A Tale of Two Eigenvalue Problems

Music of the Microspheres + Hearing the Shape of a Graph

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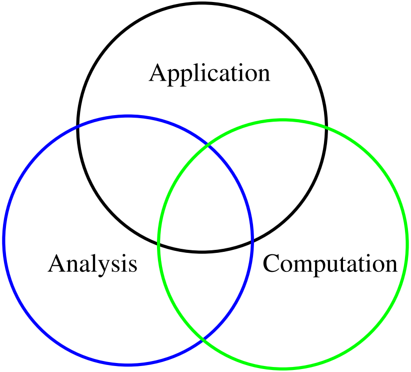
CAM Visit Talk, 28 Feb 2014

The Computational Science & Engineering Picture



- MEMS
- Smart grids
- Networks

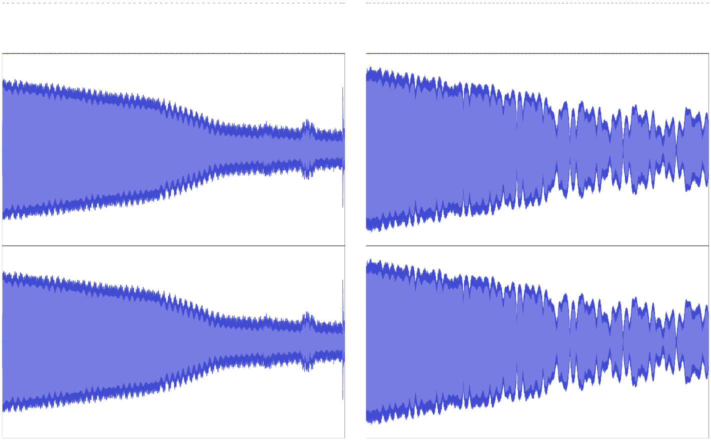
- Linear algebra
- Approximation theory
- Symmetry + structure

- HPC / cloud
- Simulators
- Solvers

Music of the Spheres!

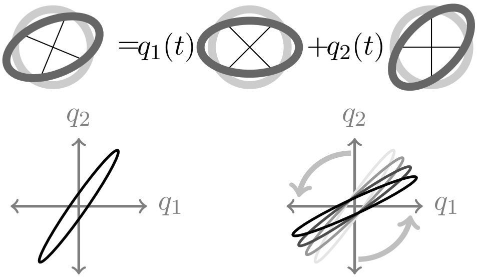
"On the beats in the vibrations of a revolving cylinder or bell"
by G. H. Bryan, 1890

