Eigenproblems in Resonant MEMS Design

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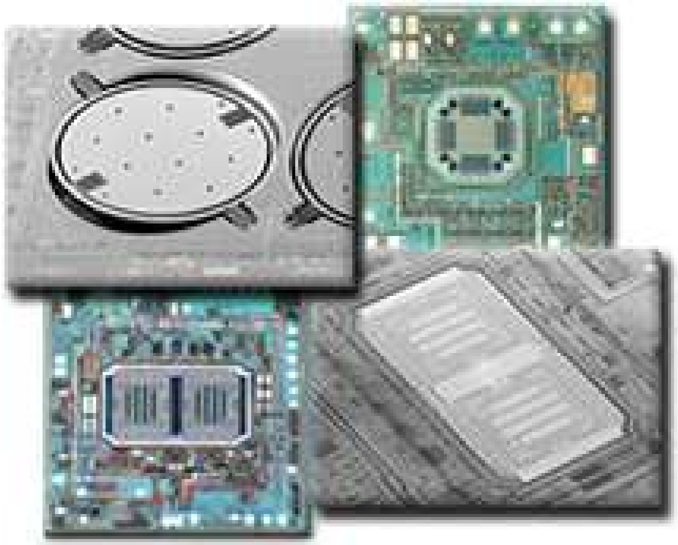
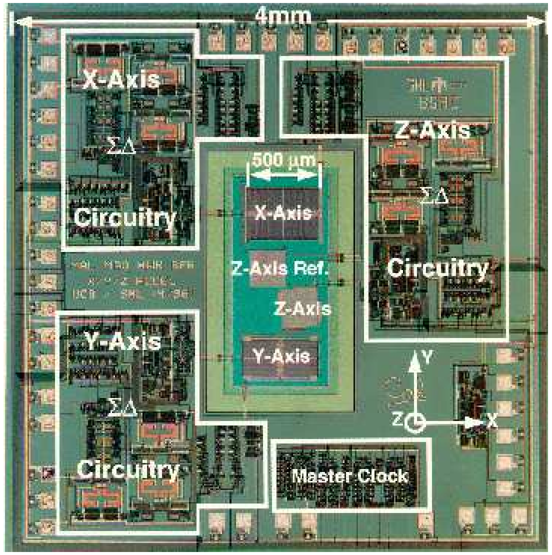
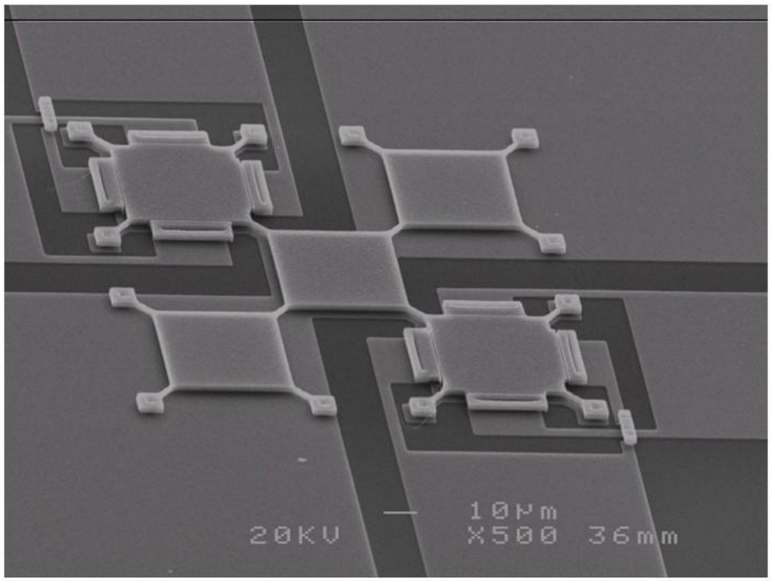
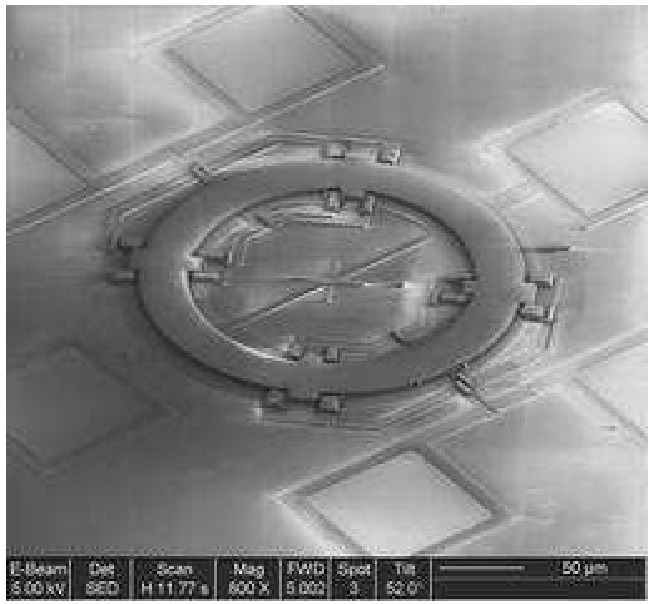
What are MEMS?

