Computer-Aided Design for Micro-Electro-Mechanical Systems

Eigenvalues, Energy Losses, and Dick Tracy Watches

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UC Davis, 9 Mar 2006

The Computational Science Picture

- Application modeling
  - Checkerboard resonator
  - Disk resonator
  - Shear ring resonator
- Mathematical analysis
  - Physical modeling and finite element technology
  - Structured eigenproblems and reduced-order models
  - Parameter-dependent eigenproblems
- Software engineering
  - HiQLab
  - SUGAR
  - FEAPMEX

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Outline

Resonant MEMS
HiQLab
Anchor Losses
Disk Resonator Analysis
Conclusions
Backup Slides

1. Resonant MEMS
2. HiQLab
3. Anchor Losses
4. Disk Resonator Analysis

Outline

1 Resonant MEMS

2 HiQLab

3 Anchor Losses

4 Disk Resonator Analysis

What are MEMS?

Resonant MEMS
MEMS Basics
RF MEMS
Checkerboard Model
HiQLab
Anchor Losses
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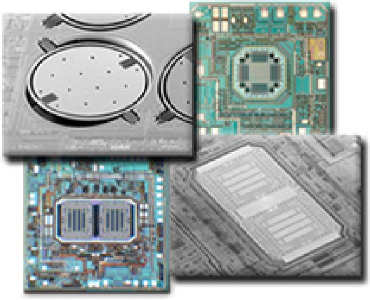
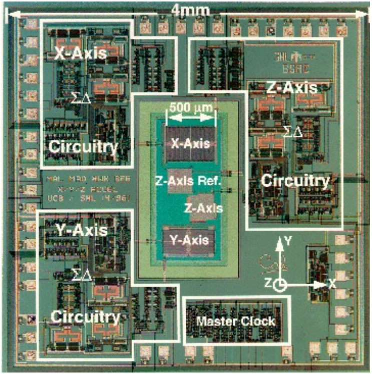
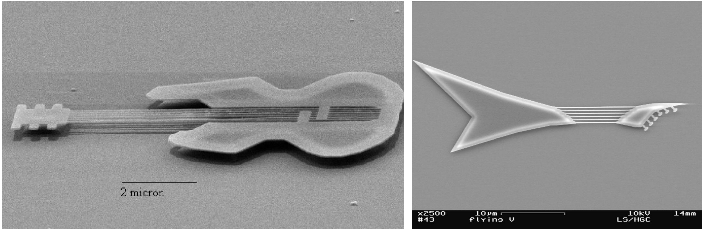




MEMS Basics

- Micro-Electro-Mechanical Systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - Radio devices: cell phones, inventory tags, pico radio
- Use integrated circuit (IC) fabrication technology
- Tiny, but still classical physics

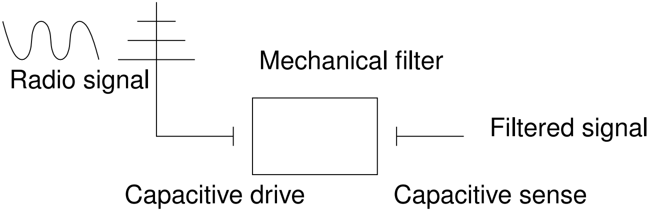
Radio-Frequency MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Impact: smaller, lower-power cell phones
- Other uses:
  - Sensing elements (e.g. chemical sensors)
  - Really high-pitch guitars

Micromechanical Filters



- Mechanical high-frequency (high MHz-GHz) filter
  - Your cell phone is mechanical!
  - New MEMS filters can be integrated with circuitry
    $\Longrightarrow$ smaller and lower power

Ultimate Success

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"Calling Dick Tracy!"

Designing Transfer Functions

Time domain:

$$\begin{aligned} Mu'' + Cu' + Ku &= b\phi(t) \\ y(t) &= p^T u \end{aligned}$$

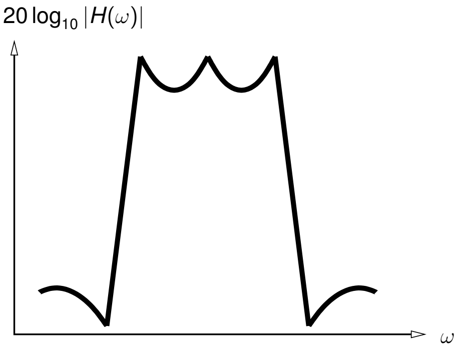
Frequency domain:

$$\begin{aligned} -\omega^2 M\hat{u} + i\omega C\hat{u} + K\hat{u} &= b\hat{\phi}(\omega) \\ \hat{y}(\omega) &= p^T \hat{u} \end{aligned}$$

Transfer function:

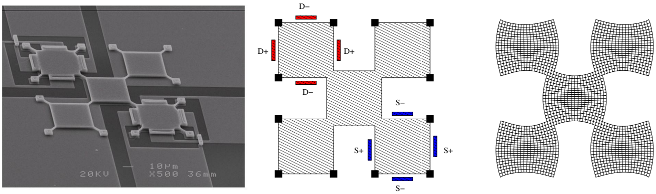
$$\begin{aligned} H(\omega) &= p^T(-\omega^2 M + i\omega C + K)^{-1} b \\ \hat{y}(\omega) &= H(\omega)\hat{\phi}(\omega) \end{aligned}$$

Narrowband Filter Needs



- Want "sharp" poles for narrowband filters
- $\Longrightarrow$ Want to minimize damping

Checkerboard Resonator

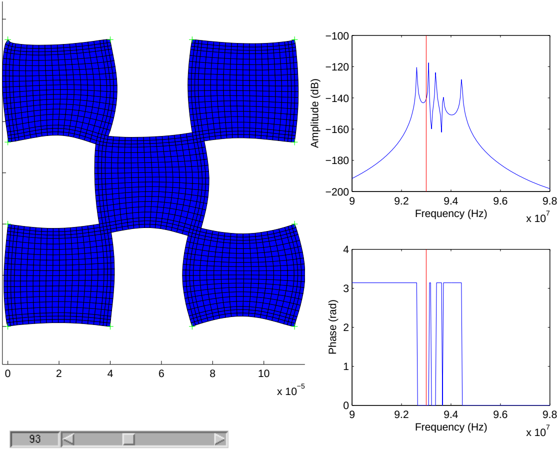


- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Checkerboard Model Reduction

- Finite element model: $N = 2154$
  - Expensive to solve for every $H(\omega)$ evaluation!
- Build a reduced-order model to approximate behavior
  - Reduced system of 80 to 100 vectors
  - Evaluate $H(\omega)$ in milliseconds instead of seconds
  - Without damping: standard Arnoldi projection
  - With damping: Second-Order ARnoldi (SOAR)

Checkerboard Simulation



Checkerboard Measurement

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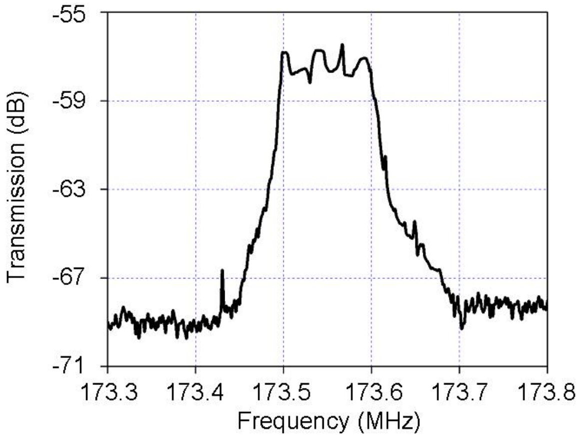

S. Bhave, MEMS 05

Contributions

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- Built predictive model used to design checkerboard
- Used model reduction to get thousand-fold speedup – fast enough for interactive use

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Damping and Q

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Designers want high *quality of resonance* (Q)

- Dimensionless damping in a one-dof system

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

Enter HiQLab

- Existing codes do not compute quality factors
- ... and awkward to prototype new solvers
- ... and awkward to programmatically define meshes
- So I wrote a new finite element code: HiQLab

Heritage of HiQLab

SUGAR: SPICE for the MEMS world

- System-level simulation using modified nodal analysis
- Flexible device description language
- C core with MATLAB interfaces and numerical routines

FEAPMEX: MATLAB + a finite element code

- MATLAB interfaces for steering, testing solvers, running parameter studies
- Time-tested finite element architecture
- But old F77, brittle in places

Other Ingredients

"Lesser artists borrow. Great artists steal."
– Picasso, Dali, Stravinsky?

