Music of the Microspheres

Eigenvalue Problems from Micro-Gyro Design

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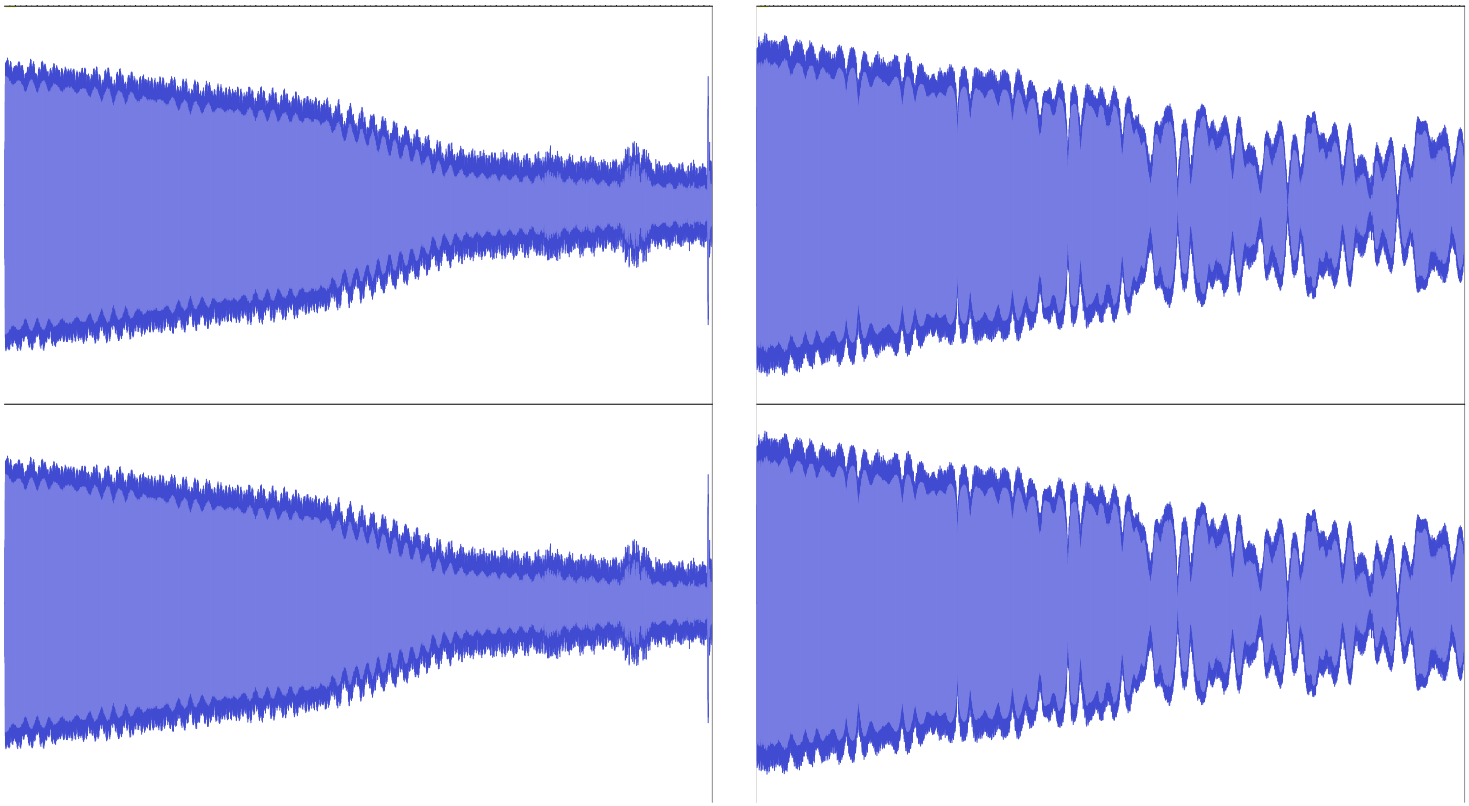
Bryan's Experiment

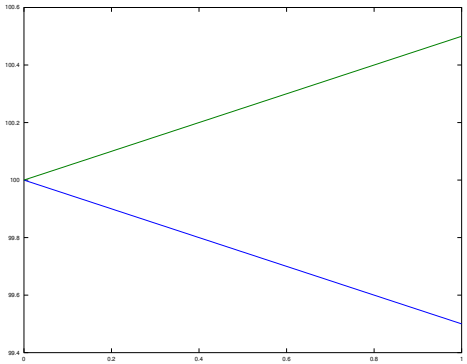
"On the beats in the vibrations of a revolving cylinder or bell"
by G. H. Bryan, 1890

