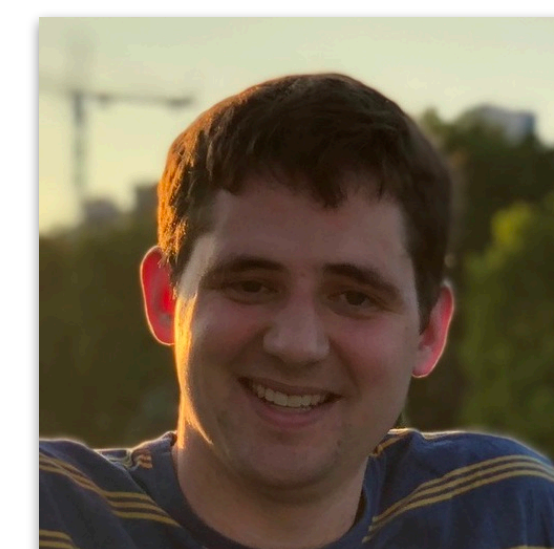


Slides: bit.ly/lcl-slides
Preprint: bit.ly/lcl-paper
Code: bit.ly/lcl-code
Data: bit.ly/lcl-data

Learning Context Effects in Triadic Closure

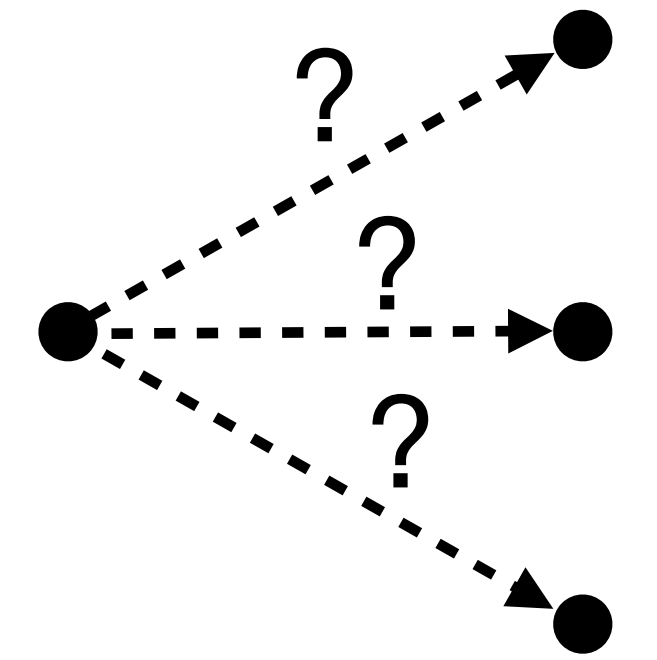
Kiran Tomlinson



SINM 2020

research with Austin R. Benson

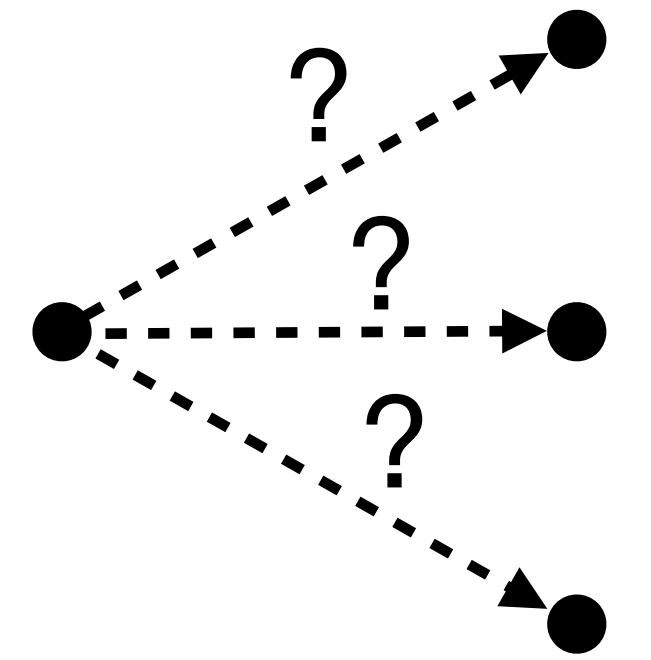
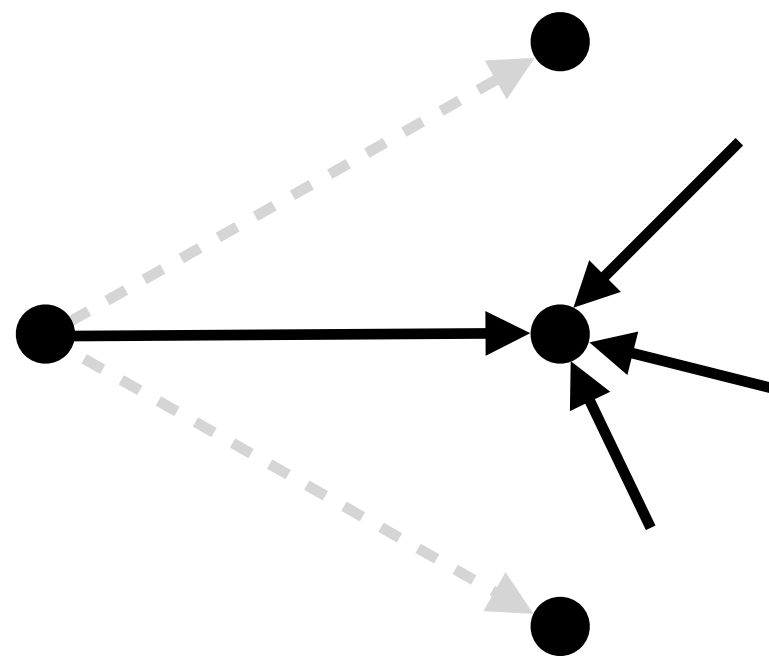
What factors drive edge formation?



What factors drive edge formation?

Preferential attachment

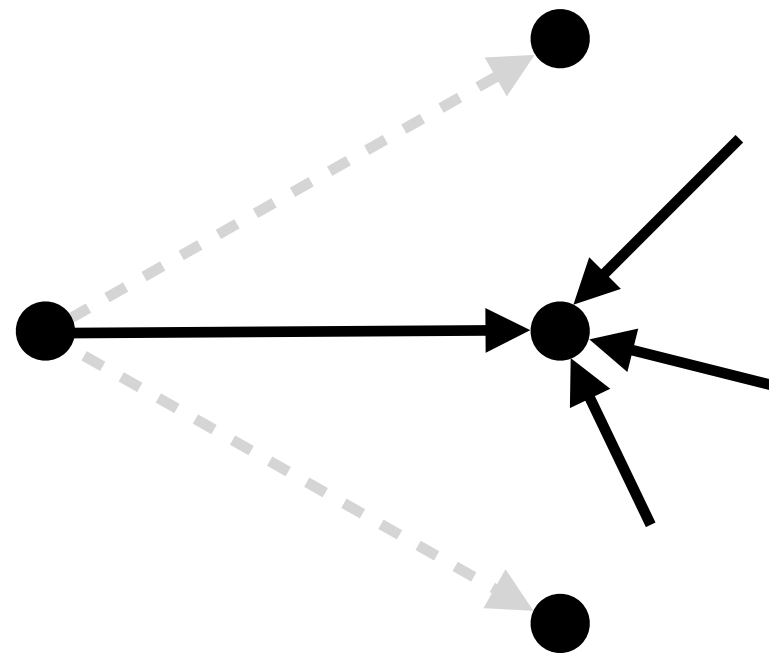
(Barabási & Albert, *Science* 1999)



What factors drive edge formation?

Preferential attachment

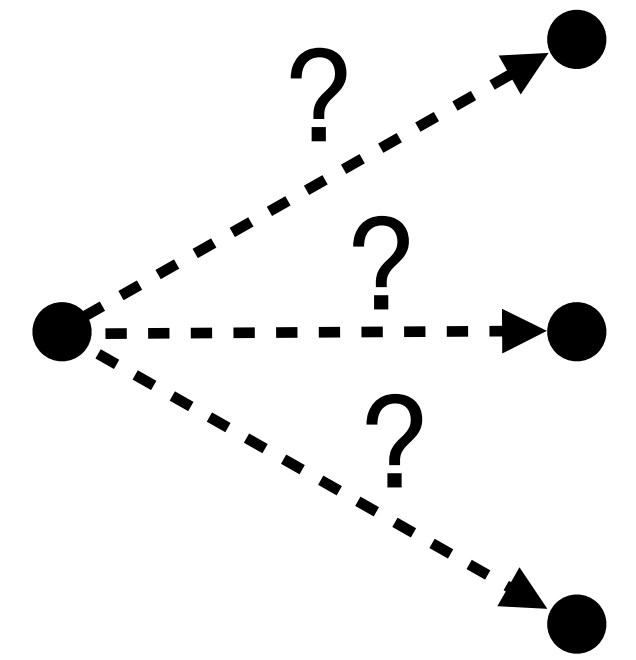
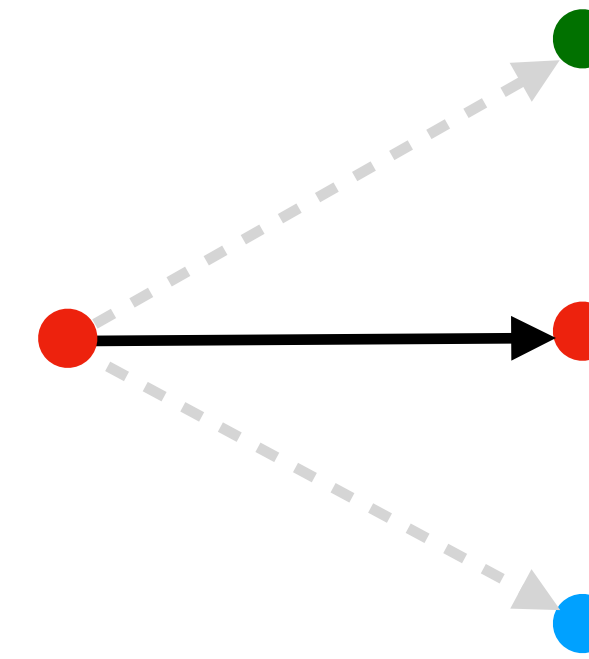
(Barabási & Albert, *Science* 1999)



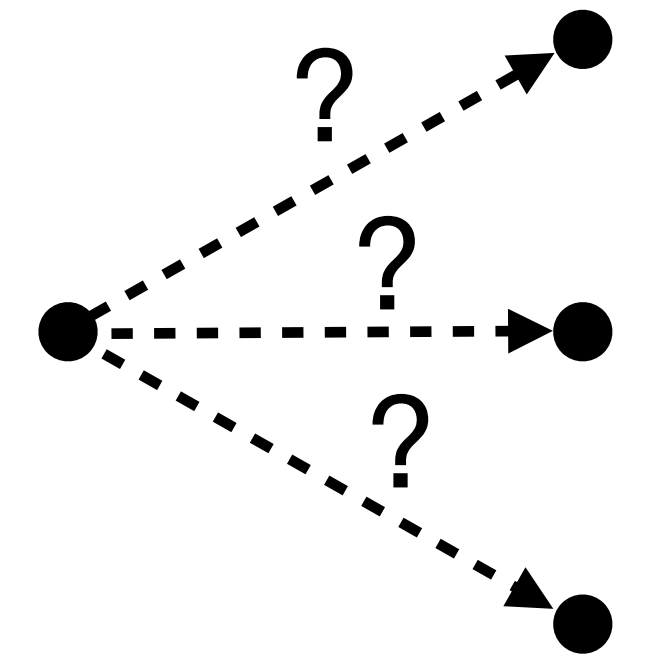
Homophily

(McPherson et al., *Annual Review of Sociology* 2001)

(Papadopoulos et al., *Nature* 2012)

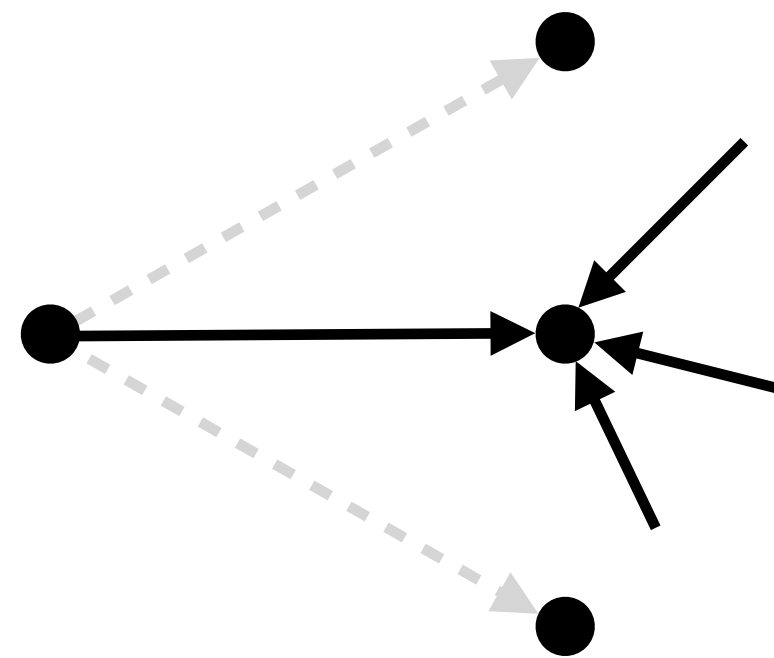


What factors drive edge formation?



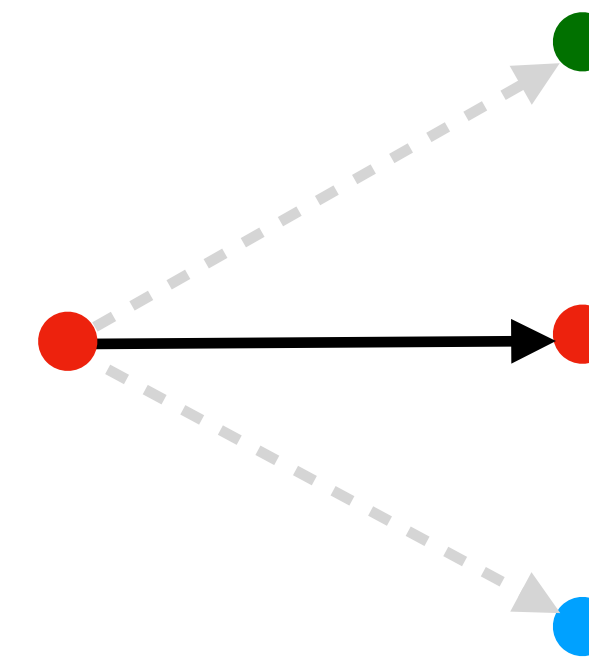
Preferential attachment

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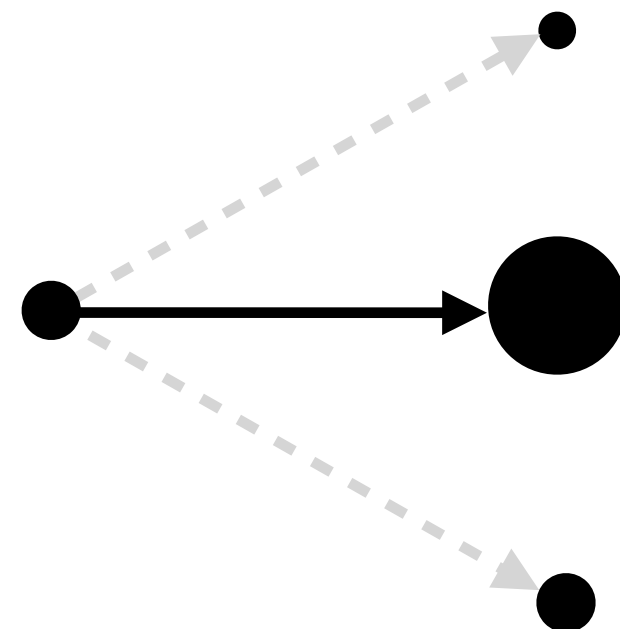
Homophily

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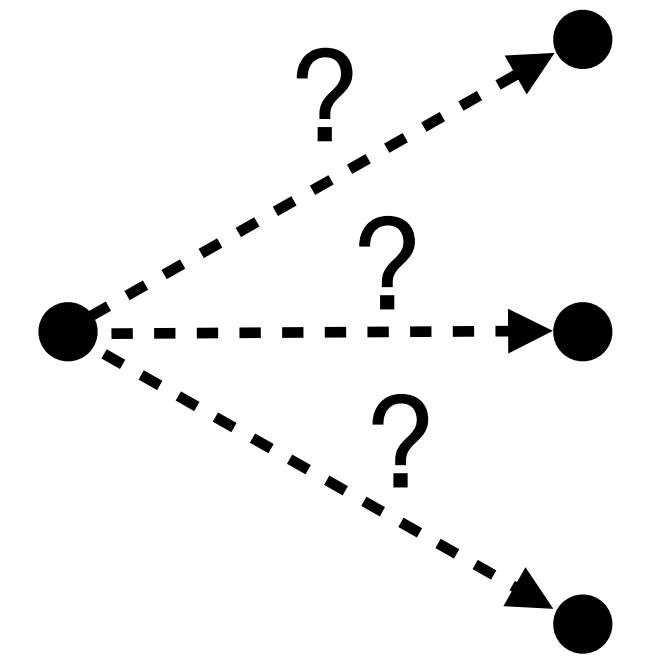


Fitness

(Bianconi & Barabási, *Europhysics Letters* 2001)
(Caldarelli et al., *Physical Review Letters* 2002)

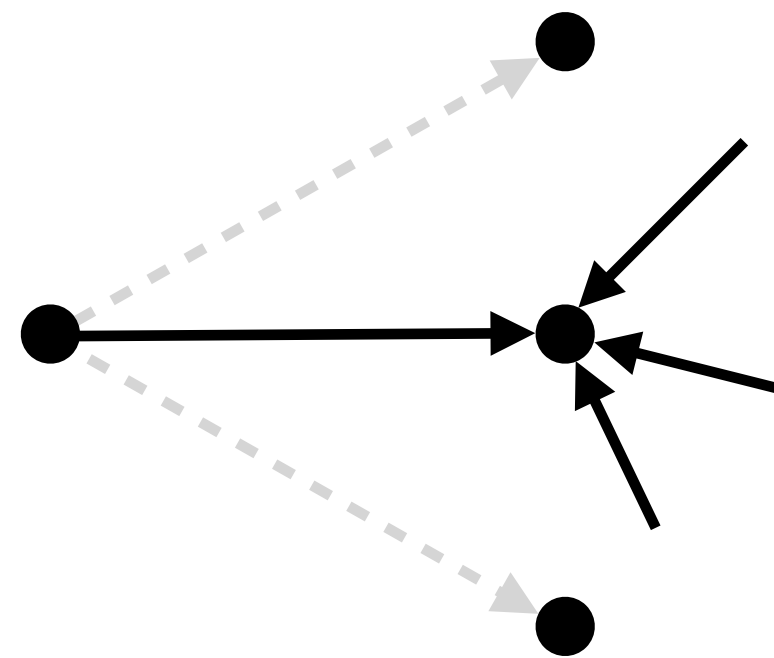


What factors drive edge formation?



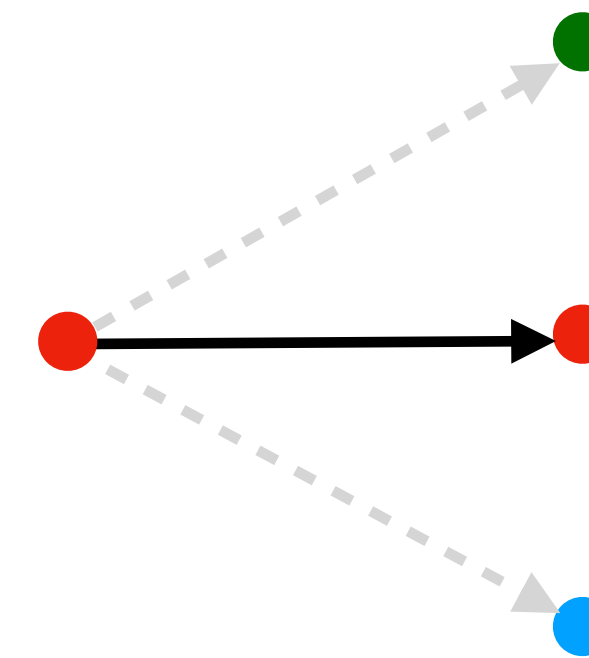
Preferential attachment

(Barabási & Albert, *Science* 1999)



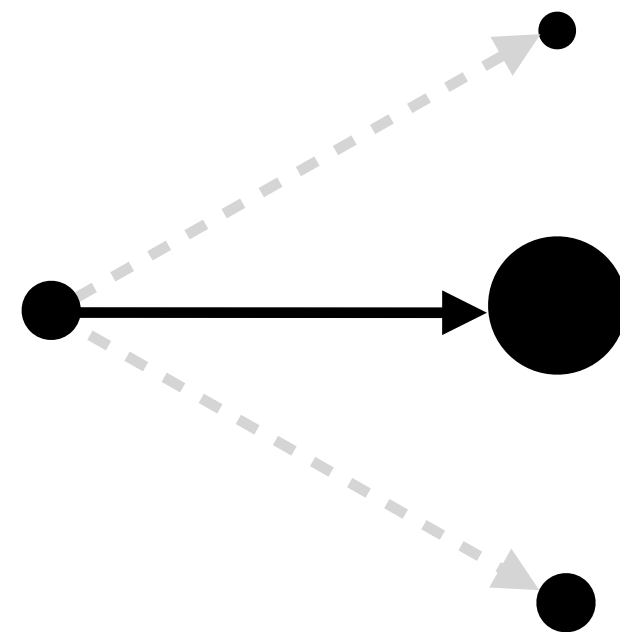
Homophily

(McPherson et al., *Annual Review of Sociology* 2001)
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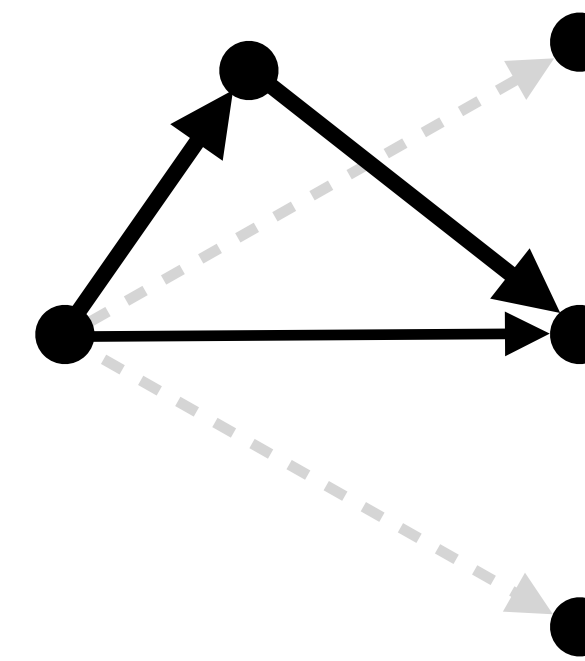
Fitness

(Bianconi & Barabási, *Europhysics Letters* 2001)
(Caldarelli et al., *Physical Review Letters* 2002)



Triadic closure

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)
(Jin et al., *Physical Review E* 2001)



“Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

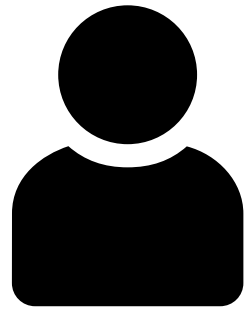
(Gupta & Porter, *arXiv* 2020)

“Choosing to grow a graph”

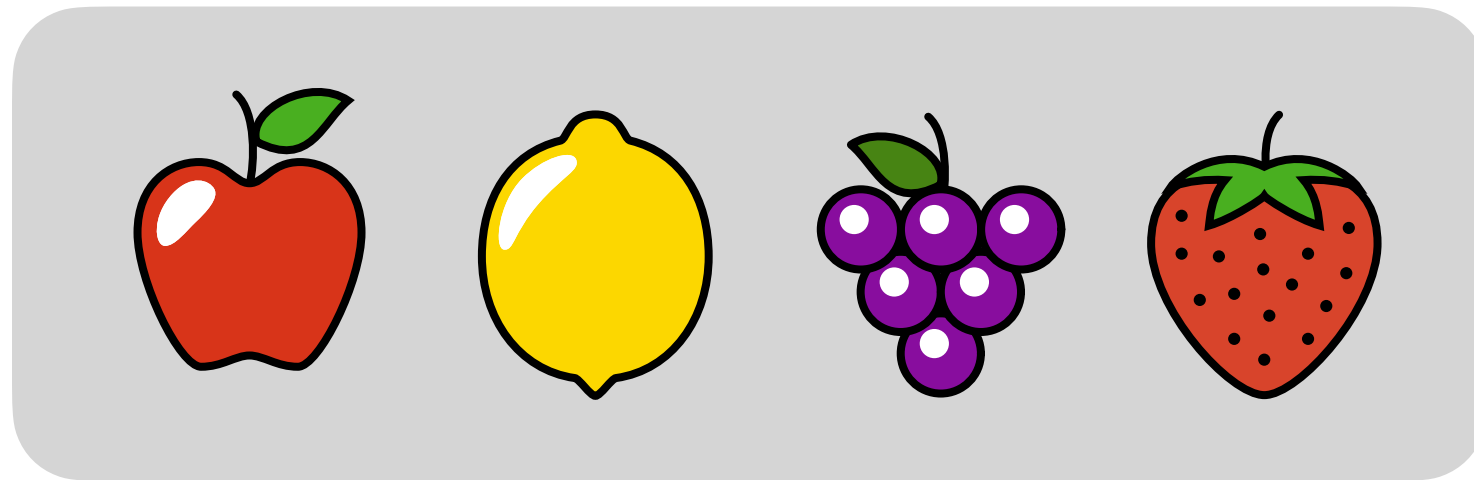
(Overgoor et al., *SINM* '19 & *WWW* '19)