- Lua: www.lua.org
  - Evolved from simulator data languages (DEL and SOL)
  - Pascal-like syntax fits on one page; complete language description is 21 pages
  - Fast, freely available, widely used in game design
- MATLAB: www.mathworks.com
  - "The Language of Technical Computing"
  - Good sparse matrix support
  - Star-P: http://www.interactivesupercomputing.com/
- Standard numerical libraries: ARPACK, UMFPACK

HiQLab Structure

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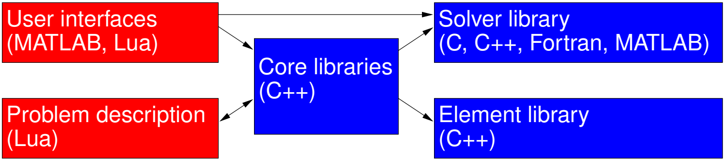
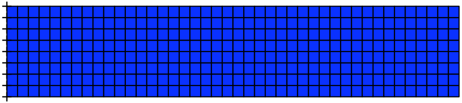


- Standard finite element structures + some new ideas
- Full scripting language for mesh input
- Callbacks for boundary conditions, material properties
- MATLAB interface for quick algorithm prototyping
- Cross-language bindings are automatically generated

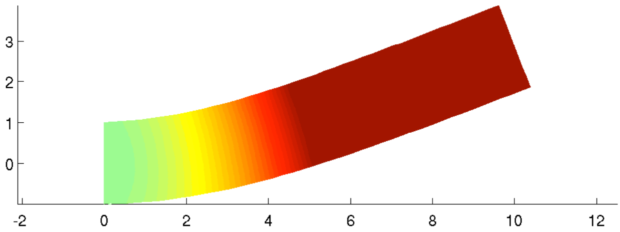
HiQLab's Hello World



```
mesh  = Mesh:new(2)
mat = make_material('silicon2', 'planestrain')
mesh:blocks2d( { 0, l }, { -w/2.0, w/2.0 },
               mat )

mesh:set_bc(function(x,y)
  if x == 0 then return 'uu', 0, 0; end
end)
```

HiQLab's Hello World




```
>> mesh = Mesh_load('beammesh.lua');
>> [M,K] = Mesh_assemble_mk(mesh);
>> [V,D] = eigs(K,M, 5, 'sm');
>> opt.axequal = 1; opt.deform = 1;
>> Mesh_scale_u(mesh, V(:,1), 2, 1e-6);
>> plotfield2d(mesh, opt);
```

Contributions

- Wrote a new code, HiQLab, to study damping
- HiQLab is based on my earlier simulators:
  - SUGAR, for system-level MEMS simulation
  - FEAPMEX, for scripting parameter studies

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Example: Disk Resonator

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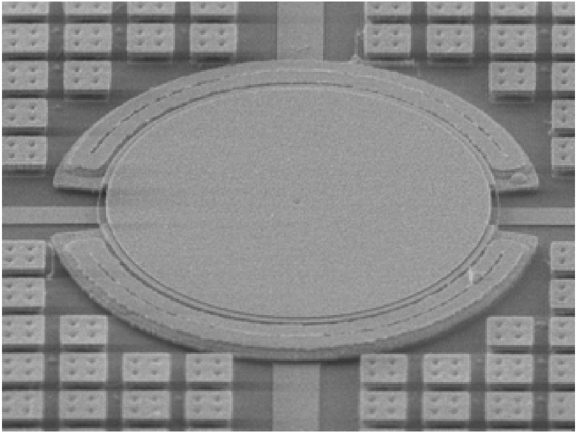

SiGe disk resonators built by E. Quévy

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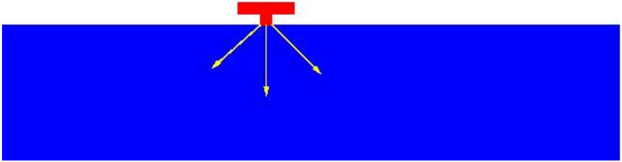
Damping Mechanisms



Possible loss mechanisms:

- Fluid damping
- Material losses
- Thermoelastic damping
- Anchor loss

Model substrate as semi-infinite with a

Perfectly Matched Layer (PML).

Perfectly Matched Layers

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- Complex coordinate transformation
- Generates a "perfectly matched" absorbing layer
- Idea works with general linear wave equations
  - Electromagnetics (Berengér, 1994)
  - Quantum mechanics – *exterior complex scaling* (Simon, 1979)
  - Elasticity in standard finite element framework (Basu and Chopra, 2003)

Model Problem

- Domain: $x \in [0, \infty)$
- Governing eq:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

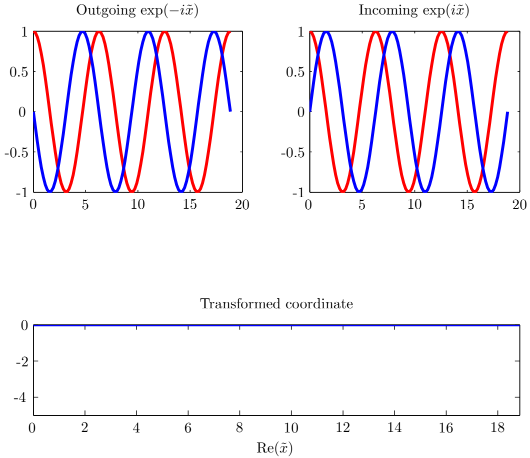
- Fourier transform:

$$\frac{d^2 \hat{u}}{dx^2} + k^2 \hat{u} = 0$$

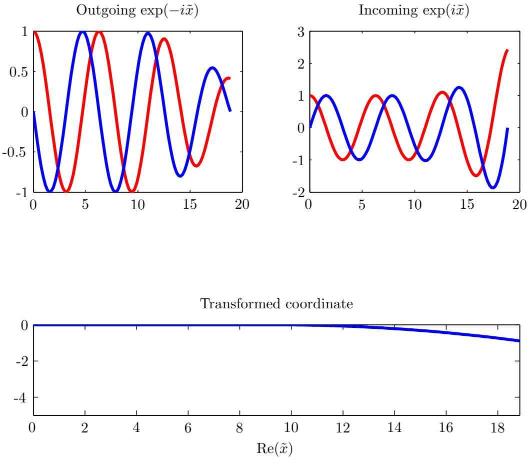
- Solution:

$$\hat{u} = c_{\text{out}} e^{-ikx} + c_{\text{in}} e^{ikx}$$

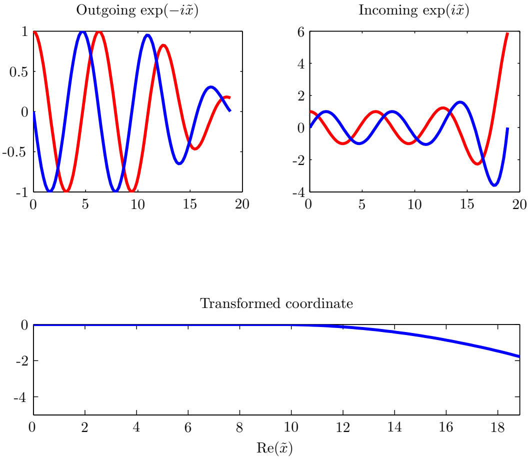
Model Problem Illustrated



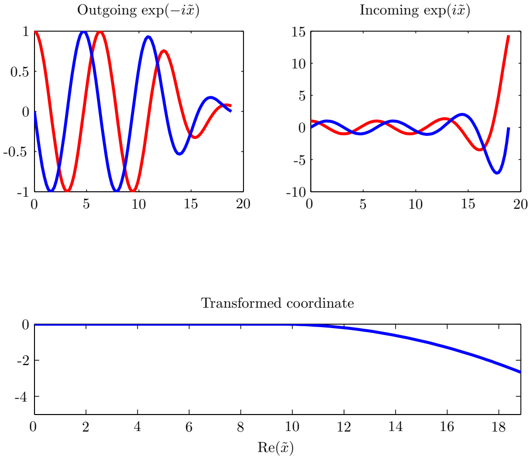
Model Problem Illustrated



Model Problem Illustrated



Model Problem Illustrated



Model Problem Illustrated

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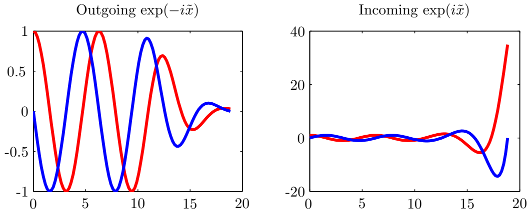
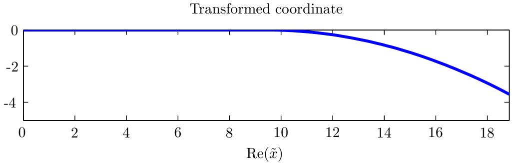
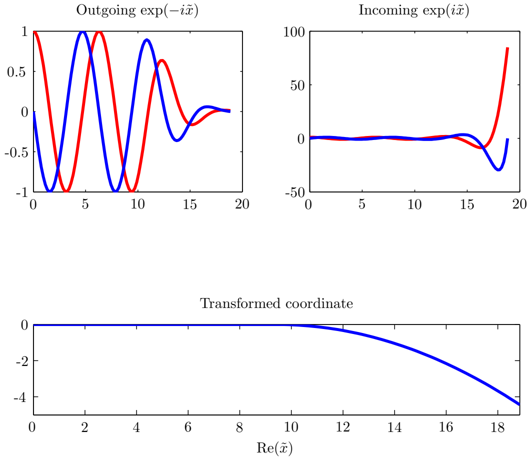
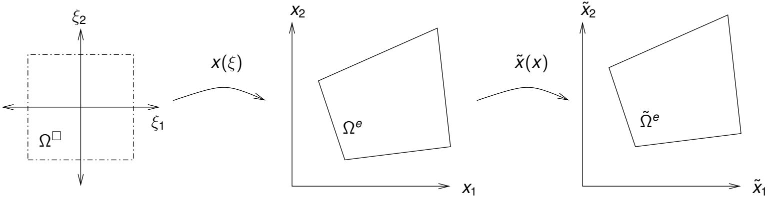




Model Problem Illustrated



Finite Element Implementation



- Combine PML and isoparametric mappings

$$\mathbf{k}^e = \int_{\Omega^\square} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \tilde{J} \, d\Omega^\square$$

$$\mathbf{m}^e = \int_{\Omega^\square} \rho \mathbf{N}^T \mathbf{N} \tilde{J} \, d\Omega^\square$$

- Matrices are *complex symmetric*

Contributions

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Backup Slides

- New formulation of perfectly matched layers
  - Easy to apply PML to axisymmetric, 2D, or 3D models
  - Same formulation applies to electromagnetics, etc.
- Analysis of discretization error for PMLs
  - Results in cheap, automatic parameter optimization
- Structure-preserving model reduction for complex symmetric systems
  - Double accuracy for same work as standard method

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Disk Resonator Simulations

CAD for MEMS

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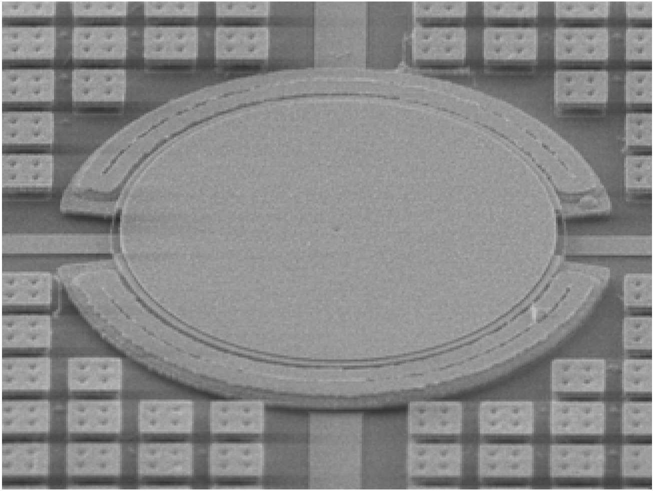
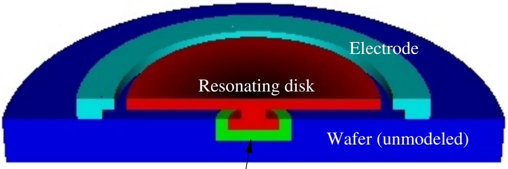
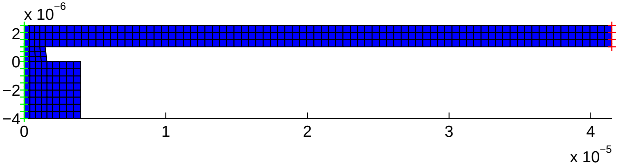


Disk Resonator Mesh





- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Mesh Convergence

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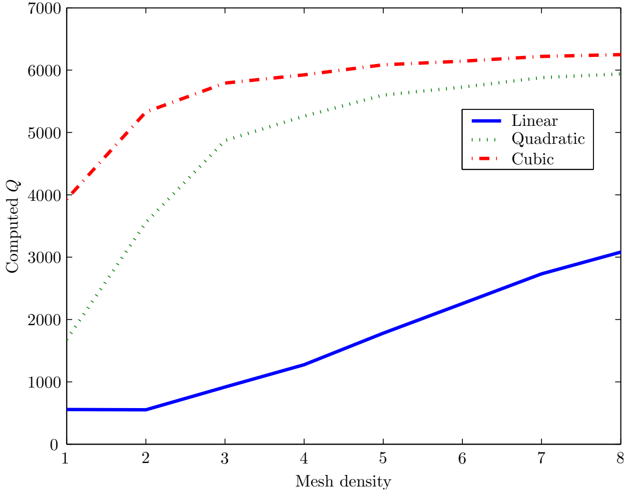
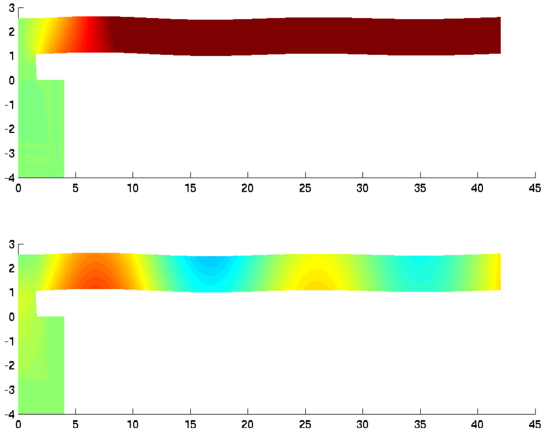
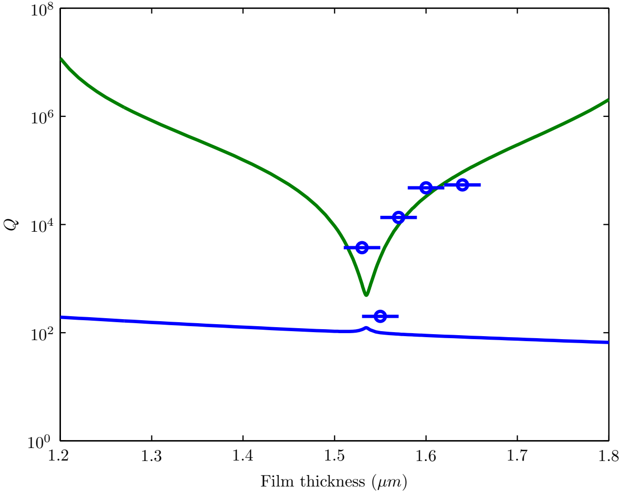