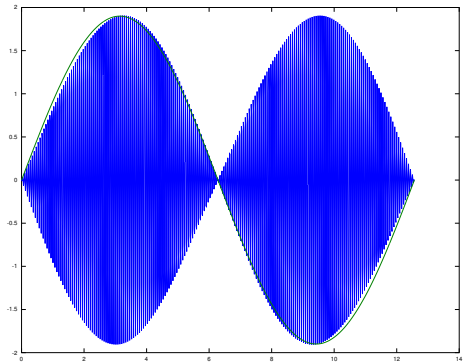
Bryan's Experiment Today

The Beat Goes On



The Beat Goes On





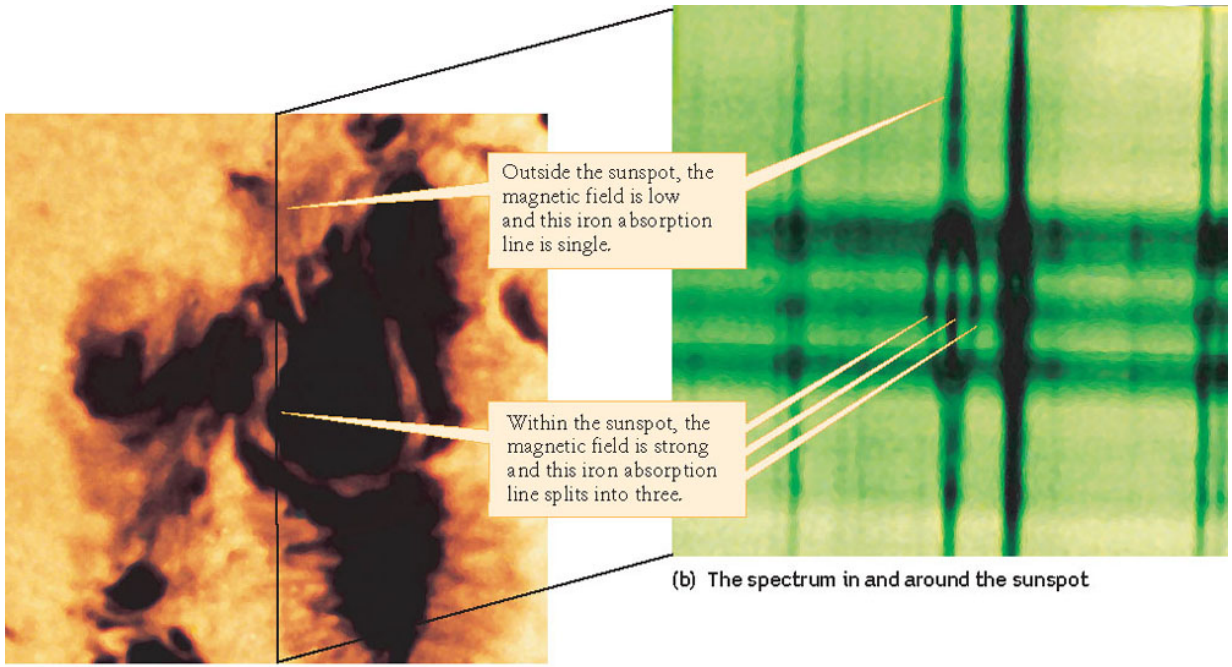
Free vibrations in a rotating frame (simplified):

$$\ddot{\mathbf{q}} + 2\beta\Omega\mathbf{J}\dot{\mathbf{q}} + \omega_0^2\mathbf{q} = 0, \qquad \mathbf{J} \equiv \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Eigenvalue problem: $\left(-\omega^2\mathbf{I} + 2i\omega\beta\Omega\mathbf{J} + \omega_0^2\right) q = 0.$

Solutions: $\omega \approx \Omega_0 \pm \beta\Omega. \implies$ beating $\propto \Omega$!

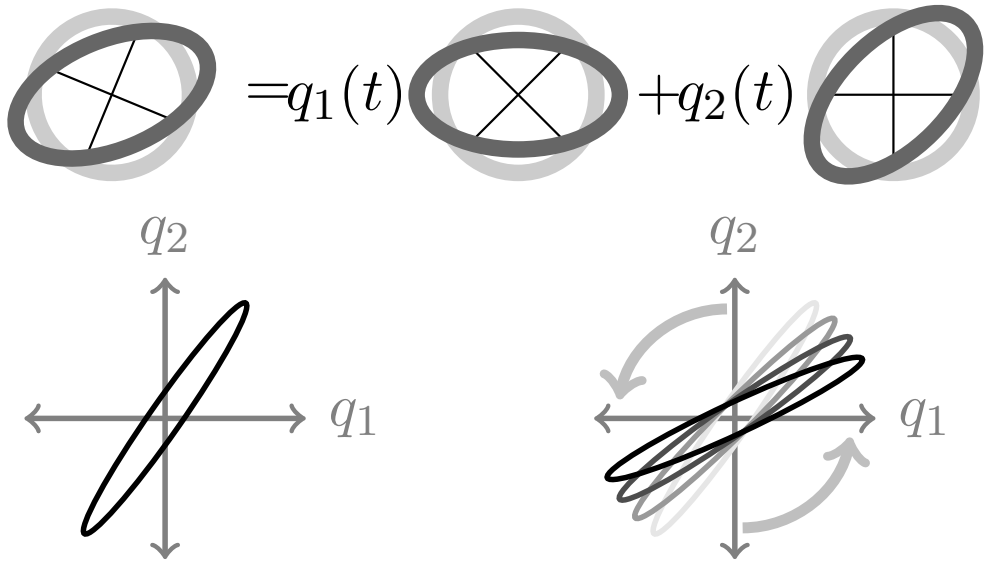
Bryan, Zeeman, Stark, ...



This is a common picture:

- Symmetry leads to degenerate modes
- Perturbations split (some) degeneracies

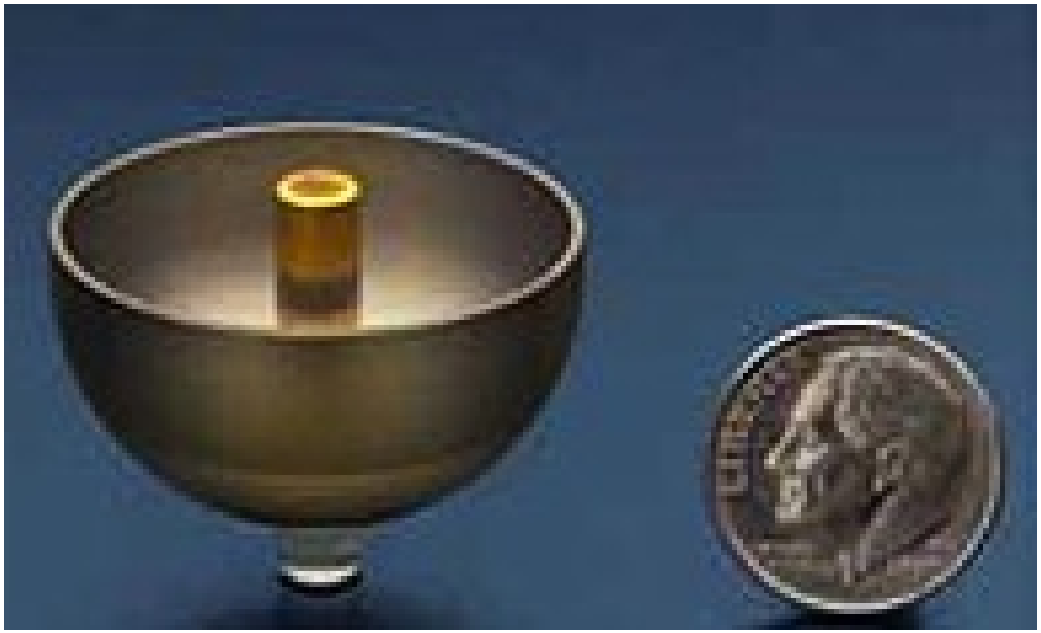
A General Picture



$$\begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \approx \begin{bmatrix} \cos(-\beta\Omega t) & -\sin(-\beta\Omega t) \\ \sin(-\beta\Omega t) & \cos(-\beta\Omega t) \end{bmatrix} \begin{bmatrix} q_1^0(t) \\ q_2^0(t) \end{bmatrix}.$$