(Gupta & Porter, *arXiv* 2020)

Traditional discrete choice:



chooser



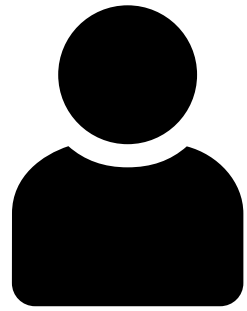
choice set

“Choosing to grow a graph”

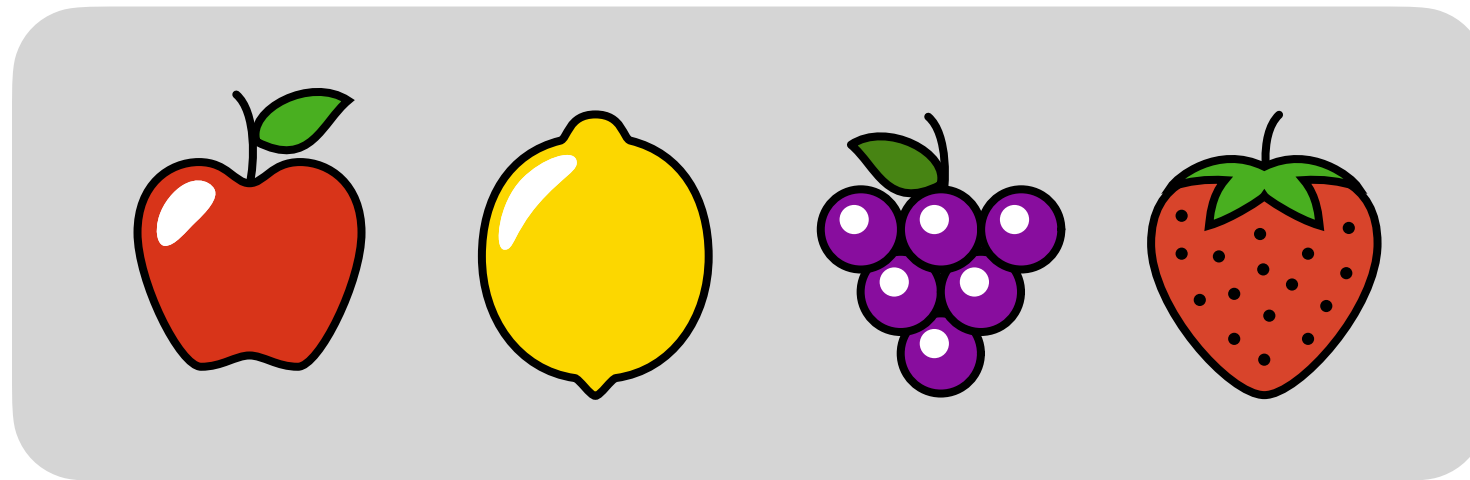
(Overgoor et al., *SINM* '19 & *WWW* '19)

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Traditional discrete choice:



chooser



choice set

(under-explored in sociology)

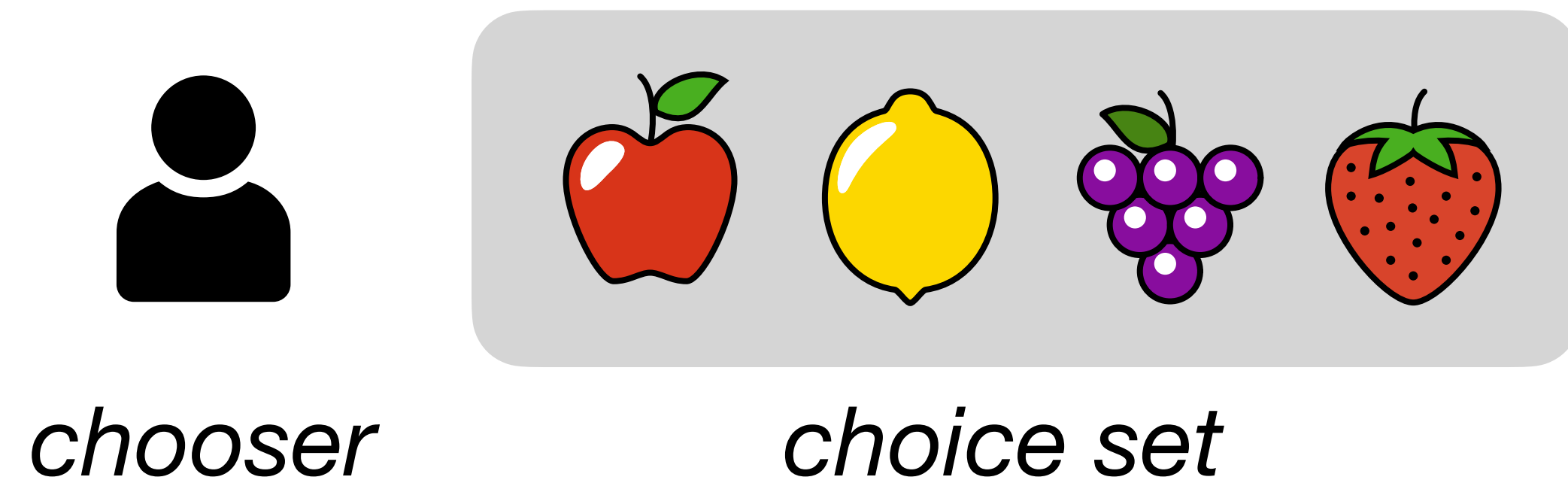
(Bruch & Feinberg, *Annual Review of Sociology* 2017)

“Choosing to grow a graph”

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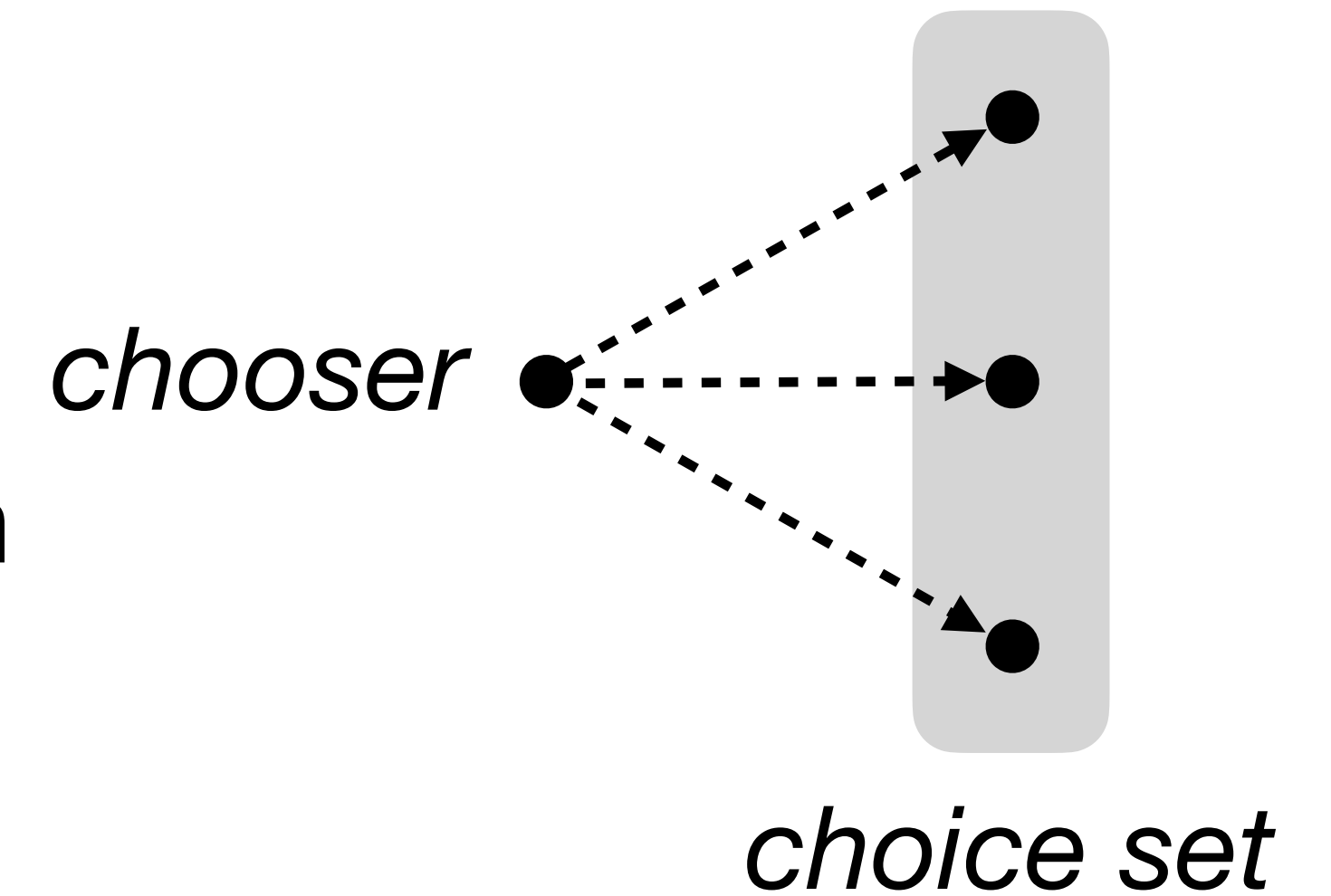
Traditional discrete choice:



(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)

→
in network growth

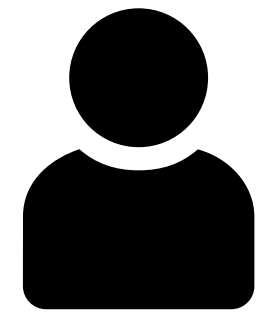


“Choosing to grow a graph”

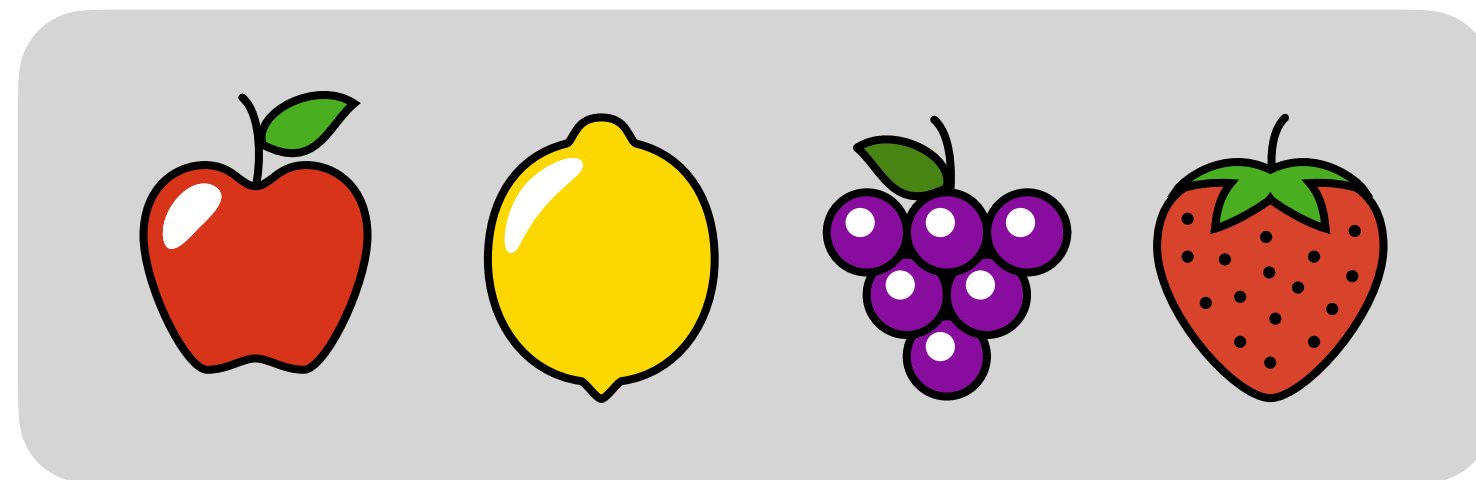
(Overgoor et al., *SINM* '19 & *WWW* '19)

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Traditional discrete choice:



chooser



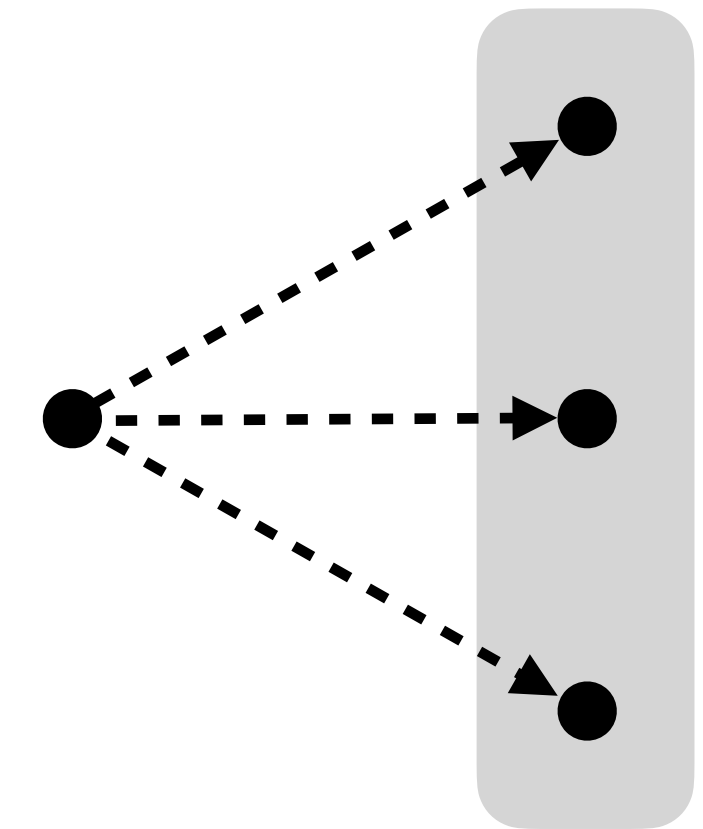
choice set

(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)

→
in network growth

chooser



choice set

Key usage

Timestamped edges

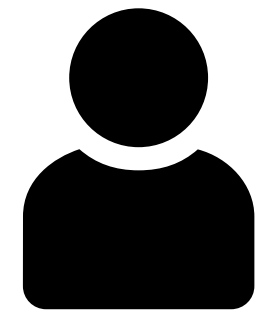
→ meaningful choice sets

“Choosing to grow a graph”

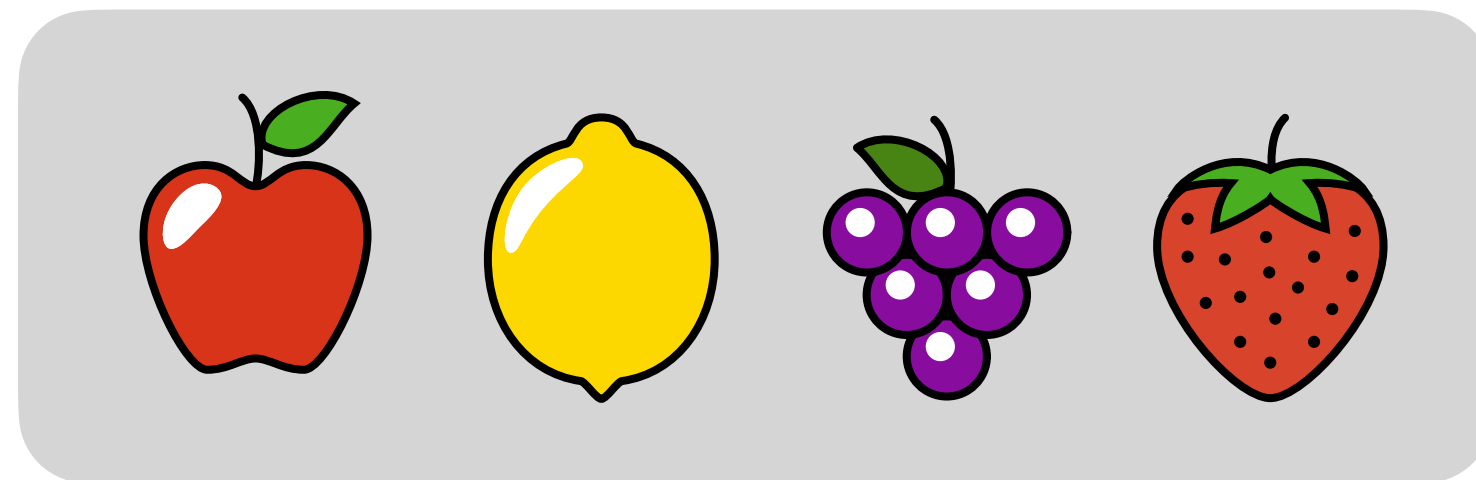
(Overgoor et al., *SINM* '19 & *WWW* '19)

(Gupta & Porter, *arXiv* 2020)

Traditional discrete choice:



chooser



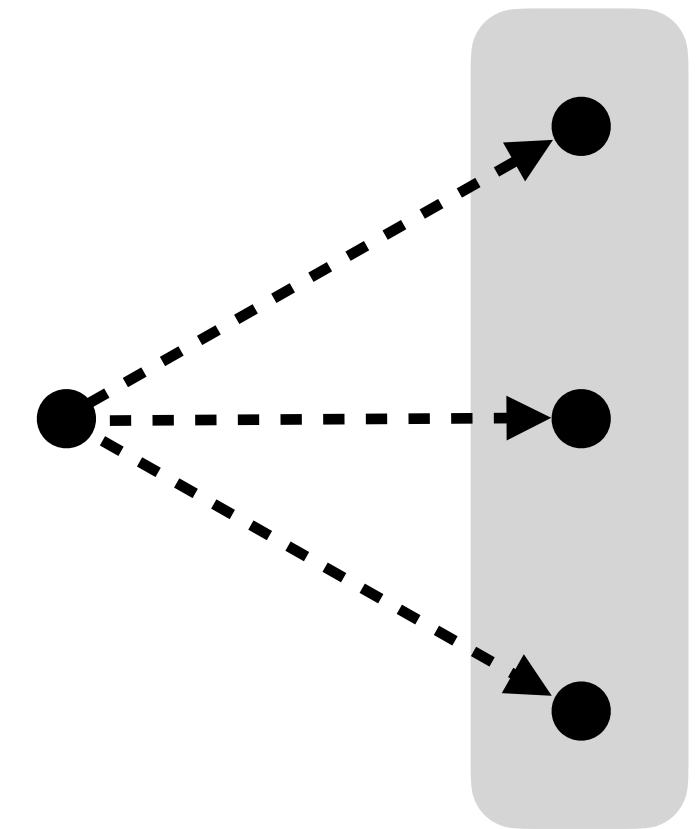
choice set

(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)

→
in network growth

chooser



choice set

Key usage

Timestamped edges

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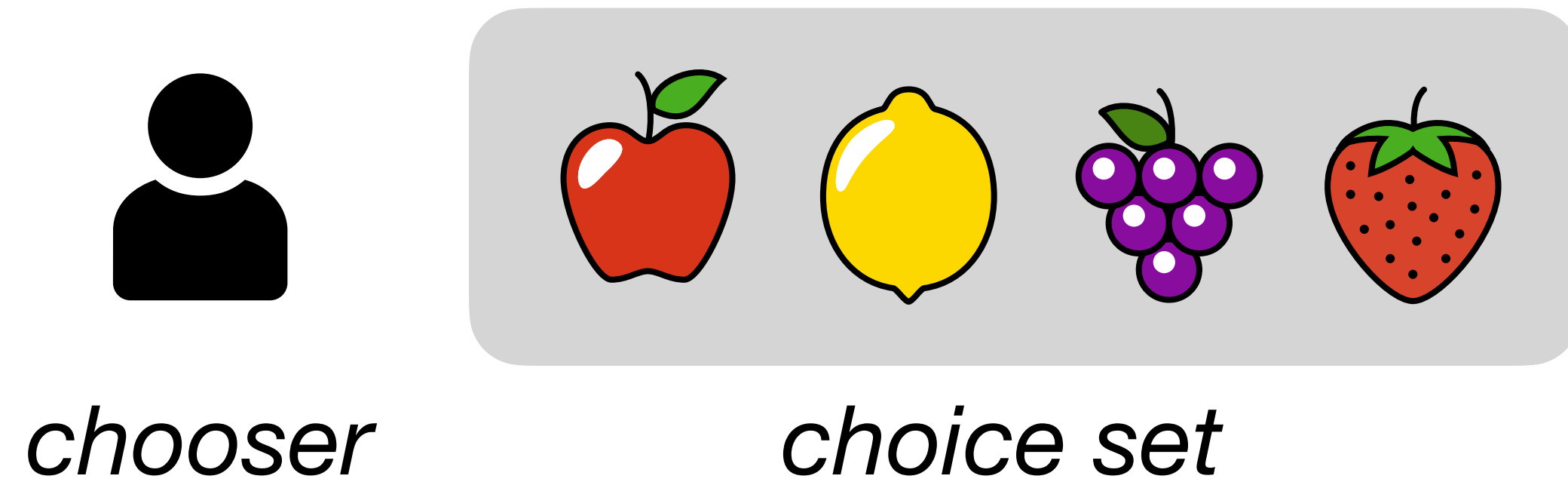
Infer relative importance of edge
formation mechanisms from data

“Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

(Gupta & Porter, *arXiv* 2020)

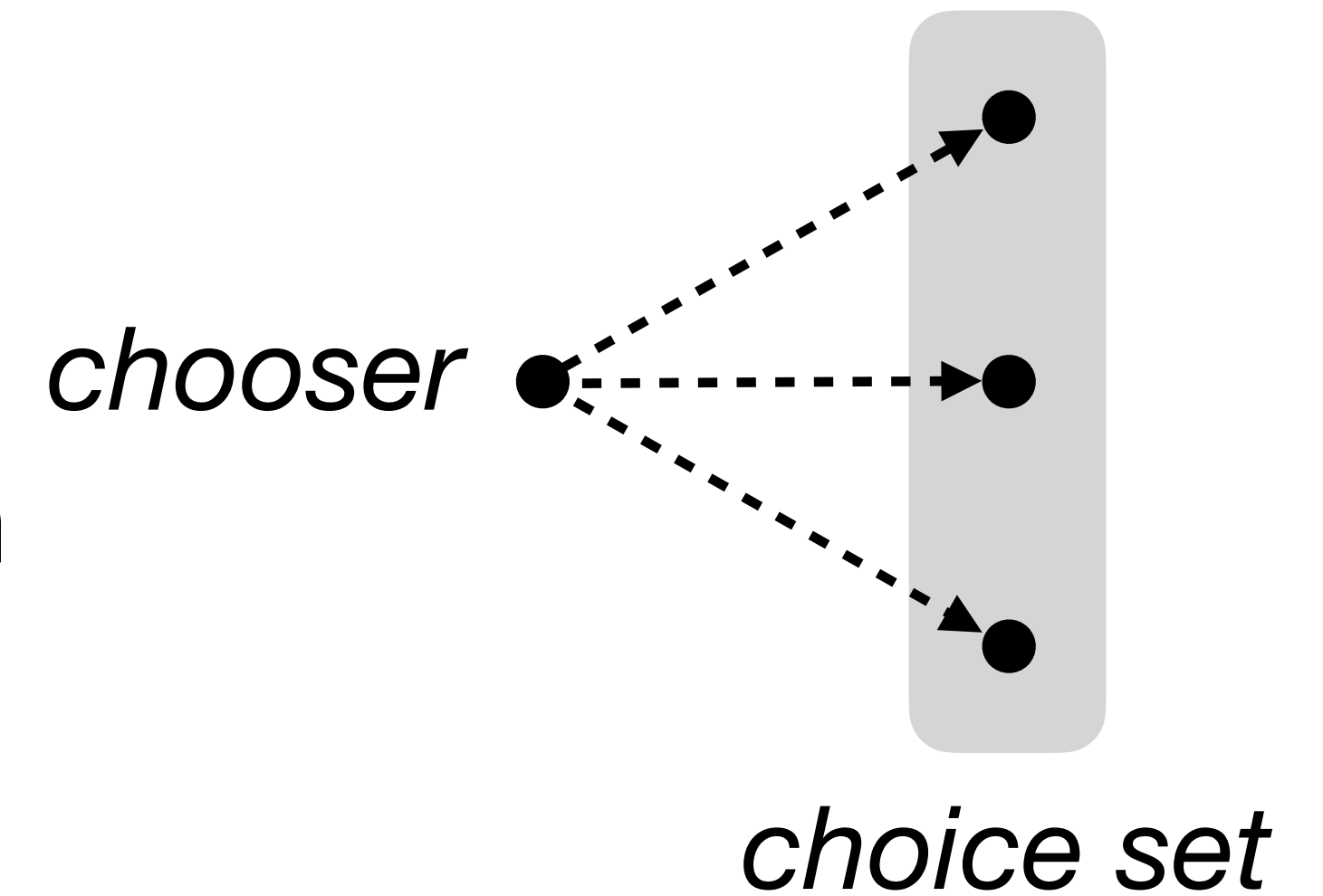
Traditional discrete choice:



(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)

→
in network growth



Key usage

Timestamped edges
→ meaningful choice sets

Infer relative importance of edge
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$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

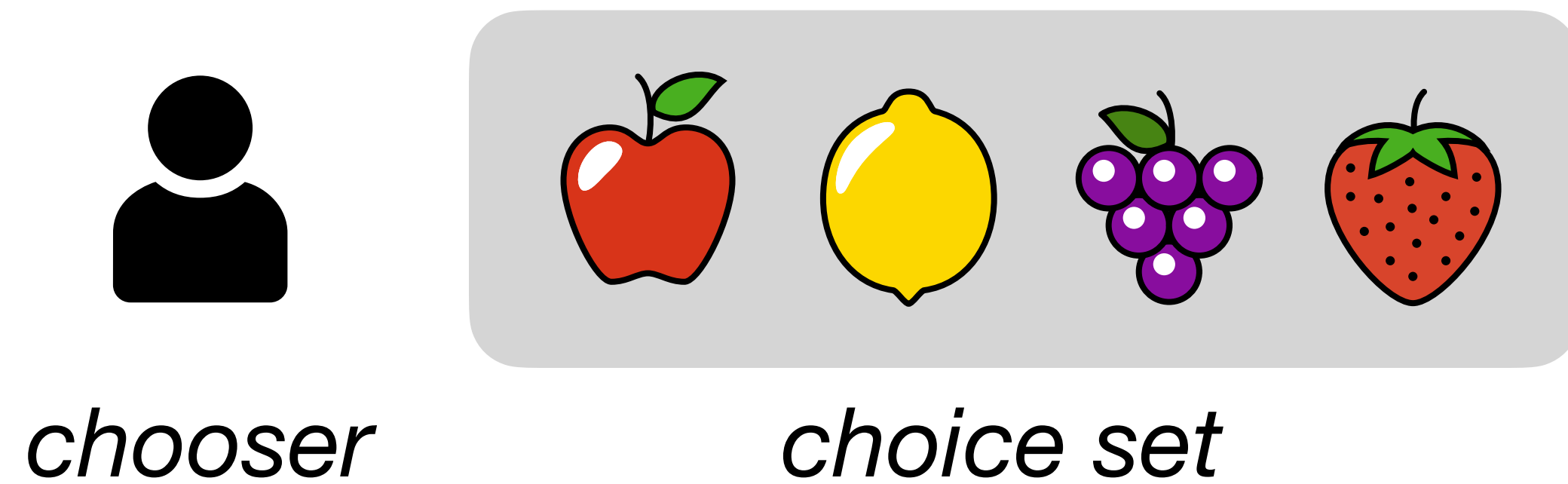
Multinomial logit
(MNL) (McFadden, 1973)

“Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

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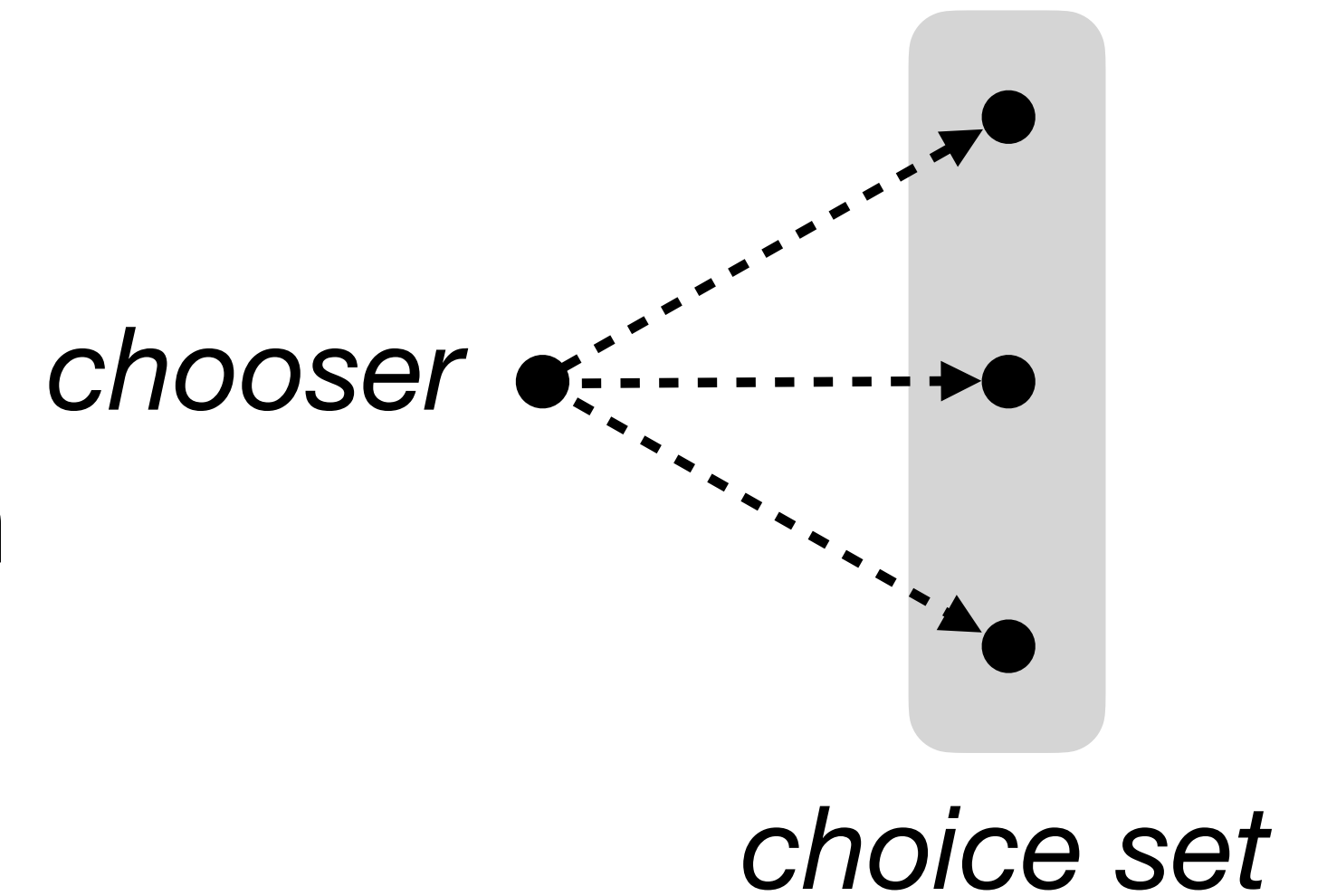
Traditional discrete choice:



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→
in network growth



Key usage

Timestamped edges
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Infer relative importance of edge
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$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

node

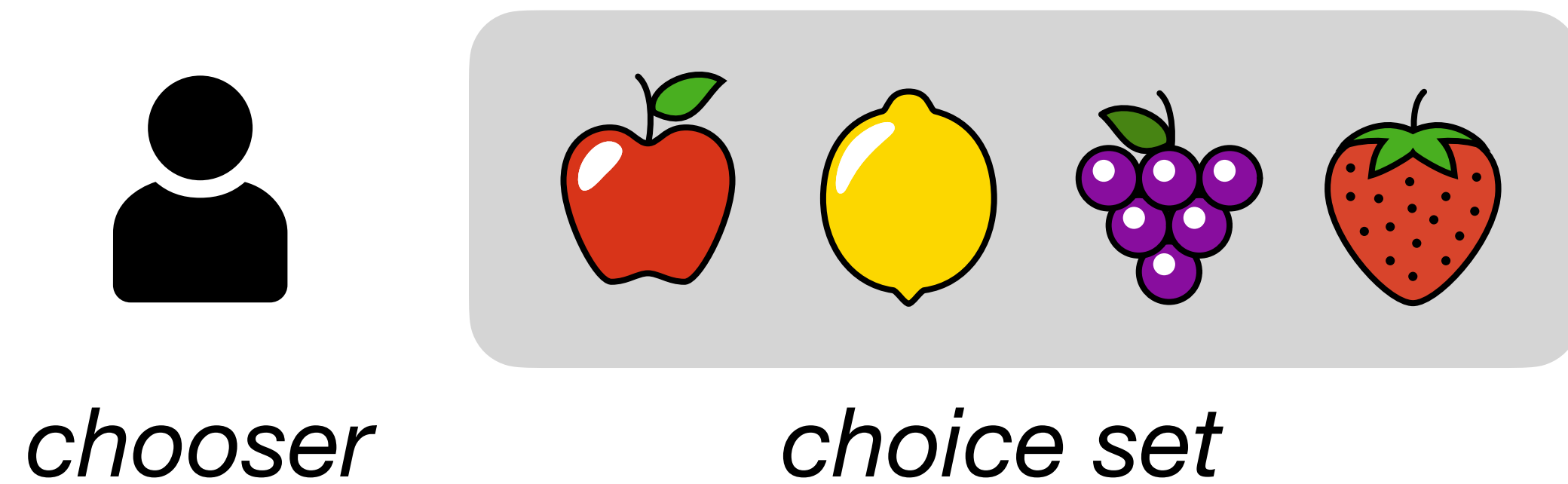
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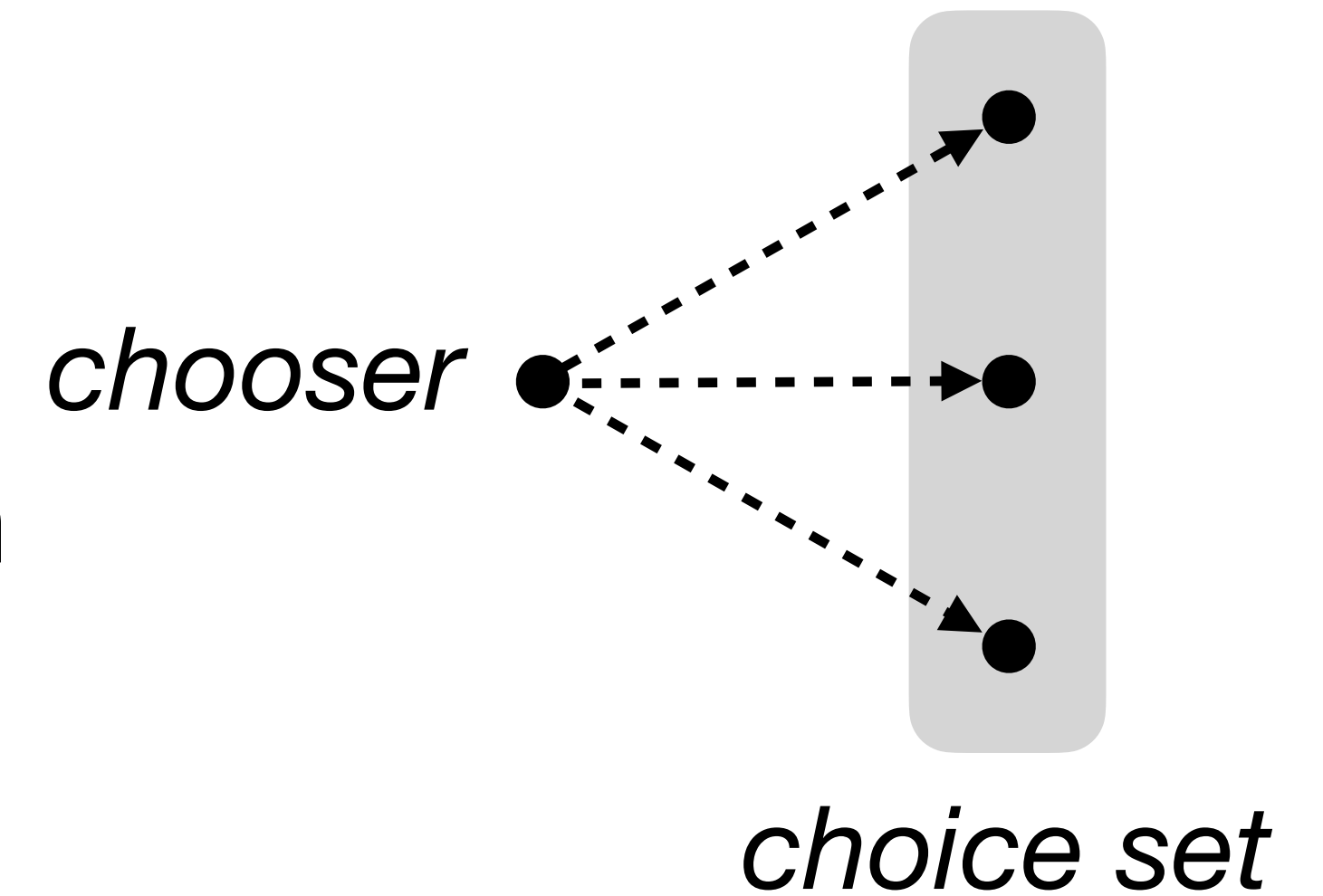
Traditional discrete choice:



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in network growth



Key usage

Timestamped edges
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Infer relative importance of edge
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$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

node choice set

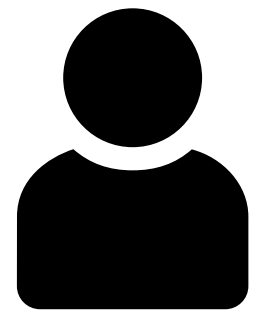
Multinomial logit
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“Choosing to grow a graph”

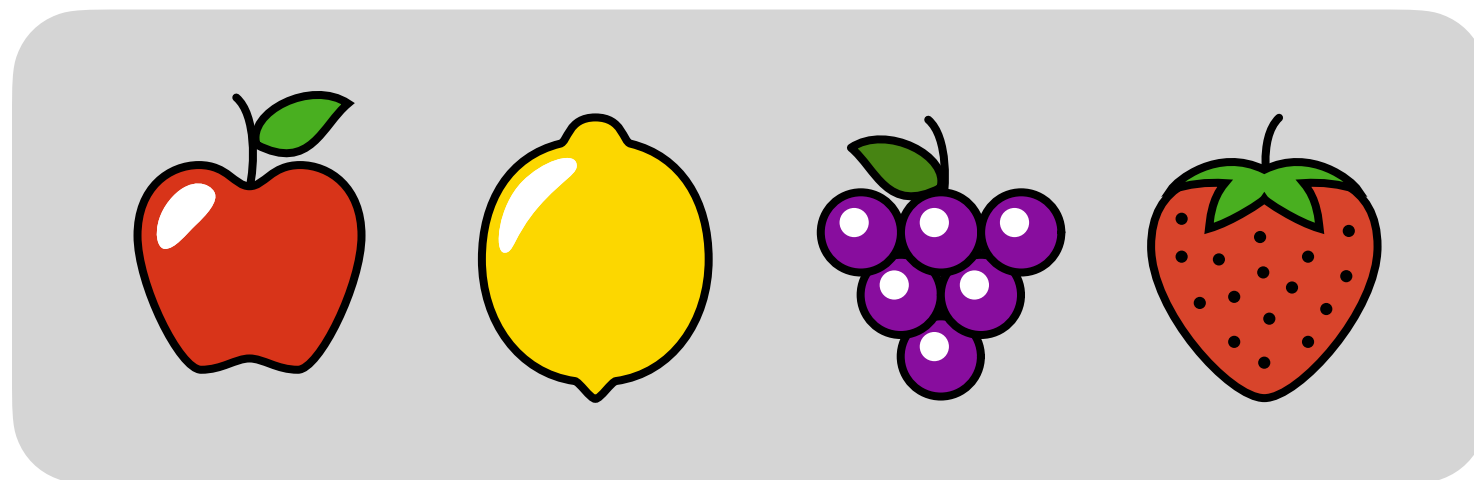
(Overgoor et al., *SINM* '19 & *WWW* '19)

(Gupta & Porter, *arXiv* 2020)

Traditional discrete choice:



chooser



choice set

(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)

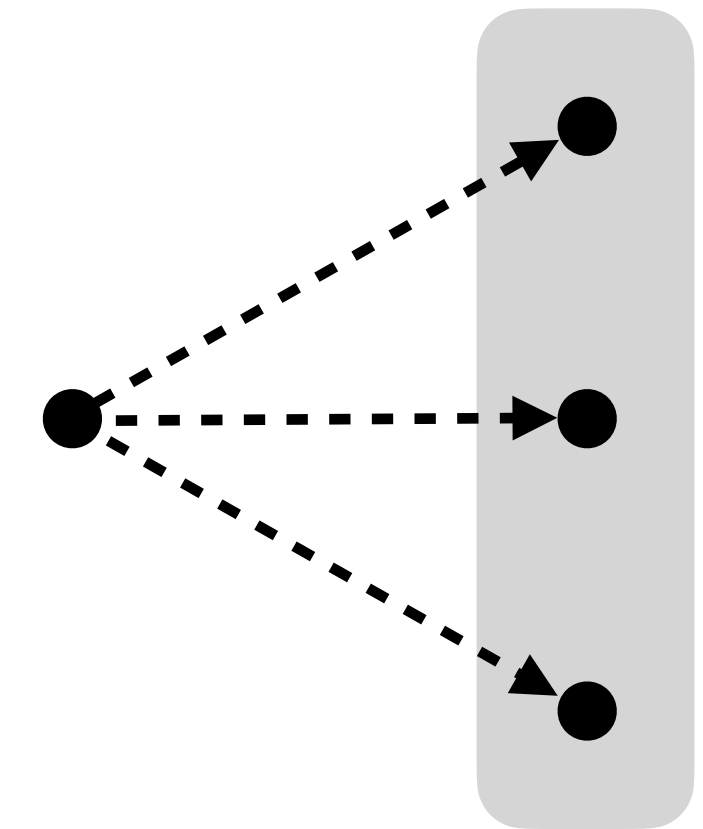
Key usage

Timestamped edges
→ meaningful choice sets

Infer relative importance of edge
formation mechanisms from data

→
in network growth

chooser



choice set

preferences

$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

node choice set

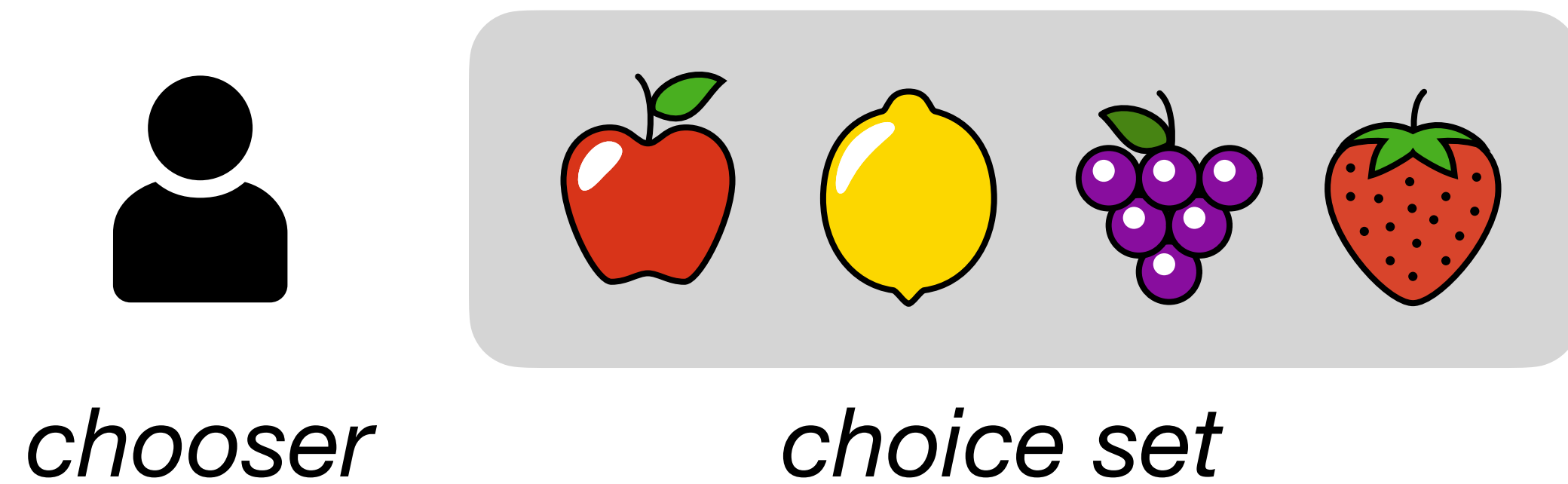
Multinomial logit
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Traditional discrete choice:



(under-explored in sociology)

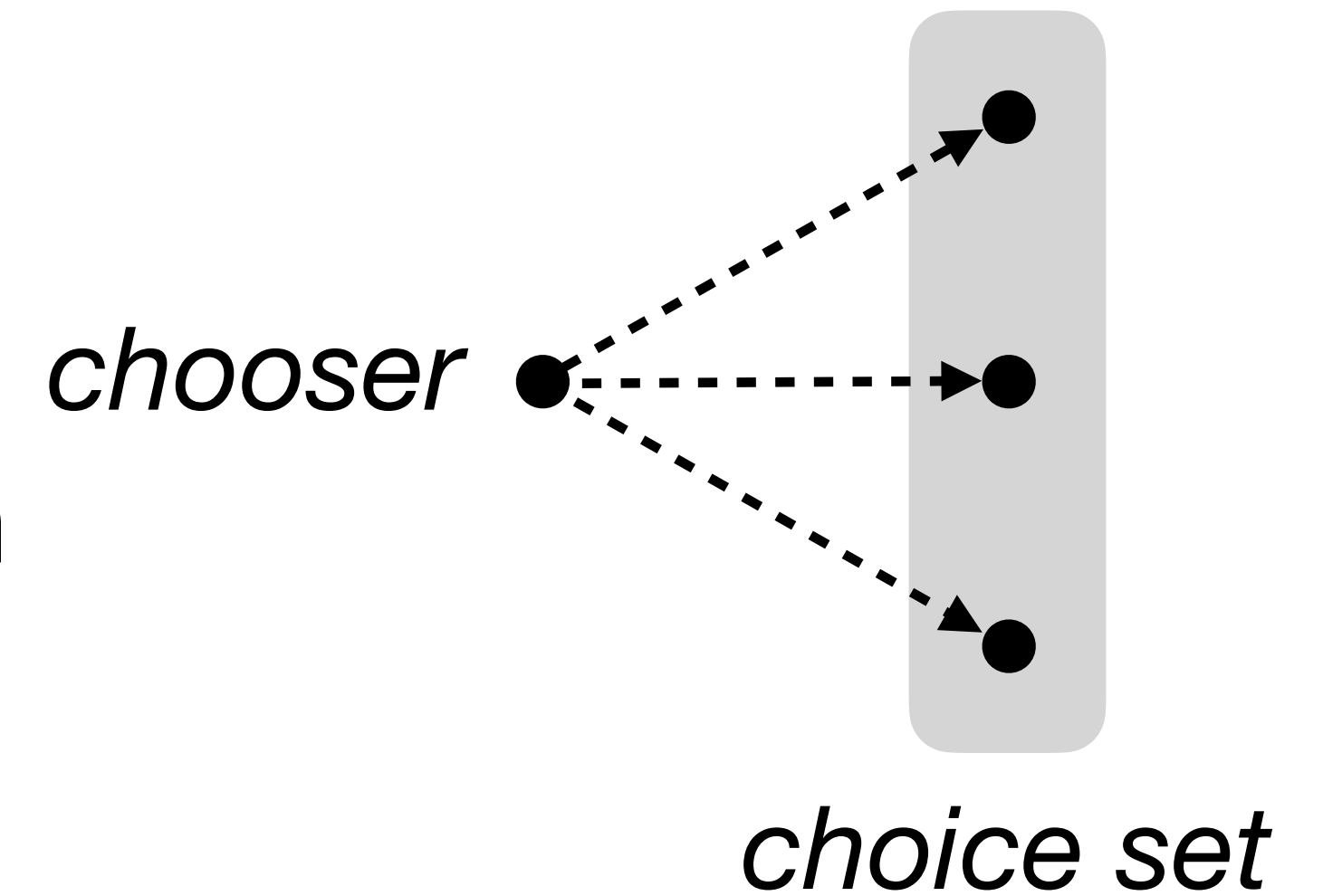
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Key usage

Timestamped edges
→ meaningful choice sets

Infer relative importance of edge
formation mechanisms from data

→
in network growth



preferences node features

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node choice set

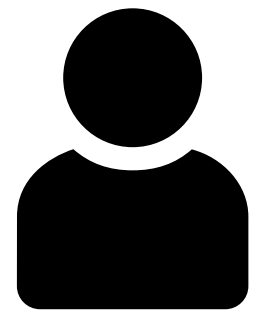
Multinomial logit
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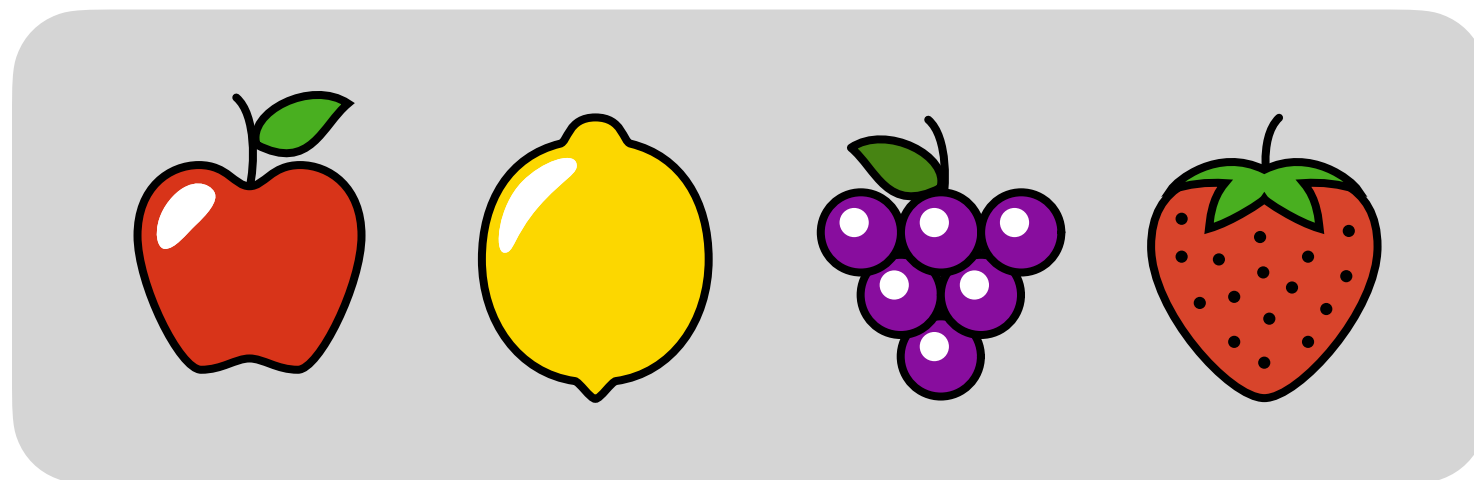
(Overgoor et al., *SINM* '19 & *WWW* '19)

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Traditional discrete choice:



chooser



choice set

(under-explored in sociology)

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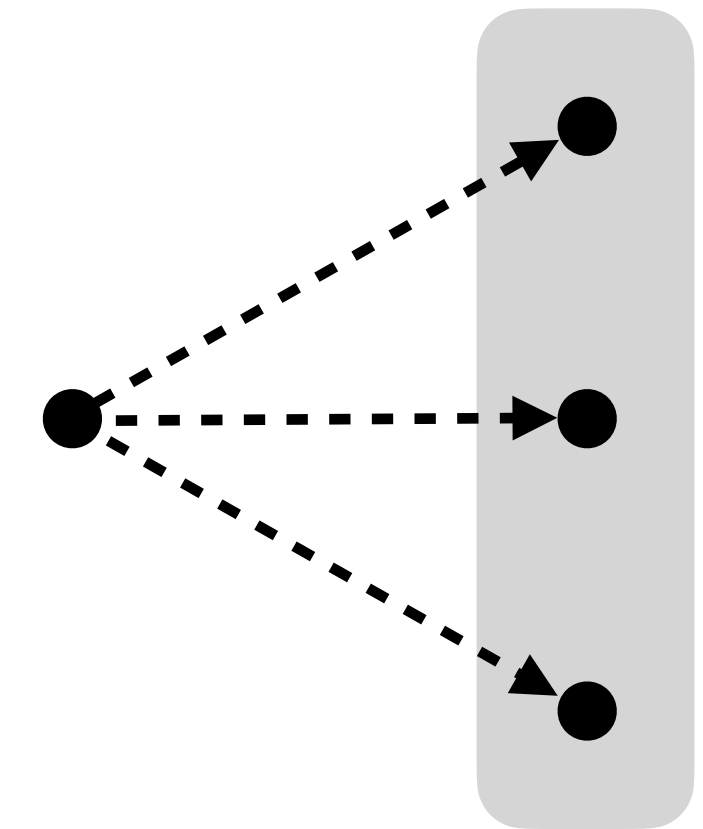
Key usage

Timestamped edges
→ meaningful choice sets

Infer relative importance of edge
formation mechanisms from data

→
in network growth

chooser



choice set

preferences

node features

(similarity, in-degree, fitness...)