Cubic elements converge with reasonable mesh density

Response of the Disk Resonator

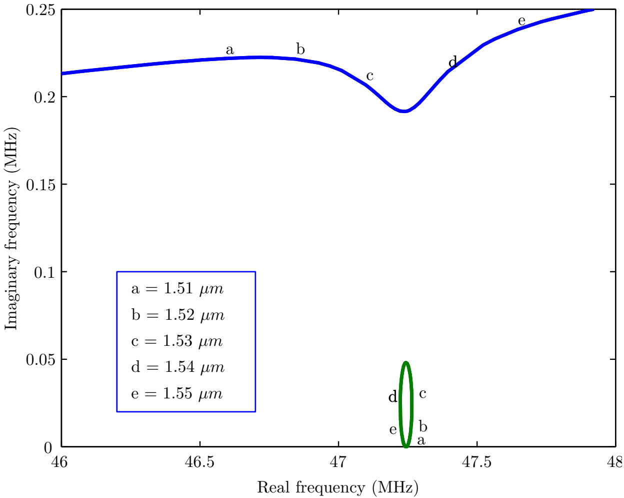


Variation in Quality of Resonance



Simulation and lab measurements vs. disk thickness

Explanation of Q Variation



Interaction of two nearby eigenmodes

Contributions

- Built disk model in HiQLab and verified against lab measurements
- Demonstrated dominance of anchor loss for this device
- Predicted geometric sensitivity of quality factor Q (which was subsequently verified in the lab)
- Explained Q sensitivity in terms of mode interference

Contributions Summary (1)

Resonant MEMS
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- Application modeling
  - Finite element models of several devices
  - Discovery of effects of mode interference
  - Importance of anchor loss vs thermoelastic damping
- Mathematical analysis
  - Reformulation of perfectly-matched layers
  - Analysis of discretization and parameter choice in PMLs
  - Complex symmetry-preserving model reduction
  - Perturbation solution for thermoelastic damping
- Software: HiQLab, FEAPMEX, SUGAR

Contributions Summary (2)

- HiQLab (about 33000 lines of code)
  - Collaborations at Berkeley, Cornell, Stanford, Bosch
- FEAPMEX (about 5000 lines of code)
  - 2400+ page views
  - Used for instrument models, stochastic structural analysis, ultrasonic nondestructive evaluation problems, material parameter identification problems
- SUGAR (about 18000 lines of code)
  - 2000+ downloads
  - Used in classes at Berkeley, Cornell, Johns Hopkins
  - Continued research use for design optimization

Other Contributions

Resonant MEMS
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- Lowered complexity of `roots` from $O(n^3)$ to $O(n^2)$
  - Code to go into next LAPACK release
- Developed first sparse subspace continuation code
  - Going into the next Matcont release
- Developed new network tomography method
- Designed initial security model for OceanStore
- Served as IEEE 754R secretary
- Responsible for last CLAPACK version

Future Work

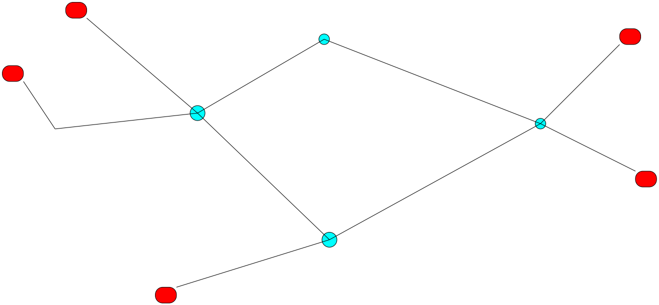
- Code development
  - Structural elements and elements for different physics
  - Design and implementation of parallelized version
- Theoretical analysis
  - More damping mechanisms
  - Sensitivity analysis and variational model reduction
- Application collaborations
  - Use of nonlinear effects (quasi-static and dynamic)
  - New designs (e.g. internal dielectric drives)
  - Continued experimental comparisons

Conclusions

- RF MEMS are a great source of problems
  - Interesting applications
  - Interesting physics (and not altogether understood)
  - Interesting numerical mathematics

`http://www.cs.berkeley.edu/~dbindel`

Application: Network monitoring



- Set of n hosts in a large network
- $n(n-1)$ directed paths between them
- Want latency and packet loss rates for each path
- Use info to choose servers, route around faults

Network “distance” metrics

CAD for MEMS

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Complex Symmetry
Damping
MEMS Fabrication
SUGAR and HiQLab
Elastic PMLs
Reflection Analysis
TED
Subspace Continuation

Linear relation between path length y_{ij} and link length x_l:

$$y_{ij} = \sum_{l \in \text{path}} x_l = \sum_l g_{ijl} x_l$$

where g_{ijl} indicates if link l is used on path $i \to j$.
Write in matrix form (one path per row): $y = Gx$.

Examples:

- Latency
- Log probability of transmission success
- Jitter (variance in latency)

Path dependencies

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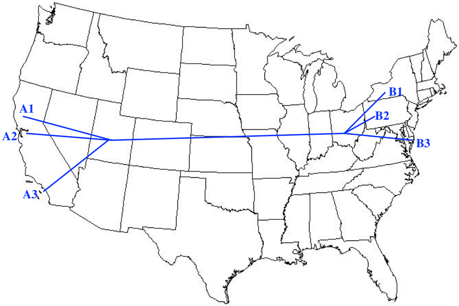
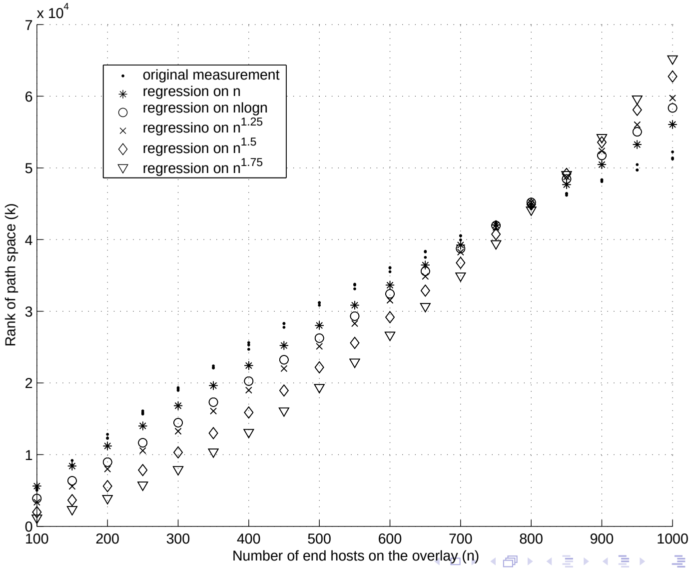