The Beat Goes On

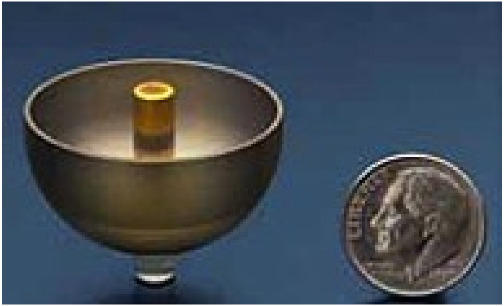


Foucault in Solid State

A General Picture

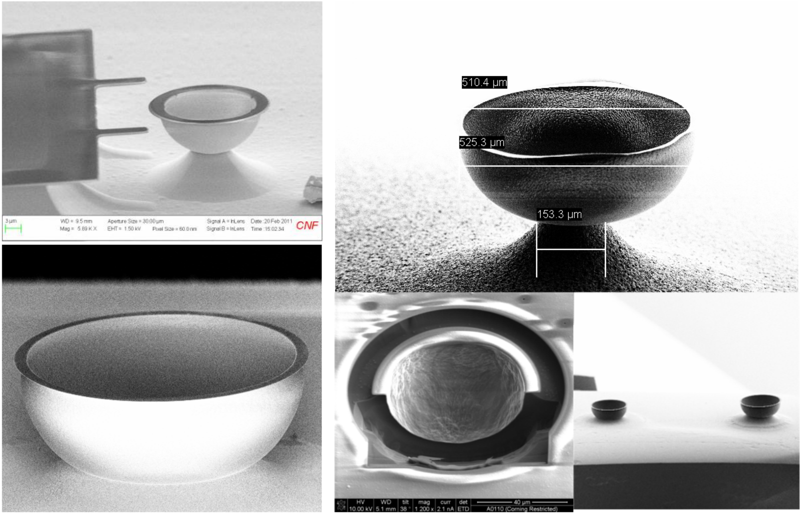


A Small Application

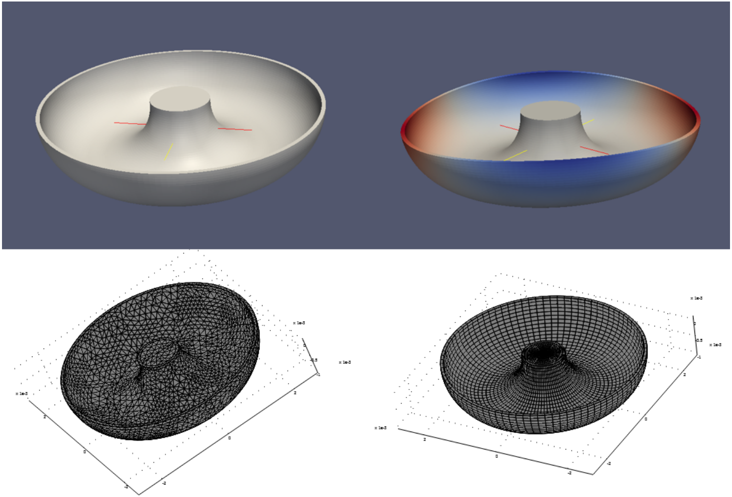


Northrup-Grumman HRG
(developed c. 1965–early 1990s)

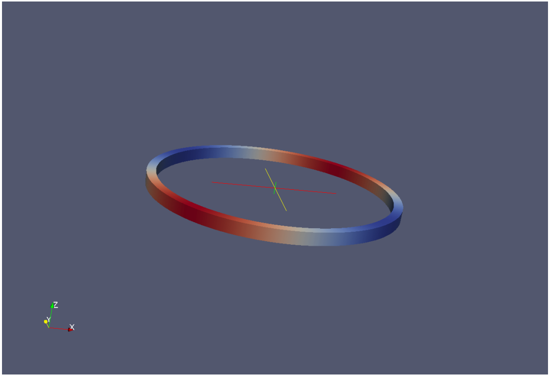
A Smaller Application (Cornell)



Uncritical FEA: Fail!



The Perturbation Picture

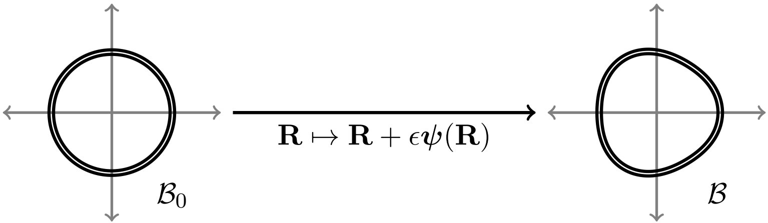


Perturbations split degenerate modes:

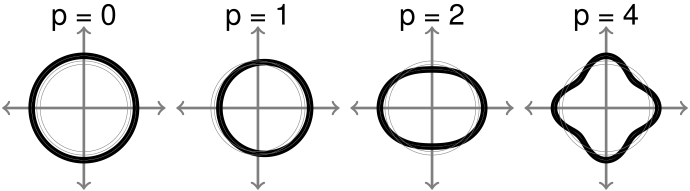
- Coriolis forces (good)
- Imperfect fab (bad, but physical)
- Discretization error (non-physical)

Fun with Fourier

- Perfect geometry: Fourier expand modal displacements
- Imperfect geometry: Fourier expand geometric distortion, too!



Typical Fabrication Imperfections

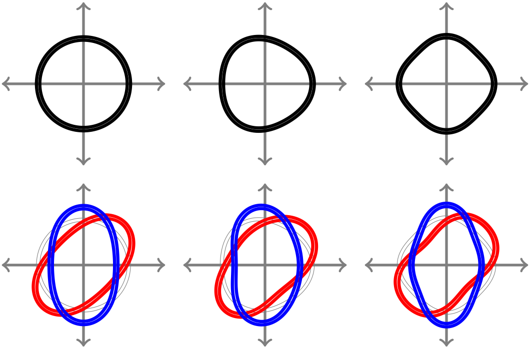


Imperfections are *small* and *structured*. Use for:

