# Computer Aided Design of Micro-Electro-Mechanical Systems

## From Energy Losses to Dick Tracy Watches

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Householder XVIII, 6 June 2008

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- Emmanuel Quévy
- ...

# The Computational Science Picture

- Application modeling
  - Checkerboard filter
  - Disk resonator and radiation loss
  - Beam resonator and thermoelastic loss
  - Shear ring resonator
- Mathematical analysis
  - Physical modeling and finite element technology
  - Structured eigenproblems and reduced-order models
  - Parameter-dependent eigenproblems
- Software engineering
  - HiQLab
  - SUGAR
  - FEAPMEX / MATFEAP

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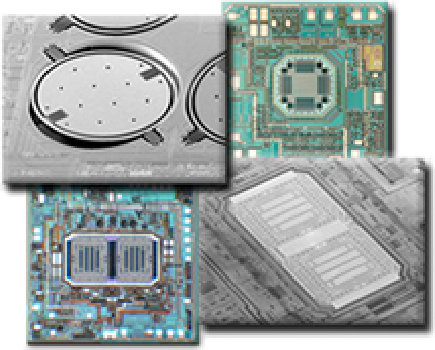
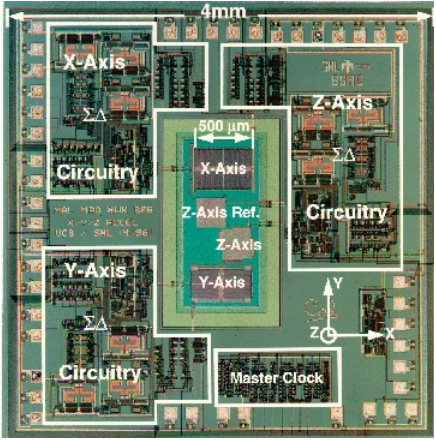
# What are MEMS?

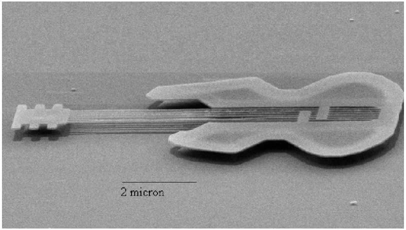


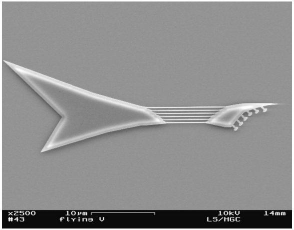


# MEMS Basics

- Micro-Electro-Mechanical Systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - Radio devices: cell phones, inventory tags, pico radio
- Use integrated circuit (IC) fabrication technology
- Tiny, but still classical physics