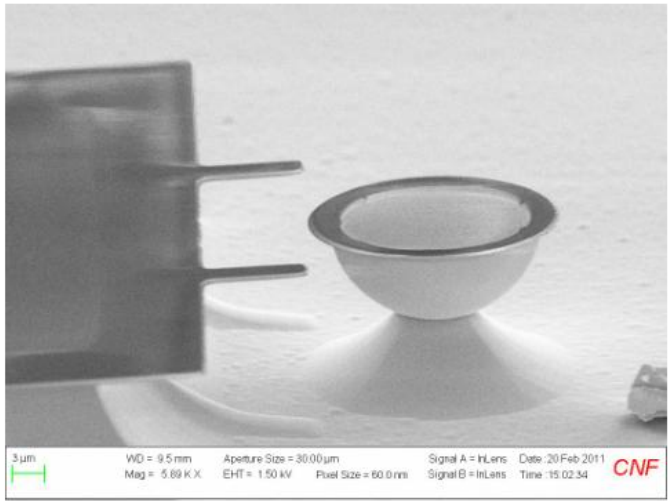
Foucault in Solid State

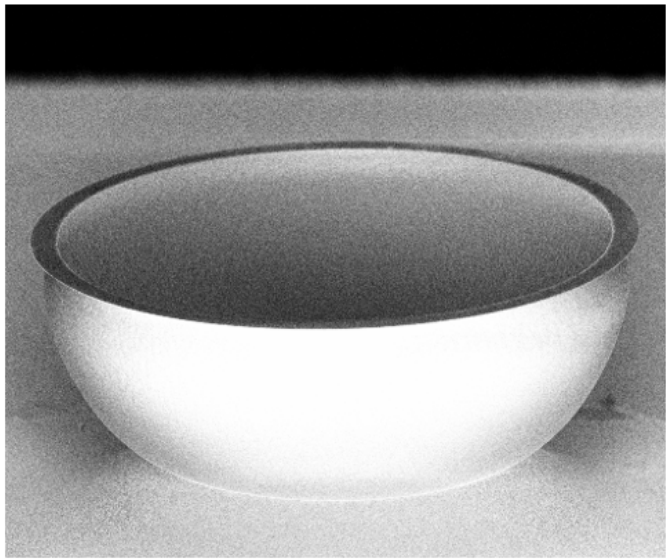
A Small Application

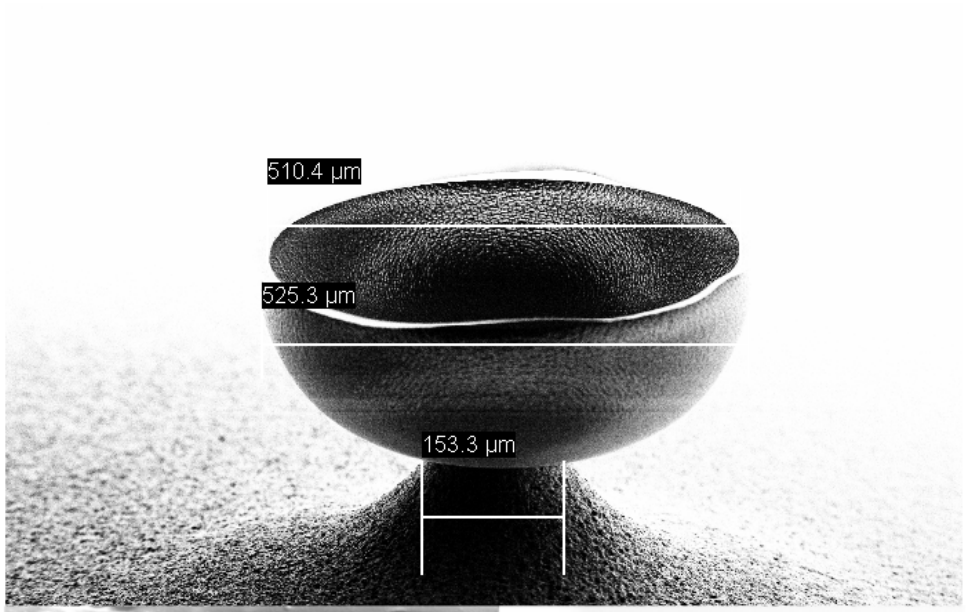


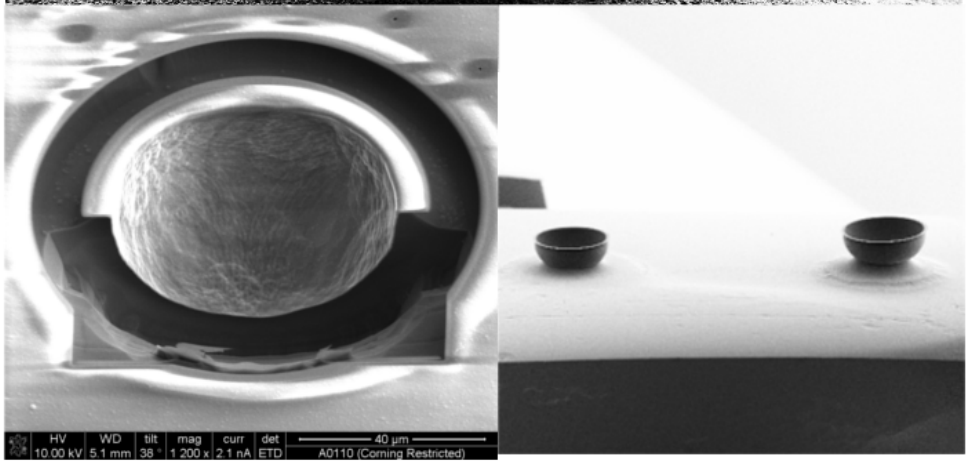
Northrup-Grummond HRG
(developed c. 1965–early 1990s)

A Smaller Application (Cornell)