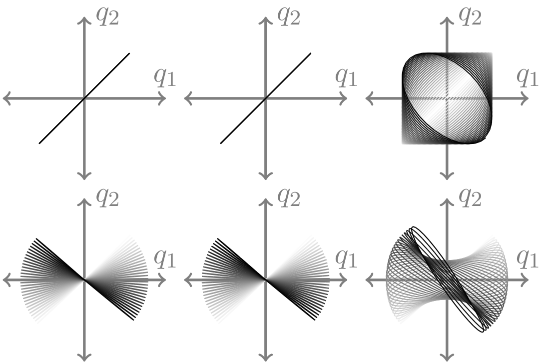
- Fast *quantitative* information via structured FEA
- *Qualitative* information for engineers in early design

Fourier analysis, group theory, eigenvalue perturbations, FEA, ...

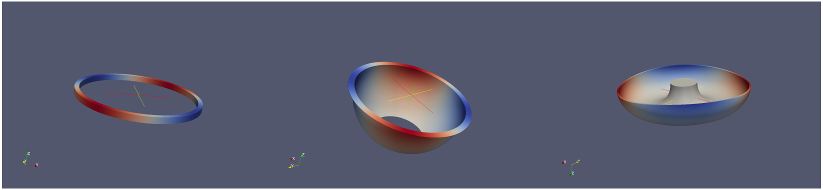
Analyzing Imperfect Rings



Analyzing Imperfect Rings



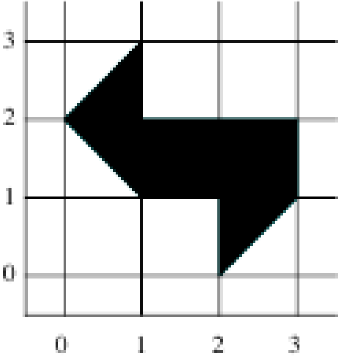
Read All About It!

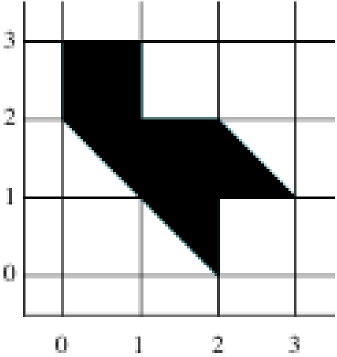


Yilmaz and Bindel
"Effects of Imperfections on Solid-Wave Gyroscope Dynamics"
Proceedings of IEEE Sensors 2013, Nov 3–6.

Thanks to DARPA MRIG + Sunil Bhave and Laura Fegely.

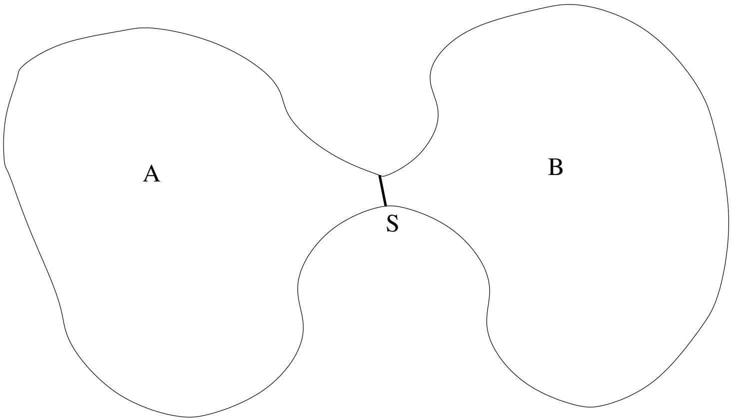
Can One Hear the Shape of a Drum?





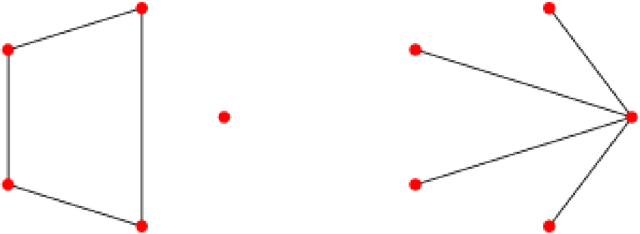
$$-\nabla^2 u = \lambda u \text{ on } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$

What Do You Hear?