# Resonant RF MEMS

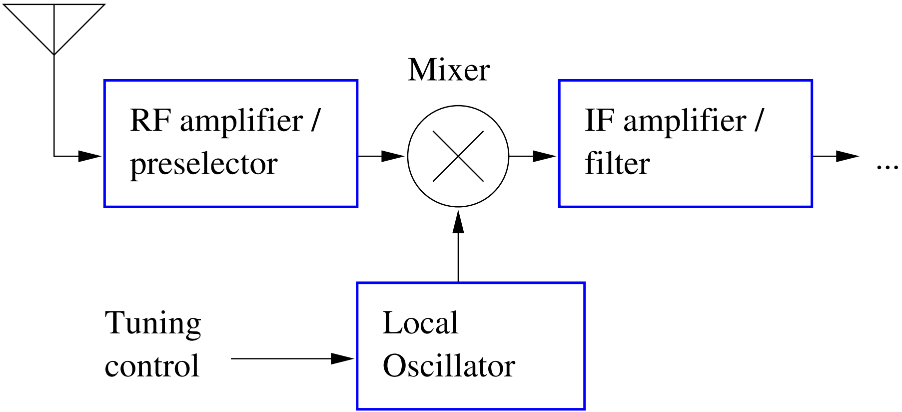



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars

# The Mechanical Cell Phone

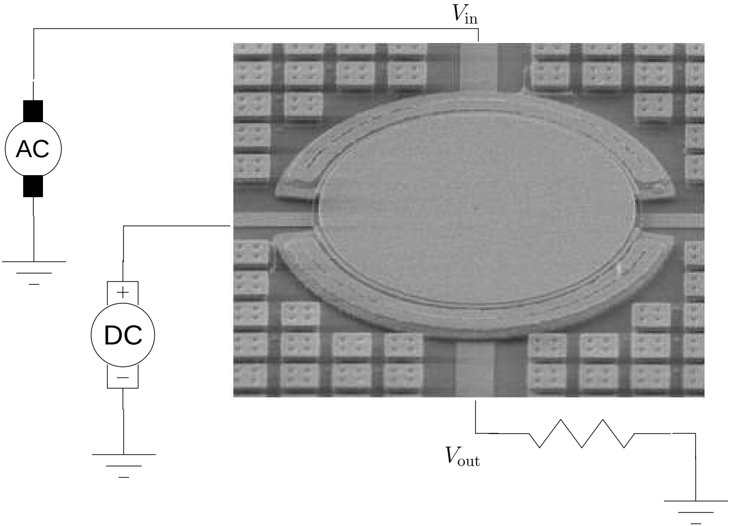



- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

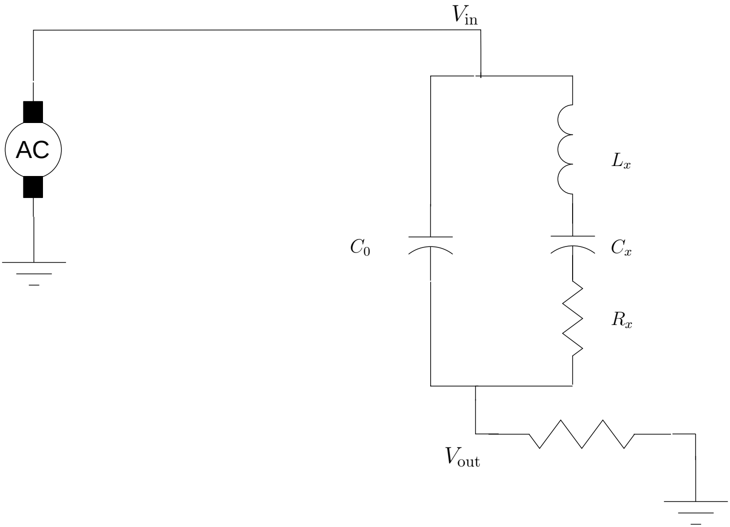
# Ultimate Success

"Calling Dick Tracy!"

# Disk Resonator



# Disk Resonator



# The Designer's Dream

Want a reduced model for the electromechanical admittance

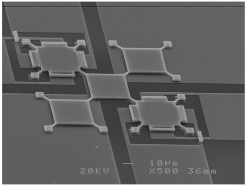
$$\begin{aligned} Y(\omega) &= i\omega C + G + i\omega H(\omega) \\ H(\omega) &= B^T(\tilde{K} - \omega^2 M)^{-1} B \end{aligned}$$

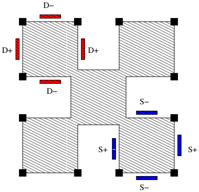
Ideally, would like

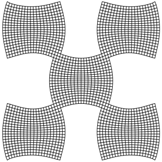
- Simple models for behavioral simulation
- Parameterized for design optimization
- With reasonably fast and accurate set-up
- Including all relevant physics

Lots of linear algebra problems left!

# Checkerboard Resonator



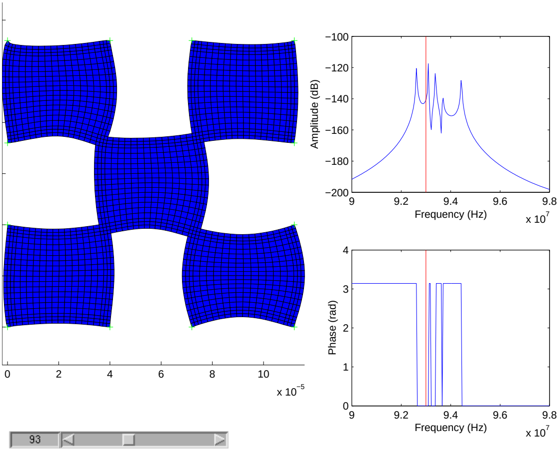



- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

# Checkerboard Model Reduction

- Finite element model: $N = 2154$
  - Expensive to solve for every $H(\omega)$ evaluation!
- Build a reduced-order model to approximate behavior
  - Reduced system of 80 to 100 vectors
  - Evaluate $H(\omega)$ in milliseconds instead of seconds
  - Without damping: standard Arnoldi projection
  - With damping: Second-Order ARnoldi (SOAR)

