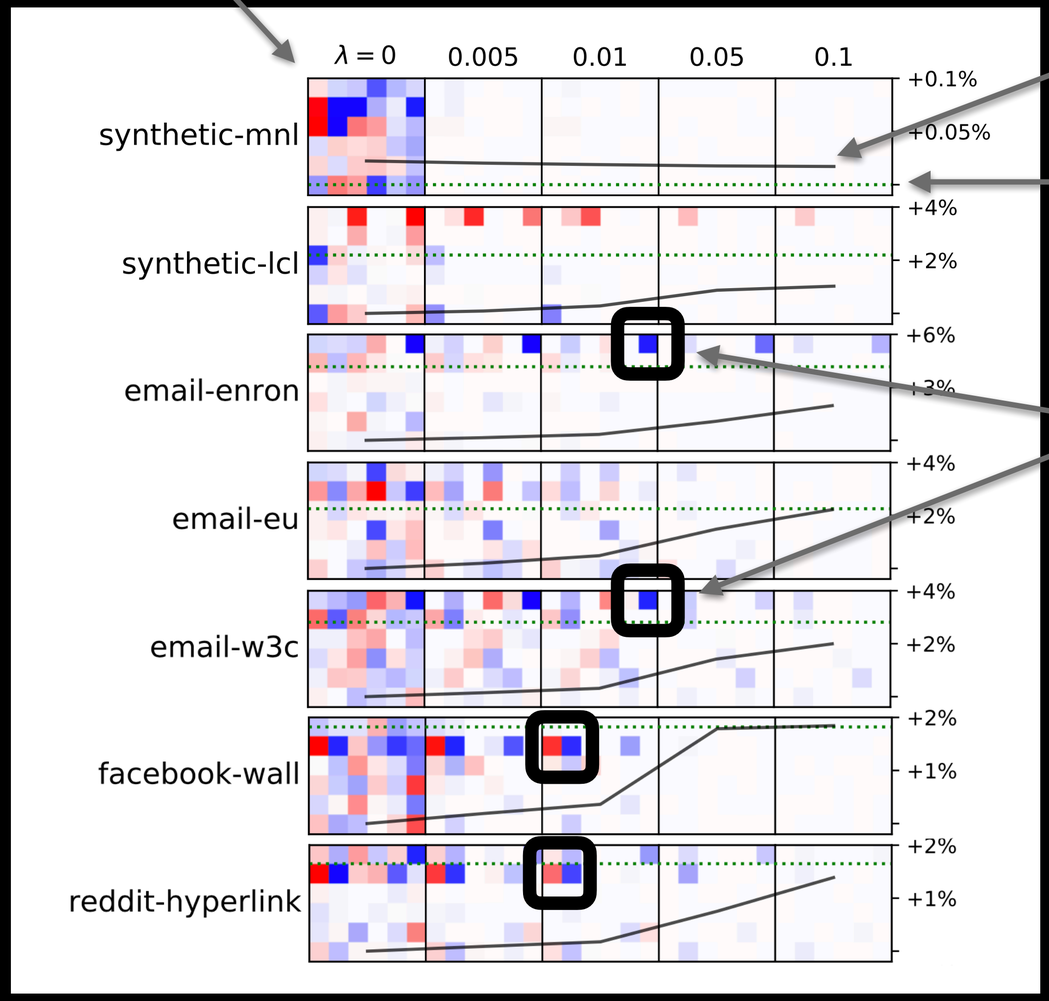

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"cluttered inbox"
high choice set reciprocal recency
→ in-degree less important