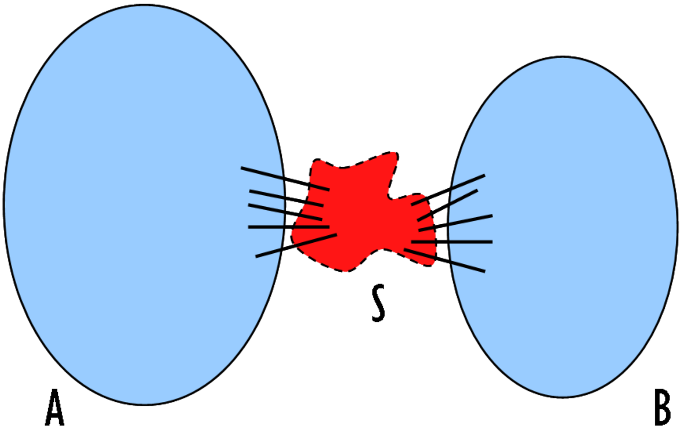
- Size of bottlenecks (Cheeger inequality) – λ_2
- Volume (Weyl law) – asymptotic distribution of λ_n

Can One Hear the Shape of a Graph?



From eigenvalues of adjacency, Laplacian, normalized Laplacian?

What Do You Hear?



- Size of separators (Cheeger inequality)
- What about analogue of Weyl's law?

What Do You Hear?

What information hides in the eigenvalue distribution?

- Discretizations of Laplacian: something like Weyl's law
- Sparse random graphs: Wigner semicircular distribution
- "Real" networks: less well understood

Computing all eigenvalues seems *expensive*!

- But estimating distributions is much less expensive
- Steal a method (KPM) from the physics literature

Exploring Spectral Densities (with David Gleich)

- Consider spectrum of normalized Laplacian (random walk matrix)
- Approximate via KPM and compare to full eigencomputation

Things we know

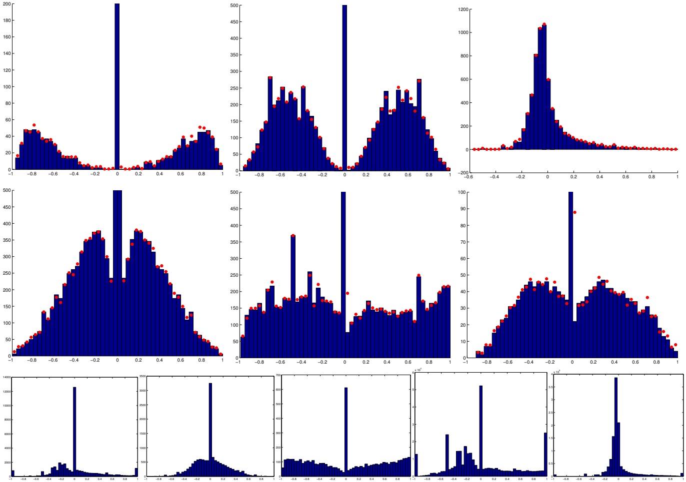
- Eigenvalues in $[-1, 1]$; nonsymmetric in general
- Stability: change d edges, have

$$\lambda_{j-d} \leq \hat{\lambda}_j \leq \lambda_{j+d}$$

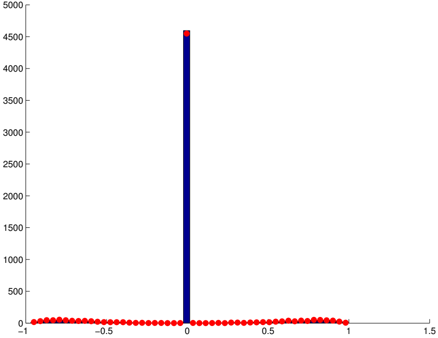
- kth moment = probability of return after k-step random walk
- Eigenvalue cluster near 1 $\sim$ well-separated clusters
- Eigenvalue cluster near 0 $\sim$ triangles connected by one node

What else can we "hear"?

What Do You Hear?