# Checkerboard Simulation



# Damping and $Q$

Designers want high *quality of resonance* ($Q$)

- Dimensionless damping in a one-dof system

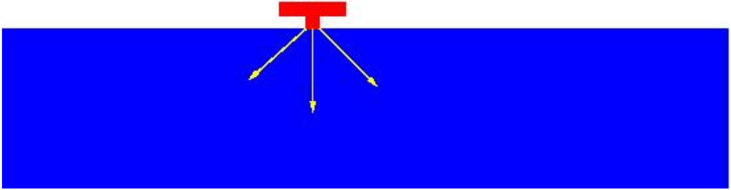
$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

To understand $Q$, we need damping models!

# Damping Mechanisms



Possible loss mechanisms:

- Fluid damping
- Material losses
- Thermoelastic damping
- Anchor loss

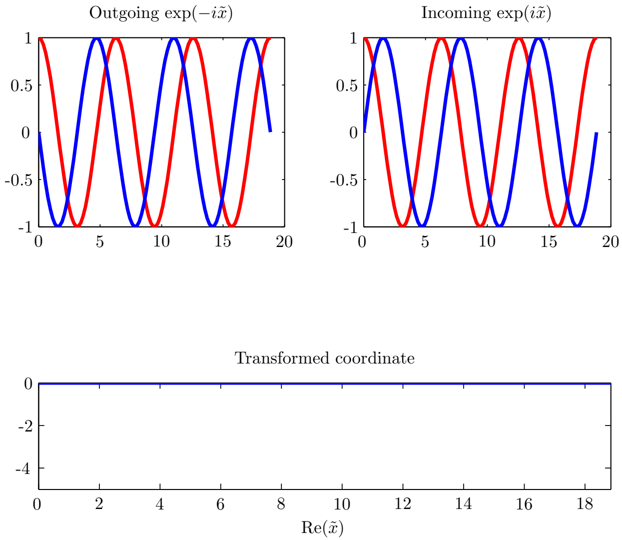
Model substrate as semi-infinite with a

Perfectly Matched Layer (PML).

# Perfectly Matched Layers

- Complex coordinate transformation
- Generates a “perfectly matched” absorbing layer
- Idea works with general linear wave equations
  - Electromagnetics (Berengér, 1994)
  - Quantum mechanics – *exterior complex scaling* (Simon, 1979)
  - Elasticity in standard finite element framework (Basu and Chopra, 2003)

# Model Problem Illustrated



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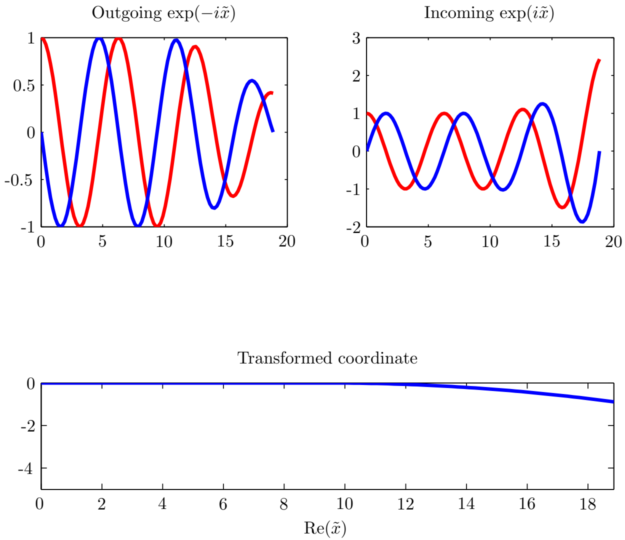

