Detecting Overlapping Communities by finding Sparse Vectors in Unconverged Invariant Subspace Approximations

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I have no idea what you're talking about...
...so here's a bunny with a pancake on its head.

Why eigenvalues?

A play in three acts.

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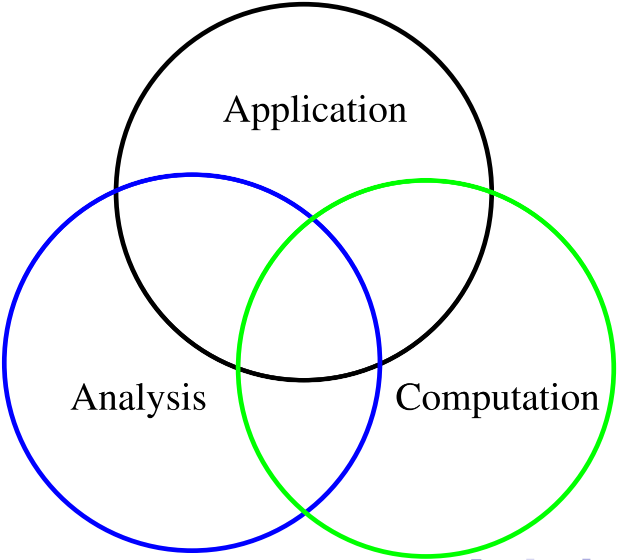
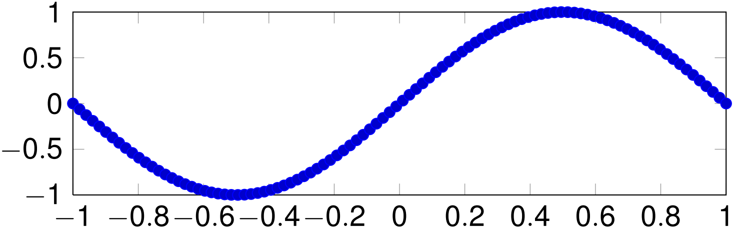
The CSE Picture



Outline

The Humble Guitar String



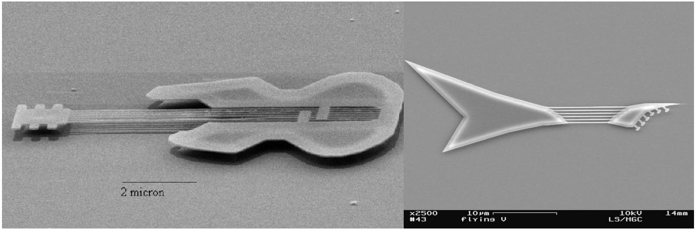
A complicated PDE

$$\rho \frac{\partial^2 u}{\partial t^2} - E \frac{\partial^2 u}{\partial x^2} = 0$$