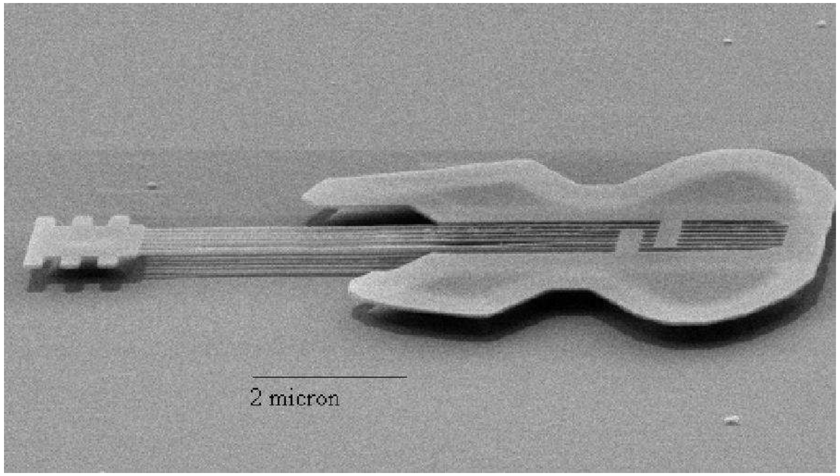


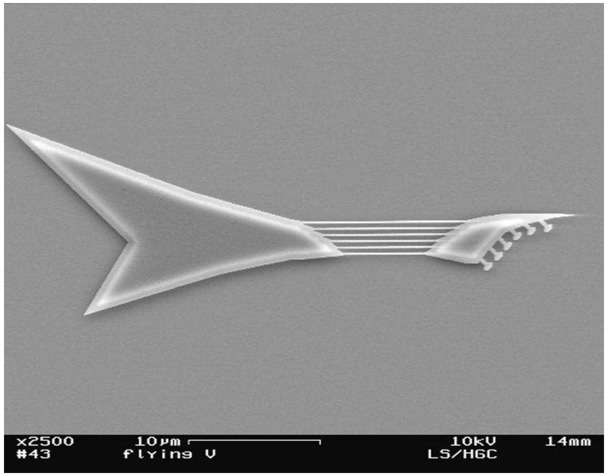






RF MEMS

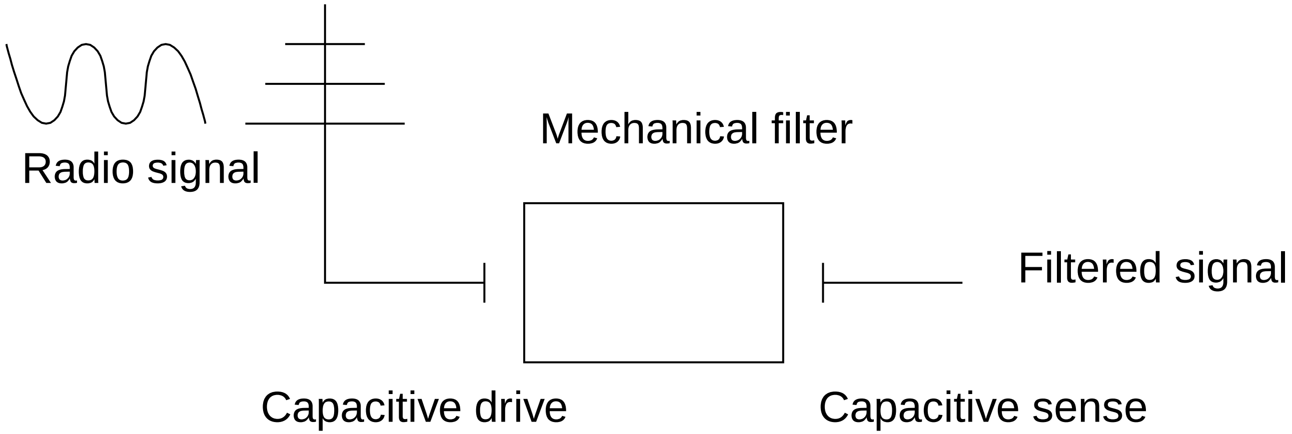




Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Uses:
  - RF signal processing (better cell phones)
  - Sensing elements (e.g. chemical sensors)
  - Really high-pitch guitars

Micromechanical filters



- Your cell phone is already mechanical!
  - Uses a quartz surface-acoustic wave (SAW) filter
- Can do better using MEMS
  - MEMS filters can be placed on-chip
  - Versus SAWs: smaller, lower power

Success $\Longrightarrow$ “Calling Dick Tracy!”

Damping

- Want to minimize damping
  - Measure by "quality of resonance"

$$Q = \frac{|\omega|}{\mathrm{Im}(\omega)}$$

  - Electronic filters have too much
  - Understanding of damping in MEMS is lacking
- Several sources of damping
  - Anchor loss
  - Thermoelastic damping
  - Fluid damping
  - Material losses

Damping

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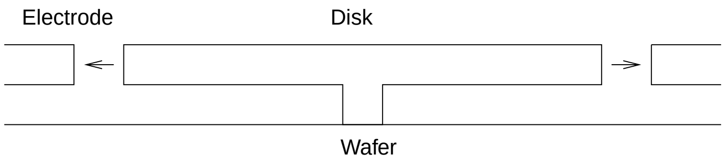
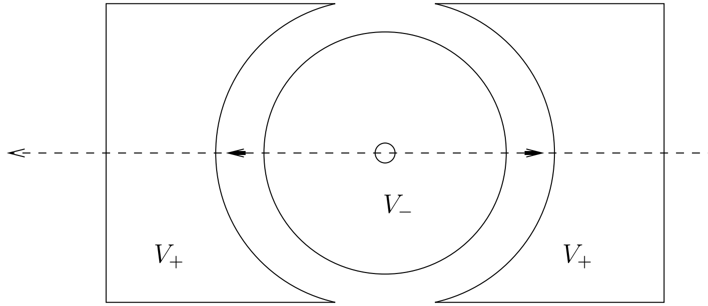
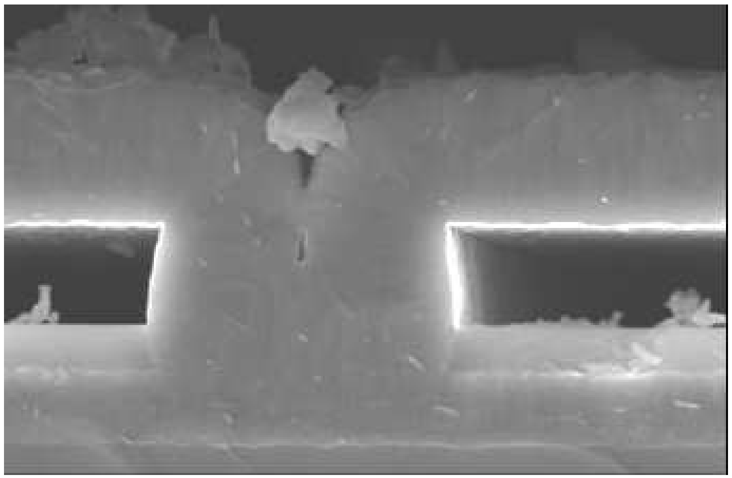
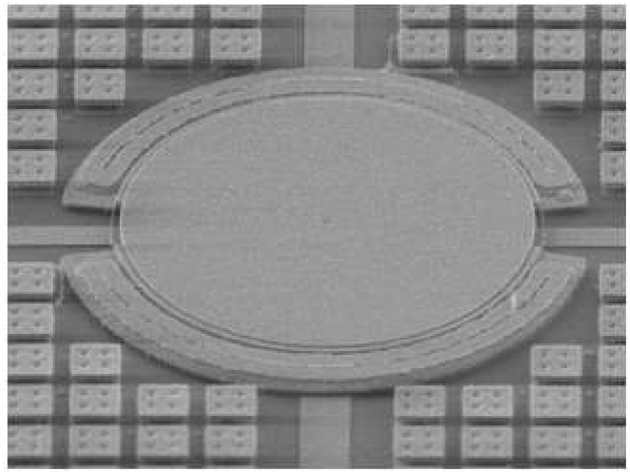
Example: Disk anchor loss