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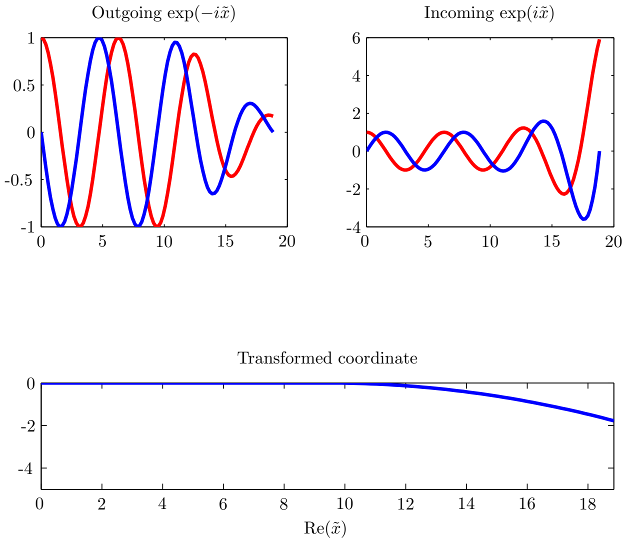

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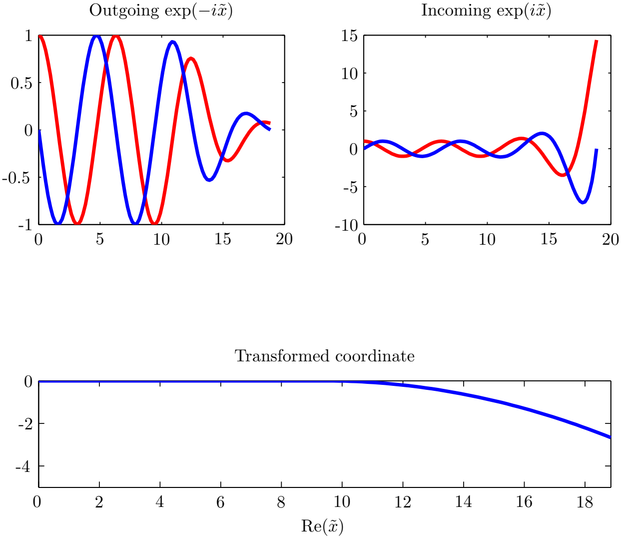

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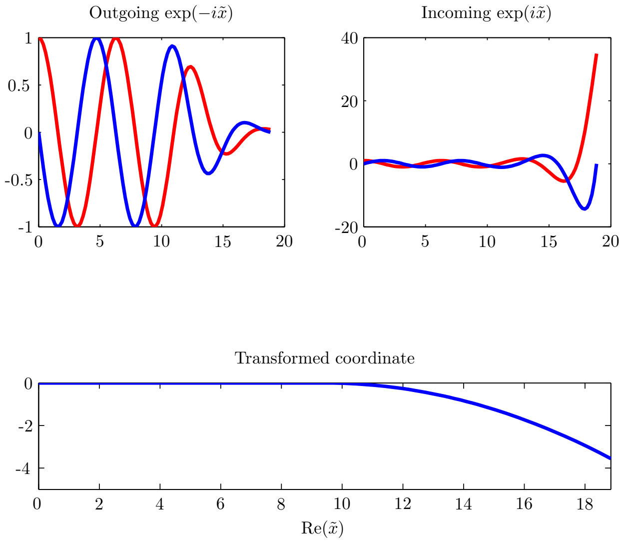

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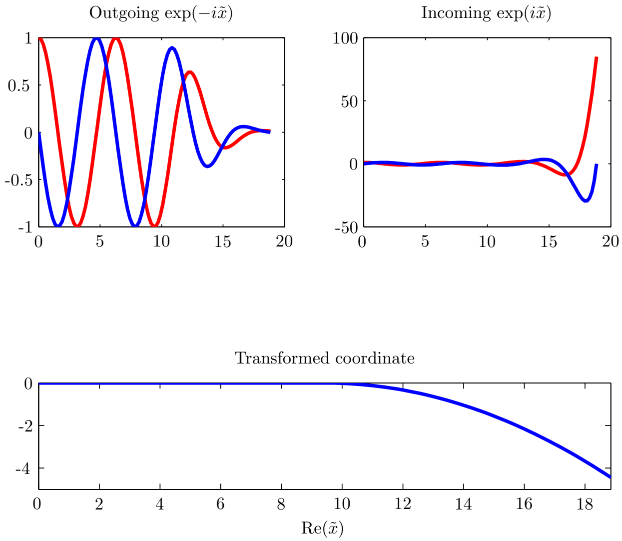
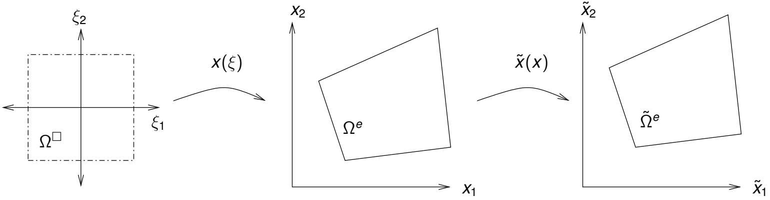