Produces a simple solution

$$u(x,t) = u_0(x)\sin(\omega t)$$

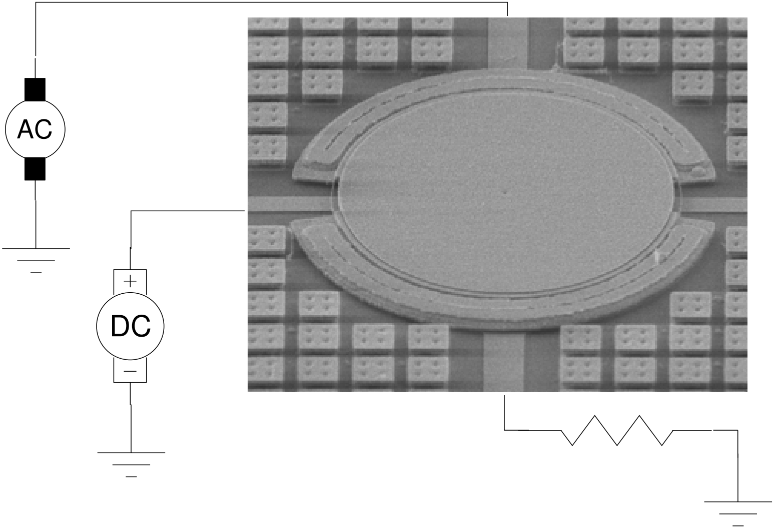
Resonating MEMS and Tiny Guitars



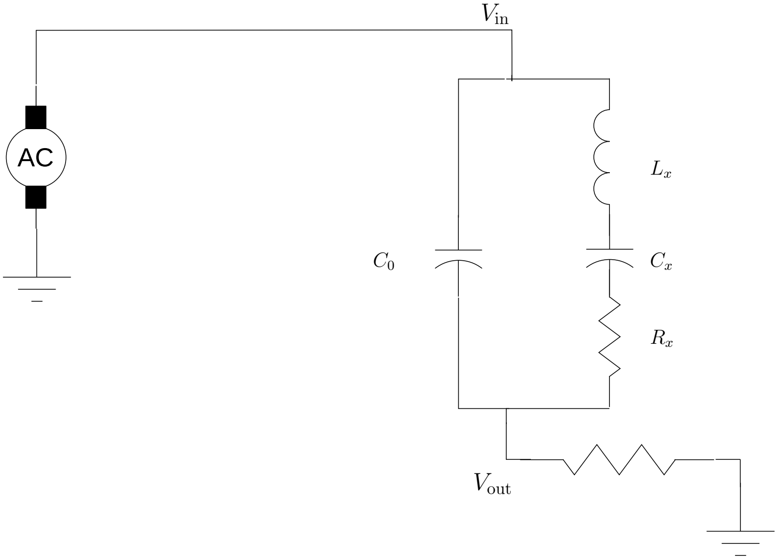
Microguitars from Cornell University (1997 and 2003)

- MEMS = Micro-Electro-Mechanical Systems
- Micron-scale *mechanical* structures with IC fab technology
- Widely used for sensing and signal processing ...
- ... and sometimes really high-pitch guitars!

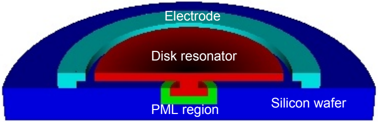
Modeling a Disk Resonator

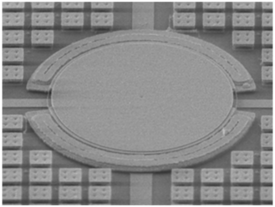


Modeling a Disk Resonator



Modeling a Ringing Disk



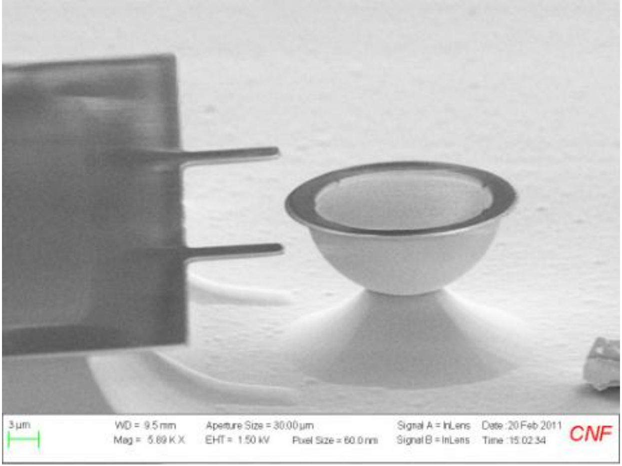


- At what frequencies does this vibrate?
- How quickly is the ringing damped?
- What about errors (in numerics or fabrication)?
- How do we answer these questions *fast*?

Current example: Micro-HRG / GOBLiT / OMG

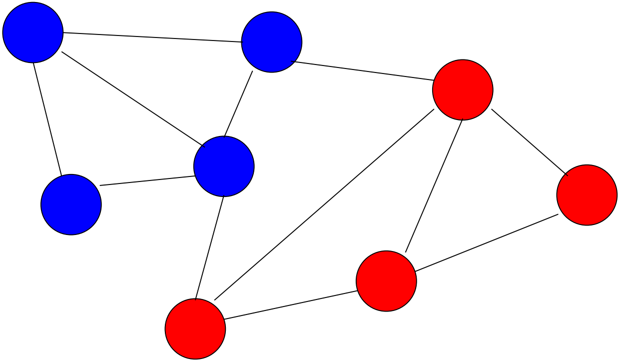
- This is a gyroscope!
- Now make it 1mm across.
- Collaborator roles:
  - Basic design
  - Fabrication
  - Measurement
- Our part:
  - Detailed physics
  - Fast software
  - Sensitivity
  - Design optimization

A little GOBLiT



Outline

Graph Bisection



Goal: Cut in half, minimize edges cut.