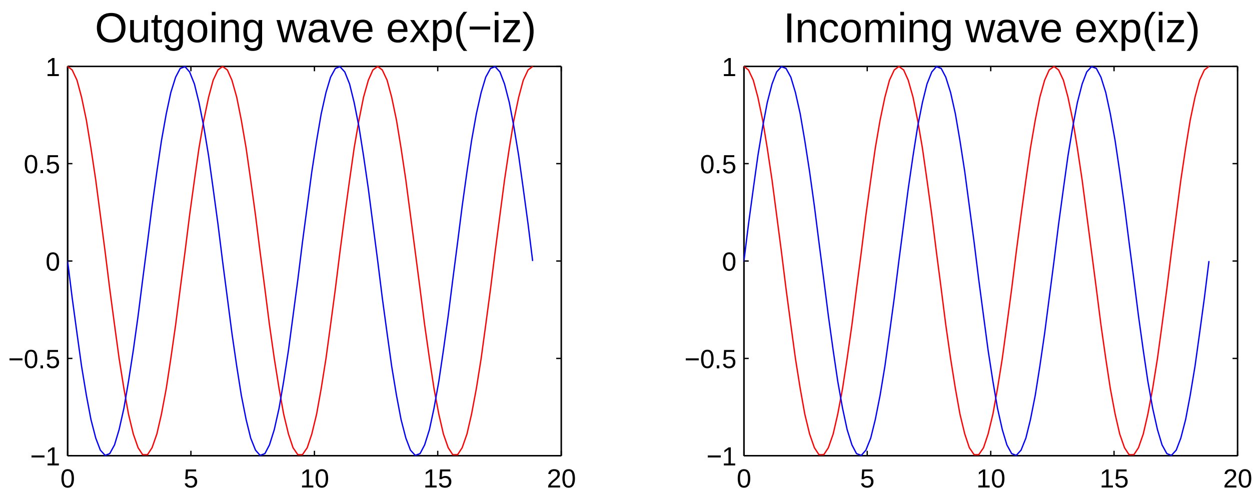
- SiGe disk resonators built by E. Quévy
- Axisymmetric model with bicubic mesh, about 10K nodal points

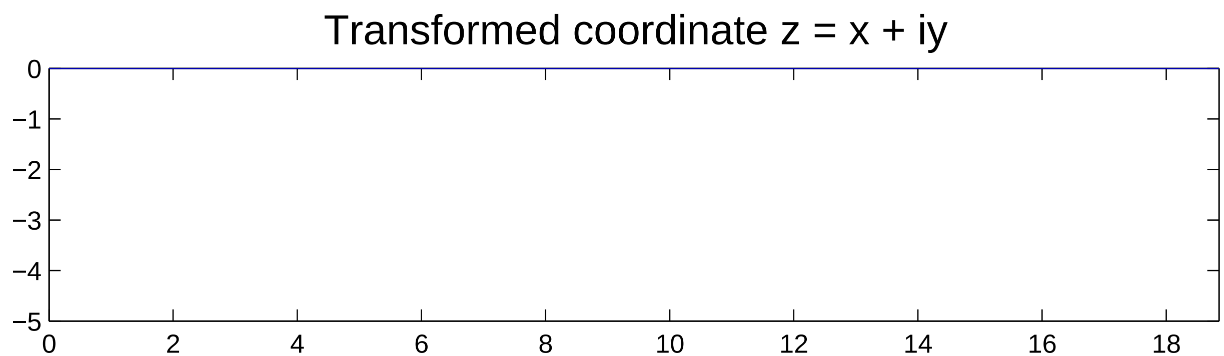
Perfectly matched layers

- Model half-space with a *perfectly matched layer*
  - Complex coordinate change $x \mapsto z(x;\omega)$
  - Apply a complex coordinate transformation
  - Generates a non-physical absorbing layer
- Idea works with general linear wave equations
  - First applied to Maxwell's equations (Berengér 95)
  - Similar idea introduced earlier in quantum mechanics (*exterior complex scaling*, Simon 79)