$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

node choice set

Multinomial logit
(MNL) (McFadden, 1973)

The choice set affects preferences

The choice set affects preferences

Context effects

(Huber et al., *Journal of Consumer Research* 1982)

(Simonson & Tversky, *Journal of Marketing Research* 1992)

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e.g., *compromise effect*:

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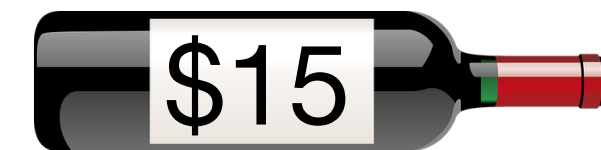
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The choice set affects preferences

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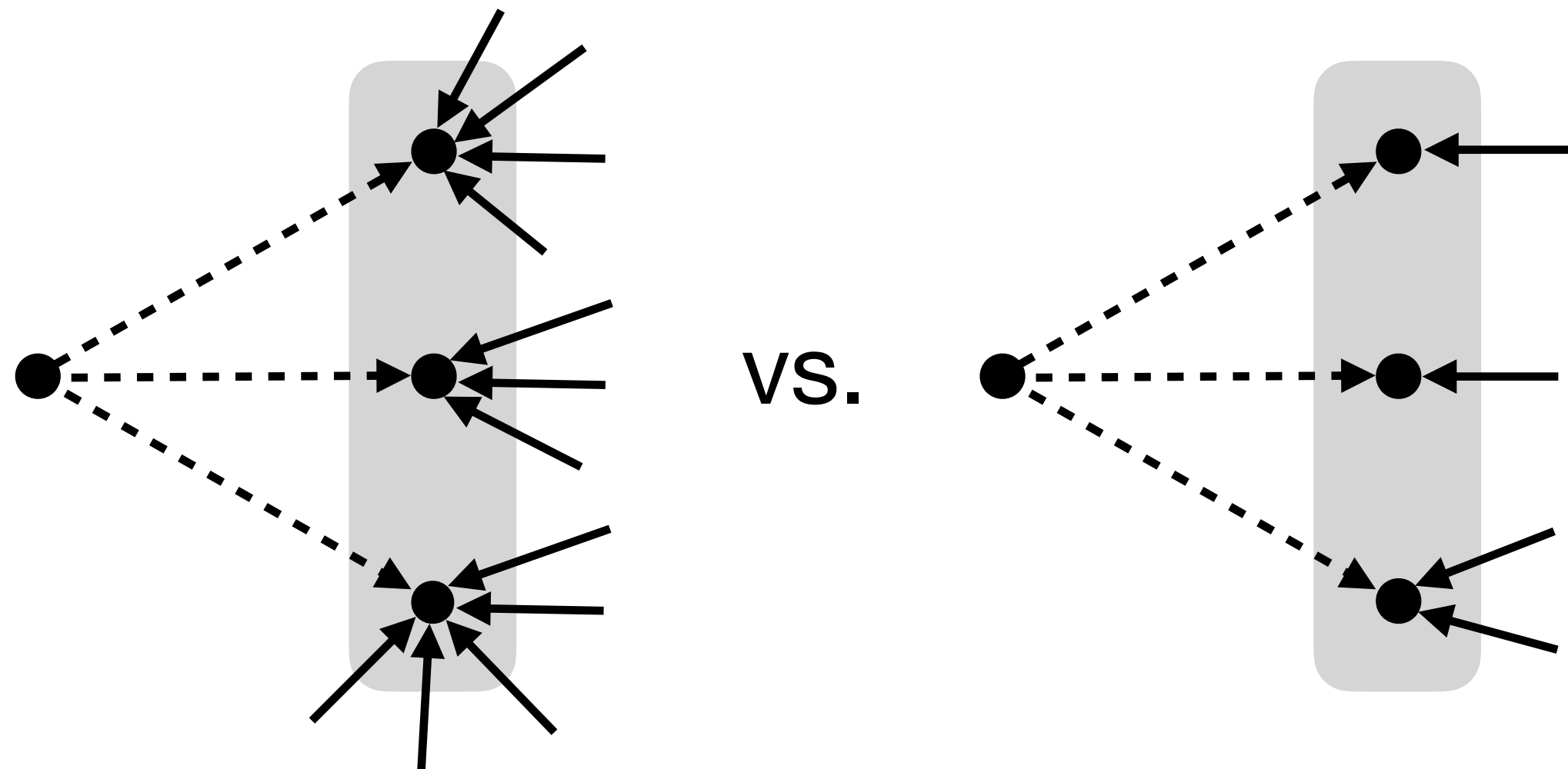
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In networks

e.g., how do preferences change
when choosing from a popular group?



The choice set affects preferences

Context effects

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e.g., *compromise effect*:

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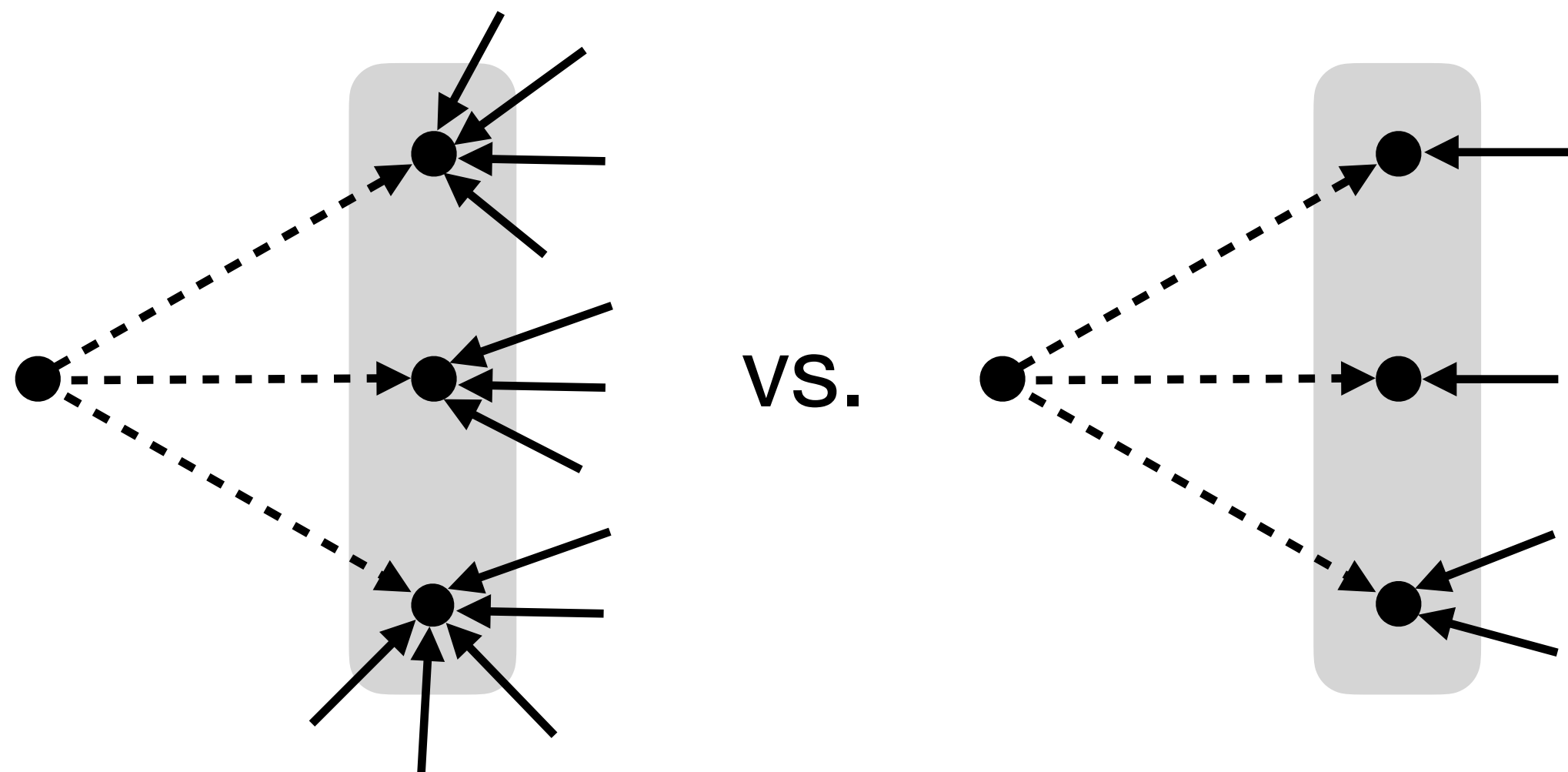
Our model:

Linear context logit (LCL)

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

In networks

e.g., how do preferences change when choosing from a popular group?



The choice set affects preferences

Context effects

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Our model:

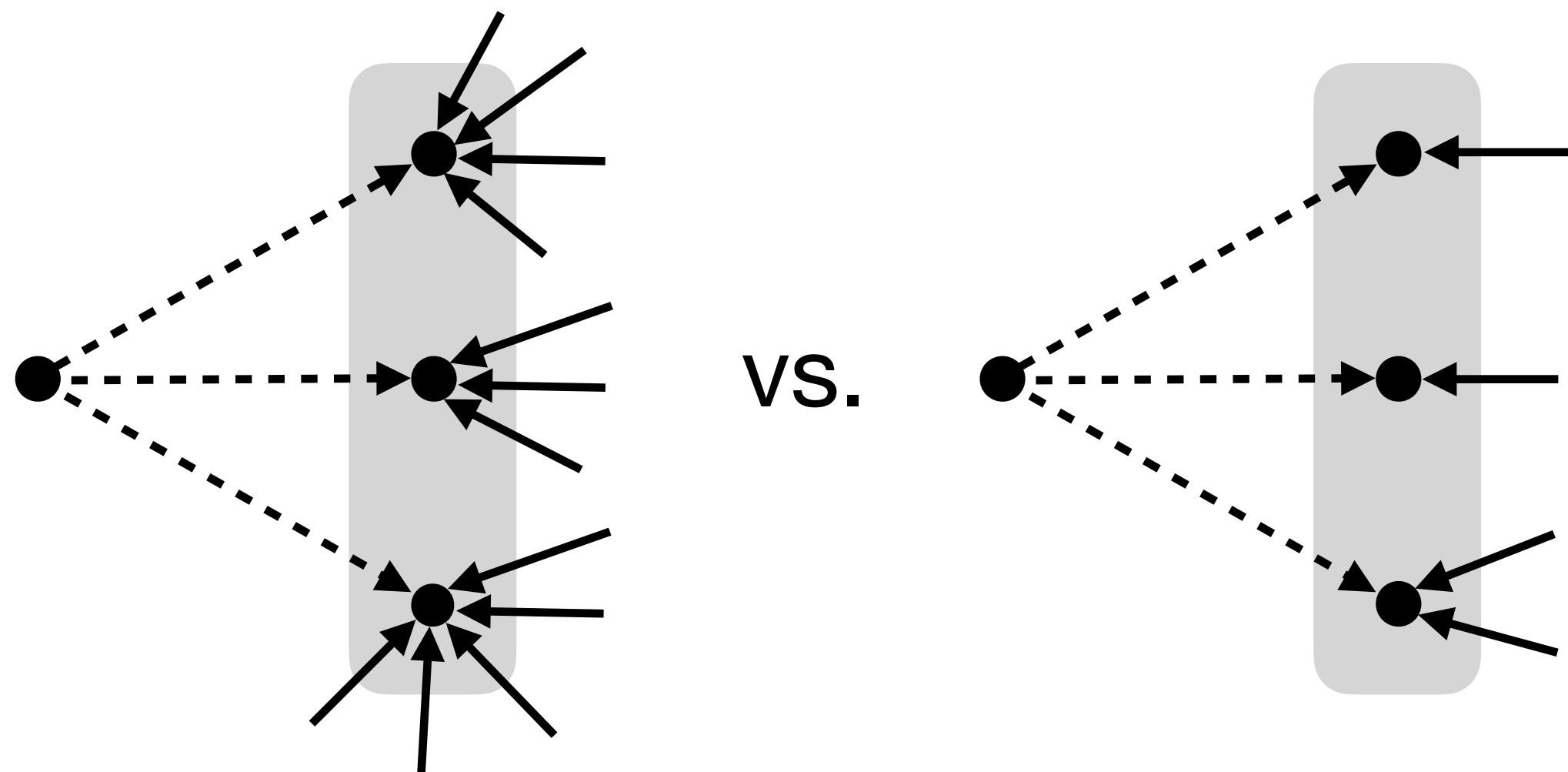
Linear context logit (LCL)

base preferences

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

In networks

e.g., how do preferences change when choosing from a popular group?



The choice set affects preferences

Context effects

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Our model:

Linear context logit (LCL)

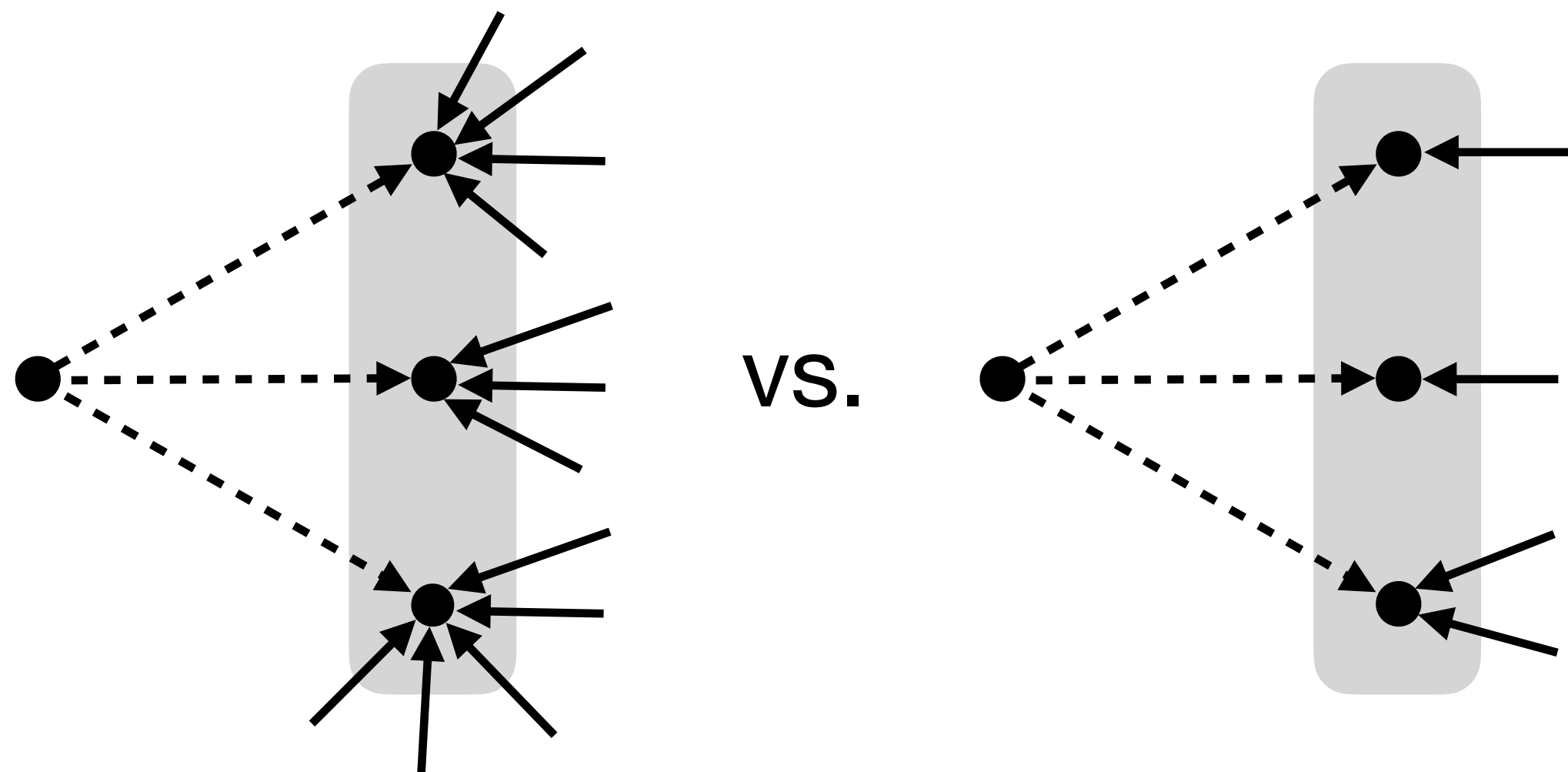
base preferences

context effect
matrix

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

In networks

e.g., how do preferences change
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The choice set affects preferences

Context effects

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Our model:

Linear context logit (LCL)

base preferences

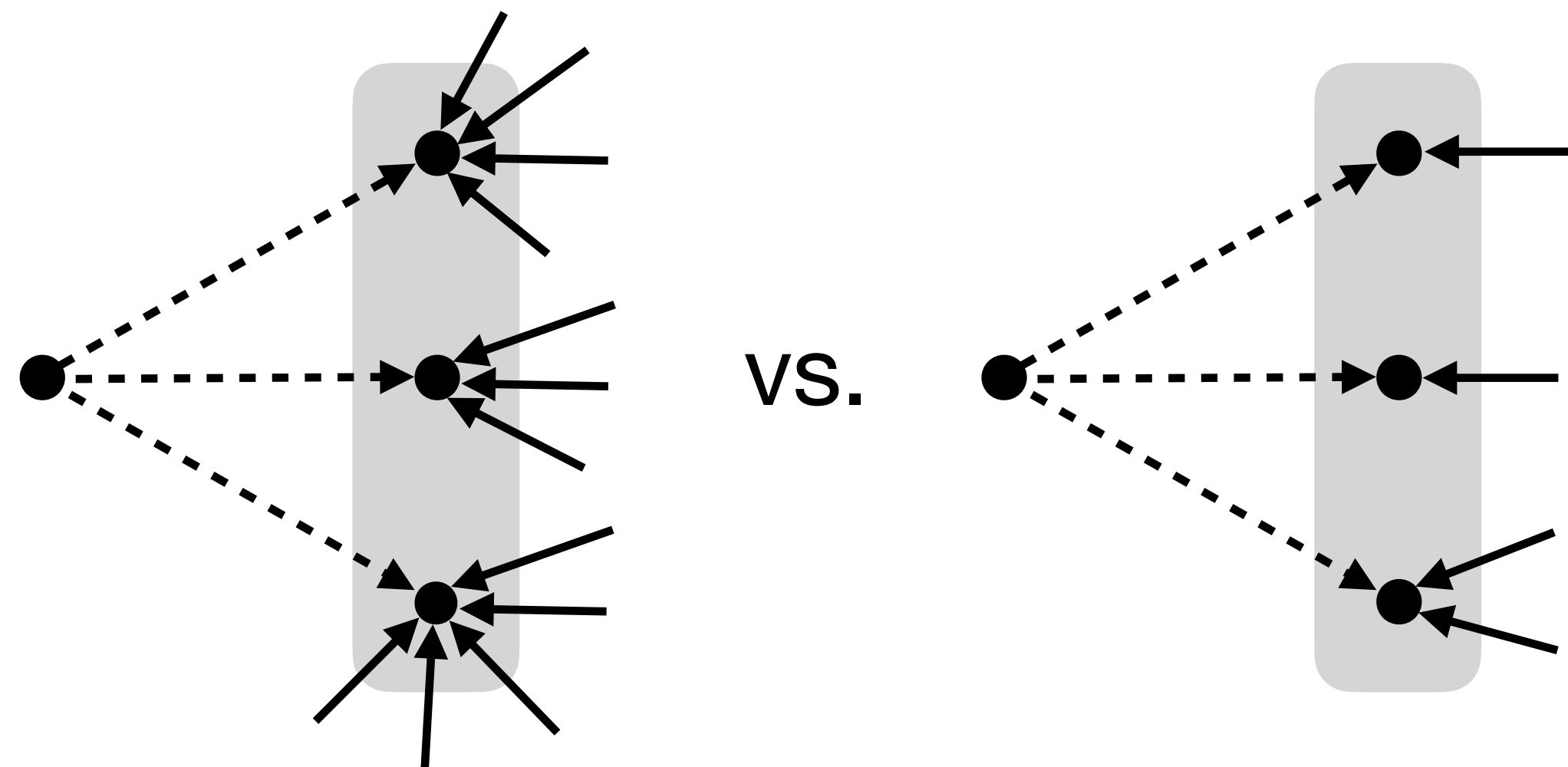
context effect
matrix

mean features
over choice set

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

In networks

e.g., how do preferences change
when choosing from a popular group?