Graphs to Quadratics

Give node i a label $x_i = \pm 1$.

- The labels cut the graph in half:

$$\sum_{i=1}^{n} x_i = 0.$$

- Count cut edges by a *quadratic form*

$$|\text{cut edges}| = \frac{1}{4} \sum_{(i,j) \in E} (x_i - x_j)^2$$

Quadratic forms and matrices

$$|\text{cut edges}| = \frac{1}{4} \sum_{(i,j)\in E} (x_i - x_j)^2$$

$$= \frac{1}{4} \sum_{(i,j)\in E} \begin{bmatrix} x_i \\ x_j \end{bmatrix}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} \quad = \frac{1}{4} x^T L x$$

where

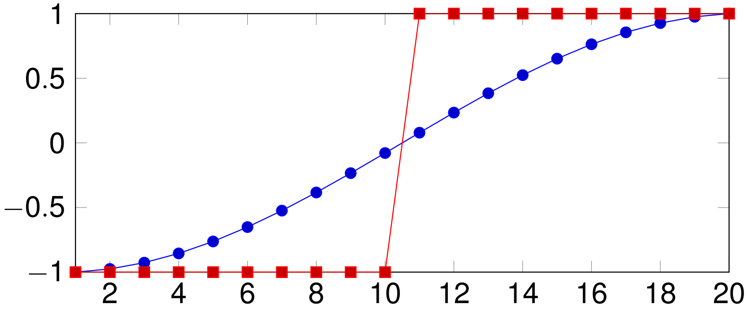
$$L_{ij} = \begin{cases} \text{degree of node } i, & i = j \\ -1, & (i,j) \in E \end{cases}$$

Graph bisection

Idea: Minimize $s^T L s$ such that $e^T s = 0$, $s \in \{\pm 1\}^n$.

Oops — NP hard!

Relax!

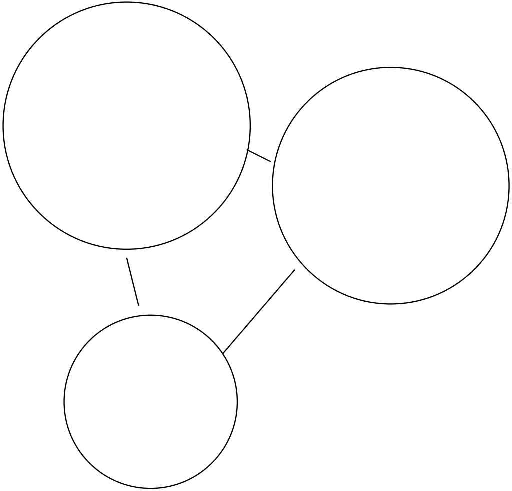


Hard: $\min s^T L s$ s.t. $e^T s = 0$, $s \in \{\pm 1\}^n$.

Easy: $\min v^T L v$ s.t. $e^T v = 0$, $v \in \mathbb{R}^n$, $\|v\|^2 = n$.

This is an eigenvalue problem!

Three cups, three straws, a drop of dye



The random walker

Basic idea: Ideas diffuse, too!

- Day 1: I came up with a funny joke!
- Day 2: I tell everyone in my family
- Day 3: My mother tells a friend?

Ideas diffuse fastest within communities (graph clusters).

Simon-Ando theory

Markov chain with loosely-coupled subchains:

- Rapid *local* mixing: after a few steps

$$p_k \approx \sum_{j=1}^{c} \alpha_{j,k} p_\infty^{(j)}$$

  where $p_\infty^{(j)}$ is a local equilibrium for the jth subchain
- Slow equilibration: $\alpha_{j,k} \to \alpha_{j,\infty}$.