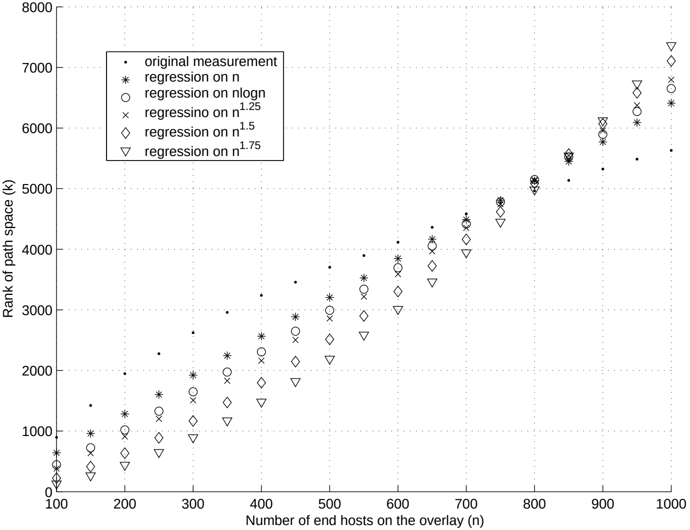
$$(A_i \to B_i) = (A_1 \to B_i) + (A_i \to B_1) - (A_1 \to B_1)$$

- Many linear dependencies among paths!
- $k := \mathrm{rank}(G) \ll n^2$
- Measure only k paths to infer properties for all paths

Rank of G: Lucent scan (bound)



Rank of *G*: AS-level Albert-Barabasi



Rank of G: AS Barabasi + RT Waxman

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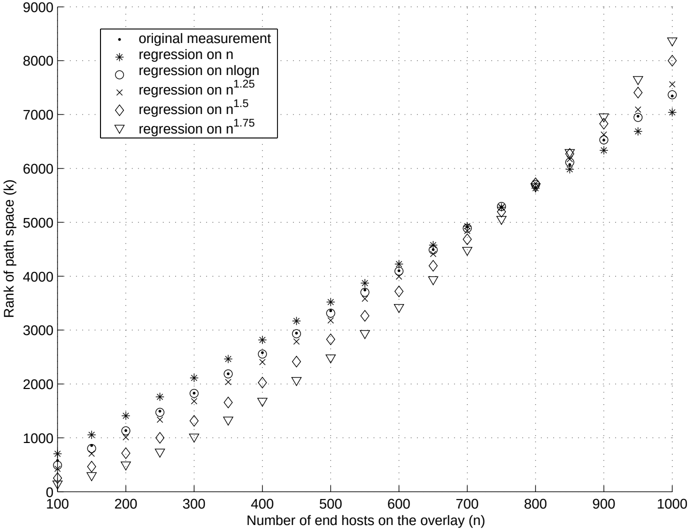

Contributions

- Accurate predictions on $O(n^2)$ paths from a small number of measurements
- Prototype monitoring system on PlanetLab
- Ongoing work on fault diagnosis using similar ideas
- See: `http://list.cs.northwestern.edu/`

Model with Perfectly Matched Layer

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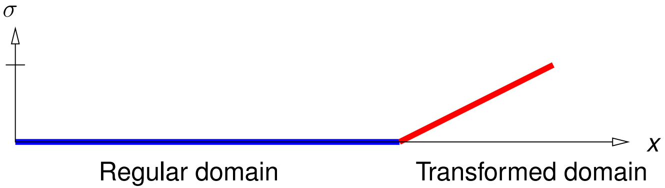

$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s)$$

$$\frac{d^2\hat{u}}{d\tilde{x}^2} + k^2\hat{u} = 0$$

$$\hat{u} = c_{\text{out}} e^{-ik\tilde{x}} + c_{\text{in}} e^{ik\tilde{x}}$$

Model with Perfectly Matched Layer

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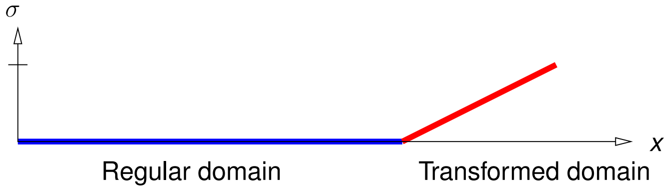


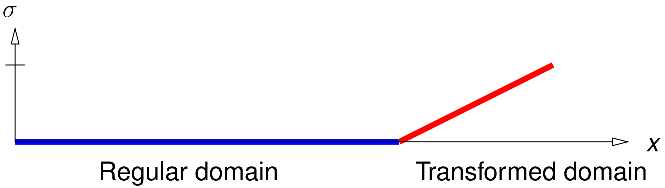
$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s),$$

$$\frac{1}{\lambda}\frac{d}{dx}\left(\frac{1}{\lambda}\frac{d\hat{u}}{dx}\right) + k^2\hat{u} = 0$$

$$\hat{u} = c_{\mathrm{out}} e^{-ikx - k\Sigma(x)} + c_{\mathrm{in}} e^{ikx + k\Sigma(x)}$$

$$\Sigma(x) = \int_0^x \sigma(s)\, ds$$

Model with Perfectly Matched Layer



If solution clamped at $x = L$ then

$$\frac{c_{\text{in}}}{c_{\text{out}}} = O(e^{-k\gamma}) \text{ where } \gamma = \Sigma(L) = \int_0^L \sigma(s)\, ds$$

Eigenvalues and Model Reduction

Want to know about the transfer function $H(\omega)$:

$$H(\omega) = p^T (K - \omega^2 M)^{-1} b$$

Can either

- Locate poles of H (eigenvalues of (K, M))
- Plot H in a frequency range (Bode plot)

Usual tactic: subspace projection

- Build an Arnoldi basis V
- Compute with much smaller $V^* K V$ and $V^* M V$

Can we do better?

Variational Principles

- Variational form for complex symmetric eigenproblems:
  - Hermitian (Rayleigh quotient):

$$\rho(v) = \frac{v^* K v}{v^* M v}$$

  - Complex symmetric (modified Rayleigh quotient):

$$\theta(v) = \frac{v^T K v}{v^T M v}$$

- First-order accurate eigenvectors $\implies$ Second-order accurate eigenvalues.
- Key: relation between left and right eigenvectors.

Accurate Model Reduction

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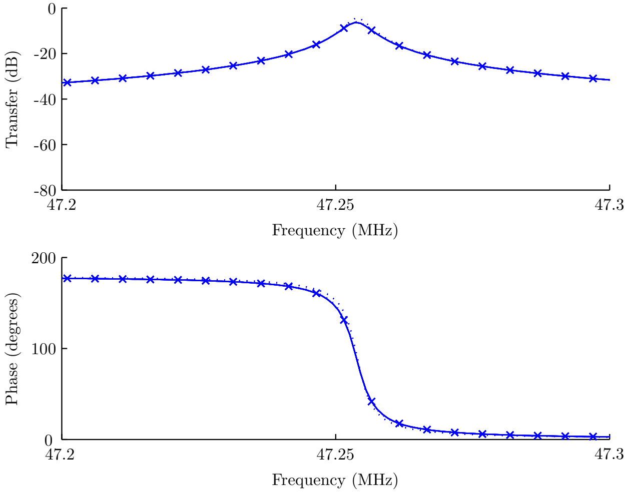
- Build new projection basis from V:

$$W = \text{orth}[\text{Re}(V), \text{Im}(V)]$$

- span(W) contains both $\mathcal{K}_n$ and $\bar{\mathcal{K}}_n$
  $\implies$ double digits correct vs. projection with V
- W is a real-valued basis
  $\implies$ projected system is complex symmetric