Choosing to close triangles

Triadic closure offers small choice sets

- tractable inference
- varied choice sets

Choosing to close triangles

Triadic closure offers small choice sets

→ tractable inference

→ varied choice sets

Our data

Timestamped edges
(including repeats)

Choosing to close triangles

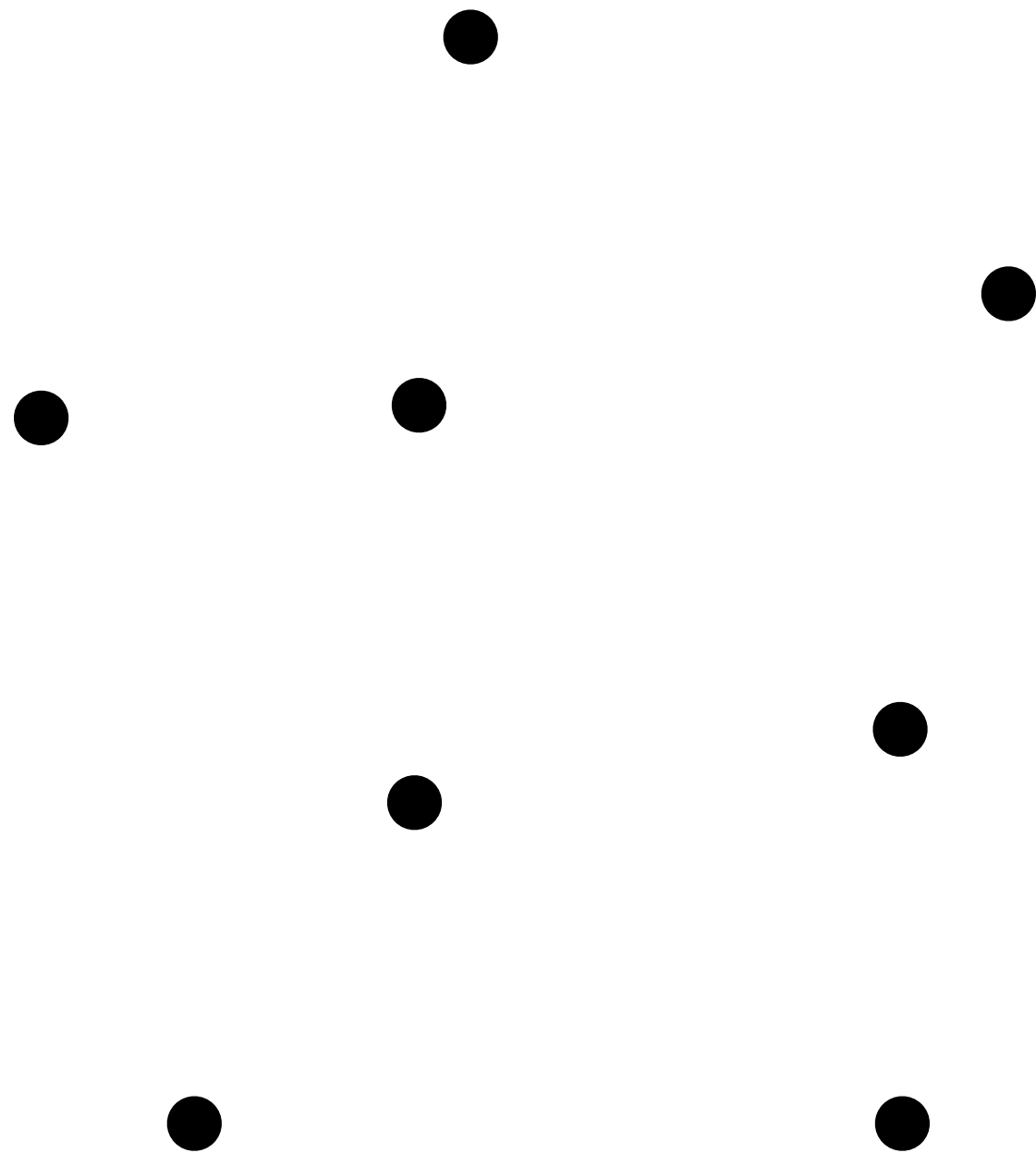
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Our data

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Choosing to close triangles

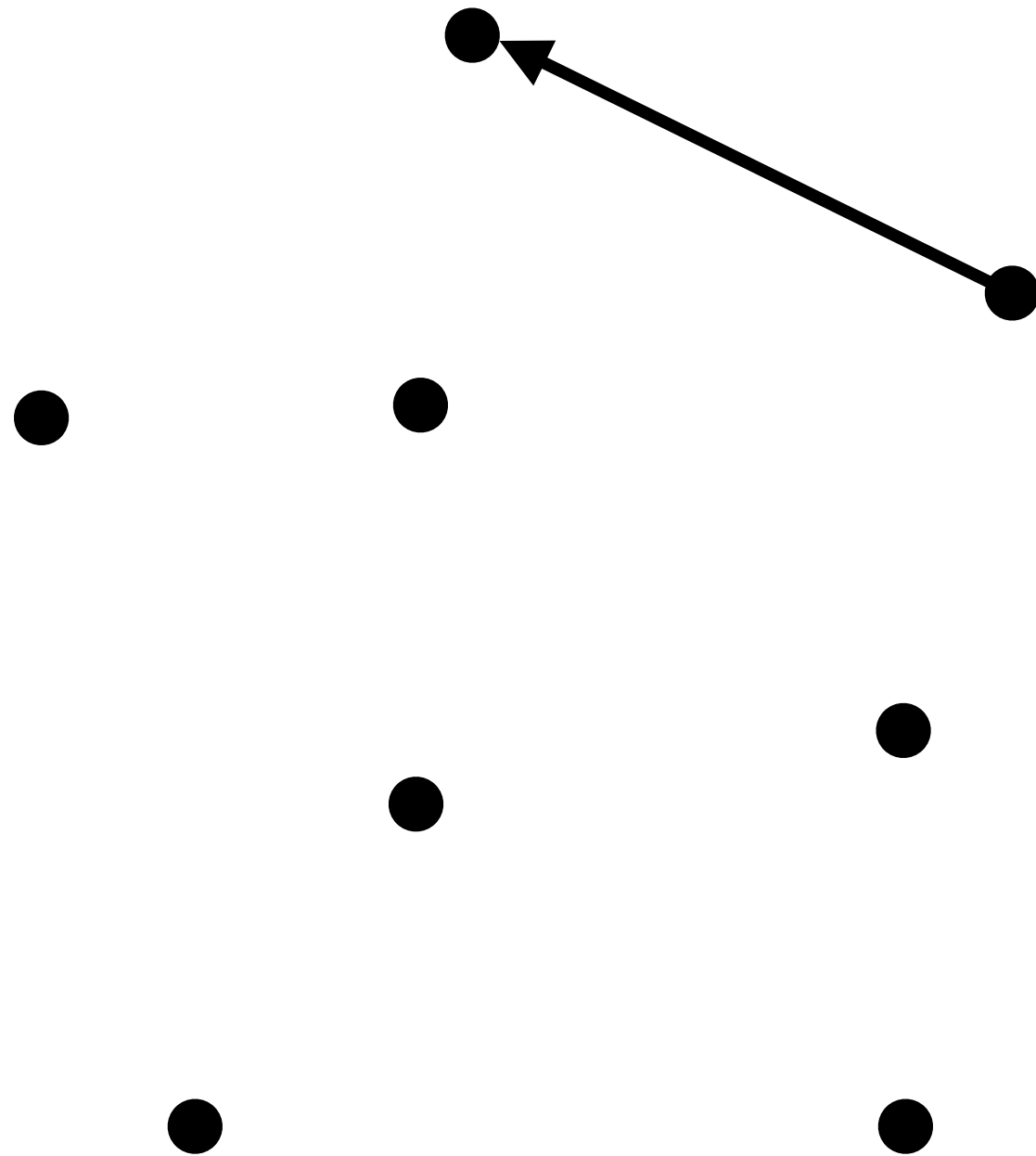
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Choosing to close triangles

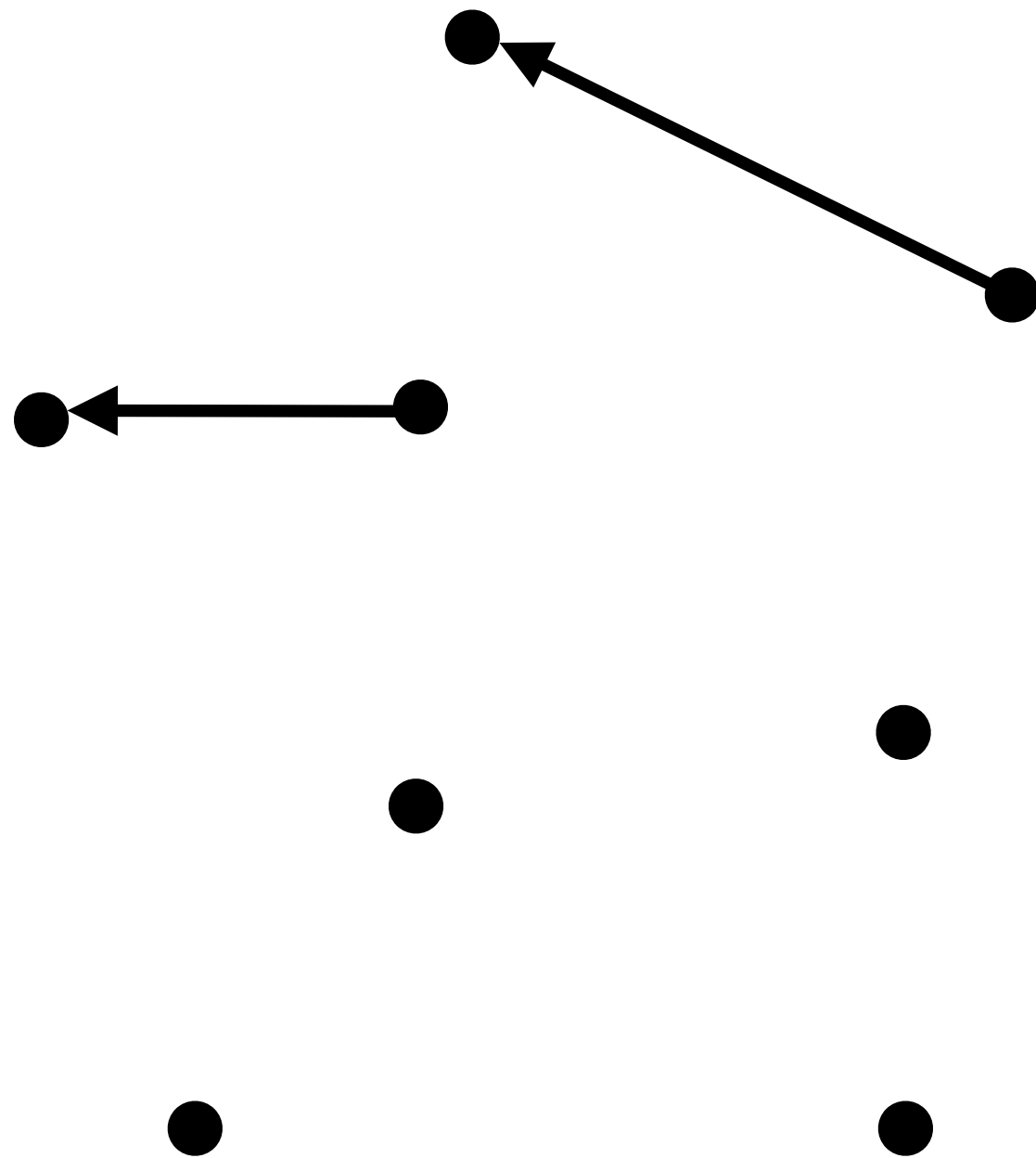
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Choosing to close triangles

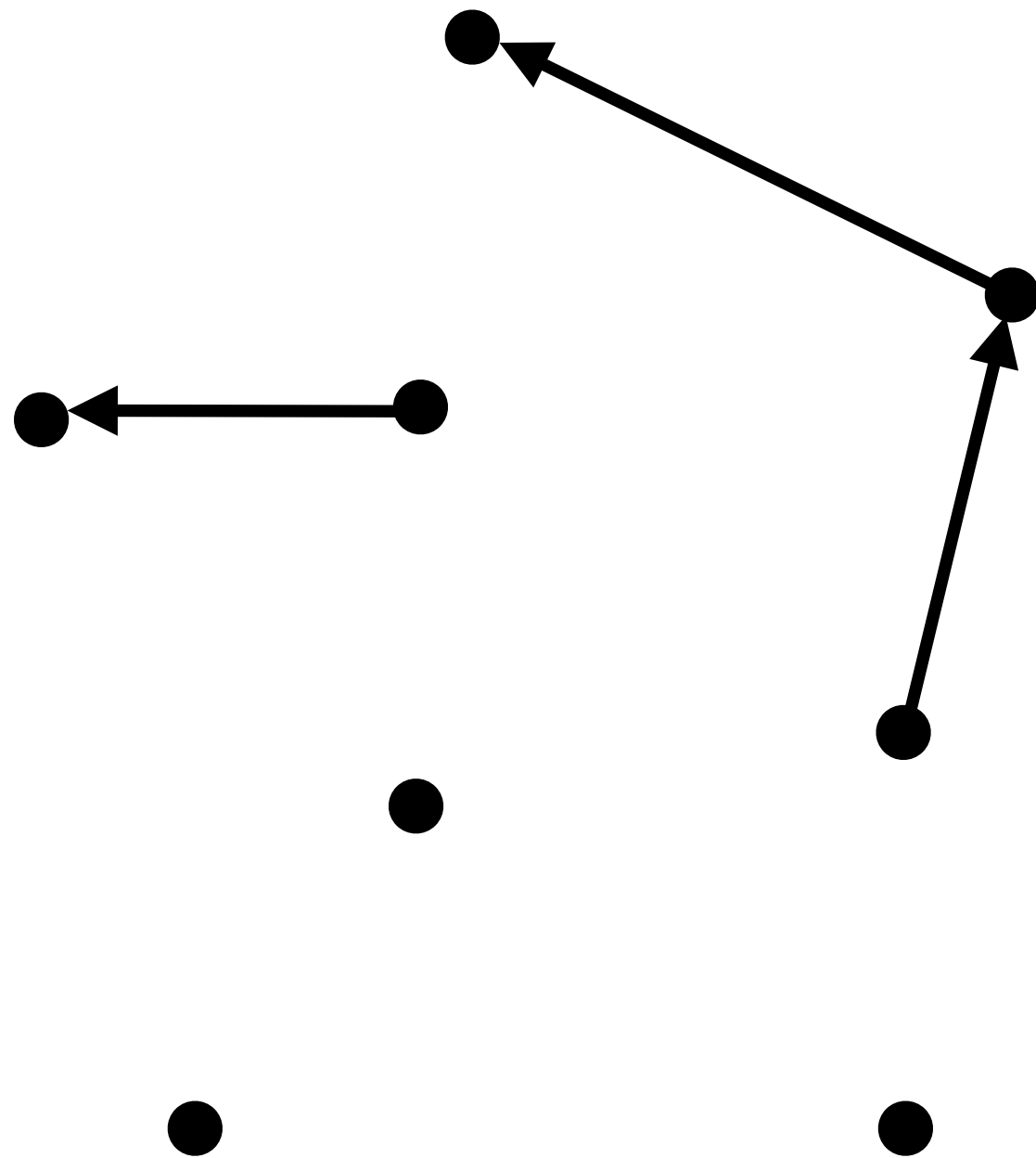
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Choosing to close triangles

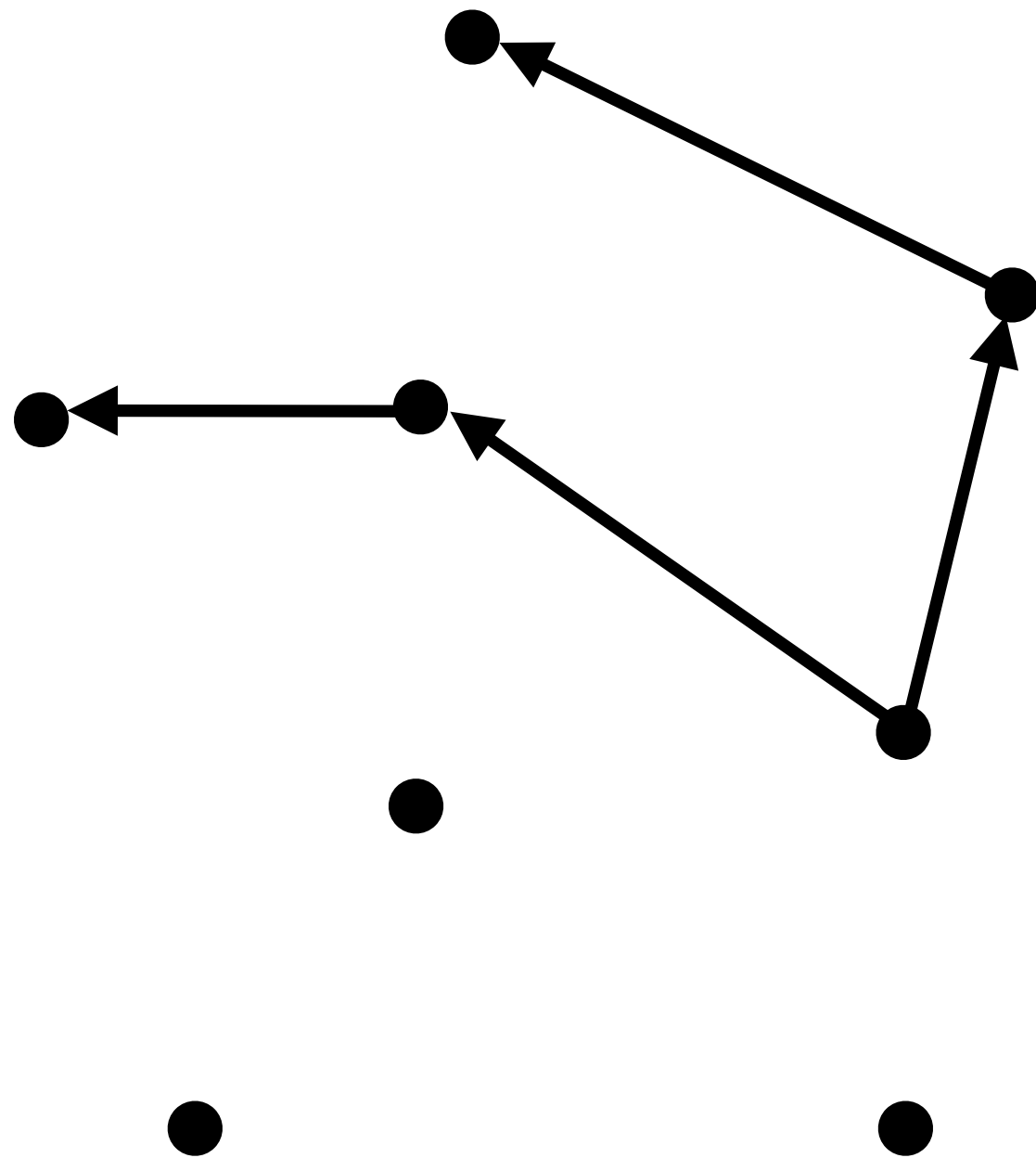
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Choosing to close triangles

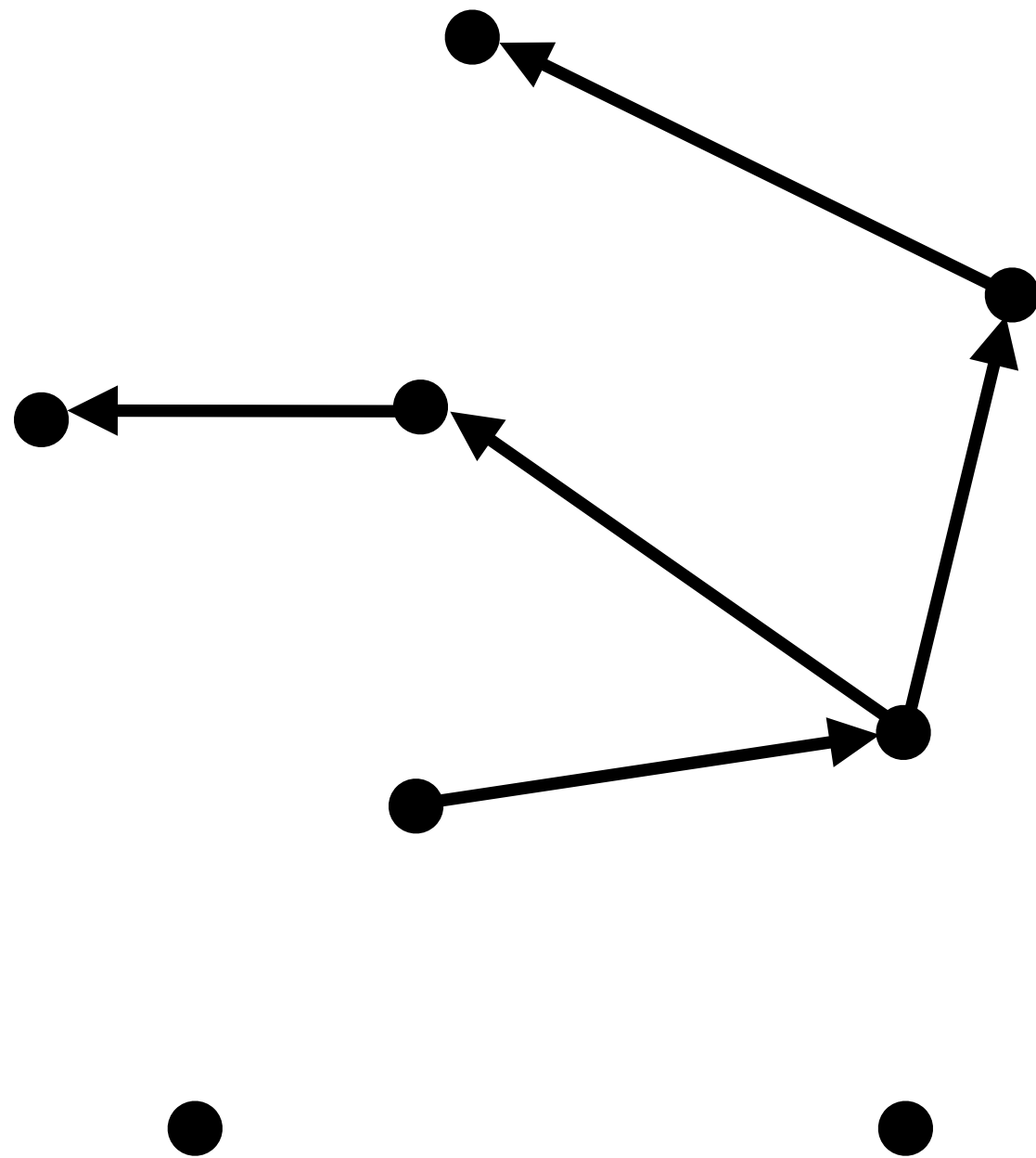
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Choosing to close triangles

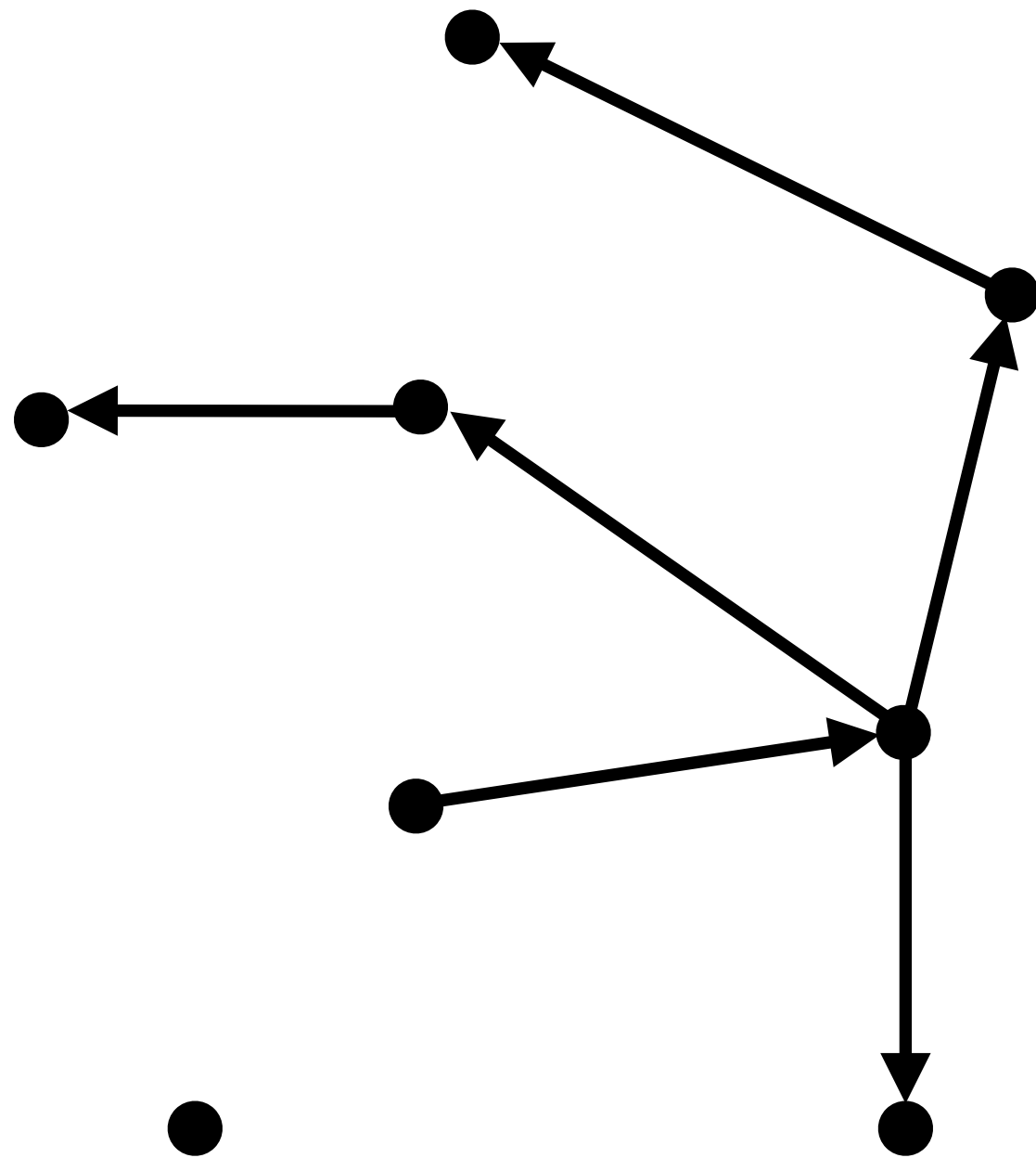
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(including repeats)