Scalar wave example

$$-c^2 u_{zz} - \omega^2 u = 0$$





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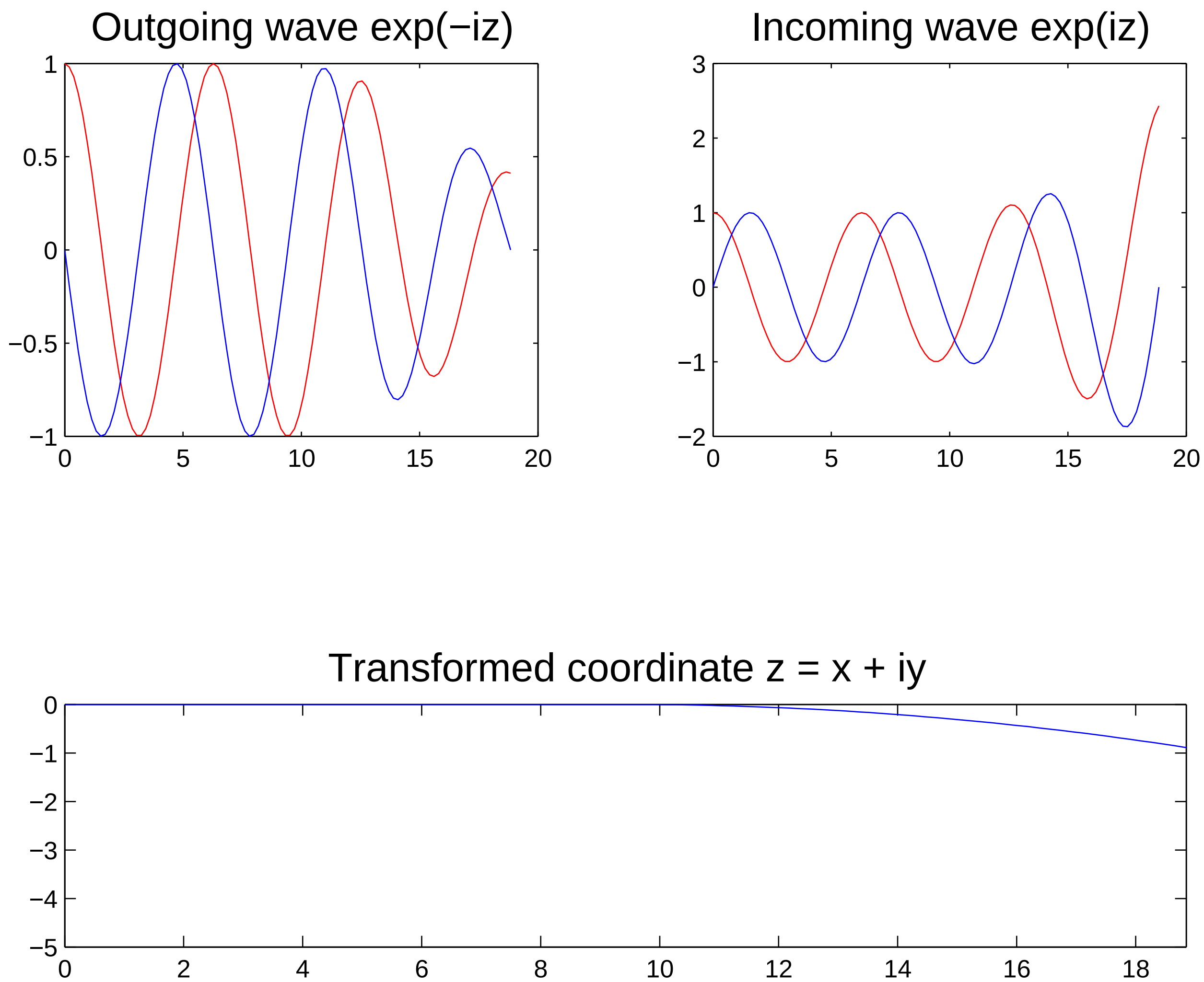

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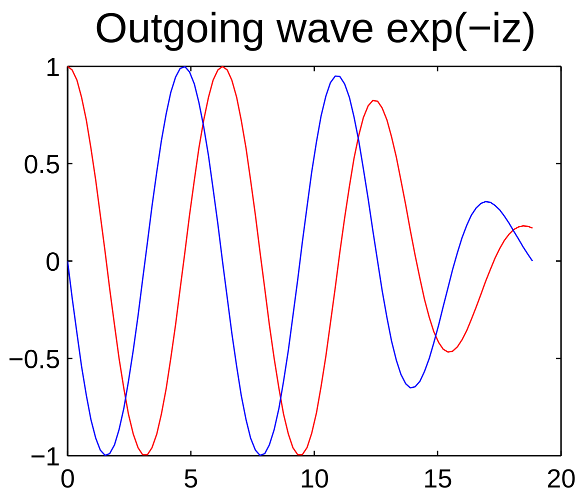

