Learning Context Effects in Triadic Closure

Kiran Tomlinson

SINM 2020 research with Austin R. Benson

Slides: bit.ly/lcl-slides
Code: bit.ly/lcl-code
Data: bit.ly/lcl-data
What factors drive edge formation?
What factors drive edge formation?

Preferential attachment
(Barabási & Albert, Science 1999)
What factors drive edge formation?

**Preferential attachment**
(Barabási & Albert, *Science* 1999)

**Homophily**
(McPherson et al., *Annual Review of Sociology* 2001)
(Papadopoulos et al., *Nature* 2012)
What factors drive edge formation?

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**Homophily**
(McPherson et al., *Annual Review of Sociology* 2001)
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**Fitness**
(Bianconi & Barabási, *Europhysics Letters* 2001)
(Caldarelli et al., *Physical Review Letters* 2002)
What factors drive edge formation?

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**Homophily**
(McPherson et al., *Annual Review of Sociology* 2001)
(Papadopoulos et al., *Nature* 2012)

**Triadic closure**
(Rapoport, *Bulletin of Mathematical Biophysics* 1953)
(Jin et al., *Physical Review E* 2001)
“Choosing to grow a graph”

(Overgoor et al., SINM '19 & WWW '19)

(Gupta & Porter, arXiv 2020)
“Choosing to grow a graph”

(Overgoor et al., SINM '19 & WWW '19)

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Traditional discrete choice:

chooser  choice set
“Choosing to grow a graph”
(Overgoor et al., SINM ‘19 & WWW ‘19)
(Gupta & Porter, arXiv 2020)

Traditional discrete choice:

chooser    choice set
(under-explored in sociology)
(Bruch & Feinberg, Annual Review of Sociology 2017)
“Choosing to grow a graph”

Traditional discrete choice:

chooser \rightarrow choice set

(under-explored in sociology)

(Overgoor et al., *SINM* '19 & *WWW* '19)

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in network growth

chooser \rightarrow choice set

(Bruch & Feinberg, *Annual Review of Sociology* 2017)
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Key usage
Timestamped edges
→ meaningful choice sets
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(Overgoor et al., SINM '19 & WWW '19)
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Key usage
Timestamped edges \rightarrow meaningful choice sets
Infer relative importance of edge formation mechanisms from data
“Choosing to grow a graph”

(Overgoor et al., SINM ’19 & WWW ’19)

Traditional discrete choice:

Infer relative importance of edge formation mechanisms from data

(Multinomial logit (MNL) (McFadden, 1973)

Key usage

Timestamped edges → meaningful choice sets

Infer relative importance of edge formation mechanisms from data

Pr(i, C) = \[ \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)} \]
"Choosing to grow a graph"

(Overgoor et al., SINM '19 & WWW '19)

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Multinomial logit (MNL) (McFadden, 1973)

(node)
“Choosing to grow a graph”

Traditional discrete choice:

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<thead>
<tr>
<th>chooser</th>
<th>choice set</th>
</tr>
</thead>
</table>

(under-explored in sociology)
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In network growth

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Key usage

Timestamped edges → meaningful choice sets

Infer relative importance of edge formation mechanisms from data

preferences

node features

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Multinomial logit (MNL) (McFadden, 1973)
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Traditional discrete choice:

Chooser

Choice set

(under-explored in sociology)

(under-explored in sociology)

Infer relative importance of edge formation mechanisms from data

Preference

Node features

(similarity, in-degree, fitness...)

Key usage

Timestamped edges → meaningful choice sets

Infer relative importance of edge formation mechanisms from data

Multinomial logit (MNL) (McFadden, 1973)
The choice set affects preferences
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Context effects
(Huber et al., Journal of Consumer Research 1982)
(Simonson & Tversky, Journal of Marketing Research 1992)
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\textit{e.g., compromise effect:}
The choice set affects preferences

**Context effects**

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*e.g., compromise effect:*

(Simonson, *Journal of Consumer Research* 1989)
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*Context effects*
(Huber et al., *Journal of Consumer Research* 1982)
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e.g., *compromise effect:*
(Simonson, *Journal of Consumer Research* 1989)
The choice set affects preferences

**Context effects**
(Huber et al., *Journal of Consumer Research* 1982)
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*E.g., compromise effect:*
(Simonson, *Journal of Consumer Research* 1989)
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* e.g., compromise effect: *

In networks
* e.g., how do preferences change when choosing from a popular group? *
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Context effects
(Huber et al., *Journal of Consumer Research* 1982)
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In networks
e.g., how do preferences change when choosing from a popular group?

Our model:

**Linear context logit (LCL)**

\[
Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}
\]
The choice set affects preferences

Context effects
(Huber et al., Journal of Consumer Research 1982)
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e.g., compromise effect:

In networks
e.g., how do preferences change when choosing from a popular group?