Spectral Simon-Ando picture

Exactly decoupled case (c decoupled chains):

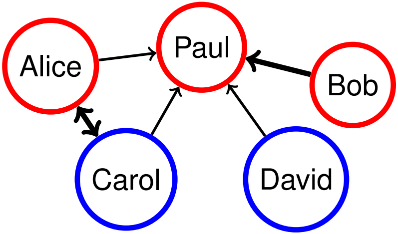
- Eigenvalue one has multiplicity c.
- Eigenvectors of T are local equilibria.
- Rapid mixing $\implies$ large gap to λ_{c+1}.

Weakly coupled case:

- Cluster of c eigenvalues near 1.
- Eigenvectors of T are combinations of local equilibria.
- Large gap between λ_c and λ_{c+1}.

Outline

How Bad is Choosing Your Own Opinion?



Modeling Opinion Formation

A basic model:

- A fixed *intrinsic* opinion s_i
- A variable *expressed* opinion x_i
- Equilibrium $x_i = \text{argmin}_{z_i} c_i(z_i)$, where

$$c_i(z_i) \equiv (s_i - z_i)^2 + \sum_{j \in N(i)} w_{ij}(z_i - x_j)^2$$

- Define a *social cost* $c(z) = \sum_i c_i(z_i)$

From Networks to Numerical Linear Algebra

Methodology: Graph problem $\mapsto$ linear algebra problem.

Nash equilibrium:	$(L+I)x = s$
Social optimum:	$(A+I)y = s$
Cost at equilibrium:	$c(x) = s^T Cs$
Optimal social cost:	$c(y) = s^T Bs$

Price of anarchy is a ratio of quadratics:

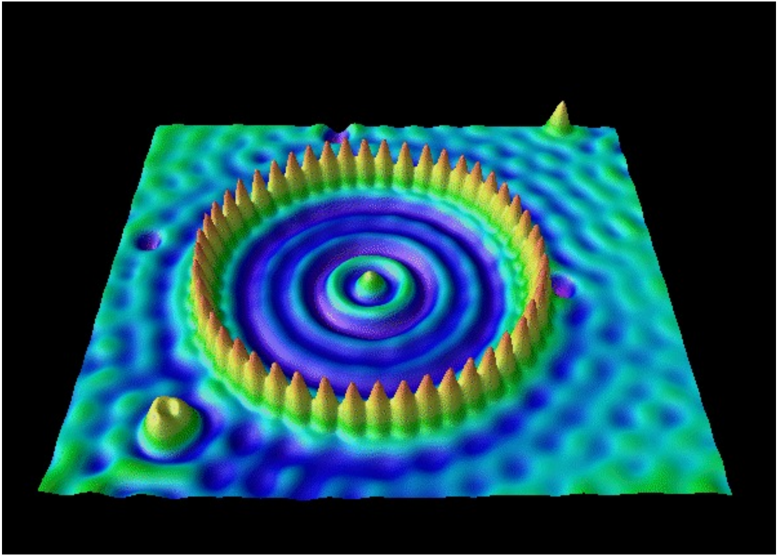
$$\mathrm{PoA}(s) = \frac{c(x)}{c(y)} = \frac{s^T Cs}{s^T Bs}$$

Find worst case through a *a generalized eigenvalue problem*:

$$Cs_* = \lambda Bs_*$$

Outline

Applications abound!

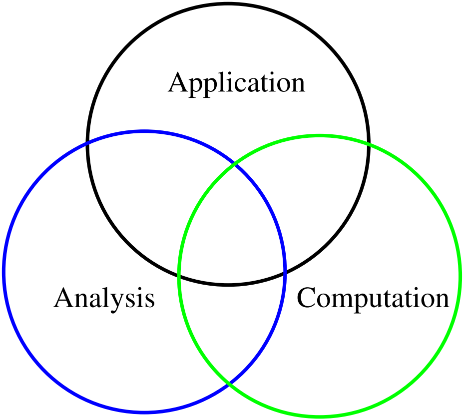


Why eigenvalue analysis?

- Because it simplifies many problems.
- It's a nonlinear equation I can solve.
- It's a nonconvex optimization I can solve.

... and because I've been thinking about it for a while!

Why scientific computing?



Because connections are fun!