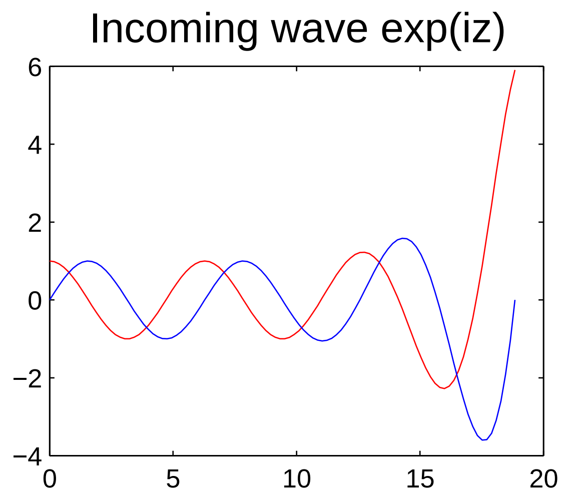

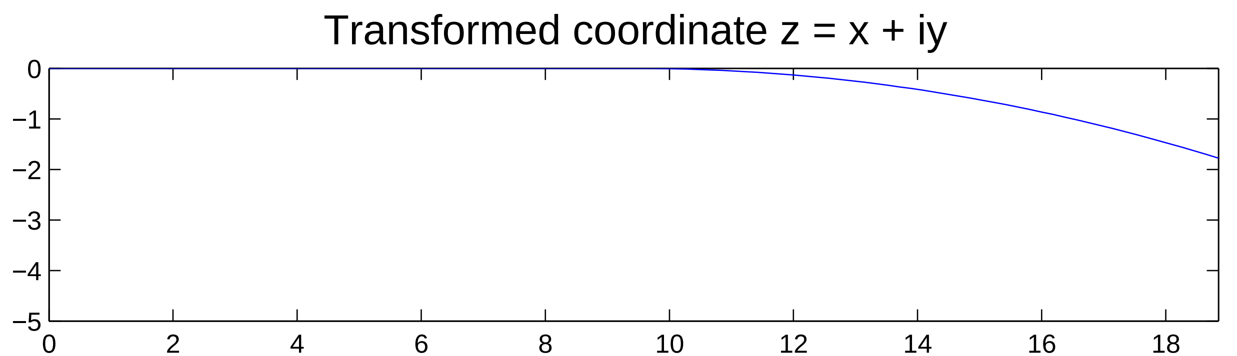

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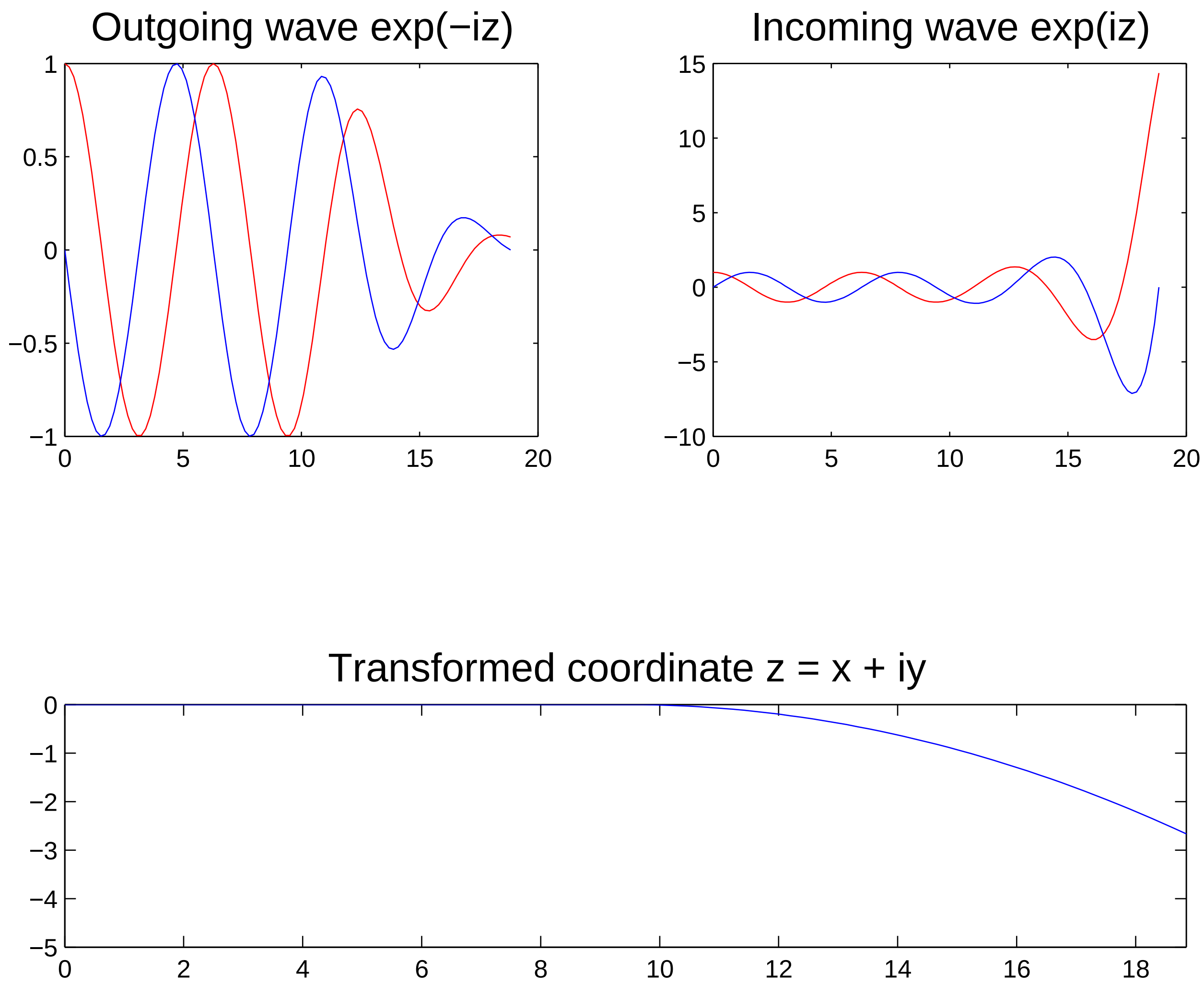

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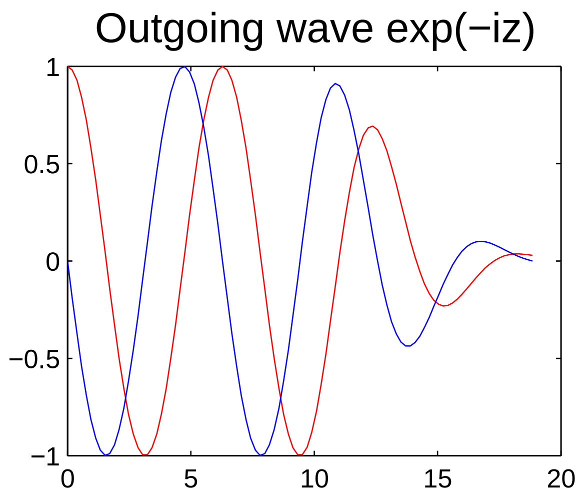
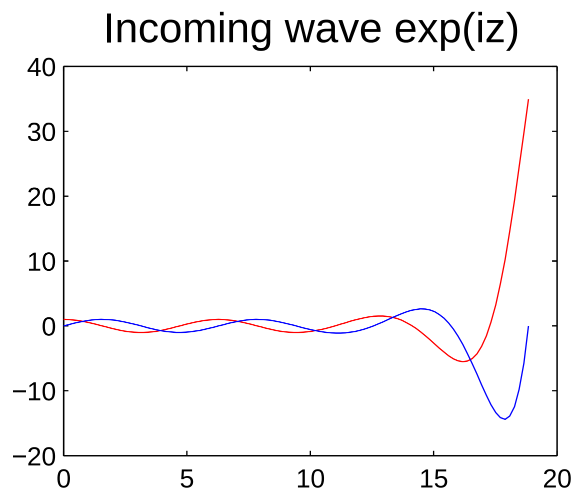
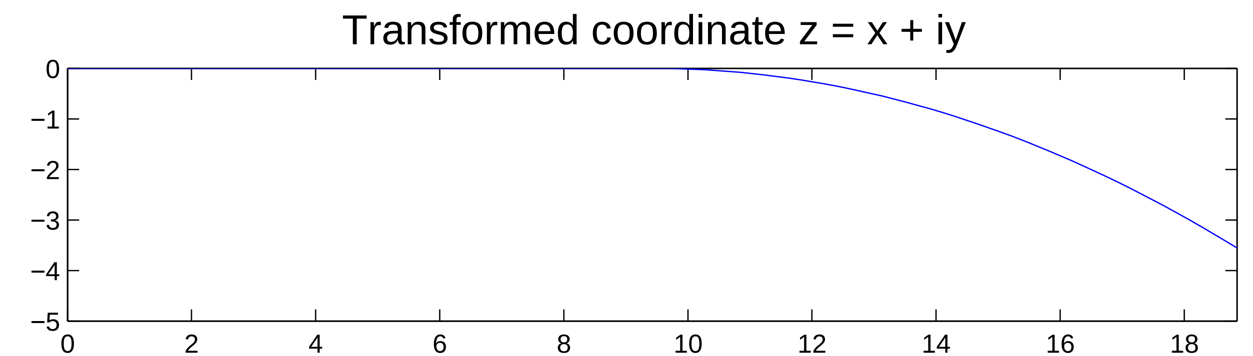
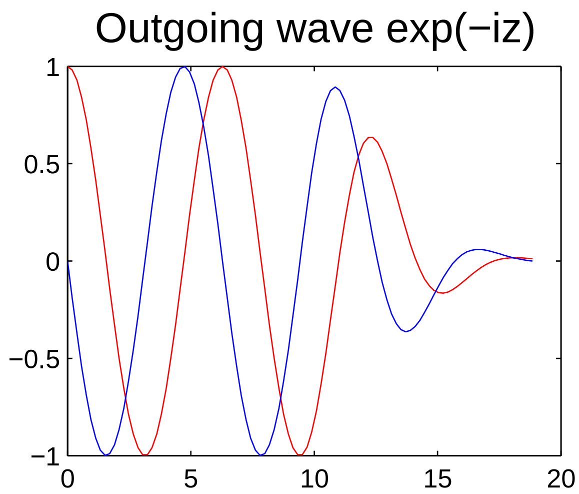