# Finite Element Implementation




- Combine PML and isoparametric mappings

$$\mathbf{k}^e = \int_{\Omega^\square} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \tilde{J} \, d\Omega^\square$$

$$\mathbf{m}^e = \int_{\Omega^\square} \rho \mathbf{N}^T \mathbf{N} \tilde{J} \, d\Omega^\square$$

- Matrices are *complex symmetric*

# Eigenvalues and Model Reduction

Want to know about the transfer function $H(\omega)$:

$$H(\omega) = B^T(K - \omega^2 M)^{-1}B$$

Can either

- Locate poles of $H$ (eigenvalues of $(K, M)$)
- Plot $H$ in a frequency range (Bode plot)

Usual tactic: subspace projection

- Build an Arnoldi basis $V$ for a Krylov subspace $\mathcal{K}_n$
- Compute with much smaller $V^*KV$ and $V^*MV$

Can we do better?

# Variational Principles

- Variational form for complex symmetric eigenproblems:
  - Hermitian (Rayleigh quotient):

$$\rho(v) = \frac{v^* K v}{v^* M v}$$

  - Complex symmetric (modified Rayleigh quotient):

$$\theta(v) = \frac{v^T K v}{v^T M v}$$

  (Hochstenbach and Arbenz, 2004)
- First-order accurate eigenvectors $\Longrightarrow$ Second-order accurate eigenvalues.
- Key: relation between left and right eigenvectors.

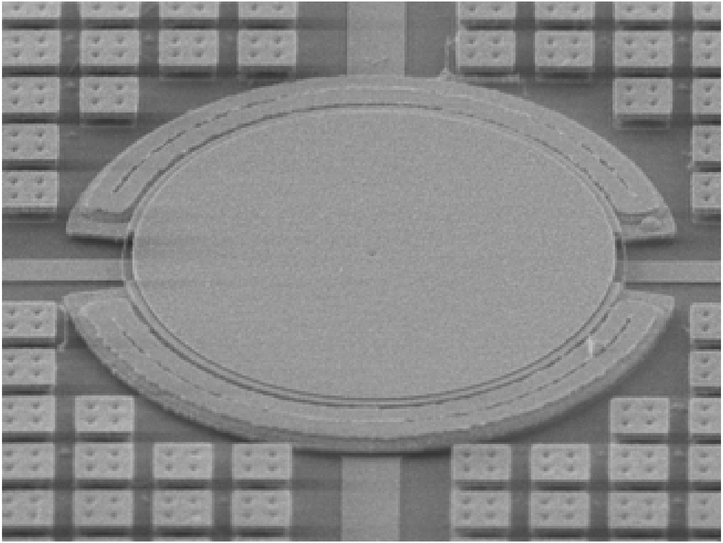
# Accurate Model Reduction

- Build new projection basis from $V$:

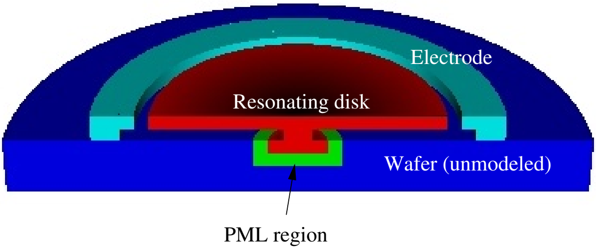
$$W = \text{orth}[\text{Re}(V), \text{Im}(V)]$$