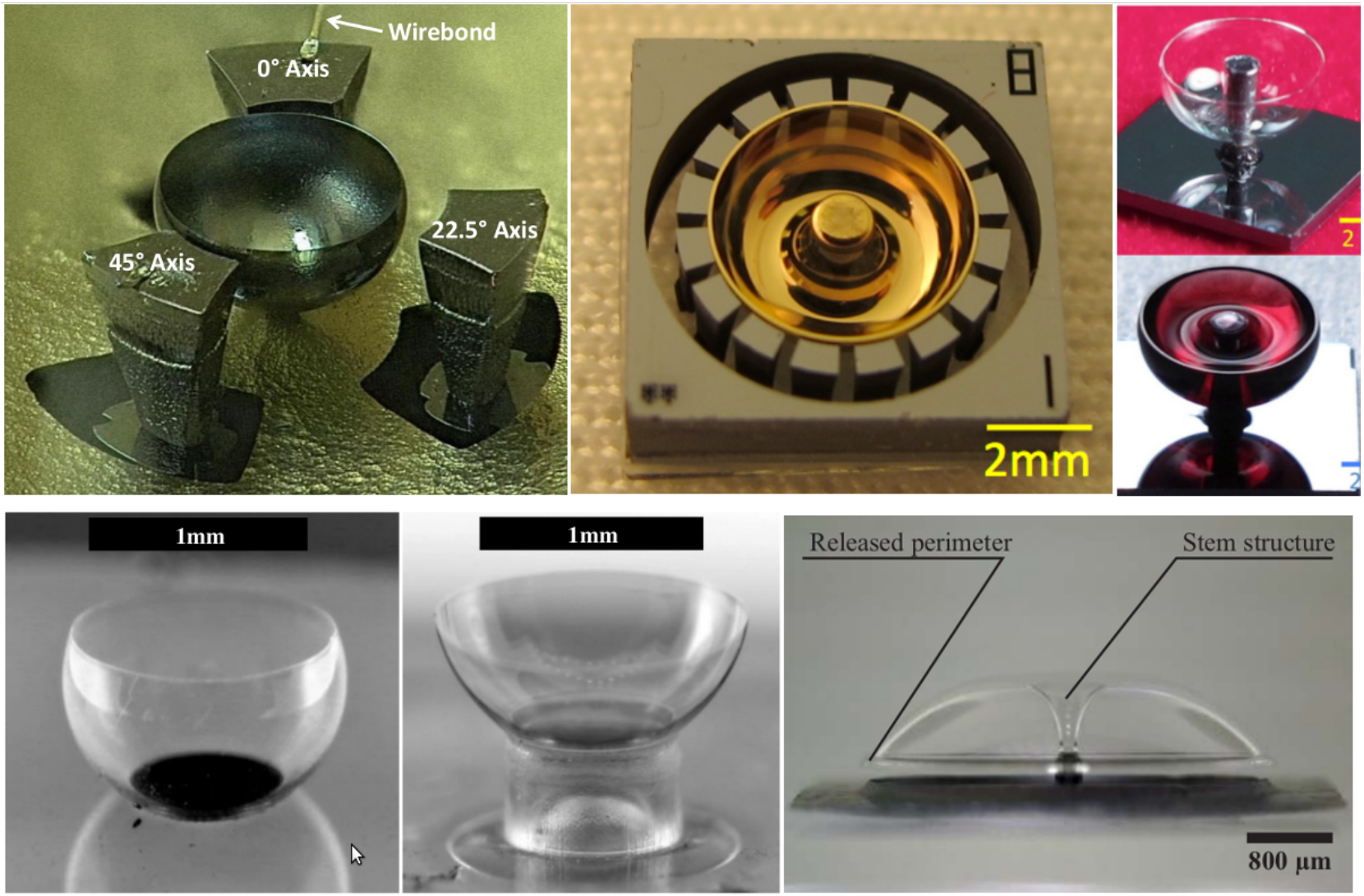




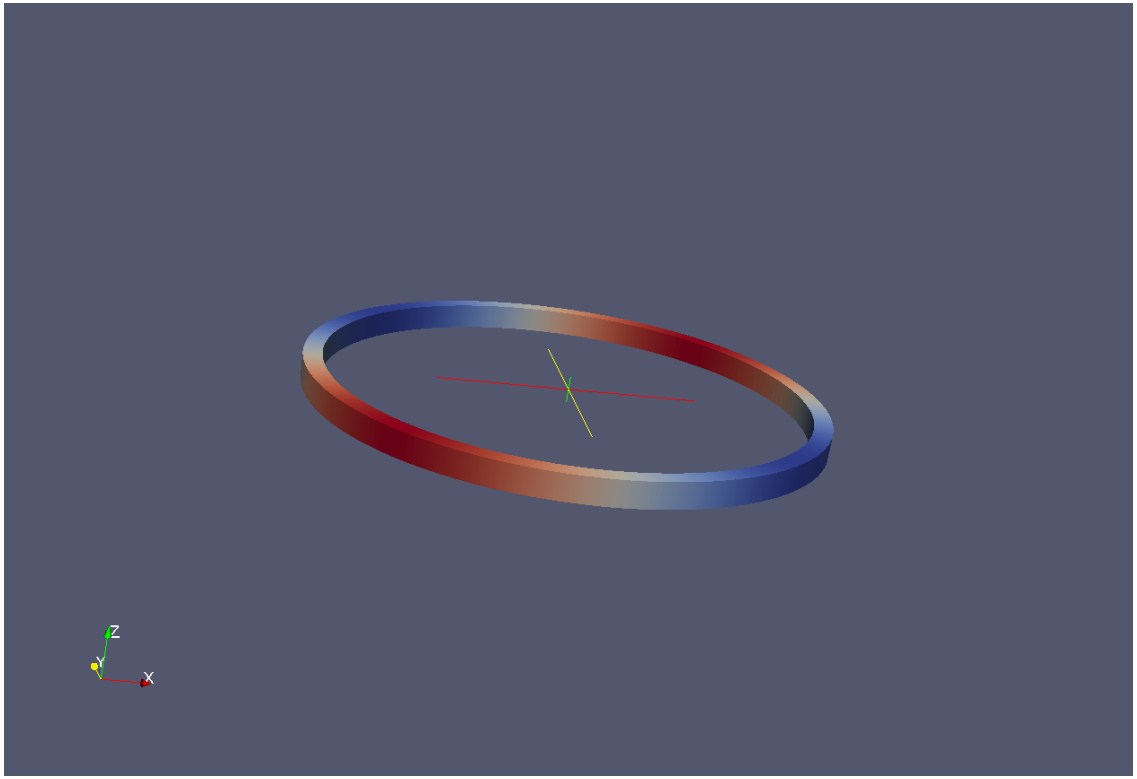




A Smaller Application (UMich, GA Tech, Irvine)