Model Reduction Accuracy



Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

Model Reduction Accuracy

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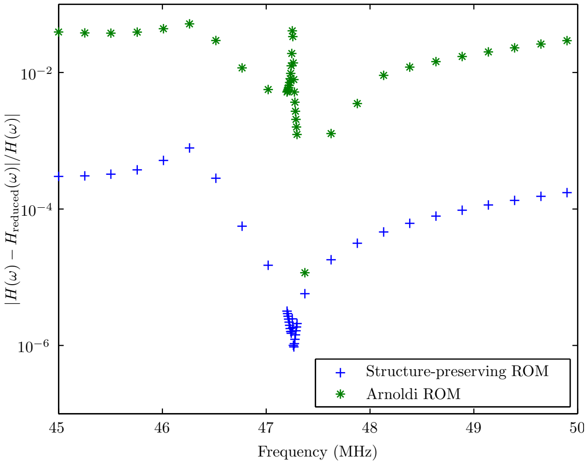
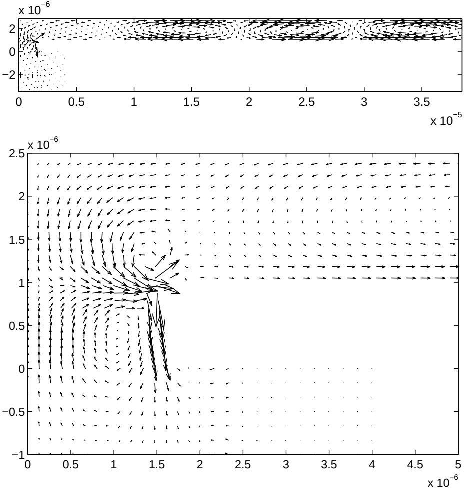


Preserve structure $\Longrightarrow$

get twice the correct digits

Time-Averaged Energy Flux



Sources of Damping

- Fluid damping
  - Air is a viscous fluid ($\mathrm{Re} \ll 1$)
  - Can operate in a vacuum
  - Shown not to dominate in many RF designs
- Material losses
  - Low intrinsic losses in silicon, diamond, germanium
  - Terrible material losses in metals
- Thermoelastic damping
  - Volume changes induce temperature change
  - Diffusion of heat leads to mechanical loss
- Anchor loss
  - Elastic waves radiate from structure

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Fabrication Outline

1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
5. Pattern and etch Si layer
6. Release etch remaining SiO2

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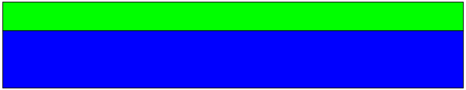

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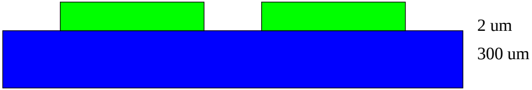

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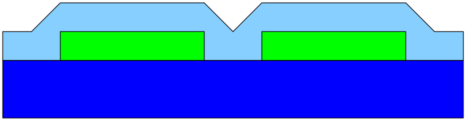

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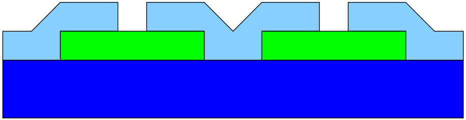

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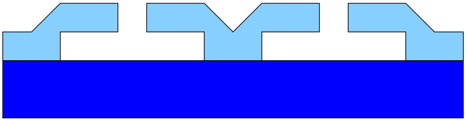

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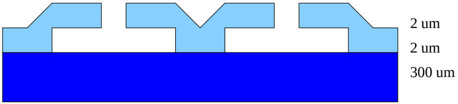
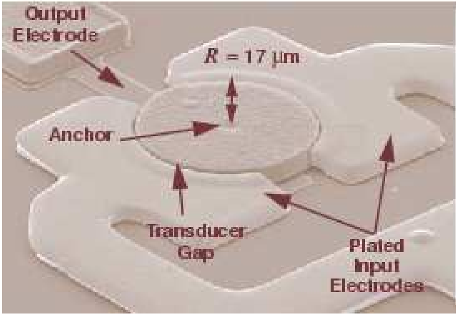
Fabrication Result





(C. Nguyen, iMEMS 02)

Role of simulation

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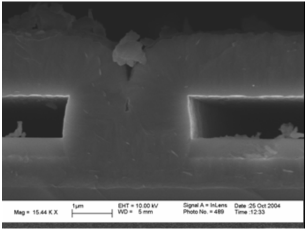
HiQLab: Modeling RF MEMS

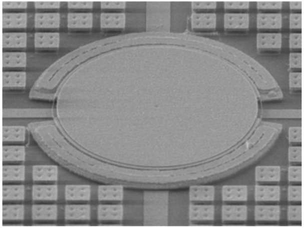
- Explore fundamental device physics
  - Particularly details of damping
- Detailed finite element modeling
- Reduced models eventually to go into SUGAR

SUGAR: "Be SPICE to the MEMS world"

- Fast enough for early design stages
- Simple enough to attract users
- Support design, analysis, optimization, synthesis
- Verify models by comparison to measurement

Why simulate?





- "Build and break" is too expensive
  - Wafer processing costs months, thousands of dollars
  - Fabrication is imprecise
  - Days or weeks to take good measurements
- Good experiments need good hypotheses
- Even when device behavior is understood, still need to understand system behavior

From simulation to synthesis

Computer can assist at many levels:

- Fundamental physics
- Detailed device models
- System-level models and macromodels
- Metrology
- Design optimization
- Design synthesis

Research thrusts

- Model development (e.g. new finite elements)
- Numerical algorithms (e.g. model reduction)
- Numerical software engineering (SUGAR, HiQLab)
- Metrology and comparison to measurement
- Optimization and design synthesis

Research group

Faculty		Students	
A. Agogino	(ME)	D. Bindel	(CS)
Z. Bai	(Math,CS,UCD)	J.V. Clark	(AS&T)
J. Demmel	(Math,CS)	C. Cobb	(ME)
S. Govindjee	(CEE)	D. Garmire	(CS)
R. Howe	(EE,ME)	T. Koyama	(CEE)
K.S.J. Pister	(EE)	J. Nie	(Math)
C. Sequin	(CS)	H. Wei	(CEE)
		Y. Zhang	(CEE)

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SUGAR

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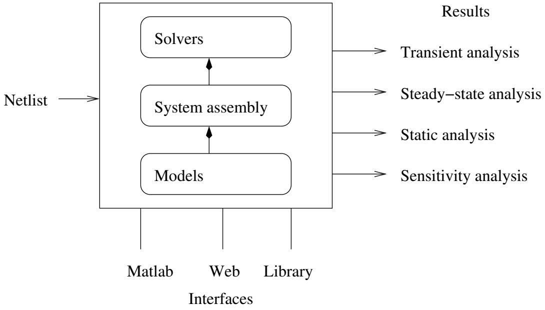

SUGAR: Analysis of a micromirror

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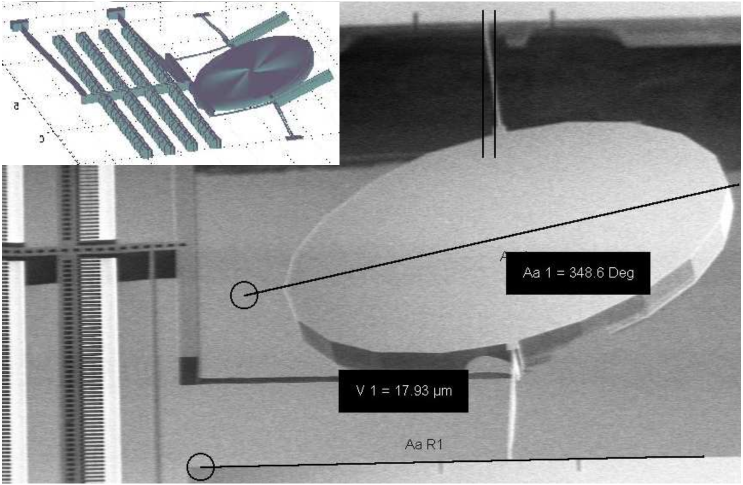
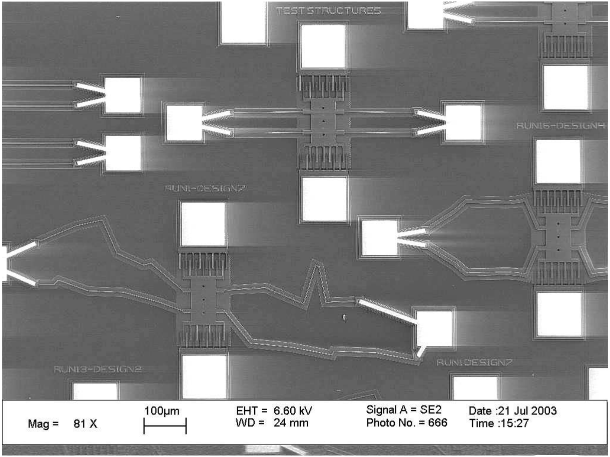
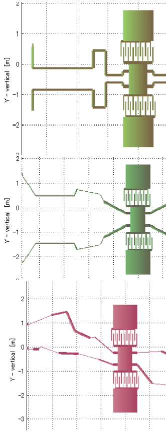