Choosing to close triangles

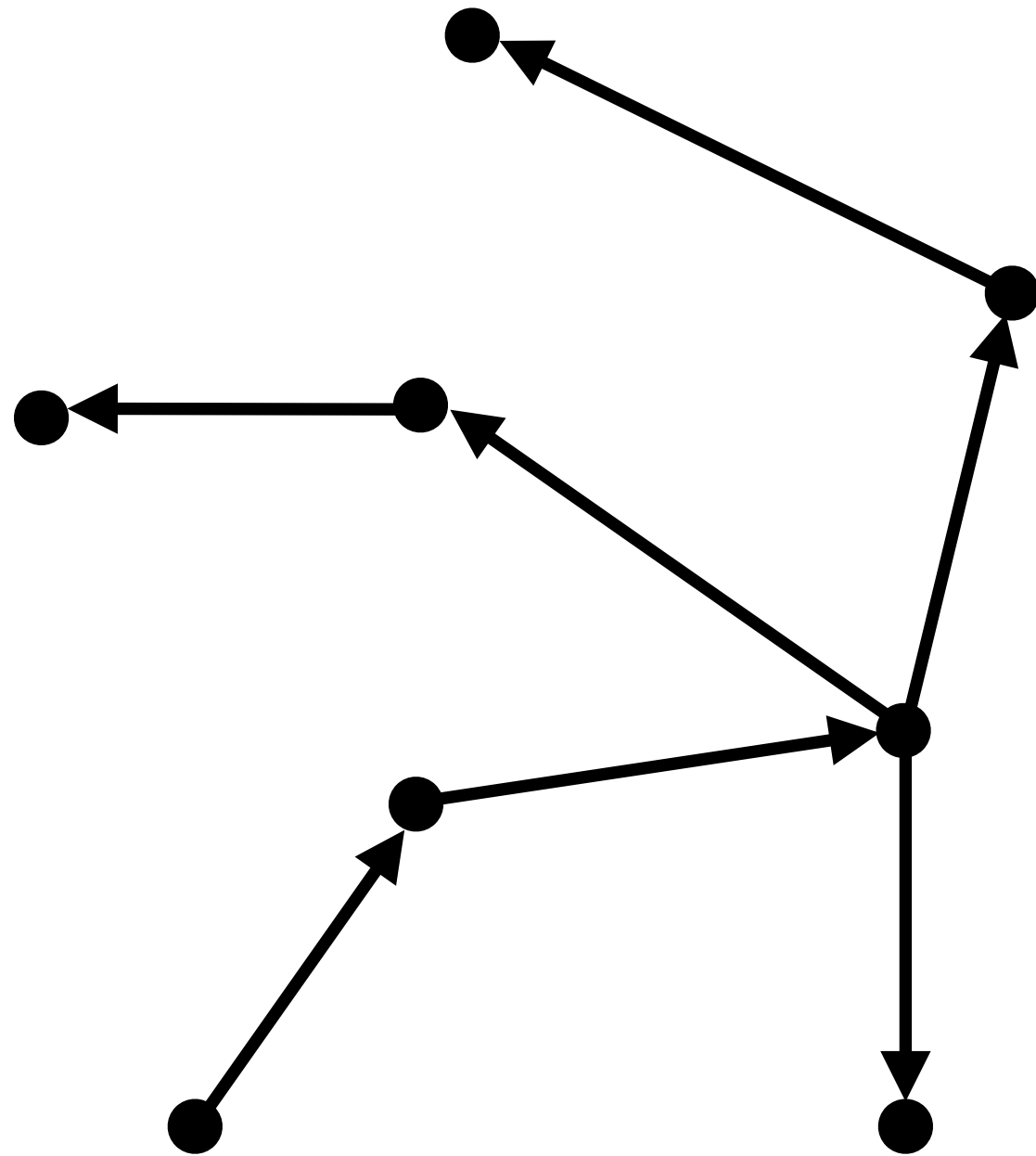
Triadic closure offers small choice sets

→ tractable inference

→ varied choice sets

Our data

Timestamped edges
(including repeats)



Choosing to close triangles

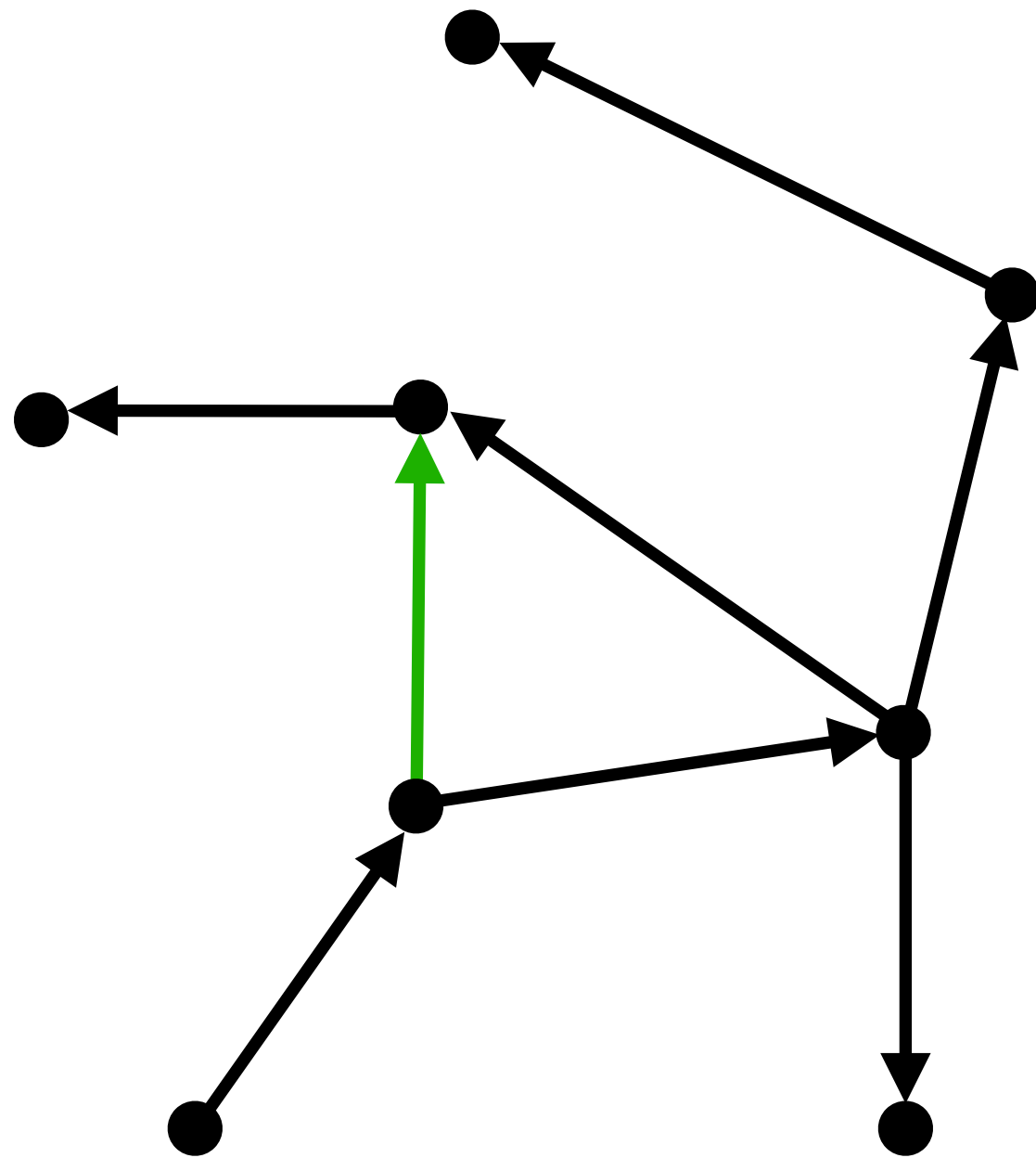
Triadic closure offers small choice sets

→ tractable inference

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Our data

Timestamped edges
(including repeats)

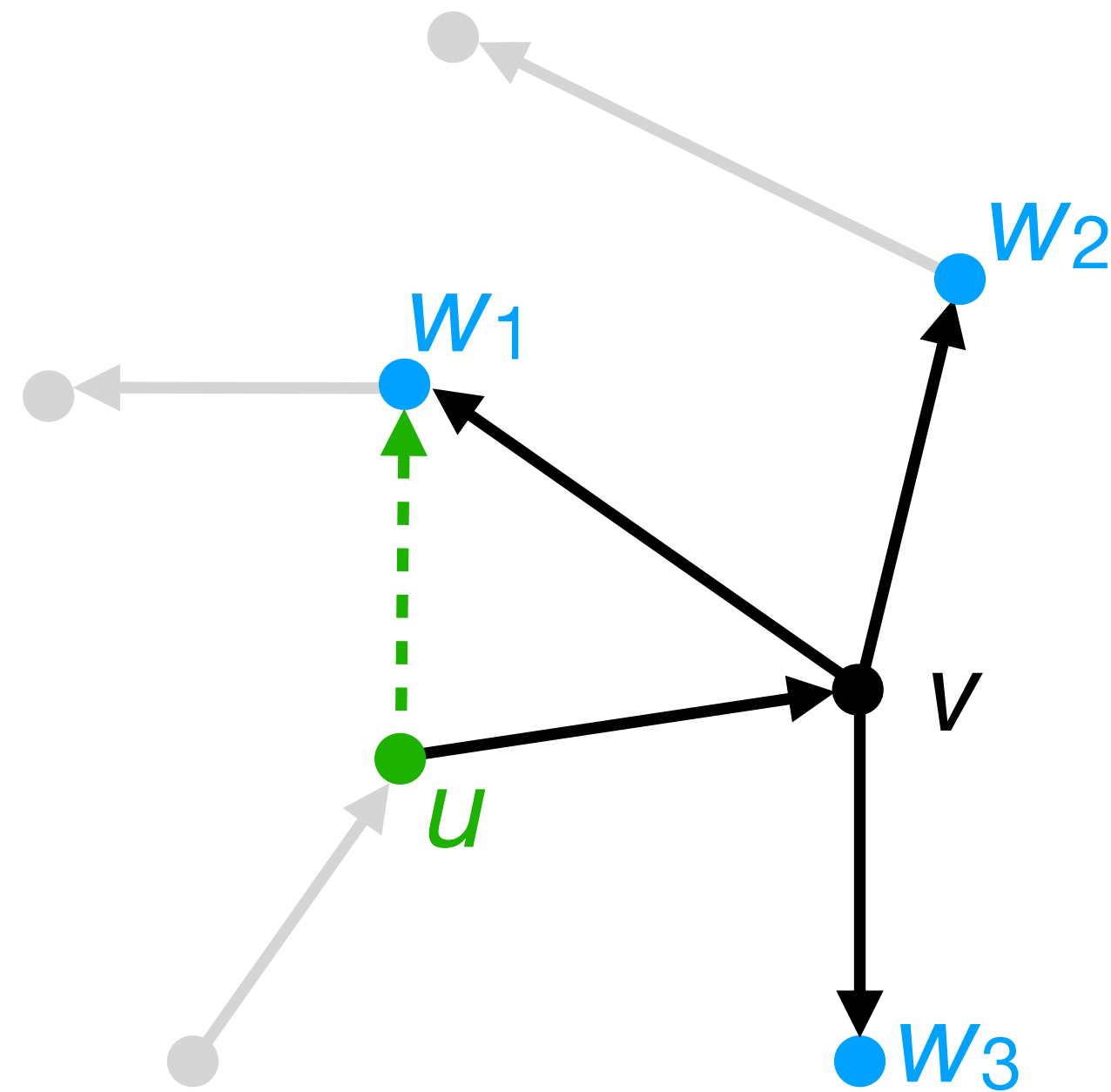
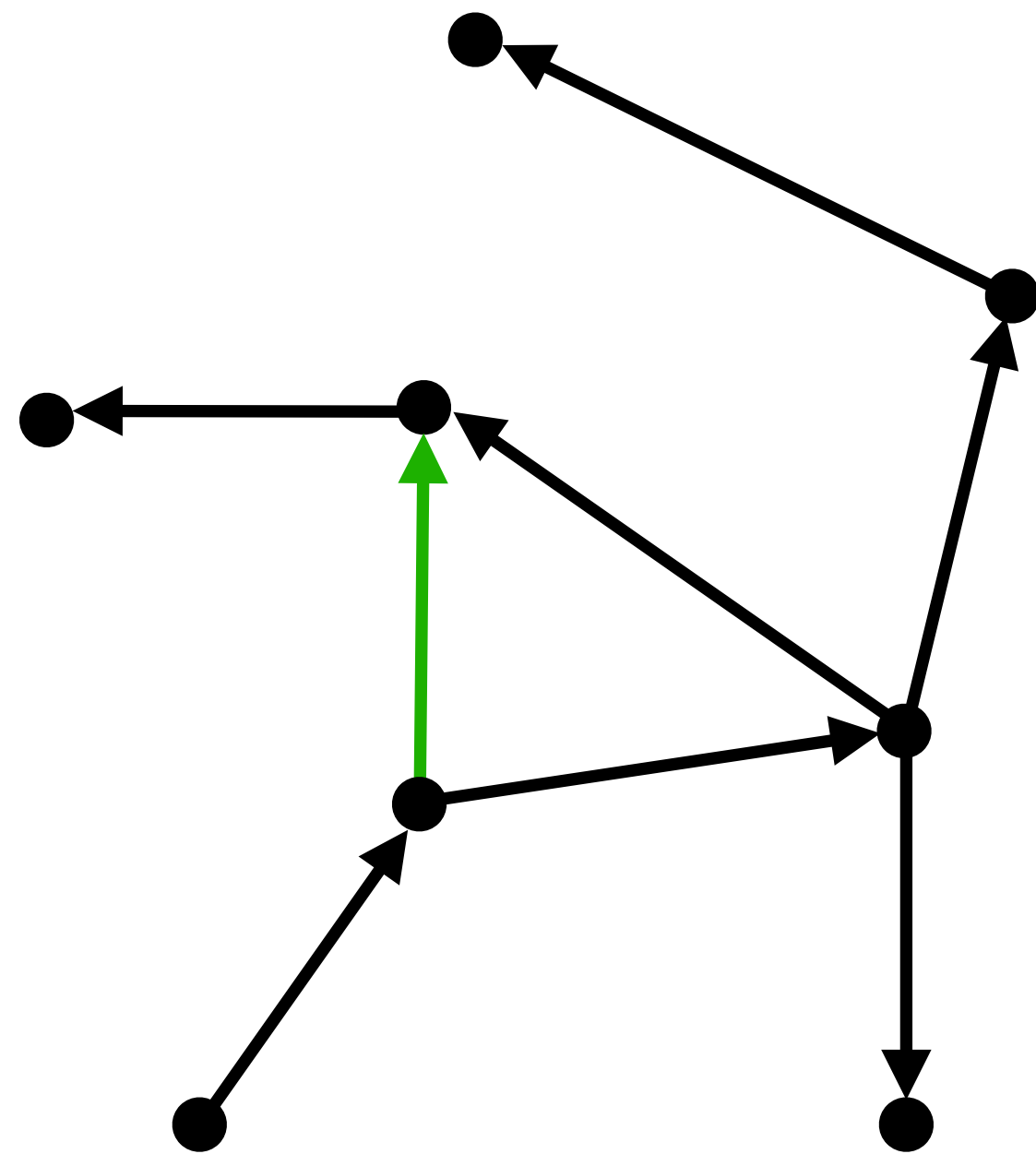


Choosing to close triangles

Triadic closure offers small choice sets

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Our data

Timestamped edges
(including repeats)

Choosing to close triangles

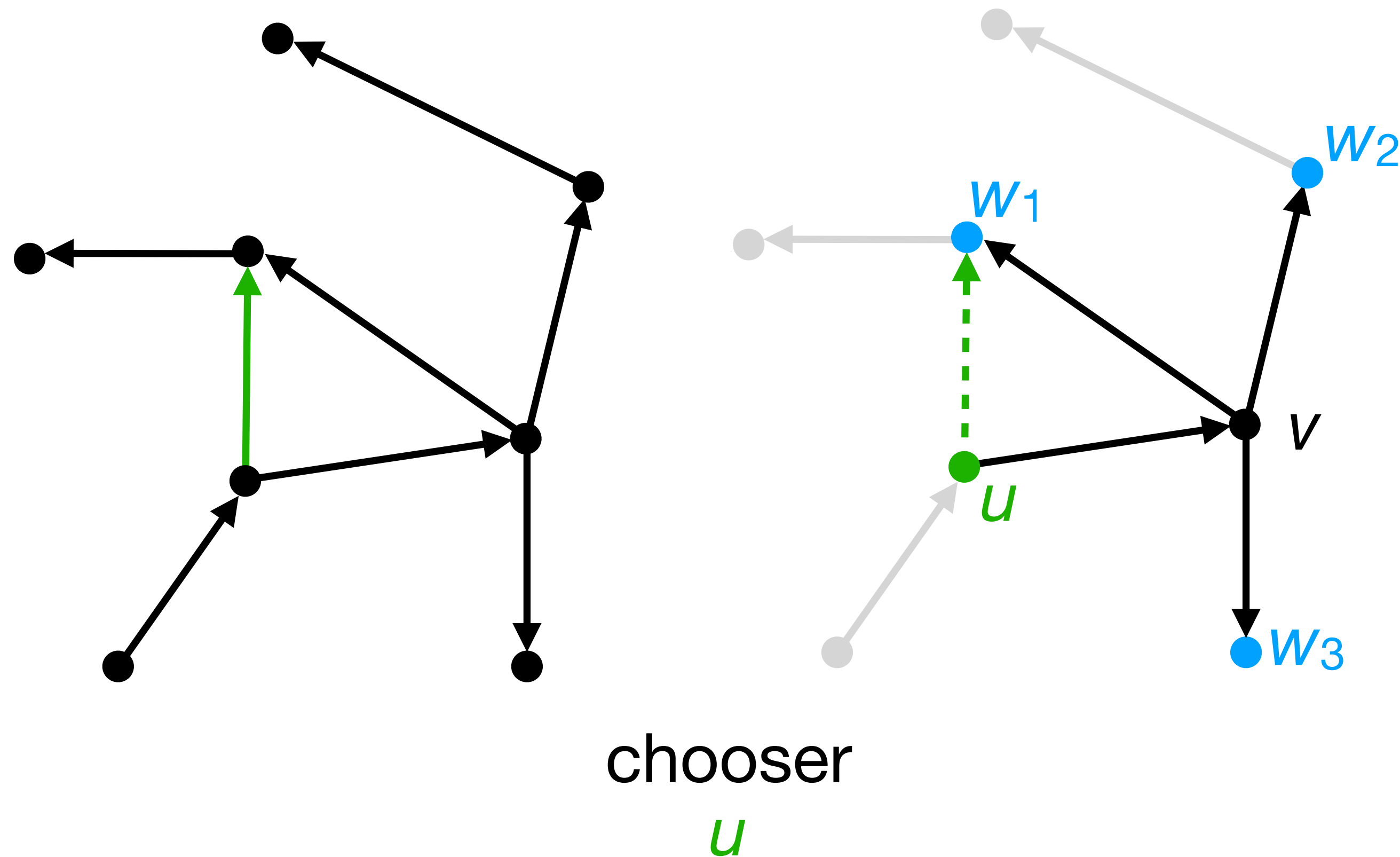
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Our data

Timestamped edges
(including repeats)



Choosing to close triangles

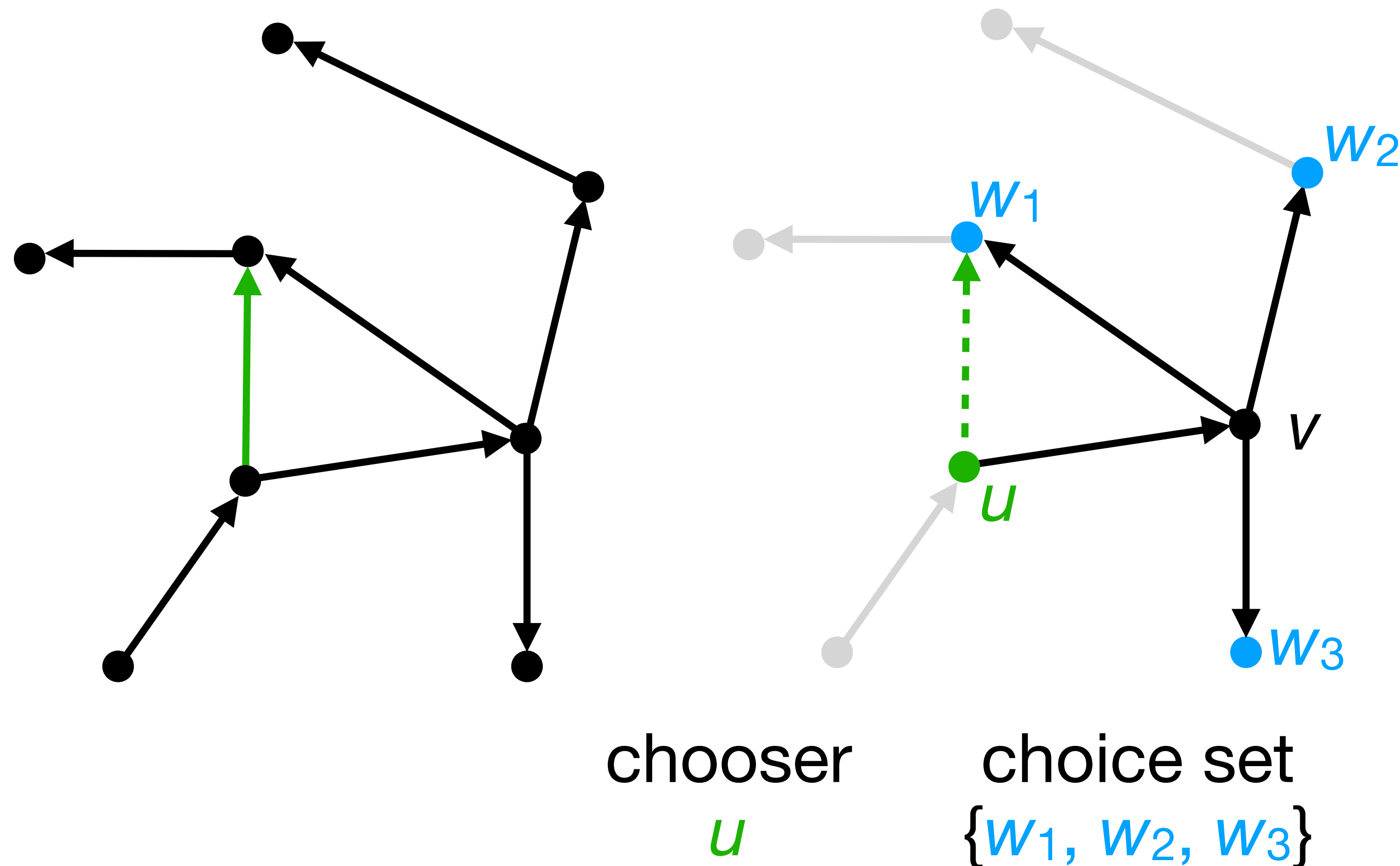
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Our data

Timestamped edges
(including repeats)



Choosing to close triangles

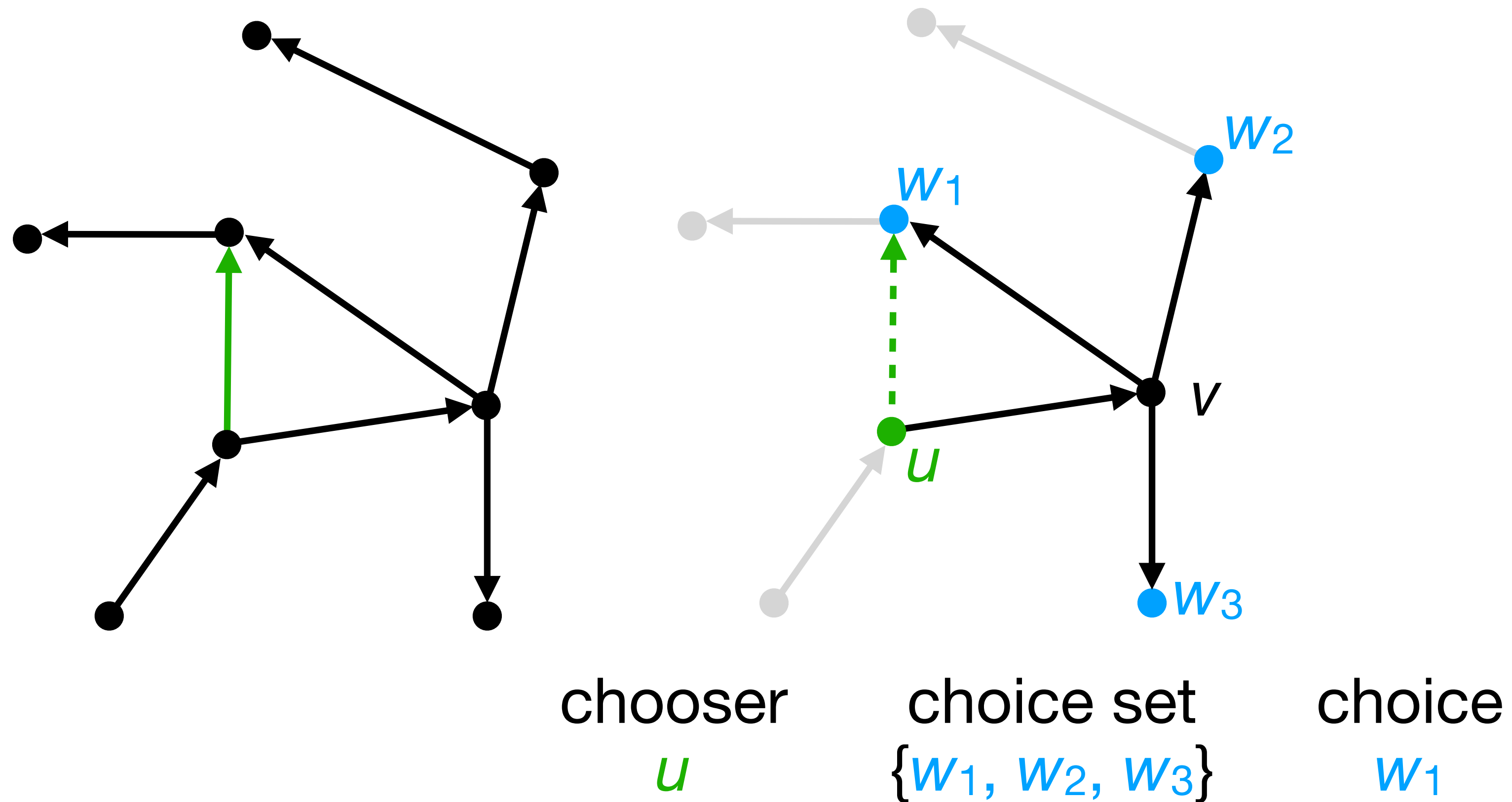
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Timestamped edges
(including repeats)