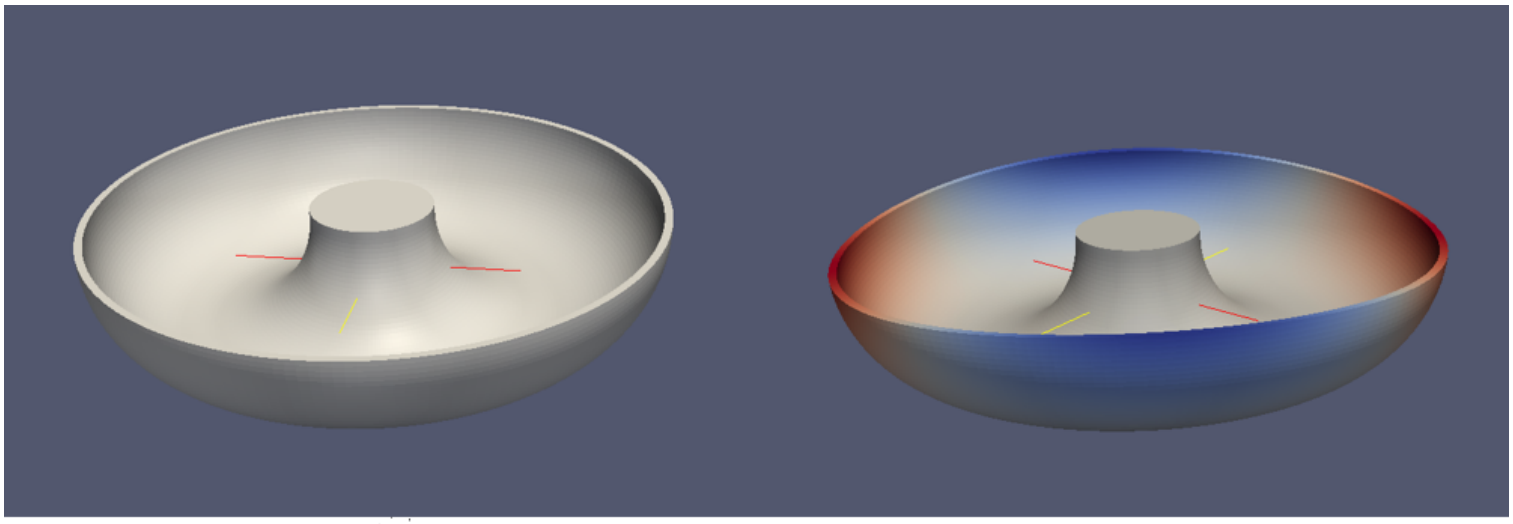
The Perturbation Picture

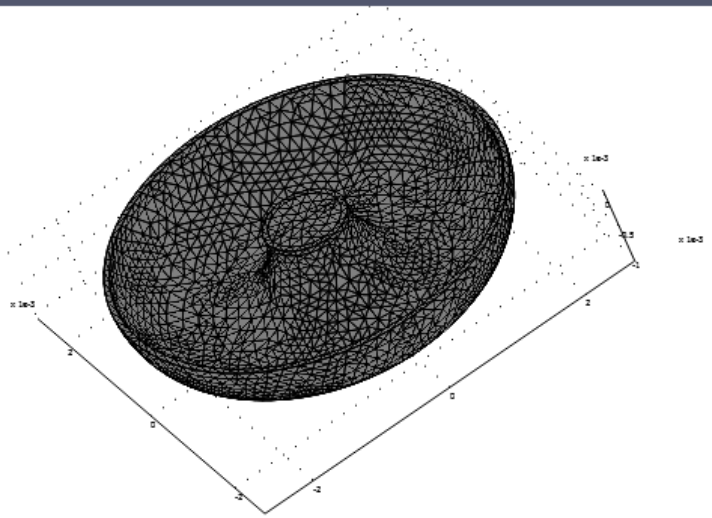


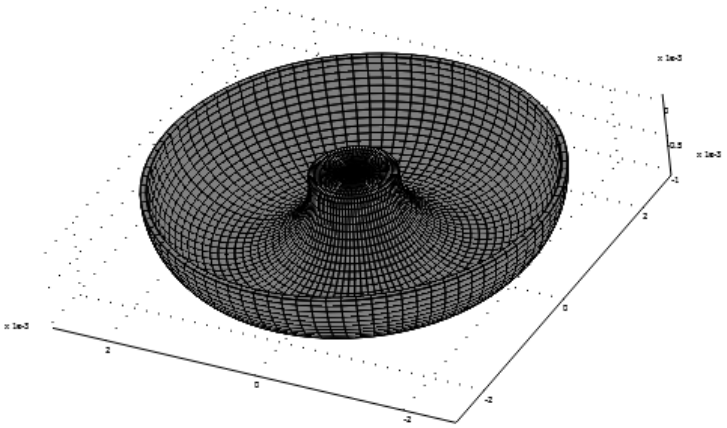
Perturbations split degenerate modes:

- Coriolis forces (good)
- Imperfect fab (bad, but physical)
- Discretization error (non-physical)

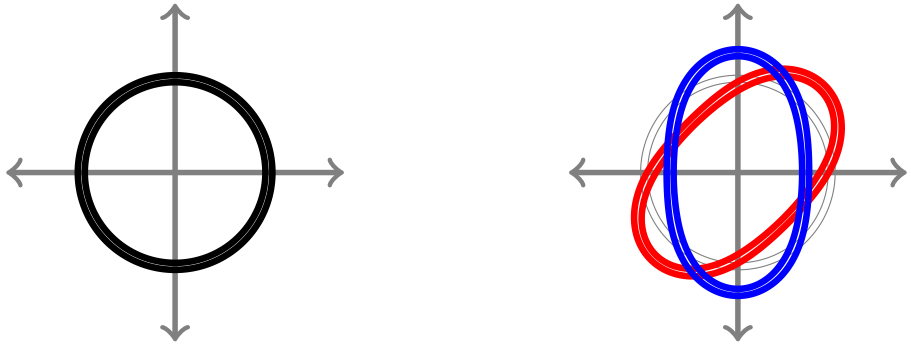
Uncritical FEA: Fail!







Perfect Geometry

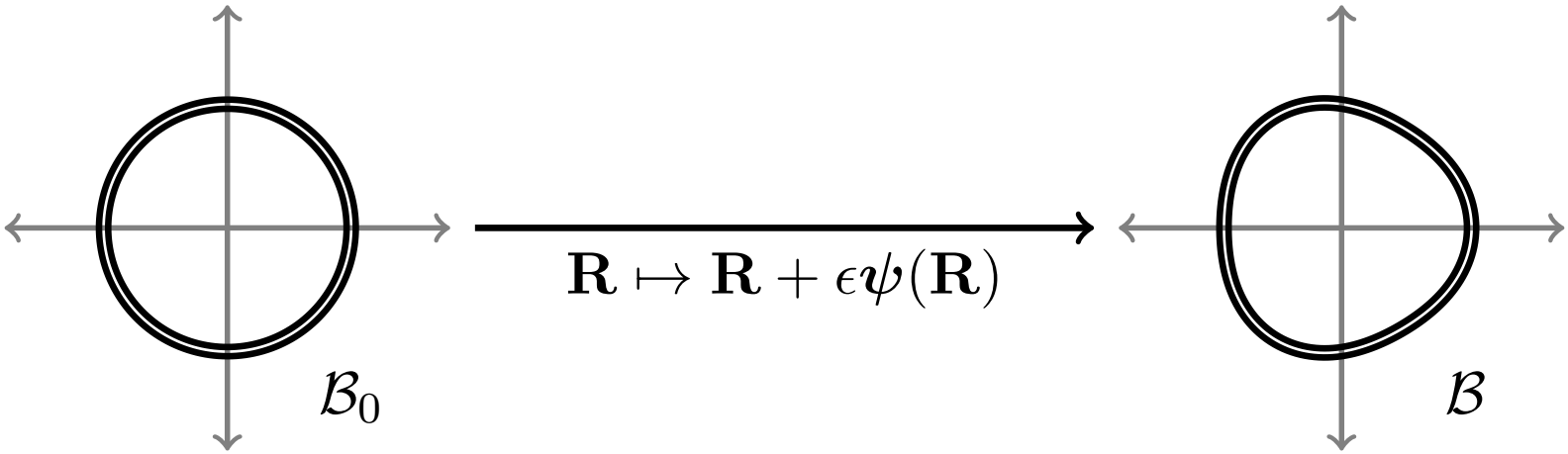


Absent rotation:

- Write motion in terms of a Fourier expansion
- Each mode involves one azimuthal number m
- Modal analysis decouples into “2.5D” subproblems