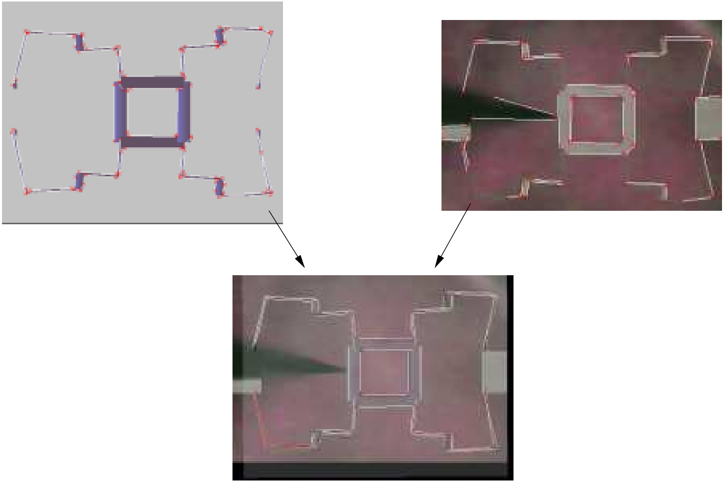
(Mirror design by M. Last)

SUGAR: Design synthesis





SUGAR: Comparison to measurement



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Elastic PMLs

$$\int_\Omega \epsilon(w) : \mathsf{C} : \epsilon(u)\, d\Omega - \omega^2 \int_\Omega \rho w \cdot u \, d\Omega = \int_\Gamma w \cdot t_n d\Gamma$$

$$\epsilon(u) = \left(\frac{\partial u}{\partial x}\right)^s$$

- Start from standard weak form
- Introduce transformed $\tilde{x}$ with $\frac{\partial \tilde{x}}{\partial x} = \Lambda$
- Map back to reference system ($J_\Lambda = \det(\Lambda)$)
- All terms are symmetric in w and u

Elastic PMLs

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$$\int_{\tilde{\Omega}} \tilde{\epsilon}(w) : \mathsf{C} : \tilde{\epsilon}(u)\, d\tilde{\Omega} - \omega^2 \int_{\tilde{\Omega}} \rho w \cdot u \, d\tilde{\Omega} = \int_{\Gamma} w \cdot t_n d\Gamma$$

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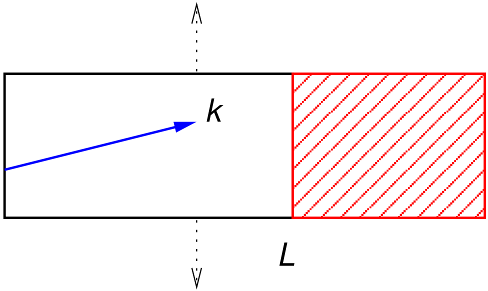
Continuum 2D model problem



$$\lambda(x) = \begin{cases} 1 - i\beta|x-L|^p, & x > L \\ 1 & x \leq L. \end{cases}$$

$$\frac{1}{\lambda}\frac{\partial}{\partial x}\left(\frac{1}{\lambda}\frac{\partial u}{\partial x}\right) + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0$$

Continuum 2D model problem

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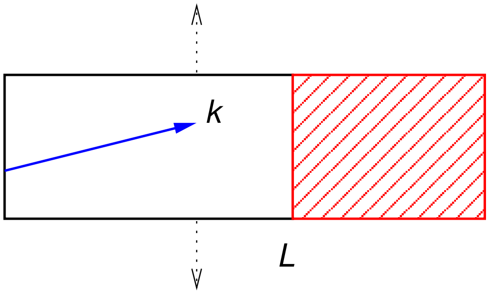

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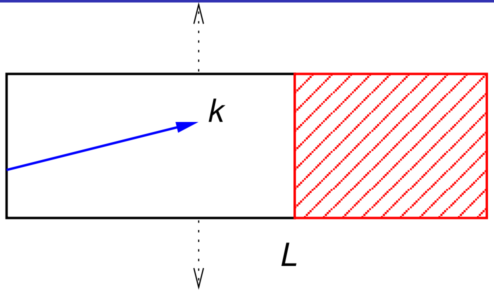
Continuum 2D model problem



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$$\frac{1}{\lambda}\frac{\partial}{\partial x}\left(\frac{1}{\lambda}\frac{\partial u}{\partial x}\right) + k_x^2 u = 0$$

1D problem, reflection of $O(e^{-k_x \gamma})$

Discrete 2D model problem

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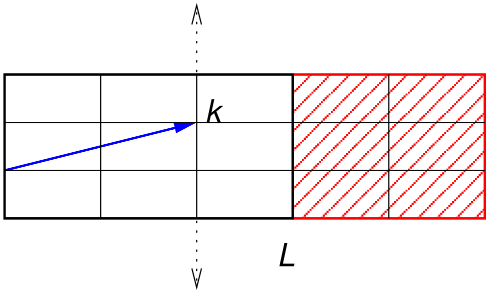
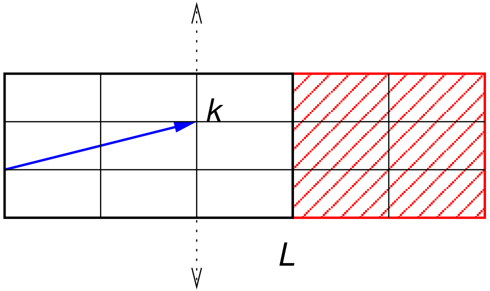


- Discrete Fourier transform in y
- Solve numerically in x
- Project solution onto infinite space traveling modes
- Extension of Collino and Monk (1998)

Nondimensionalization



$$\lambda(x) = \begin{cases} 1 - i\beta|x-L|^p, & x > L \\ 1 & x \le L. \end{cases}$$

Rate of stretching:	βh^p
Elements per wave:	$(k_x h)^{-1}$ and $(k_y h)^{-1}$
Elements through the PML:	N

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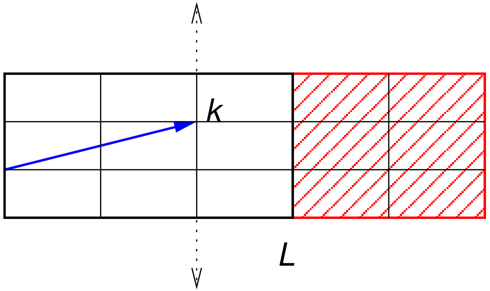