Choosing to close triangles

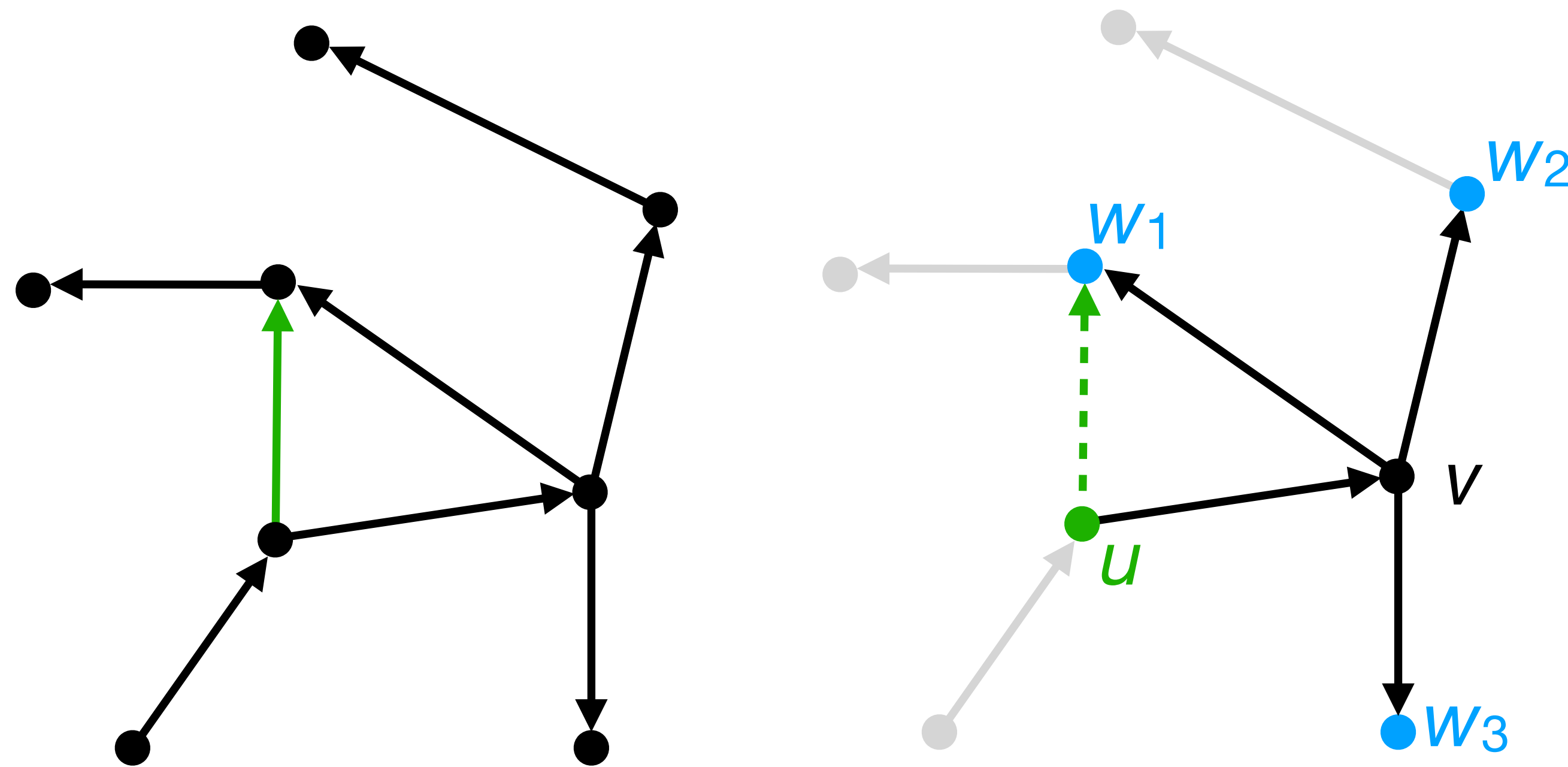
Triadic closure offers small choice sets

→ tractable inference

→ varied choice sets

Our data

Timestamped edges
(including repeats)



chooser

u

choice set

$\{w_1, w_2, w_3\}$

choice

w_1

Node features

1. in-degree of w
2. # shared neighbors of u, w
3. weight of edge $w \rightarrow u$
4. time since last edge into w
5. time since last edge out of w
6. time since last $w \rightarrow u$ edge

Context matters in triadic closure

Context matters in triadic closure

Datasets

email-enron

email-eu

email-w3c

wiki-talk

reddit-hyperlink

bitcoin-alpha

bitcoin-otc

mathoverflow

college-msg

facebook-wall

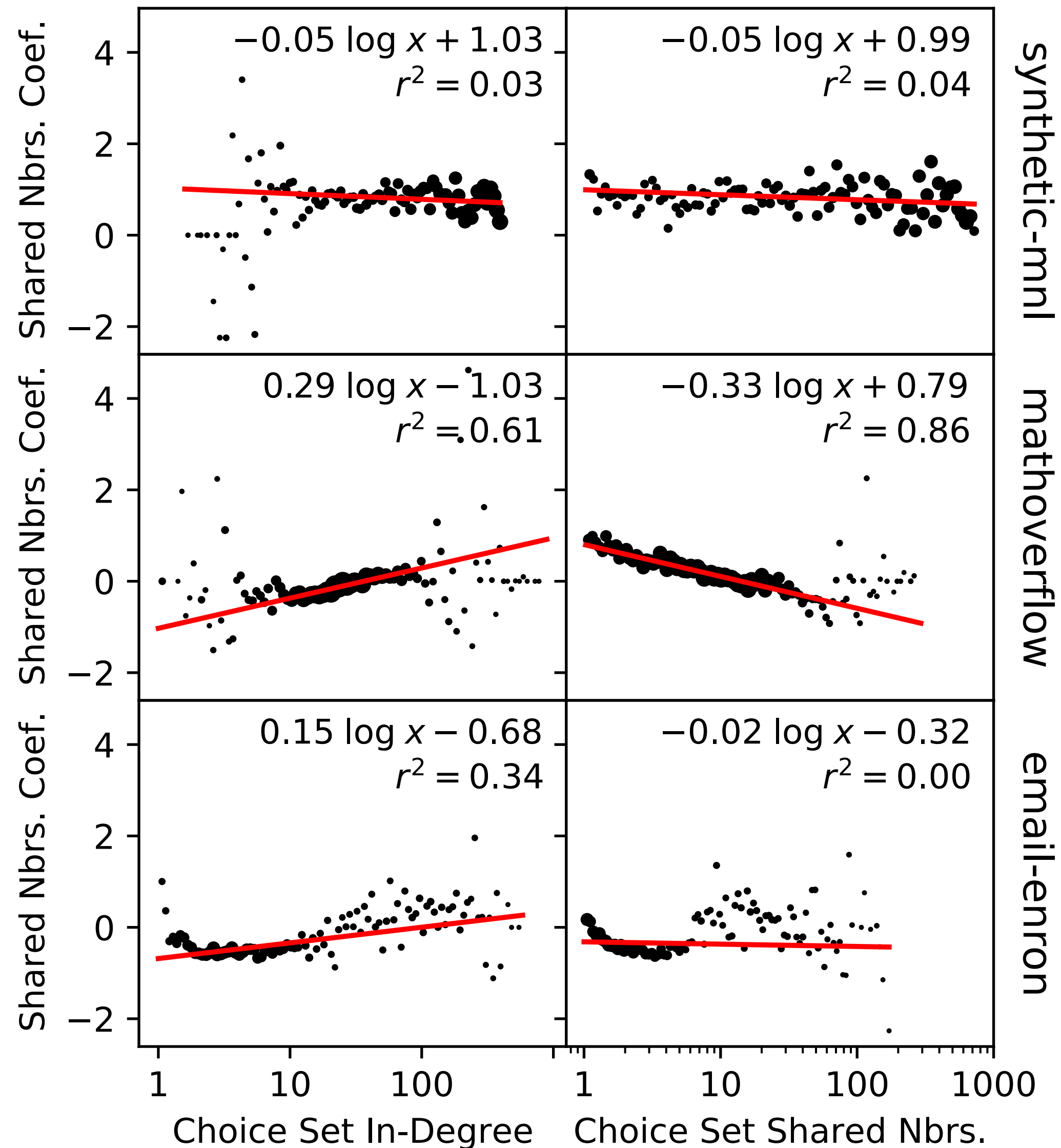
sms-a

sms-b

sms-c

bit.ly/lcl-data

Context matters in triadic closure

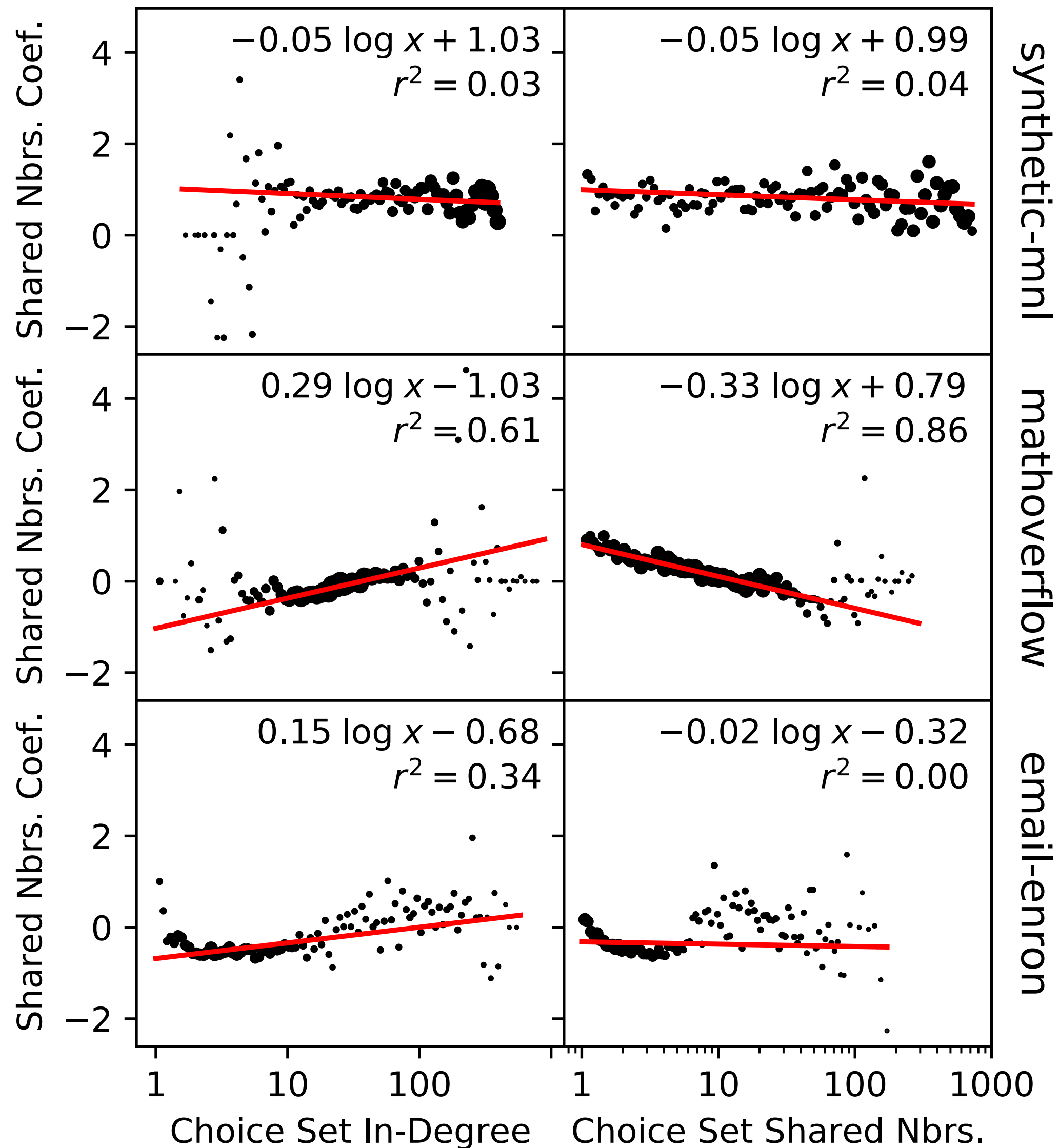


Datasets

email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

bit.ly/lcl-data

Context matters in triadic closure



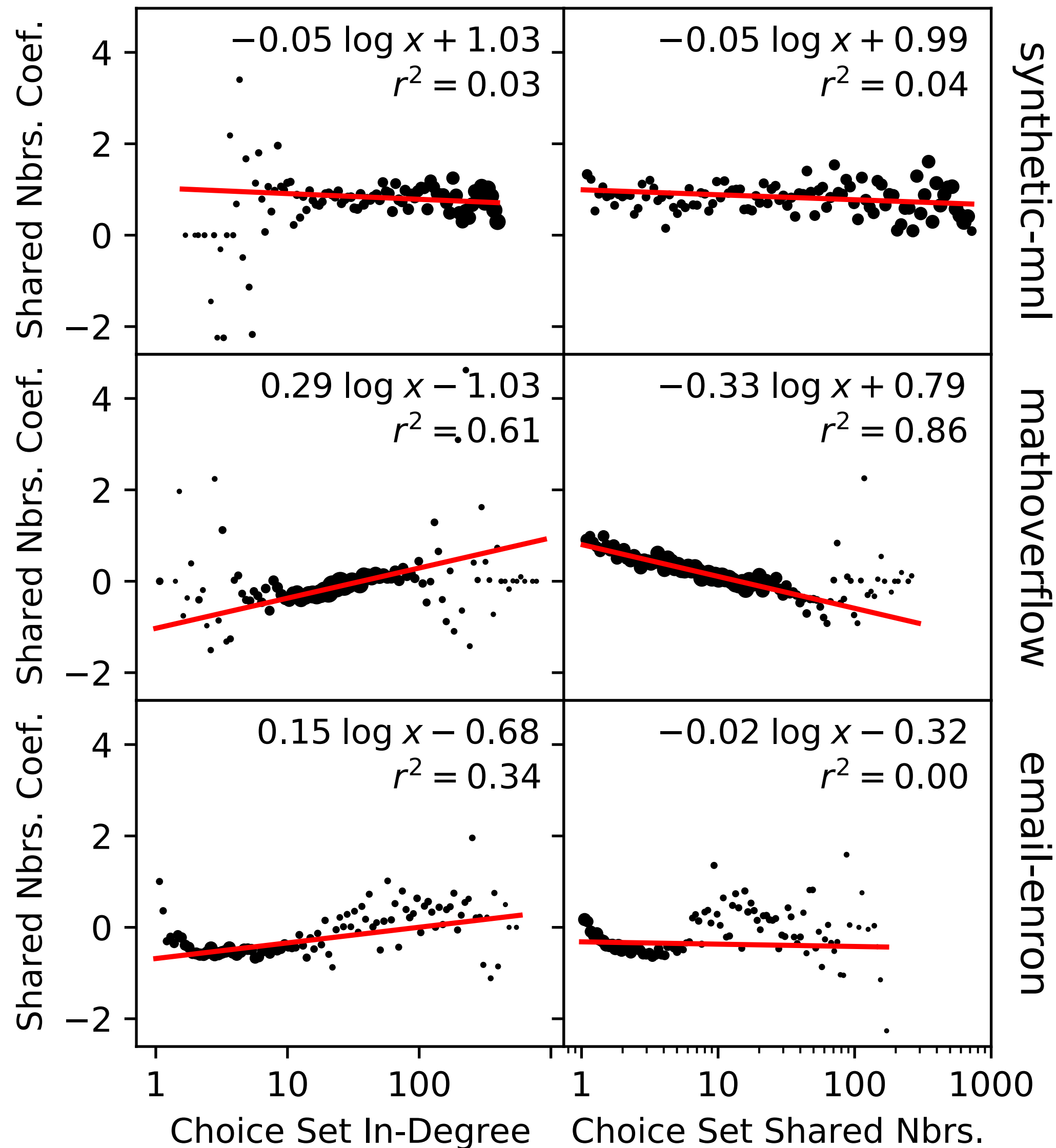
Synthetic data,
no context effects

Datasets

email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

bit.ly/lcl-data

Context matters in triadic closure



Synthetic data,
no context effects

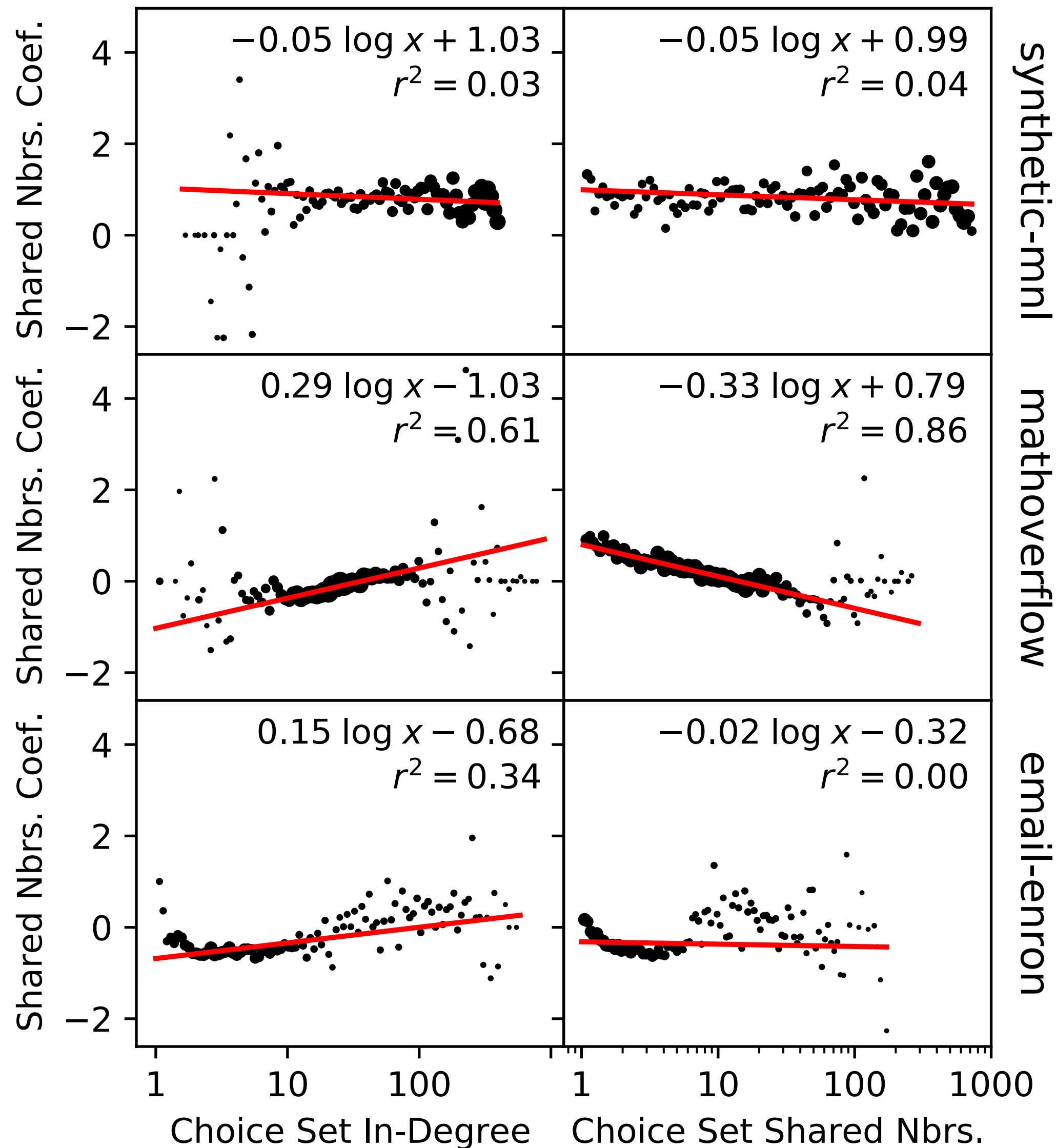
Commenting network,
linear context effects

Datasets

email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

bit.ly/lcl-data

Context matters in triadic closure



Synthetic data,
no context effects

Commenting network,
linear context effects

Email network,
nonlinear context effects?

Datasets

email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c

bit.ly/lcl-data

LCL reveals interpretable context effects

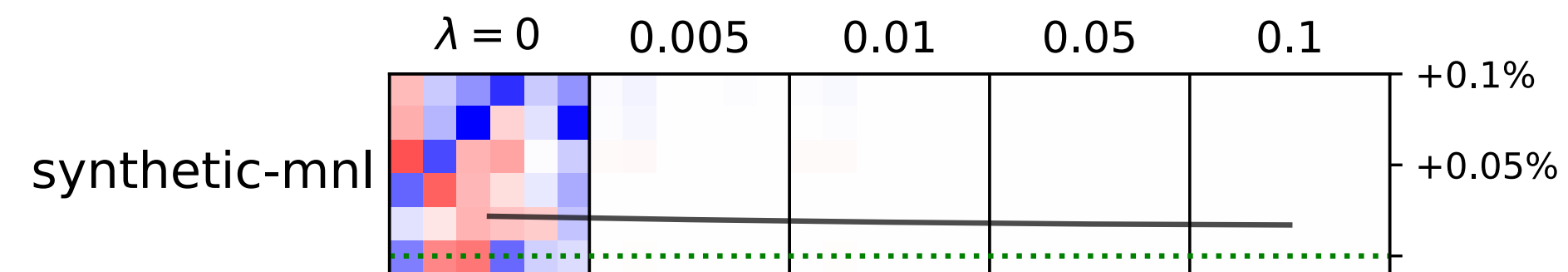
LCL reveals interpretable context effects

Estimation

MLE to infer LCL

$$\begin{aligned}\ell(\theta, A; \mathcal{D}) = & \sum_{(i,C) \in \mathcal{D}} (\theta + Ax_C)^T x_i \\ & - \log \sum_{j \in C} \exp([\theta + Ax_C]^T x_j) \\ & \text{(concave)}\end{aligned}$$

LCL reveals interpretable context effects



Estimation

MLE to infer LCL

$$\ell(\theta, A; \mathcal{D}) = \sum_{(i, C) \in \mathcal{D}} (\theta + Ax_C)^T x_i - \log \sum_{j \in C} \exp([\theta + Ax_C]^T x_j)$$

(concave)

LCL reveals interpretable context effects

Node features

(left-right, top-bottom)

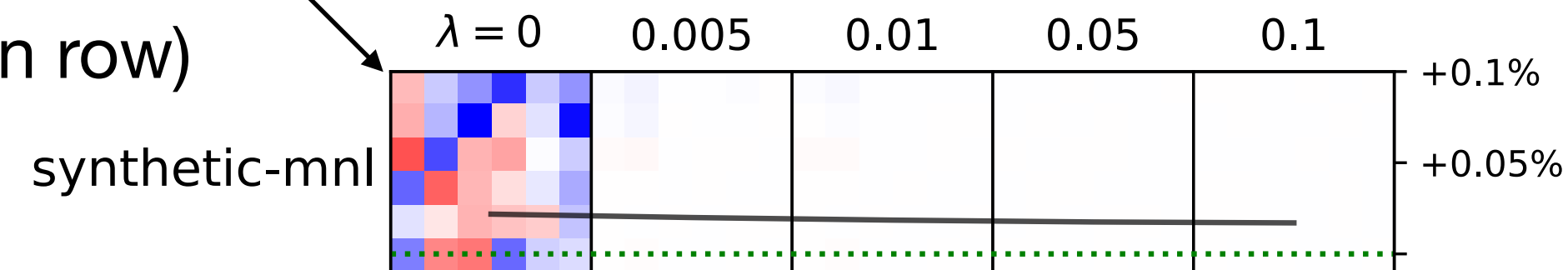
1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

Estimation

MLE to infer LCL

$$\begin{aligned} \ell(\theta, A; \mathcal{D}) = & \sum_{(i,C) \in \mathcal{D}} (\theta + Ax_C)^T x_i \\ & - \log \sum_{j \in C} \exp([\theta + Ax_C]^T x_j) \\ & \text{(concave)} \end{aligned}$$

context effect matrix A
red: +, blue: -, white: 0
(column acts on row)