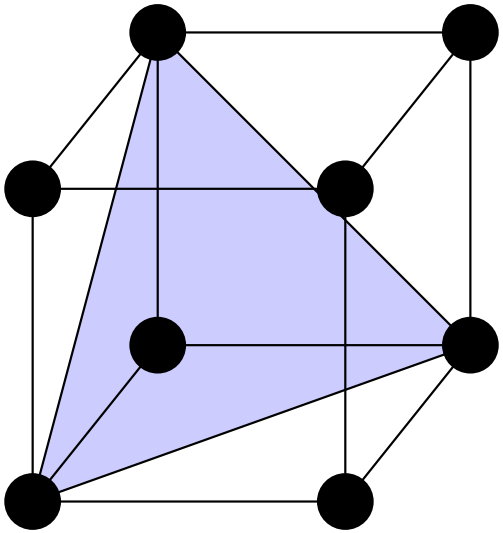
Rotating frame requires a minor tweak (perturbation theory).

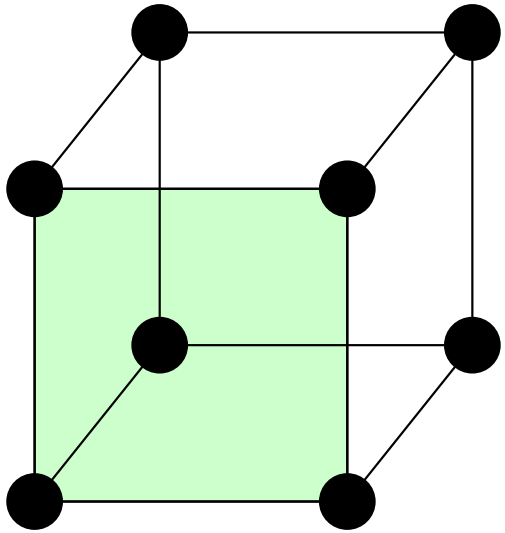
Representing Imperfection



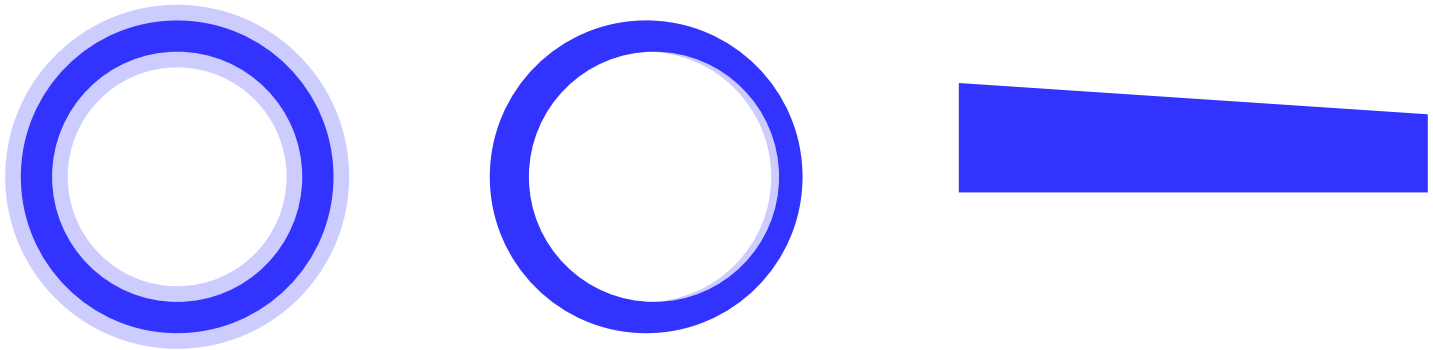
Write Fourier series for $\boldsymbol{\psi}$, too!

Imperfections: Etch Anisotropy





Imperfections: Processing Effects



Analyzing Imperfections

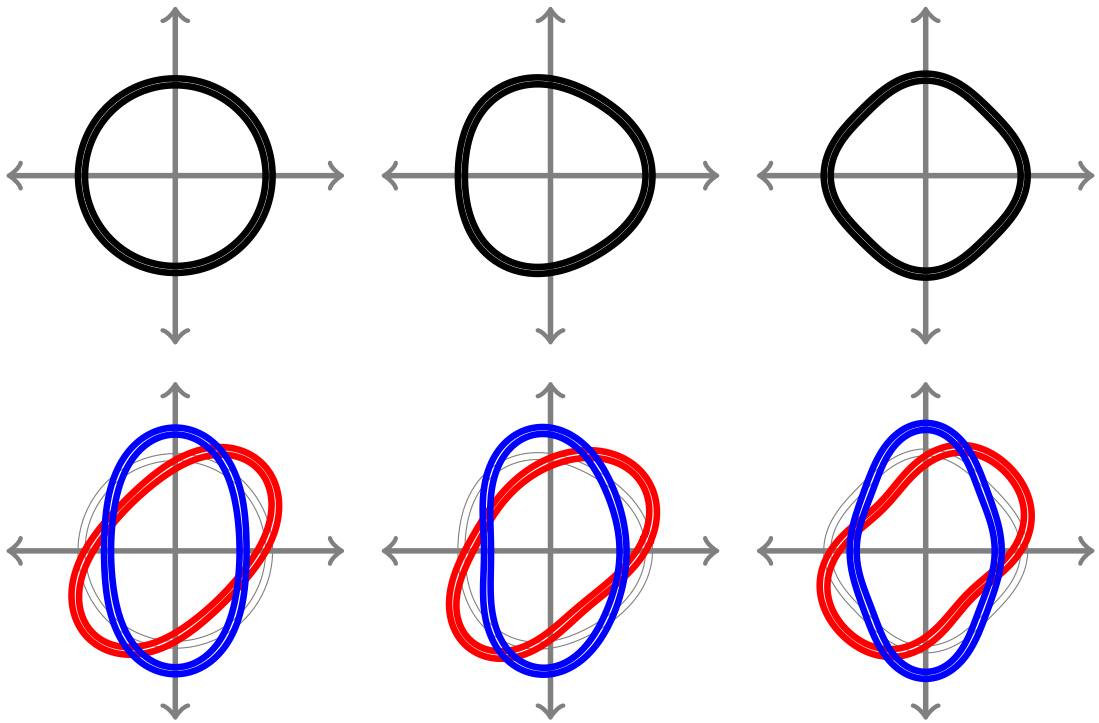
Basic framework:

- Represent geometry and imperfections in Fourier series
- Treat imperfections as perturbations

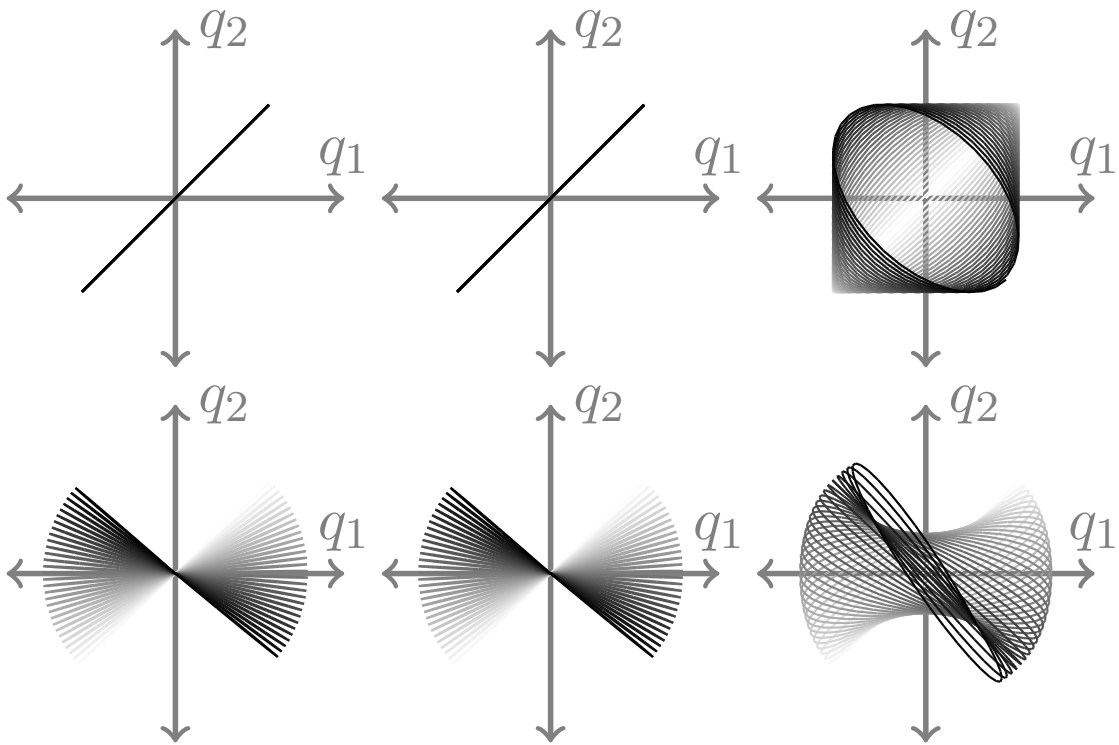
Payoff:

- Quantitative: Fast and accurate “2.5D” simulations
- Qualitative: *Selection rules* identify “dangerous” imperfections

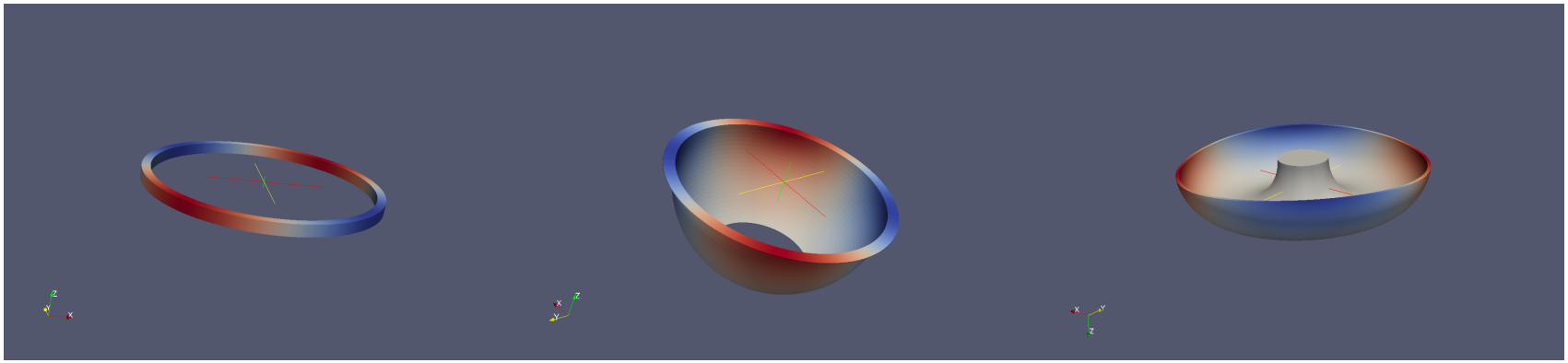
Analyzing Imperfect Rings



Analyzing Imperfect Rings



Read All About It!



Yilmaz and Bindel
“Effects of Imperfections on Solid-Wave Gyroscope Dynamics”
Proceedings of IEEE Sensors 2013, Nov 3–6.

Thanks to DARPA MRIG + Sunil Bhave and Laura Fegely.