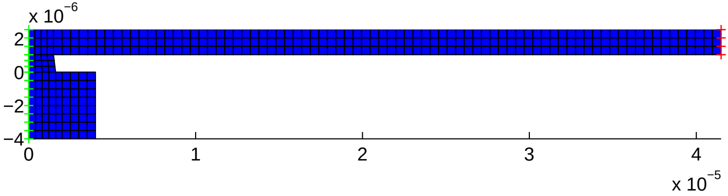
- $\text{span}(W)$ contains both $\mathcal{K}_n$ and $\bar{\mathcal{K}}_n$
  $\implies$ double digits correct vs. projection with $V$
- $W$ is a real-valued basis
  $\implies$ projected system is complex symmetric

# Disk Resonator Simulations



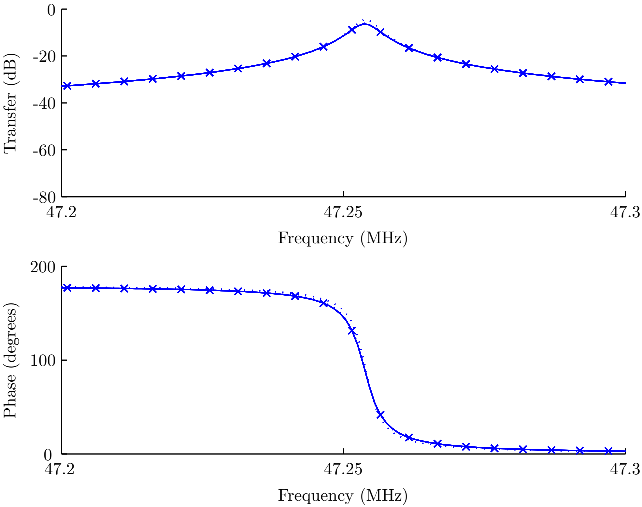
# Disk Resonator Mesh



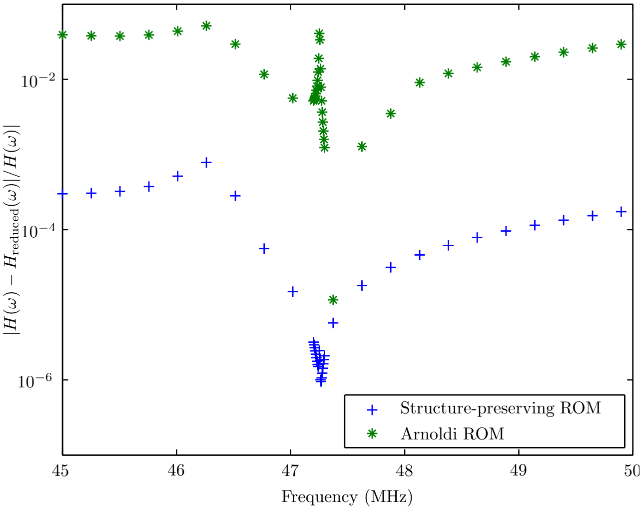

- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

# Model Reduction Accuracy



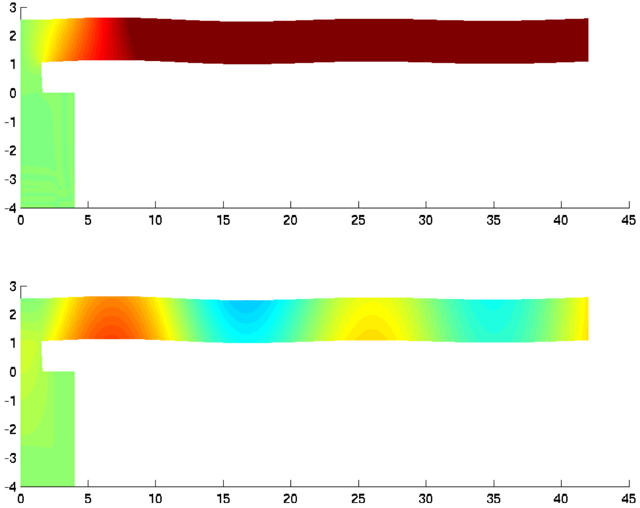

Results from ROM (solid and dotted lines) nearly indistinguishable from full model (crosses)