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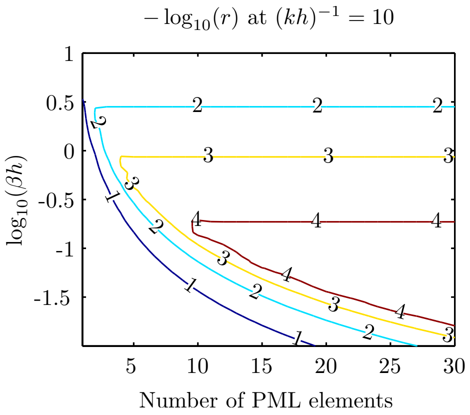
Discrete reflection behavior



Quadratic elements, $p = 1$, $(k_x h)^{-1} = 10$

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Discrete reflection decomposition

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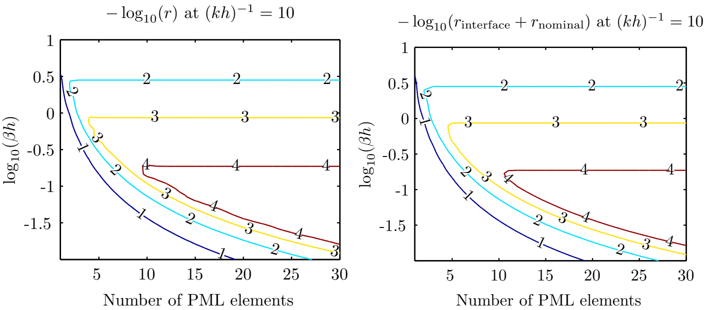
Model discrete reflection as two parts:

- Far-end reflection (clamping reflection)
  - Approximated well by continuum calculation
  - Grows as $(k_x h)^{-1}$ grows
- Interface reflection
  - Discrete effect: mesh does not resolve decay
  - Does not depend on N
  - Grows as $(k_x h)^{-1}$ shrinks

Discrete reflection behavior



Quadratic elements, $p = 1$, $(k_x h)^{-1} = 10$

- Model does well at predicting actual reflection
- Similar picture for other wavelengths, element types, stretch functions

Choosing PML parameters

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- Discrete reflection dominated by
  - Interface reflection when k_x large
  - Far-end reflection when k_x small
- Heuristic for PML parameter choice
  - Choose an acceptable reflection level
  - Choose β based on interface reflection at $k_x^{\max}$
  - Choose length based on far-end reflection at $k_x^{\min}$

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Thermoelastic damping (TED)

u is displacement and $T = T_0 + \theta$ is temperature

$$\begin{aligned} \sigma &= C\epsilon - \beta\theta 1 \\ \rho u_{tt} &= \nabla \cdot \sigma \\ \rho c_v \theta_t &= \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \operatorname{tr}(\epsilon_t) \end{aligned}$$

- Volumetric strain rate drives energy transfer from mechanical to thermal domain
  - Irreversible diffusion $\implies$ mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators
- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system

Nondimensionalization

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$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
u_{tt} &= \nabla \cdot \sigma \\
\theta_t &= \eta\nabla^2\theta - \mathrm{tr}(\epsilon_t)
\end{aligned}
$$

$$
\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}
$$

$$
\begin{aligned}
\text{Length} &\sim L \\
\text{Time} &\sim L/c, \text{ where } c = \sqrt{E/\rho} \\
\text{Temperature} &\sim T_0 \frac{\beta}{\rho c_v}
\end{aligned}
$$

Scaling analysis

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ u_{tt} &= \nabla \cdot \sigma \\ \theta_t &= \eta \nabla^2 \theta - \mathrm{tr}(\epsilon_t) \end{aligned}$$

$$\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}$$

- Micron-scale poly-Si devices: ξ and η are $\sim 10^{-4}$.
- Small η leads to thermal boundary layers
- Linearize about $\xi = 0$

Discrete mode equations

$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
u_{tt} &= \nabla \cdot \sigma \\
\theta_t &= \eta\nabla^2\theta - \mathrm{tr}(\epsilon_t)
\end{aligned}
$$

$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
-\omega^2 u &= \nabla \cdot \sigma \\
i\omega\theta &= \eta\nabla^2\theta - i\omega\,\mathrm{tr}(\epsilon)
\end{aligned}
$$

$$
\begin{aligned}
-\omega^2 M_{uu}u + K_{uu}u + K_{ut}\theta &= 0 \\
i\omega D_{tt}\theta + K_{tt}\theta + i\omega D_{tu}u &= 0
\end{aligned}
$$

Perturbation computation

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$$\begin{aligned} -\omega^2 M_{uu} u + K_{uu} u + K_{ut} \theta &= 0 \\ i\omega D_{tt} \theta + K_{tt} \theta + i\omega D_{tu} u &= 0 \end{aligned}$$

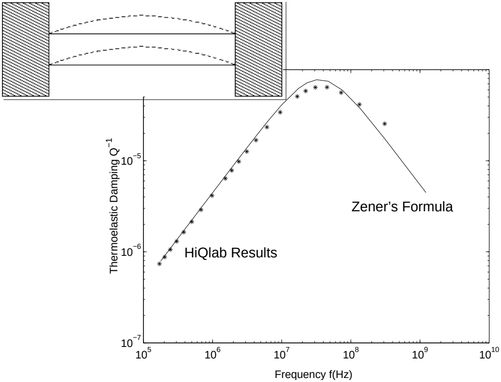
Approximate ω by perturbation about $K_{ut} = 0$:

$$\begin{aligned} -\omega_0^2 M_{uu} u_0 + K_{uu} u_0 &= 0 \\ i\omega_0 D_{tt} \theta_0 + K_{tt} \theta_0 + i\omega_0 D_{tu} u_0 &= 0 \end{aligned}$$

Choose $v : v^T u_0 \neq 0$ and compute

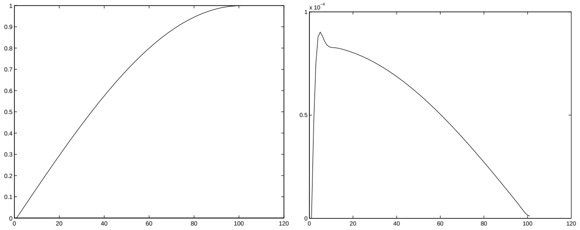
$$\begin{bmatrix} (-\omega_0^2 M_{uu} + K_{uu}) & -2\omega_0 M_{uu} u_0 \\ v^T & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta \omega \end{bmatrix} = \begin{bmatrix} -K_{ut} \theta_0 \\ 0 \end{bmatrix}$$

Comparison to Zener's model



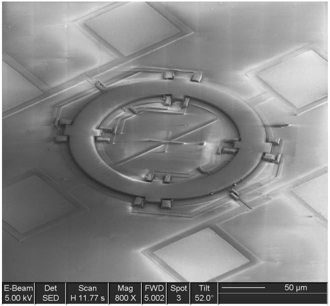
- Comparison of fully coupled simulation to Zener approximation over a range of frequencies
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi

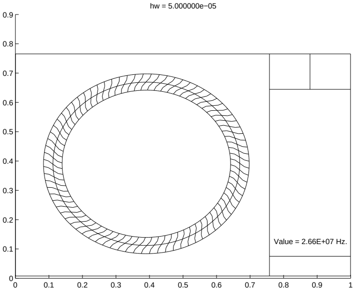
Thermoelastic boundary layer



- One-dimensional test problem (longitudinal mode in a bar)
- Fixed temperature and displacement at left
- Free at right

Shear ring resonator





- Ring is driven in a shearing motion
- Can couple ring to other resonators
- How do we track the desired mode?

Mode tracking

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Find a continuous solution to

$$\left(K(s) - \omega(s)^2 M(s)\right) u(s) = 0.$$

- K and M are symmetric and $M > 0$
- Eigenvectors are M-orthogonal
- Perturbation theory gives good shifts
- Look if $u(s+h)$ and $u(s)$ are on the same path by looking at $u(s+h)^T M(s+h) u(s)$
- Many more subtleties in the nonsymmetric case
  - Focus of the *CIS algorithm*

Mode tracking in a shear resonator