Structure Preserving Model Reduction for Damped Resonant MEMS

David Bindel

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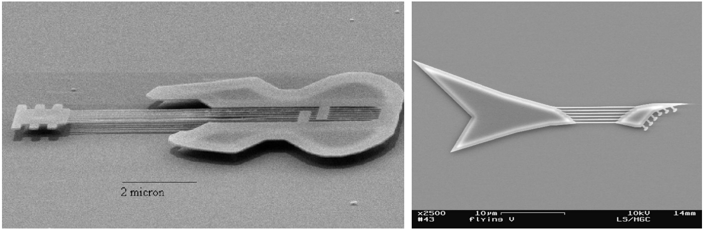
Collaborators

- Tsuyoshi Koyama
- Sanjay Govindjee
- Sunil Bhave
- Emmanuel Quévy
- Zhaojun Bai

Resonant MEMS

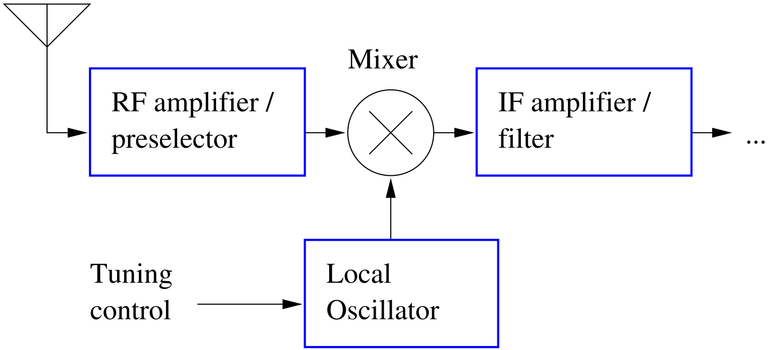
USNCCM07



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars

The Mechanical Cell Phone

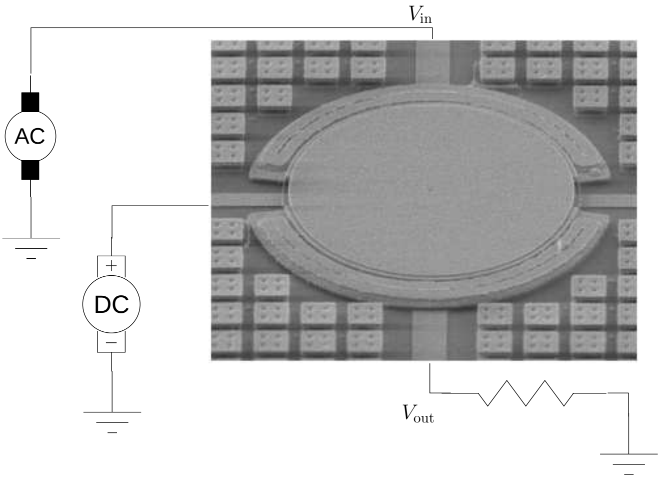


- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

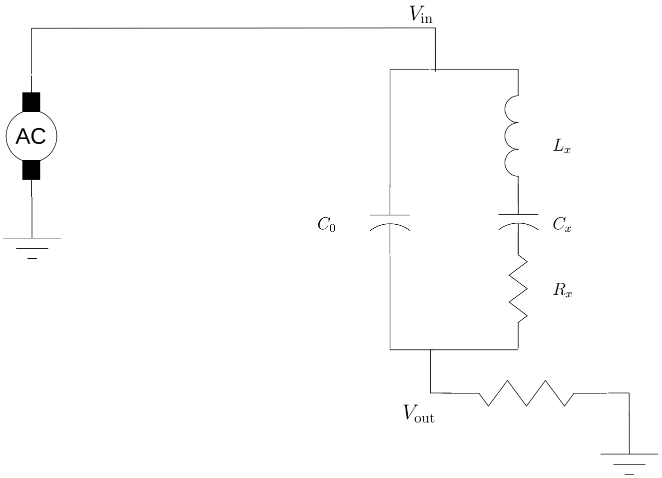
Ultimate Success

"Calling Dick Tracy!"

Example Resonant System



Example Resonant System



Model Reduction: Basic set-up

Linear time-invariant system:

$$\begin{aligned} Mu'' + Ku &= b\phi(t) \\ y(t) &= p^T u \end{aligned}$$

Frequency domain:

$$\begin{aligned} -\omega^2 M\hat{u} + K\hat{u} &= b\hat{\phi}(\omega) \\ \hat{y}(\omega) &= p^T \hat{u} \end{aligned}$$

Transfer function:

$$\begin{aligned} H(\omega) &= p^T(-\omega^2 M + K)^{-1} b \\ \hat{y}(\omega) &= H(\omega)\hat{\phi}(\omega) \end{aligned}$$

Model Reduction: Basic set-up

Have a *rational* transfer function relating input and output:

$$H(\omega) = p^T(-\omega^2 M + K)^{-1} b$$

Can approximate H by Galerkin projection:

$$\hat{H}(\omega) = (Vp)^T(-\omega^2 V^T M V + V^T K V)^{-1}(Vb)$$

Could also try to approximate H directly (often equivalent).