# Model Reduction Accuracy

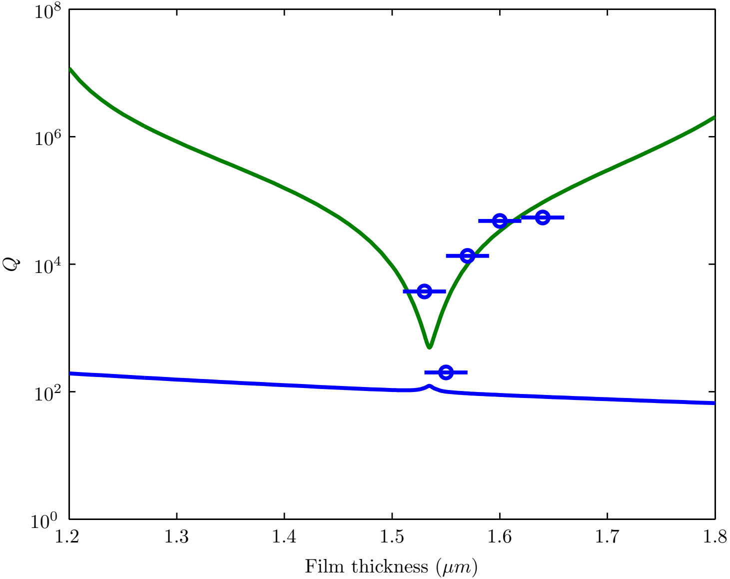



Preserve structure $\Longrightarrow$ get twice the correct digits

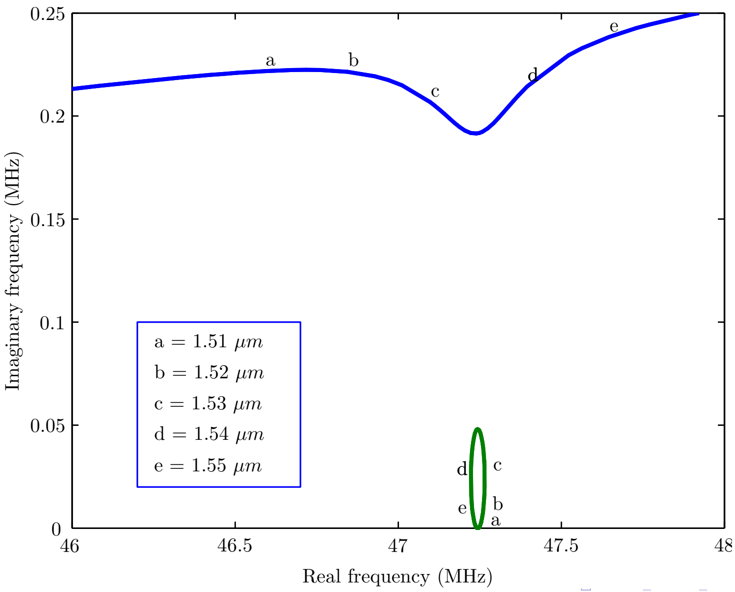
# Response of the Disk Resonator



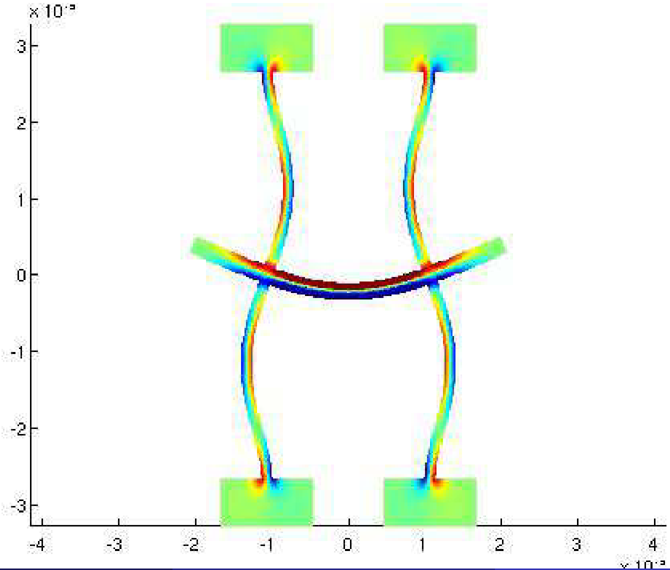
# Variation in Quality of Resonance



# Explanation of $Q$ Variation



# Thermoelastic Damping (TED)



# Thermoelastic Damping (TED)

$u$ is displacement and $T = T_0 + \theta$ is temperature

$$
\begin{aligned}
\sigma &= C\epsilon - \beta\theta 1 \\
\rho\ddot{u} &= \nabla \cdot \sigma \\
\rho c_v \dot{\theta} &= \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \operatorname{tr}(\dot{\epsilon})
\end{aligned}
$$