Linear context logit (LCL)

$$\Pr(i, C) = \frac{\exp(\theta + A x_{C}^{T} x_{i})}{\sum_{j \in C} \exp(\theta + A x_{C}^{T} x_{j})}$$
The choice set affects preferences

**Context effects**

(Huber et al., *Journal of Consumer Research* 1982)
(Simonson & Tversky, *Journal of Marketing Research* 1992)

*Example: compromising effect:*
(Simonson, *Journal of Consumer Research* 1989)

**In networks**

e.g., how do preferences change when choosing from a popular group?

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**Linear context logit (LCL)**

\[
\text{Pr}(i, C) = \frac{\exp(\left[\theta + Ax_C\right]^T x_i)}{\sum_{j \in C} \exp(\left[\theta + Ax_C\right]^T x_j)}
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**In networks**
e.g., how do preferences change when choosing from a popular group?

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**Linear context logit (LCL)**

\[
\text{Pr}(i, C) = \frac{\exp(\left[\theta + Ax_C\right]^T x_i)}{\sum_{j \in C} \exp(\left[\theta + Ax_C\right]^T x_j)}
\]

- base preferences
- context effect matrix
- mean features over choice set
Choosing to close triangles

Triadic closure offers small choice sets
→ tractable inference
→ varied choice sets
Choosing to close triangles

Triadic closure offers small choice sets
→ tractable inference
→ varied choice sets

Our data
Timestamped edges
(including repeats)
Choosing to close triangles

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Our data
Timestamped edges (including repeats)

Node features
1. in-degree of \( w \)
2. # shared neighbors of \( u, w \)
3. weight of edge \( w \rightarrow u \)
4. time since last edge into \( w \)
5. time since last edge out of \( w \)
6. time since last \( w \rightarrow u \) edge

chooser
\( u \)

choice set \( \{w_1, w_2, w_3\} \)

choice \( w_1 \)
Context matters in triadic closure
Context matters in triadic closure

Datasets
email-enron
email-eu
email-w3c
wiki-talk
reddit-hyperlink
bitcoin-alpha
bitcoin-otc
mathoverflow
college-msg
facebook-wall
sms-a
sms-b
sms-c
bit.ly/lcl-data
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Synthetic data, no context effects

Bit.ly/lcl-data
Context matters in triadic closure

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Synthetic data, no context effects

Commenting network, linear context effects

Bitly/lcl-data
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facebook-wall
sms-a
sms-b
sms-c

Synthetic data, no context effects
Commenting network, linear context effects
Email network, nonlinear context effects?
LCL reveals interpretable context effects
LCL reveals interpretable context effects

Estimation

MLE to infer LCL

\[ \ell(\theta, A; \mathcal{D}) = \sum_{(i,C) \in \mathcal{D}} (\theta + A x_C)^T x_i \]

\[-\log \sum_{j \in C} \exp[(\theta + A x_C)^T x_j) \] (concave)
LCL reveals interpretable context effects

Estimation
MLE to infer LCL

$$\ell(\theta, A; \mathcal{D}) = \sum_{(i,C) \in \mathcal{D}} \left( \theta + A x_C \right)^T x_i$$

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(concave)
LCL reveals interpretable context effects

Node features
(left-right, top-bottom)
1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

context effect matrix $A$
red: +, blue: -, white: 0
(column acts on row)

Estimation
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$L_1$ regularization level

LCL negative log-likelihood
(lower = better)
LCL reveals interpretable context effects

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Estimation
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\ell(\theta, A; \mathcal{D}) = \sum_{(i, C) \in \mathcal{D}} \left( \theta + A x_C \right)^T x_i - \log \sum_{j \in C} \exp \left( \left[ \theta + A x_C \right]^T x_j \right) \]  
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LCL negative log-likelihood
(lower = better)
likelihood-ratio test vs MNL
significance threshold \( (p < 0.001) \)

context effect matrix \( A \)
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\( L_1 \) regularization level
\( \lambda = 0 \)
0.005
0.01
0.05
0.1

+0.1%
+0.05%
LCL reveals interpretable context effects

Node features
(left-right, top-bottom)
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context effect matrix A
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LCL negative log-likelihood (lower = better)

likelihood-ratio test vs MNL
significance threshold ($p < 0.001$)

“popularity matters less when choosing from close connections”
“close connections matter more when choosing from the popular”
LCL reveals interpretable context effects

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significance threshold \( p < 0.001 \)

“popularity matters less when choosing from close connections”
“close connections matter more when choosing from the popular”
“popularity matters less when your inbox is full of recent emails”
Other things in our paper

Kiran Tomlinson and Austin R. Benson
Learning Interpretable Feature Context Effects in Discrete Choice
bit.ly/lcl-paper
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Kiran Tomlinson and Austin R. Benson
Learning Interpretable Feature Context Effects in Discrete Choice
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• LCL derivation from simple assumptions
Other things in our paper