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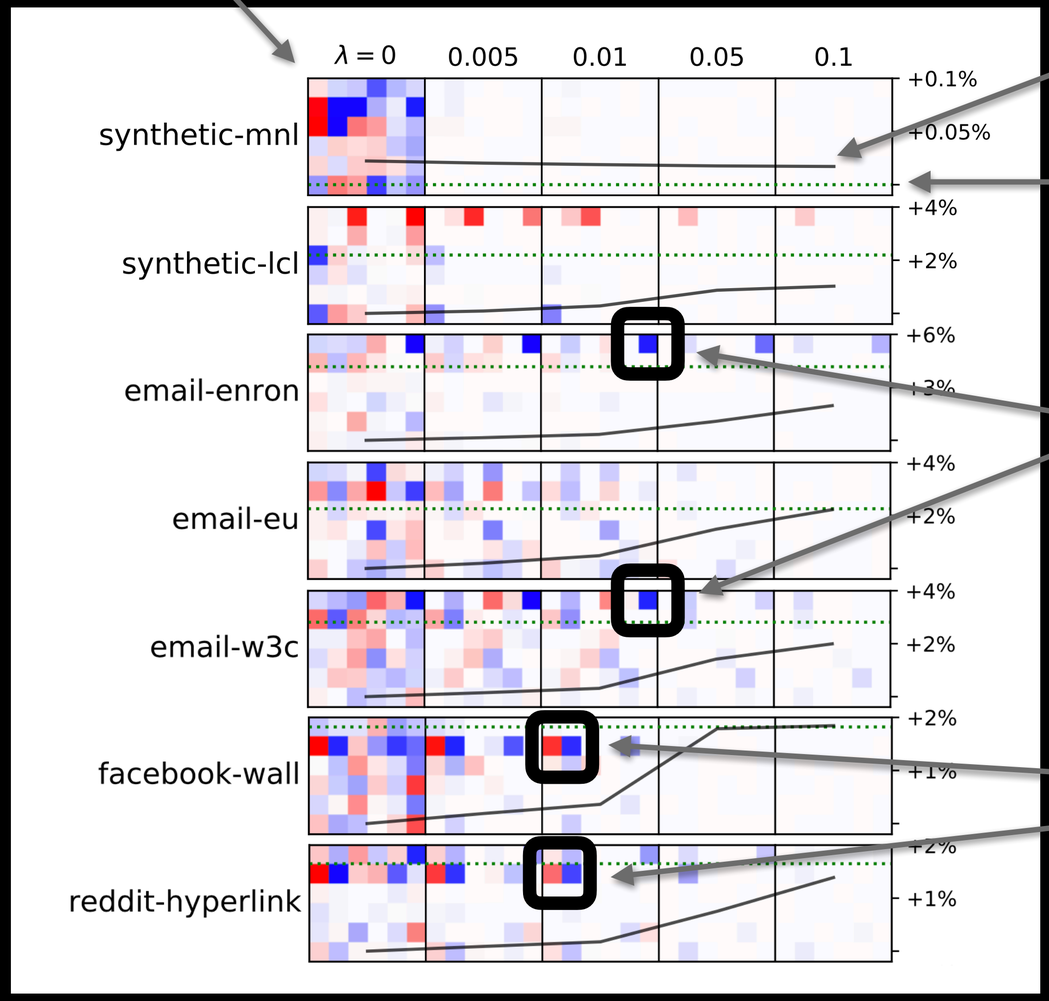