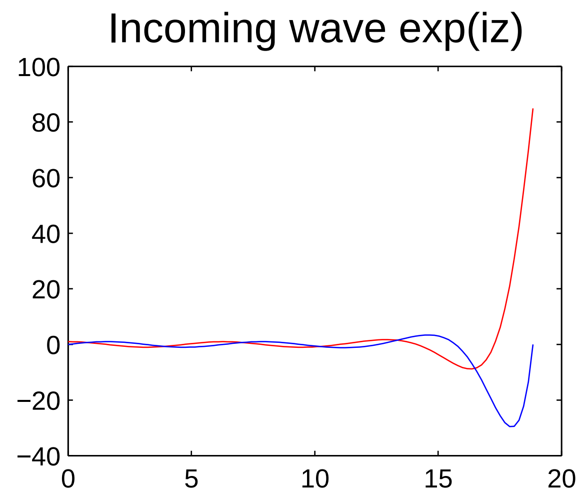


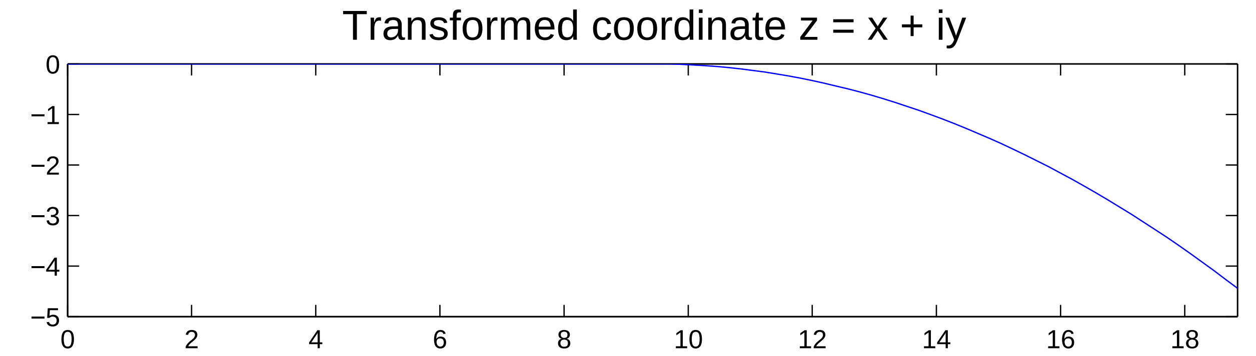


Scalar wave example

$$-c^2 u_{zz} - \omega^2 u = 0$$







Clamp solution at transformed end to isolate outgoing wave.

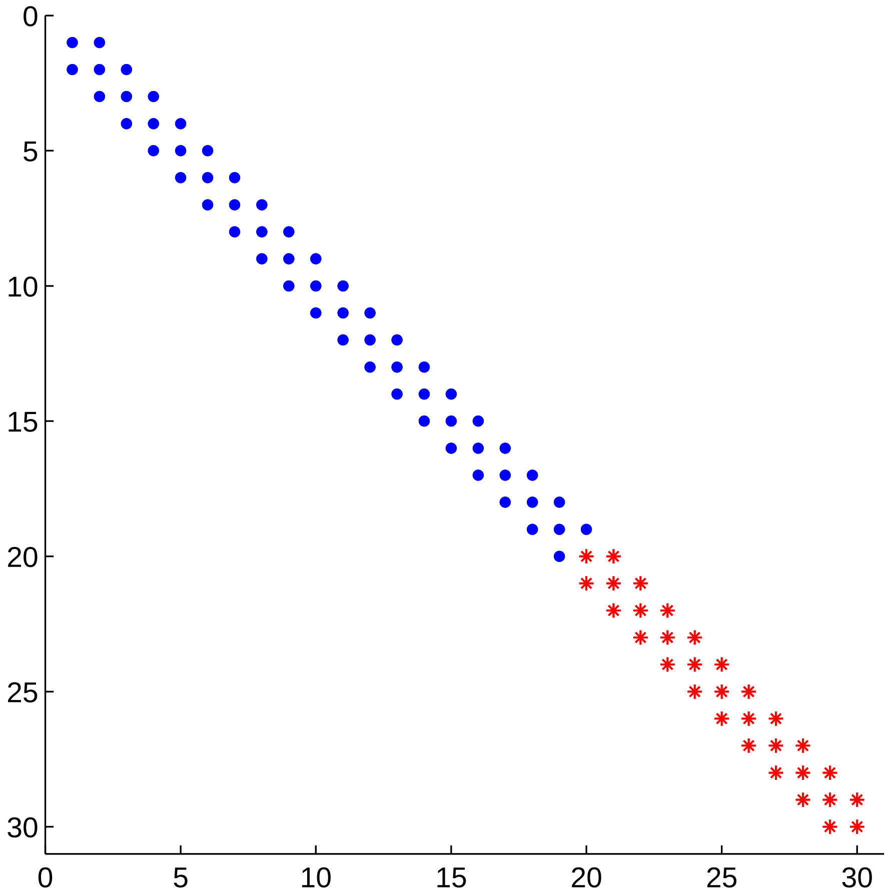
Choice of transformations

- Generally z depends nontrivially on ω
  - Needed for frequency-independent attenuation
  - Common choice is