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- LCL derivation from simple assumptions
- More flexible model: decomposed LCL

\[ \Pr(i, C) = \sum_{k=1}^{d} \pi_k \frac{\exp \left( [B_k + A_k(x_C)_k]^T x_i \right)}{\sum_{j \in C} \exp \left( [B_k + A_k(x_C)_k]^T x_j \right)} \]
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• LCL derivation from simple assumptions
• More flexible model: decomposed LCL
• LCL identifiability condition

\[
Pr(i, C) = \sum_{k=1}^{d} \pi_k \frac{\exp \left( [B_k + A_k(x_C)k]^T x_i \right)}{\sum_{j \in C} \exp \left( [B_k + A_k(x_C)k]^T x_j \right)}
\]

Theorem 1. A \( d \)-feature linear context logit is identifiable from a dataset \( D \) if and only if

\[
\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in C_D, i \in C \right\} = \mathbb{R}^{d^2+d}.
\] (6)
Other things in our paper

Kiran Tomlinson and Austin R. Benson
Learning Interpretable Feature Context Effects in Discrete Choice
bit.ly/lcl-paper

• LCL derivation from simple assumptions
• More flexible model: decomposed LCL
• LCL identifiability condition
• Application to general choice data
Other things in our paper

Kiran Tomlinson and Austin R. Benson
Learning Interpretable Feature Context Effects in Discrete Choice
bit.ly/lcl-paper

- LCL derivation from simple assumptions
- More flexible model: decomposed LCL
- LCL identifiability condition
- Application to general choice data
- Accounting for context improves prediction

\[
\Pr(i, C) = \sum_{k=1}^{d} \pi_k \frac{\exp \left( [B_k + A_k(x_C)_k]^T x_i \right)}{\sum_{j \in C} \exp \left( [B_k + A_k(x_C)_k]^T x_j \right)}
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\[
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\]

<table>
<thead>
<tr>
<th>Dataset</th>
<th>MNL</th>
<th>LCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISTRICT</td>
<td>3.680 (4.823)</td>
<td>.3327 (4.712)</td>
</tr>
<tr>
<td>DISTRICT-SMART</td>
<td>4.006 (4.900)</td>
<td>.3894 (4.876)</td>
</tr>
<tr>
<td>EXPEDIA</td>
<td>3.859 (2.954)</td>
<td>.3666 (2.926)</td>
</tr>
<tr>
<td>SUSHI</td>
<td>2.727 (2.751)</td>
<td>.2741 (2.771)</td>
</tr>
<tr>
<td>CAR-A</td>
<td>3.570 (4.791)</td>
<td>.3514 (4.774)</td>
</tr>
<tr>
<td>CAR-B</td>
<td>3.326 (4.711)</td>
<td>.3326 (4.711)</td>
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<tr>
<td>CAR-ALT</td>
<td>2.944 (2.875)</td>
<td>.2650 (2.804)</td>
</tr>
<tr>
<td>SYNTHETIC-MNL</td>
<td>.1513 (.1865)</td>
<td>.1512 (.1864)</td>
</tr>
<tr>
<td>SYNTHETIC-LCL</td>
<td>.1360 (.1864)</td>
<td>.1357 (.1883)</td>
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<tr>
<td>WIKI-TALK</td>
<td>2.946 (2.916)</td>
<td>.2666 (2.773)</td>
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<td>REDDIT-HYPERLINK</td>
<td>2.859 (2.611)</td>
<td>.2761 (2.606)</td>
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<td>BITCOIN-ALPHA</td>
<td>2.724 (3.266)</td>
<td>.2591 (3.178)</td>
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<td>BITCOIN-OTC</td>
<td>1.891 (2.756)</td>
<td>.1529 (2.468)</td>
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<td>SMS-A</td>
<td>2.825 (3.250)</td>
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<tr>
<td>SMS-B</td>
<td>3.045 (3.419)</td>
<td>.2848 (3.372)</td>
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<tr>
<td>SMS-C</td>
<td>3.115 (3.455)</td>
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<td>EMAIL-ENRON</td>
<td>1.285 (2.086)</td>
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<td>EMAIL-ED</td>
<td>2.683 (3.021)</td>
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<td>EMAIL-W3C</td>
<td>1.332 (2.070)</td>
<td>.1210 (1.845)</td>
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<td>FACEBOOK-WALL</td>
<td>2.176 (2.895)</td>
<td>.2109 (2.871)</td>
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<tr>
<td>COLLEGE-MSG</td>
<td>1.850 (2.726)</td>
<td>.1723 (2.655)</td>
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<tr>
<td>MATHOVERFLOW</td>
<td>1.385 (2.503)</td>
<td>.1153 (2.200)</td>
</tr>
</tbody>
</table>
Concluding thoughts

Key takeaway
Context effects matter in triadic closure

Challenges
Features correlate
Causal context effects?
Handling nonlinearity?
Global edge formation modes?
Missing timestamps?
Concluding thoughts

Key takeaway
Context effects matter in triadic closure

Challenges
Features correlate
Causal context effects?
Handling nonlinearity?
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Missing timestamps?

Thank you!
More questions or ideas?
Email me: kt@cs.cornell.edu

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