Erdos



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40
20
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−0.6
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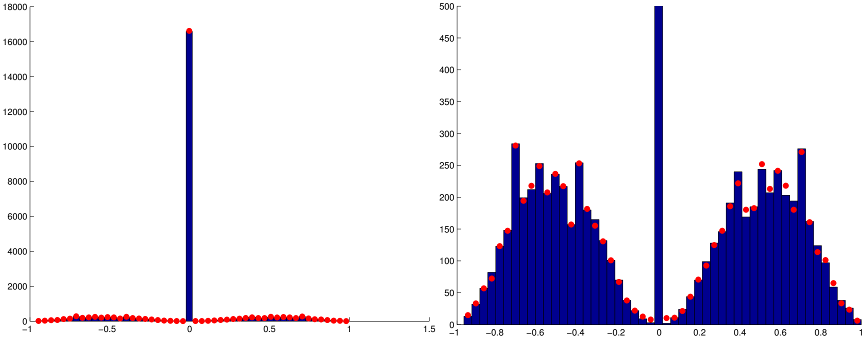
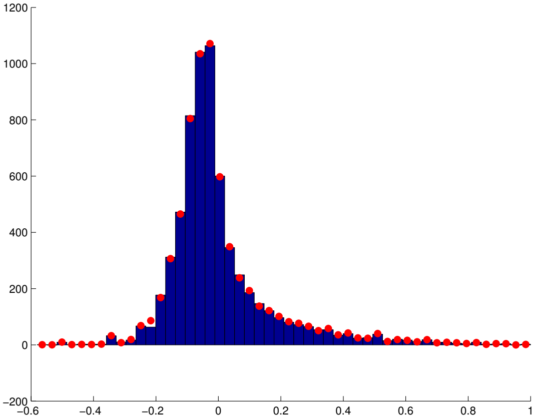
1

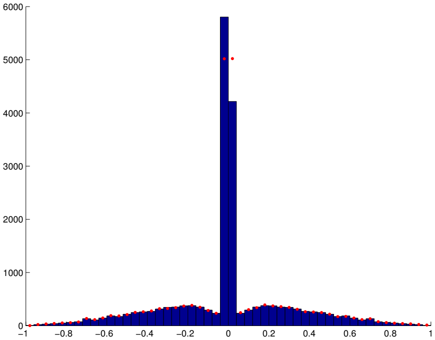
Internet topology

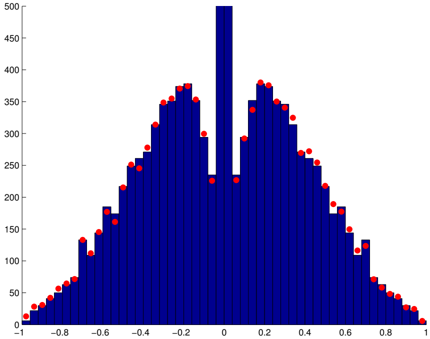


Marvel characters

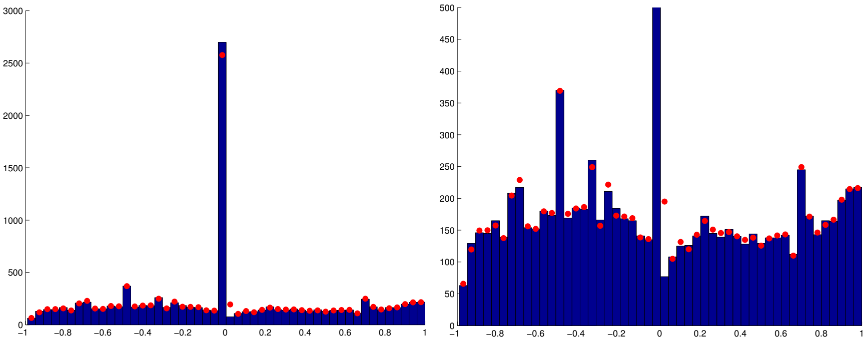


Marvel comics





PGP



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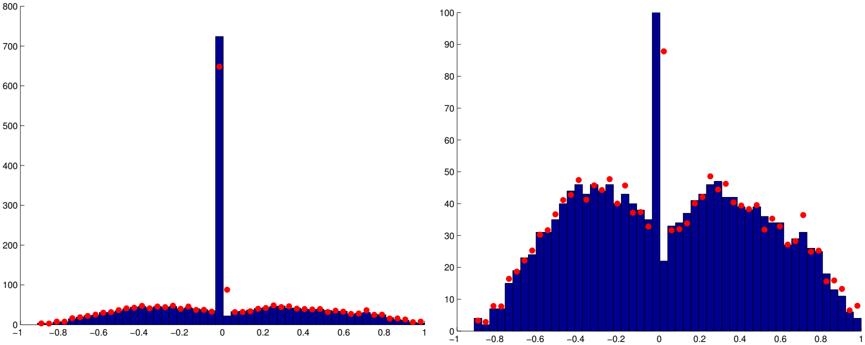
Yeast