The Designer's Dream

Ideally, would like

- Compact models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up

We aren't there yet.

Standard Projection Model Reduction

Approximate H by Galerkin projection:

$$\hat{H}(i\omega) = (Vp)^T(-\omega^2 V^T M V + V^T K V)^{-1}(Vb)$$

1. Define $K_\sigma := K - \sigma^2 M$; build a Krylov subspace

   $$\text{span}(V) = \mathcal{K}_n(K_\sigma^{-1}M, K_\sigma^{-1}b) = \text{span}\{(K_\sigma^{-1}M)^j K_\sigma^{-1}b\}_{j=0}^{n}$$

   Has the *moment-matching property*:

   $$H^{(k)}(i\sigma) = \hat{H}^{(k)}(i\sigma), \quad k = 0, \ldots, n$$

   Get $2n$ moments for symmetric systems (or for separate left and right subspaces).
2. Project onto one or more modal vectors.

The Hero of the Hour

Major theme: use problem structure for better reduced models

- ODE structure
- Complex symmetric structure
- Perturbative structure
- Geometric structure

SOAR and ODE structure

Damped second-order system:

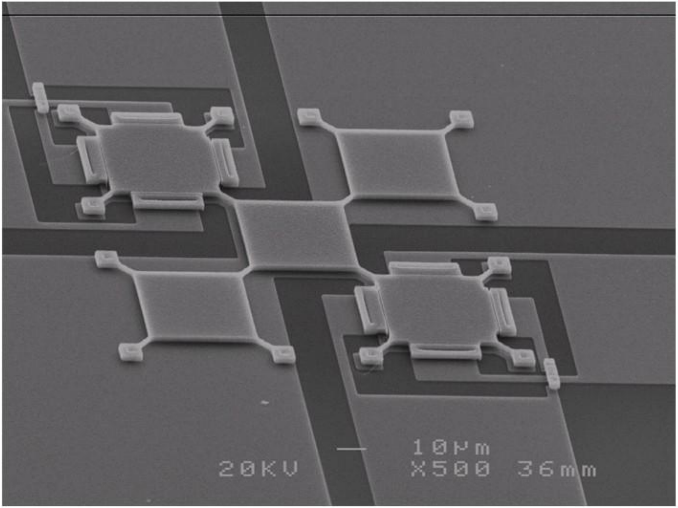
$$\begin{aligned} Mu'' + Cu' + Ku &= P\phi \\ y &= V^T u. \end{aligned}$$

Projection basis Q_n with Second Order ARnoldi (SOAR):

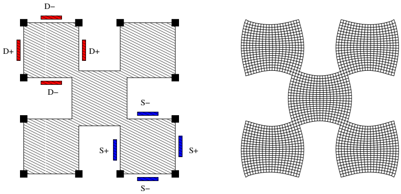
$$\begin{aligned} M_n u_n'' + C_n u_n' + K_n u_n &= P_n \phi \\ y &= V_n^T u \end{aligned}$$

where $P_n = Q_n^T P$, $V_n = Q_n^T V$, $M_n = Q_n^T M Q_n, \ldots$

Checkerboard Resonator



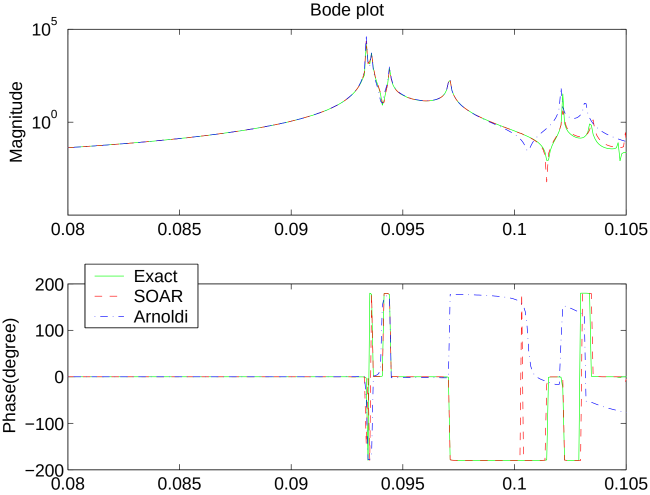
Checkerboard Resonator



- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Performance of SOAR vs Arnoldi