$$\frac{dz}{dx} = 1 - \sigma(x)/k$$

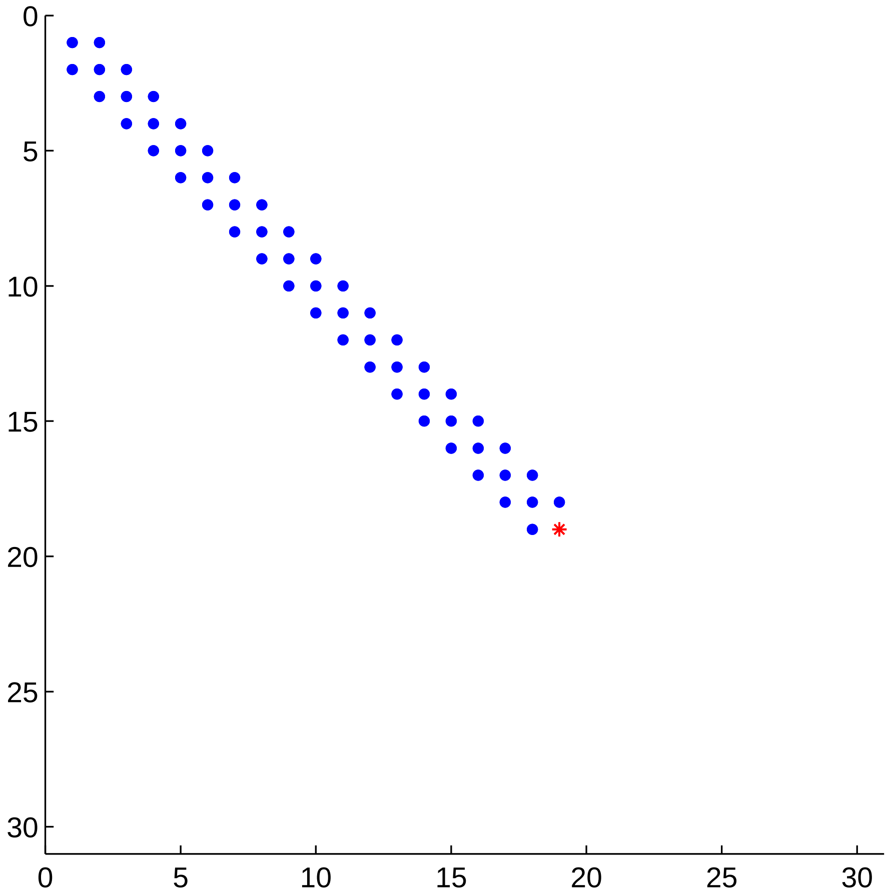
- What if we use a fixed transformation?
  - Can choose to absorb well over finite ω range
  - Solve a *linear* eigenvalue problem
  - Amounts to rational approx of true radiation condition (in discrete case)