- Coupling between temperature and volumetric strain:
  - Compression and expansion $\implies$ heating and cooling
  - Heat diffusion $\implies$ mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators
- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system

# Nondimensionalized Equations

Continuum equations:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ \ddot{u} &= \nabla \cdot \sigma \\ \dot{\theta} &= \eta\nabla^2\theta - \mathrm{tr}(\dot{\epsilon}) \end{aligned}$$

Discrete equations:

$$\begin{aligned} M_{uu}\ddot{u} + K_{uu}u &= \xi K_{u\theta}\theta + f \\ C_{\theta\theta}\dot{\theta} + \eta K_{\theta\theta}\theta &= -C_{\theta u}\dot{u} \end{aligned}$$

- Micron-scale poly-Si devices: $\xi$ and $\eta$ are $\sim 10^{-4}$.
- Linearize about $\xi = 0$

# Perturbative Mode Calculation

Discretized mode equation:

$$
\begin{aligned}
(-\omega^2 M_{uu} + K_{uu})u &= \xi K_{u\theta}\theta \\
(i\omega C_{\theta\theta} + \eta K_{\theta\theta})\theta &= -i\omega C_{\theta u}u
\end{aligned}
$$

# Perturbative Mode Calculation

First approximation about $\xi = 0$:

$$\begin{aligned}(-\omega_0^2 M_{uu} + K_{uu})u_0 &= 0 \\ (i\omega_0 C_{\theta\theta} + \eta K_{\theta\theta})\theta_0 &= -i\omega_0 C_{\theta u} u_0\end{aligned}$$

First-order correction in $\xi$:

$$-\delta(\omega^2) M_{uu} u_0 + (-\omega_0^2 M_{uu} + K_{uu})\delta u = \xi K_{u\theta}\theta_0$$

Multiply by $u_0^T$:

$$\delta(\omega^2) = -\xi \left( \frac{u_0^T K_{u\theta}\theta_0}{u_0^T M_{uu} u_0} \right)$$

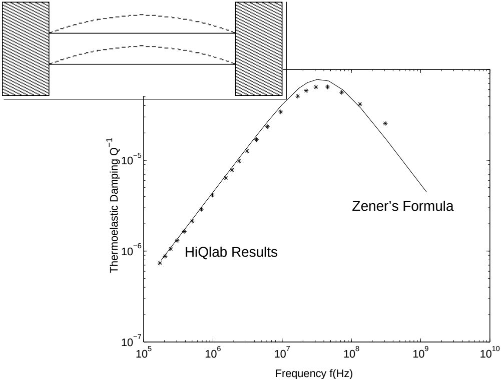
# The Power of Perturbation

- Full method: nonsymmetric eigensolve of size
- Perturbation method:
  1. Purely mechanical symmetric eigensolve
  2. Linear solve for corresponding thermal field
  3. A couple dot products for frequency correction

# Zener's Model

- Clarence Zener investigated TED in late 30s-early 40s.
- Model for beams common in MEMS literature.
- "Method of orthogonal thermodynamic potentials" == perturbation method + a variational method.

# Comparison to Zener's Model




- Comparison of fully coupled simulation to Zener approximation over a range of frequencies
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi

# Onward!

*The purpose of computing is insight, not numbers.*
*Richard Hamming*

What about:

- Modeling more geometrically complex devices?
- Modeling general dependence on geometry?
- Modeling general dependence on operating point?
- Computing nonlinear dynamics?
- Digesting all this to help designers?

# Conclusions

- RF MEMS are still a great source of problems
  - Still many unresolved physical modeling problems
  - Need better parameterized and nonlinear model reduction methods
  - System scale simulations mean parallel computing challenges

`http://www.cims.nyu.edu/~dbindel`