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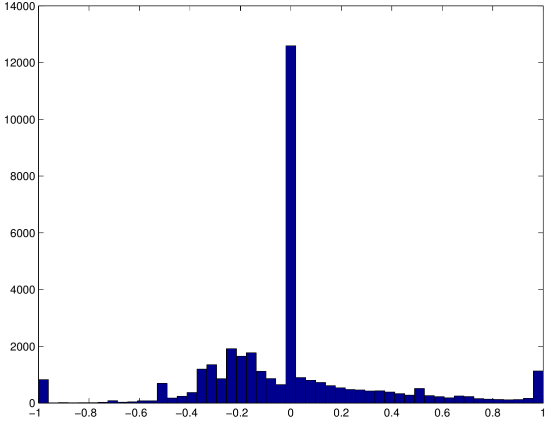
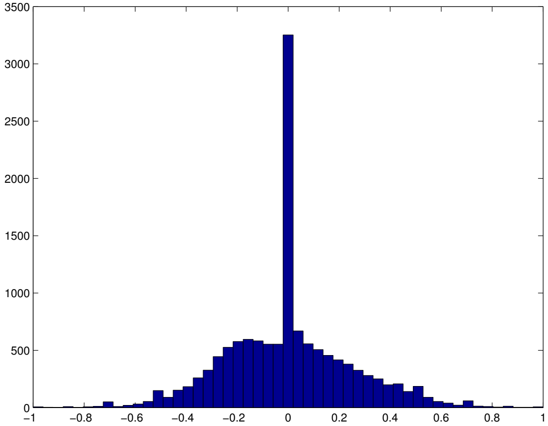
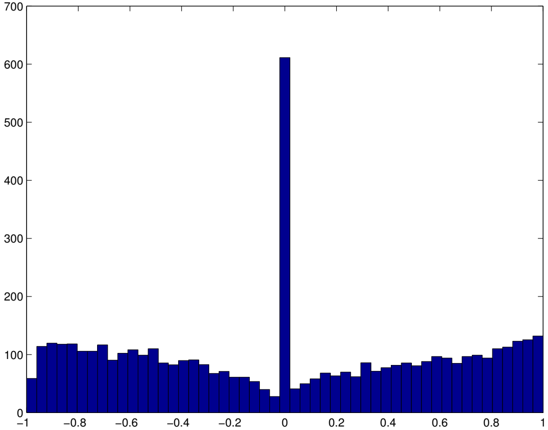
Enron emails (SNAP)



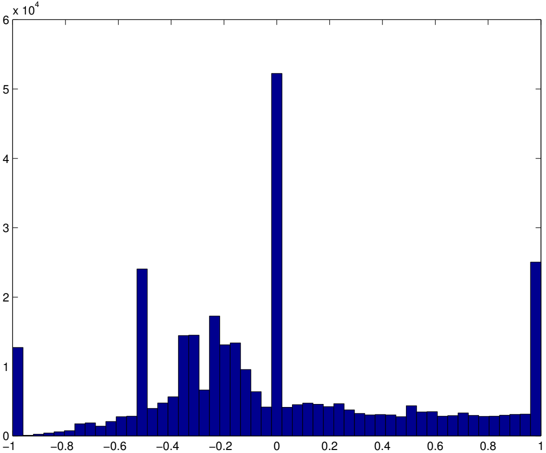
Reuters911 (Pajek)



US power grid (Pajek)

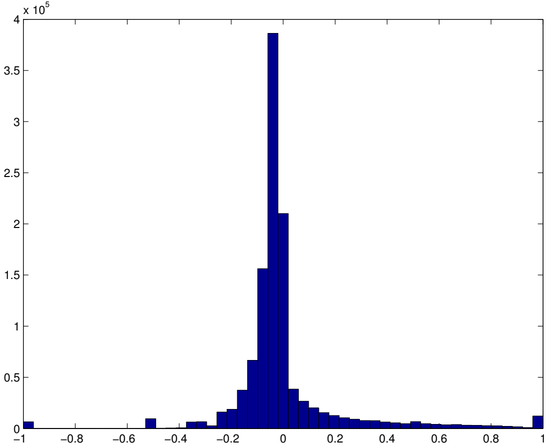


DBLP 2010 (LAW)



$N = 326186$, $nnz = 1615400$, 80 s (1000 moments, 10 probes)

Hollywood 2009 (LAW)



$N = 1139905$, $nnz = 113891327$, 2093 s (1000 moments, 10 probes)