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6. reciprocal recency

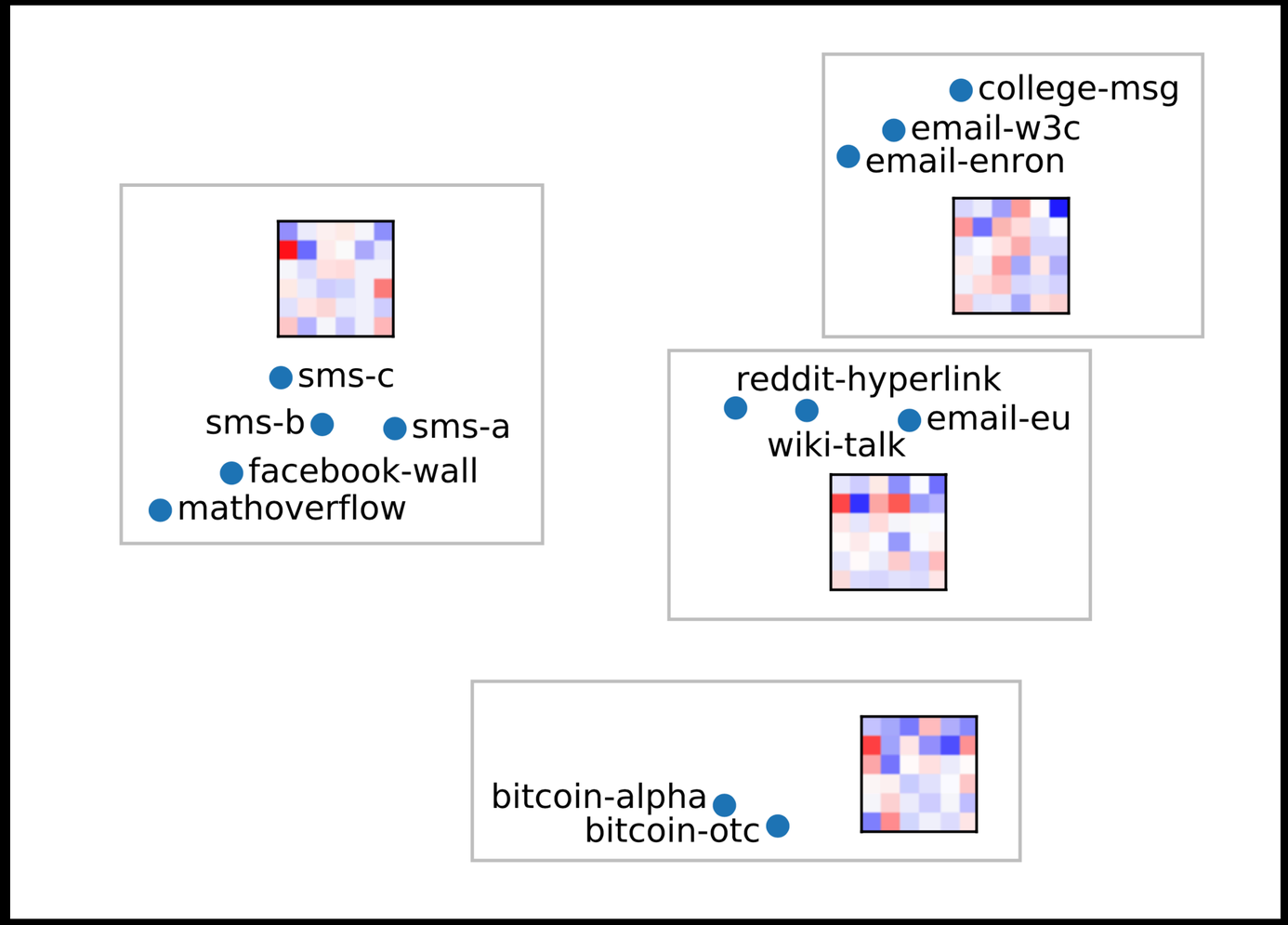
"cluttered inbox"
high choice set reciprocal recency
→ in-degree less important

red: *"cocktail party introduction"*
high choice set in-degree
→ shared neighbors more important

blue: *"familiarity saturation"*
high choice set shared neighbors
→ shared neighbors less important

Similar datasets have similar feature context effects

T-SNE embedding of learned A matrices



Concluding thoughts

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Feature context effects extend item-level effects
LCL offers an interpretable and tractable way to reveal them

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Features correlate
Causal context effects?
Handling nonlinearity?

Concluding thoughts

Slides: bit.ly/lcl-slides-2
Preprint: bit.ly/lcl-paper
Code: bit.ly/lcl-code
Data: bit.ly/lcl-data

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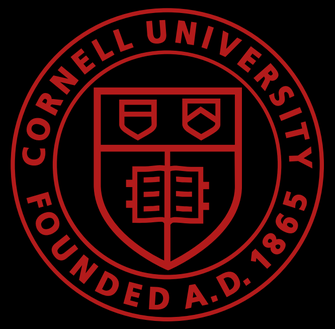

Thank you!

More questions or ideas?
Email me: kt@cs.cornell.edu
@kiran_tomlinson

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