$$N = 2154 \rightarrow n = 80$$



Complex Symmetry

Model with radiation damping (PML) gives complex problem:

$$(K - \omega^2 M)u = f, \text{ where } K = K^T, M = M^T$$

Forced solution u is a stationary point of

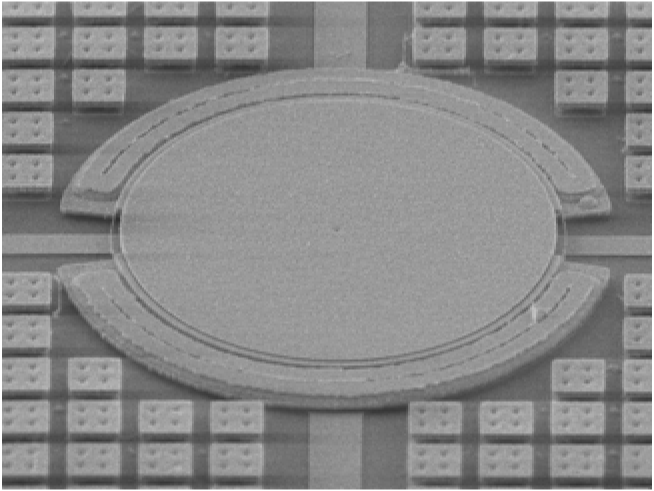
$$I(u) = \frac{1}{2}u^T(K - \omega^2 M)u - u^T f.$$

Eigenvalues of (K, M) are stationary points of

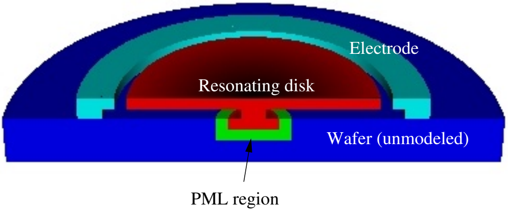
$$\rho(u) = \frac{u^T K u}{u^T M u}$$

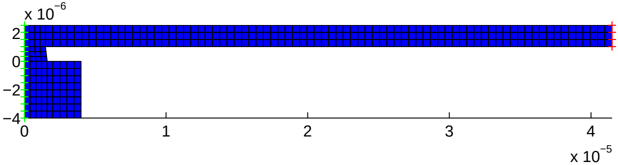
First-order accurate vectors $\implies$
second-order accurate eigenvalues.

Disk Resonator Simulations



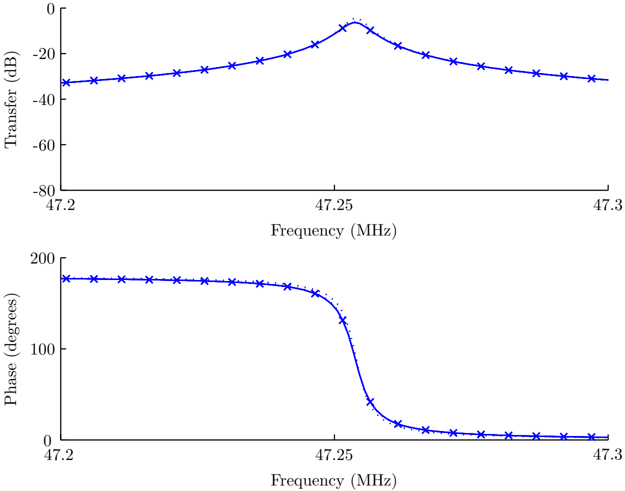
Disk Resonator Mesh





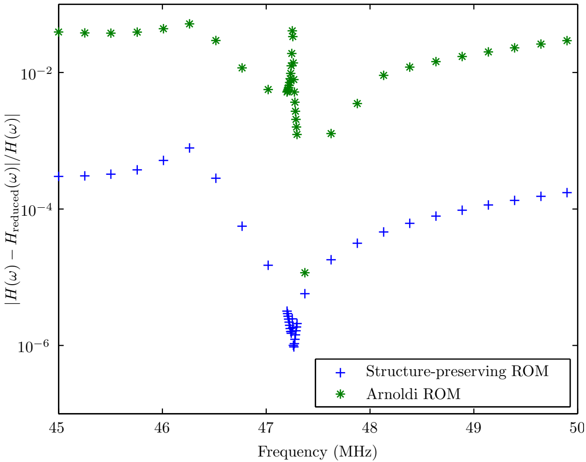
- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Symmetric ROM Accuracy



Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

Symmetric ROM Accuracy



Preserve structure $\Longrightarrow$
get twice the correct digits

Perturbative Structure

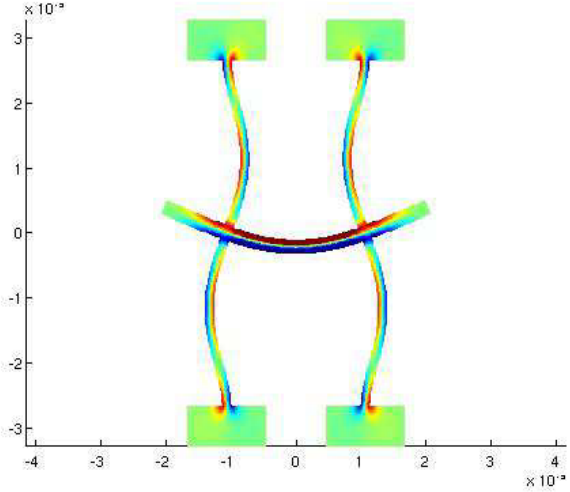
Dimensionless continuum equations for thermoelastic damping:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ \ddot{u} &= \nabla \cdot \sigma \\ \dot{\theta} &= \eta\nabla^2\theta - \mathrm{tr}(\dot{\epsilon}) \end{aligned}$$

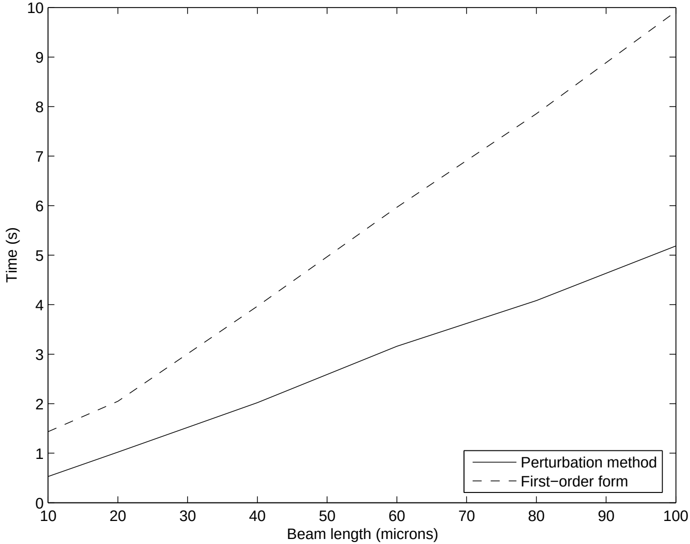
Dimensionless coupling ξ and heat diffusivity η are 10^{-4} $\Longrightarrow$ perturubation method (about $\xi = 0$).

Large, non-self-adjoint, first-order coupled problem $\rightarrow$ Smaller, self-adjoint, mechanical eigenproblem + symmetric linear solve.

Thermoelastic Damping Example



Performance for Beam Example



Aside: Effect of Nondimensionalization

100 μm beam example, first-order form.

Before nondimensionalization

- Time: 180 s
- $\mathrm{nnz}(L) = 11\mathrm{M}$

After nondimensionalization

- Time: 10 s
- $\mathrm{nnz}(L) = 380\mathrm{K}$

Semi-Analytical Model Reduction

We work with hand-build model reduction all the time!

- Circuit elements: Maxwell equation + field assumptions
- Beam theory: Elasticity + kinematic assumptions
- Axisymmetry: 3D problem + kinematic assumption

Idea: Provide *global shapes*

- User defines shapes through a callback
- Mesh serves defines a quadrature rule
- Reduced equations fit known abstractions

Global Shape Functions

Normally:

$$u(X) = \sum_j N_j(X)\hat{u}_j$$

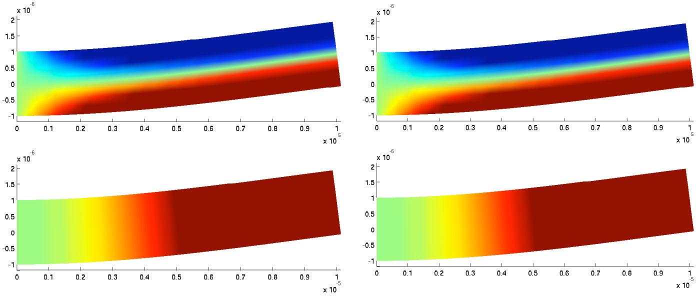
Global shape functions:

$$\hat{u} = \hat{u}^l + G(\hat{u}^g)$$

Then constrain values of some components of $\hat{u}^l$, $\hat{u}^g$.

“Hello, World!”

Which mode shape comes from the reduced model (3 dof)?



(Left: 28 MHz; Right: 31 MHz)

Conclusions

Respecting problem structure is a Good Thing!

- ODE structure
- Complex symmetric structure
- Perturbative structure
- Geometric structure

Result:
Better accuracy, faster set-up, better understanding.