LCL reveals interpretable context effects

Node features

(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

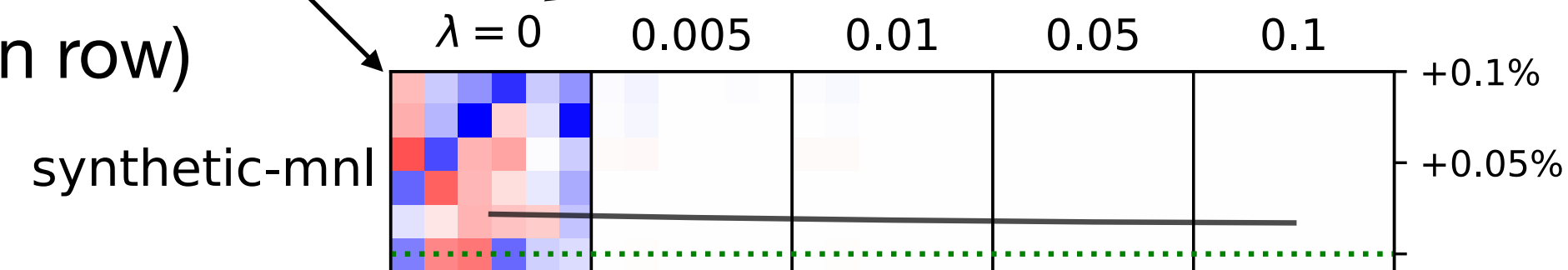
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MLE to infer LCL

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context effect matrix A
red: +, blue: -, white: 0
(column acts on row)

L_1 regularization level



LCL reveals interpretable context effects

Node features

(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

Estimation

MLE to infer LCL

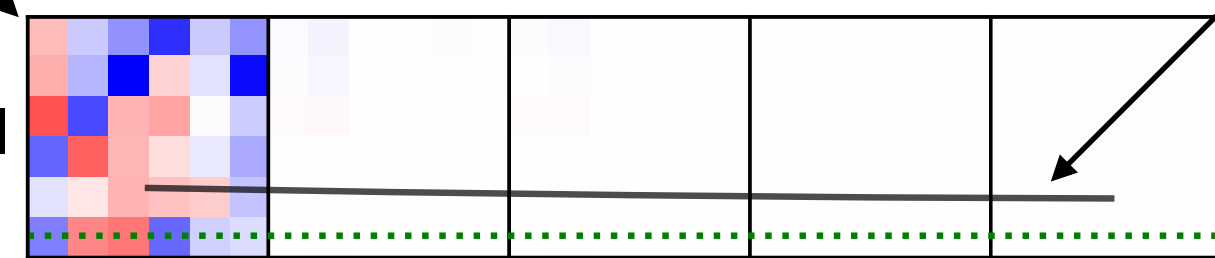
$$\begin{aligned} \ell(\theta, A; \mathcal{D}) = & \sum_{(i,C) \in \mathcal{D}} (\theta + Ax_C)^T x_i \\ & - \log \sum_{j \in C} \exp([\theta + Ax_C]^T x_j) \\ & \text{(concave)} \end{aligned}$$

context effect matrix A
red: +, blue: -, white: 0
(column acts on row)

L_1 regularization level

$\lambda = 0$ 0.005 0.01 0.05 0.1

synthetic-mnl



LCL negative log-likelihood
(lower = better)

+0.1%
+0.05%

LCL reveals interpretable context effects

Node features

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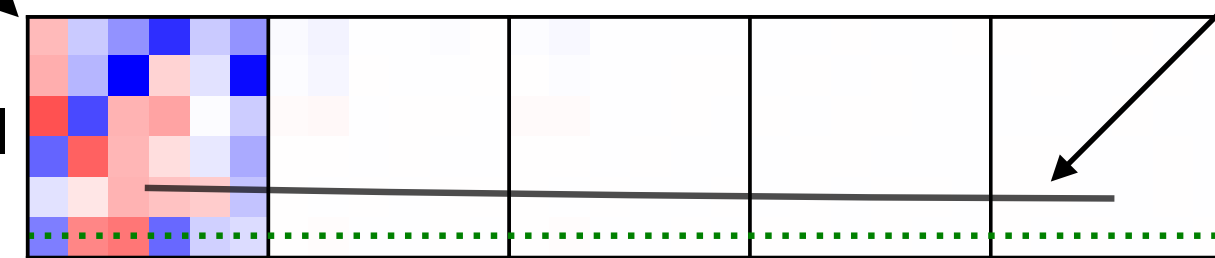
(concave)

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(column acts on row)

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synthetic-mnl



LCL negative log-likelihood
(lower = better)

+0.1%

+0.05%

likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)

LCL reveals interpretable context effects

Node features

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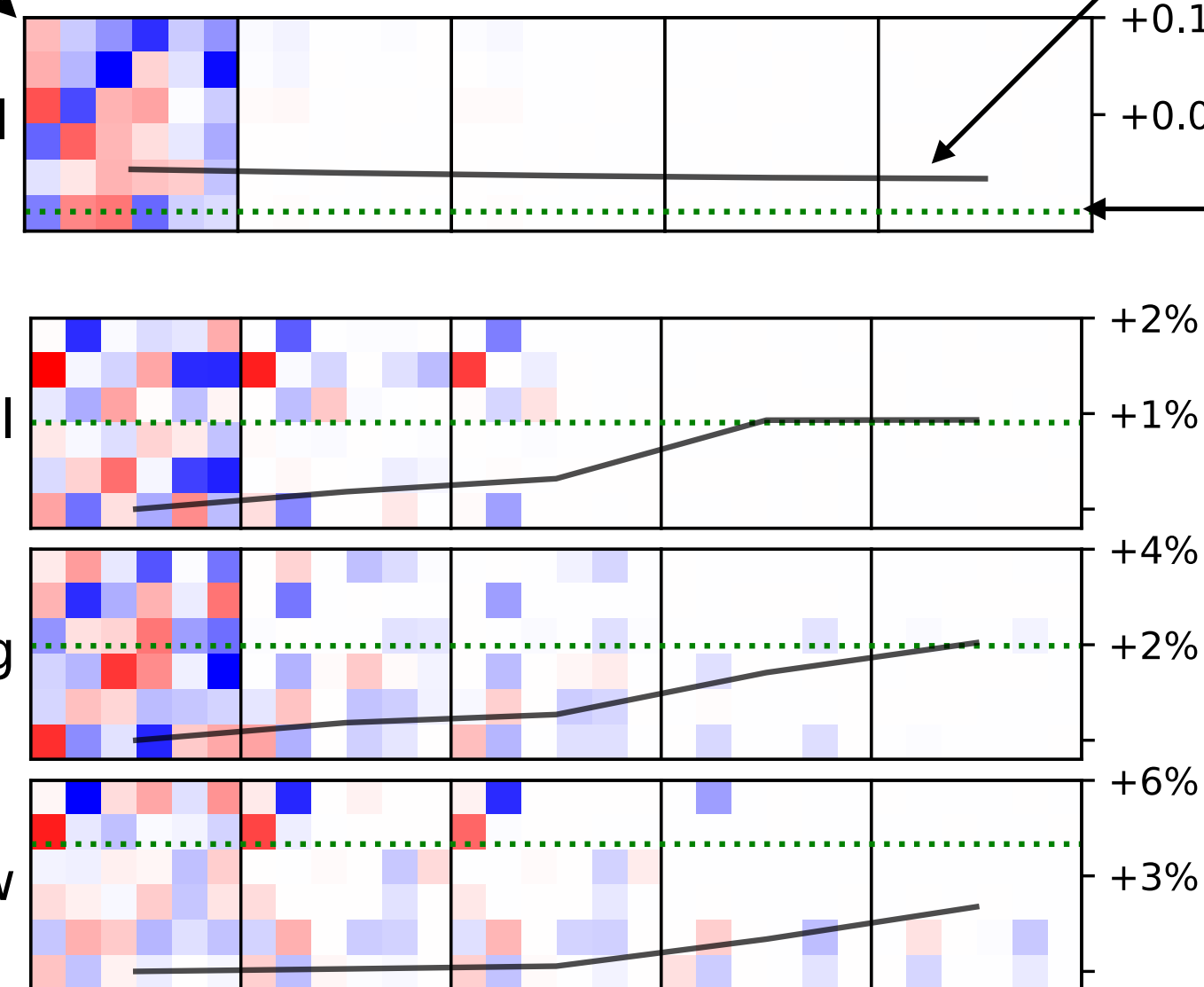
synthetic-mnl

facebook-wall

college-msg

mathoverflow

likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)



LCL reveals interpretable context effects

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LCL negative log-likelihood
(lower = better)

synthetic-mnl

facebook-wall

college-msg

mathoverflow

+0.1%

+0.05%

+2%

+1%

+4%

+2%

+6%

+3%

likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)

LCL reveals interpretable context effects

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facebook-wall

college-msg

mathoverflow

+0.1%

+0.05%

+2%

+1%

+4%

+2%

+6%

+3%

likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)

“popularity matters less when
choosing from close connections”

“close connections matter more
when choosing from the popular”

LCL reveals interpretable context effects

Node features

(left-right, top-bottom)

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likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)

synthetic-mnl

facebook-wall

college-msg

mathoverflow

email-enron

email-eu

email-w3c

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(concave)

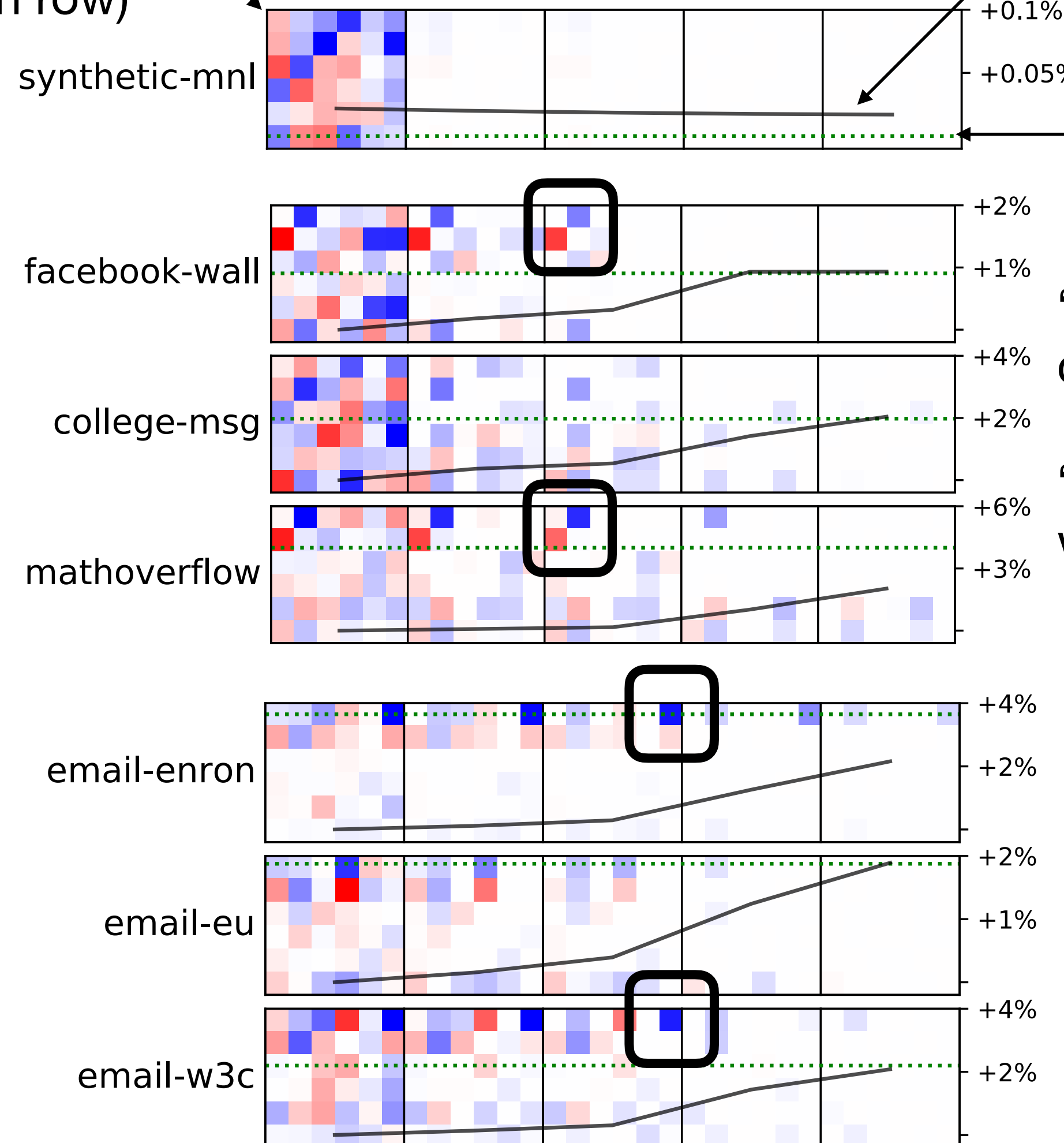
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L_1 regularization level

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LCL negative log-likelihood
(lower = better)

likelihood-ratio test vs MNL
significance threshold
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(concave)

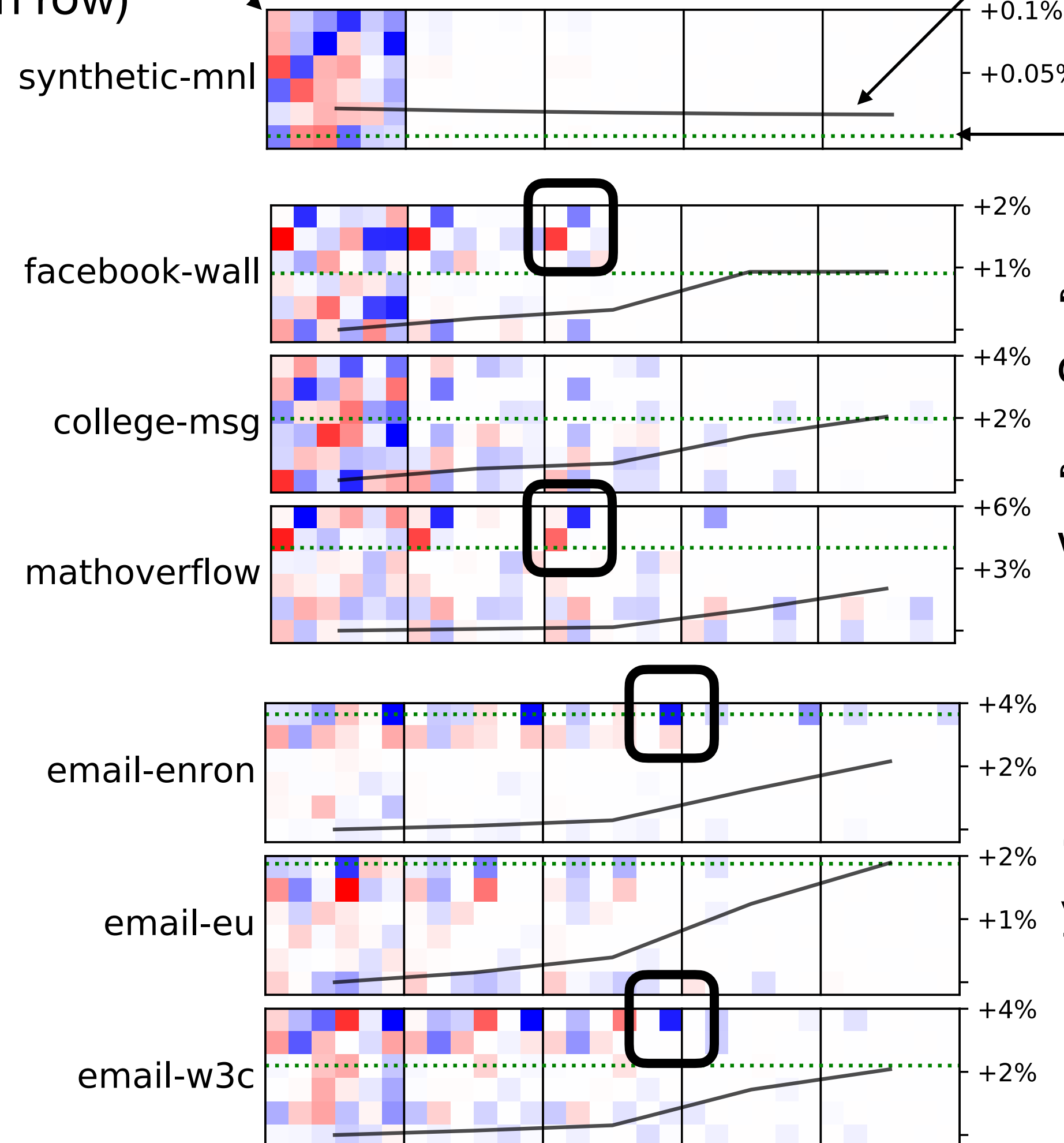
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(column acts on row)

L_1 regularization level

$\lambda = 0$ 0.005 0.01 0.05 0.1

LCL negative log-likelihood
(lower = better)

likelihood-ratio test vs MNL
significance threshold
($p < 0.001$)



“popularity matters less when
choosing from close connections”

“close connections matter more
when choosing from the popular”

“popularity matters less when
your inbox is full of recent emails”

Other things in our paper

Kiran Tomlinson and Austin R. Benson

Learning Interpretable Feature Context Effects in Discrete Choice

arXiv: 2009.03417, September 2020

bit.ly/lcl-paper

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- LCL derivation from simple assumptions

Other things in our paper

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bit.ly/lcl-paper

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

- LCL derivation from simple assumptions
- More flexible model: decomposed LCL

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- LCL derivation from simple assumptions
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Theorem 1. A d -feature linear context logit is identifiable from a dataset \mathcal{D} if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

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bit.ly/lcl-paper

- LCL derivation from simple assumptions
- More flexible model: decomposed LCL
- LCL identifiability condition
- Application to general choice data

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

Theorem 1. A d -feature linear context logit is identifiable from a dataset \mathcal{D} if and only if

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Dataset

DISTRICT

DISTRICT-SMART

SUSHI

EXPEDIA

CAR-A

CAR-B

CAR-ALT

Other things in our paper

Kiran Tomlinson and Austin R. Benson

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arXiv: 2009.03417, September 2020

bit.ly/lcl-paper

- LCL derivation from simple assumptions
- More flexible model: decomposed LCL
- LCL identifiability condition
- Application to general choice data
- Accounting for context improves prediction

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

Theorem 1. A d -feature linear context logit is identifiable from a dataset \mathcal{D} if and only if

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	MNL	LCL
Dataset		
DISTRICT	.3680 (.4823)	.3327 (.4712)
DISTRICT-SMART	.4006 (.4900)	.3894 (.4876)
EXPEDIA	.3859 (.2954)	.3696* (.2926)
SUSHI	.2727 (.2751)	.2741 (.2771)
CAR-A	.3570 (.4791)	.3514 (.4774)
CAR-B	.3326 (.4711)	.3326 (.4711)
CAR-ALT	.2944 (.2875)	.2650* (.2804)
SYNTHETIC-MNL	.1513 (.1865)	.1512 (.1864)
SYNTHETIC-LCL	.1360 (.1684)	.1357* (.1683)
WIKI-TALK	.2946 (.2916)	.2666* (.2773)
REDDIT-HYPERLINK	.2859 (.2611)	.2761* (.2606)
BITCOIN-ALPHA	.2724 (.3246)	.2591* (.3178)
BITCOIN-OTC	.1891 (.2756)	.1529* (.2468)
SMS-A	.2825 (.3250)	.2661* (.3193)
SMS-B	.3045 (.3419)	.2848* (.3273)
SMS-C	.3115 (.3455)	.3070 (.3477)
EMAIL-ENRON	.1265 (.2068)	.1244* (.2115)
EMAIL-EU	.2683 (.3021)	.2665 (.3037)
EMAIL-W3C	.1332 (.2070)	.1210* (.1845)
FACEBOOK-WALL	.2176 (.2895)	.2109* (.2871)
COLLEGE-MSG	.1850 (.2726)	.1723* (.2655)
MATHTOERFLOW	.1385 (.2503)	.1153* (.2200)

Concluding thoughts

Key takeaway

Context effects matter in triadic closure

Challenges

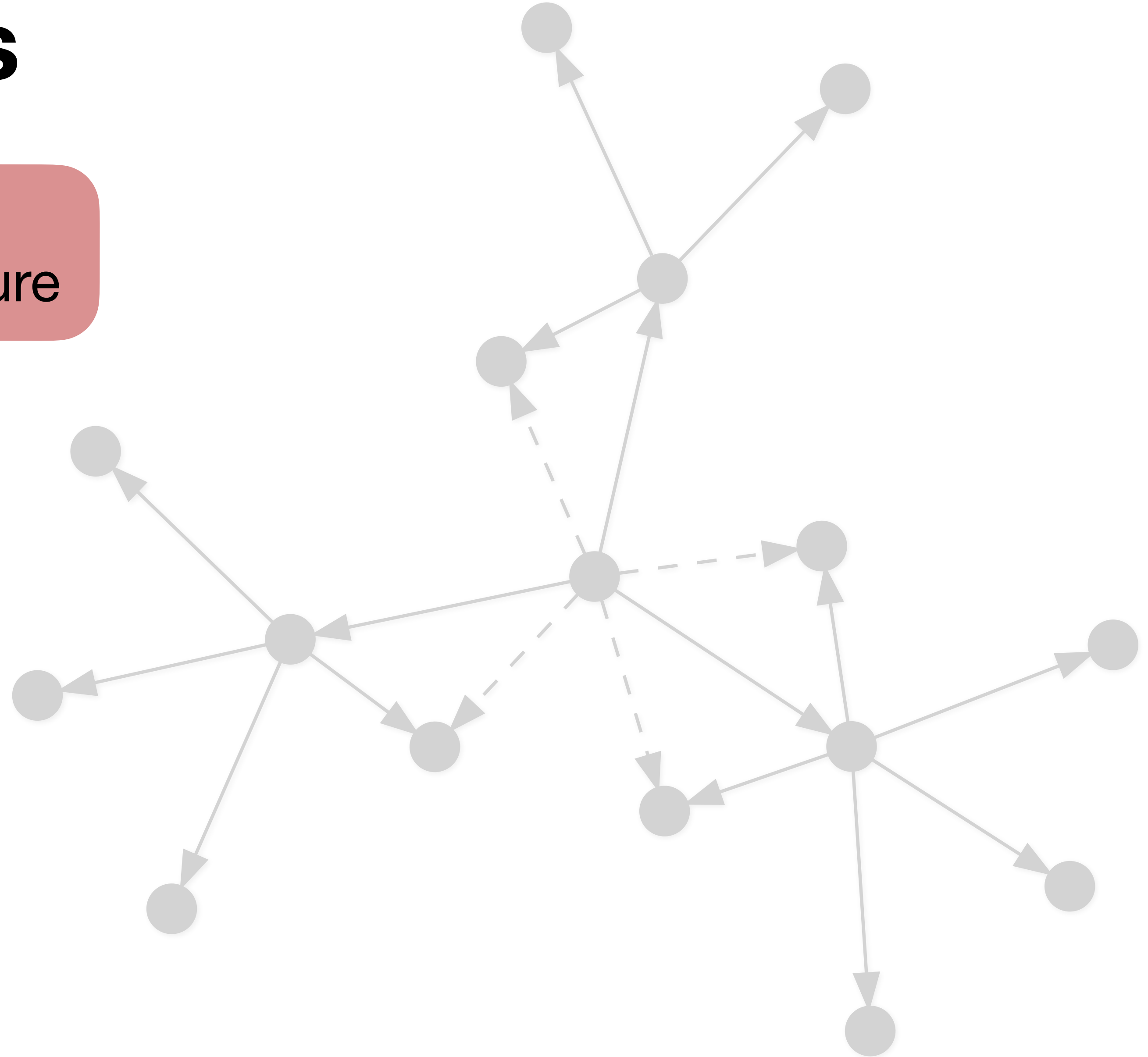
Features correlate

Causal context effects?

Handling nonlinearity?

Global edge formation modes?

Missing timestamps?



Concluding thoughts

Key takeaway

Context effects matter in triadic closure

Challenges

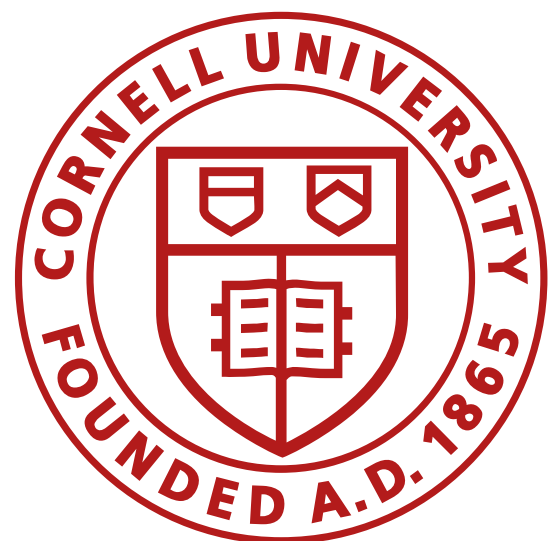
Features correlate

Causal context effects?

Handling nonlinearity?

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Missing timestamps?



Thank you!

More questions or ideas?

Email me: kt@cs.cornell.edu

Slides: bit.ly/lcl-slides
Preprint: bit.ly/lcl-paper
Code: bit.ly/lcl-code
Data: bit.ly/lcl-data



Acknowledgments

Funding from NSF, ARO

Thanks to Johan Ugander
and Jan Overgoor