Behavior with fixed transformations



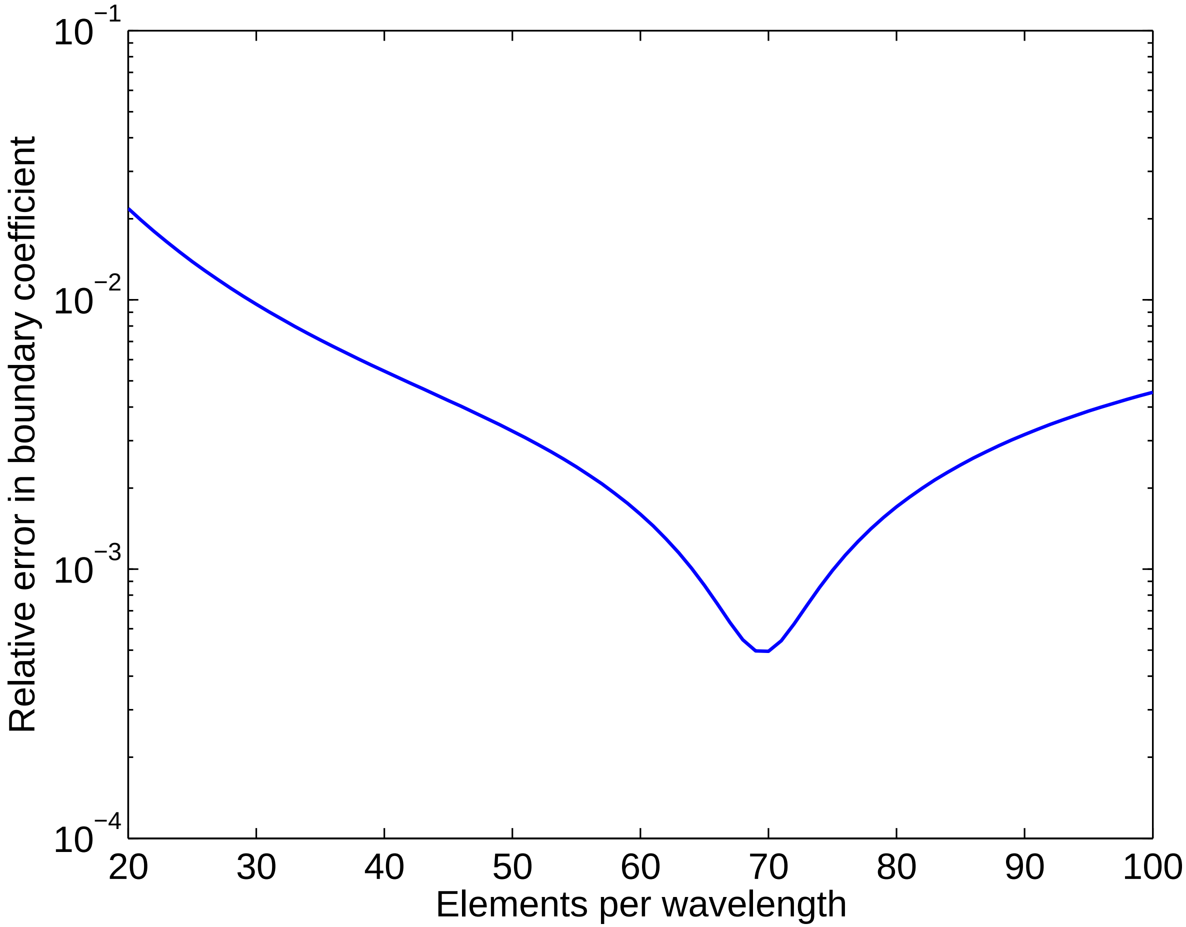
- Start with $(K - \omega^2 M)u = e_1$

Behavior with fixed transformations



- Schur complement to eliminate PML unknowns

Behavior with fixed transformations



- Compare last coefficient with exact (discrete) BC

Complex symmetry

Finite element equations (forced vibration) are

$$-\omega^2 Mu + Ku = F$$

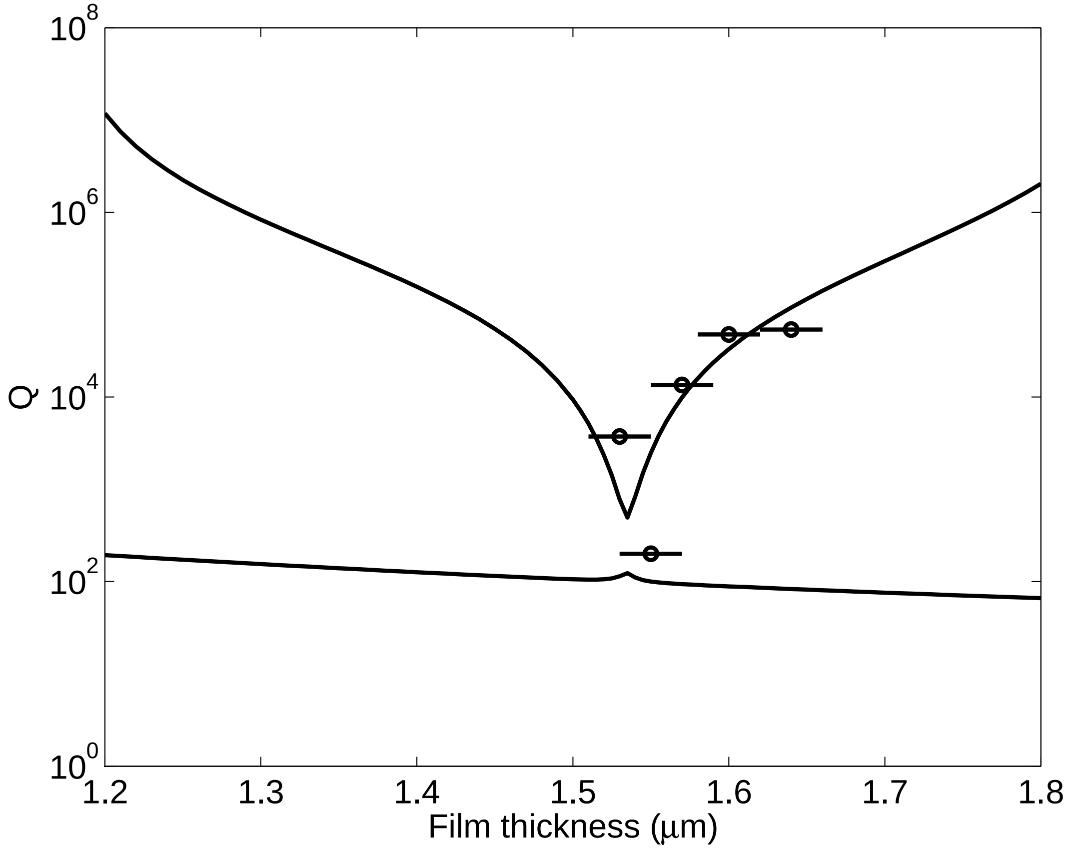
where M and K are *complex symmetric*.

- Row and column eigenvectors are transposes
- Second-order accuracy with modified Rayleigh quotient:

$$\theta(v) = (v^T Kv)/(v^T Mv)$$

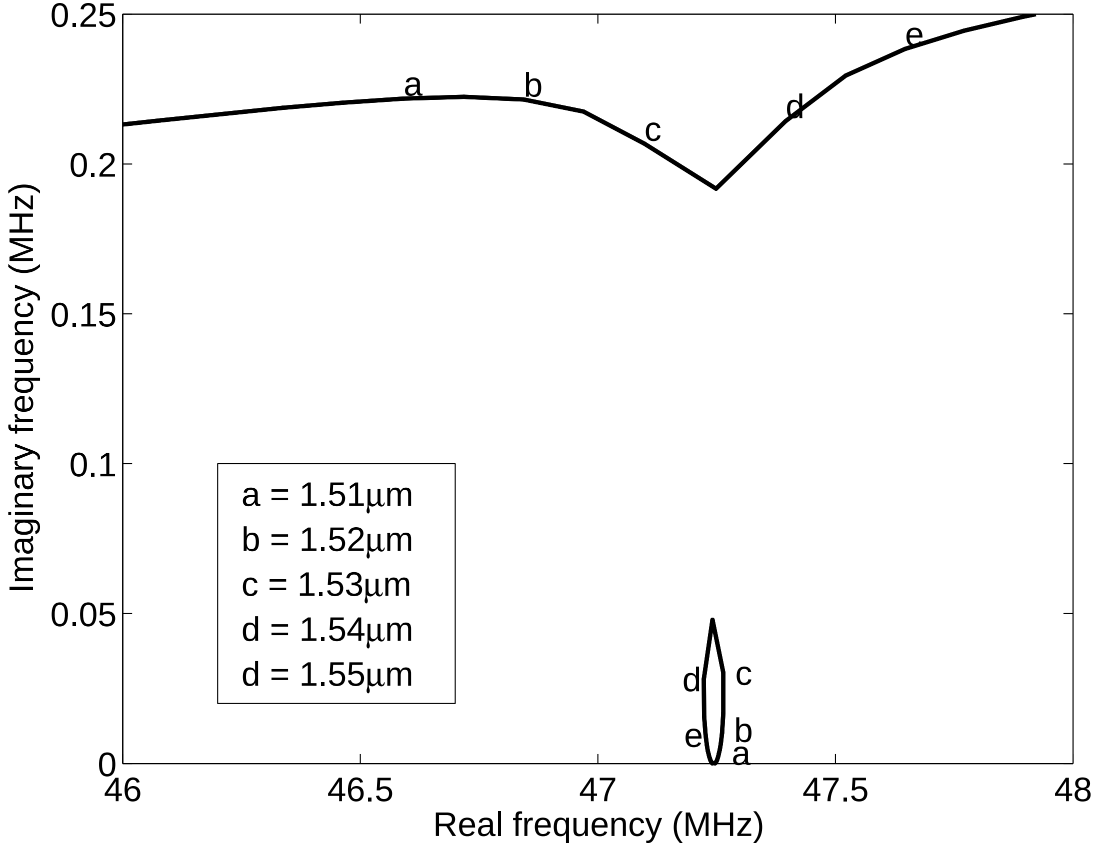
- *Can* have $v^T Mv \approx 0$
  - Propagating modes (continuous spectrum)
  - Not the modes of interest for resonators

Q variation



- Small geometry variation $\Longrightarrow$ large damping variation
- Solid line is simulated; dots are measured

Effect of varying film thickness



- Sudden dip in Q comes from an interaction between a (mostly) bending mode and a (mostly) radial mode

Model reduction

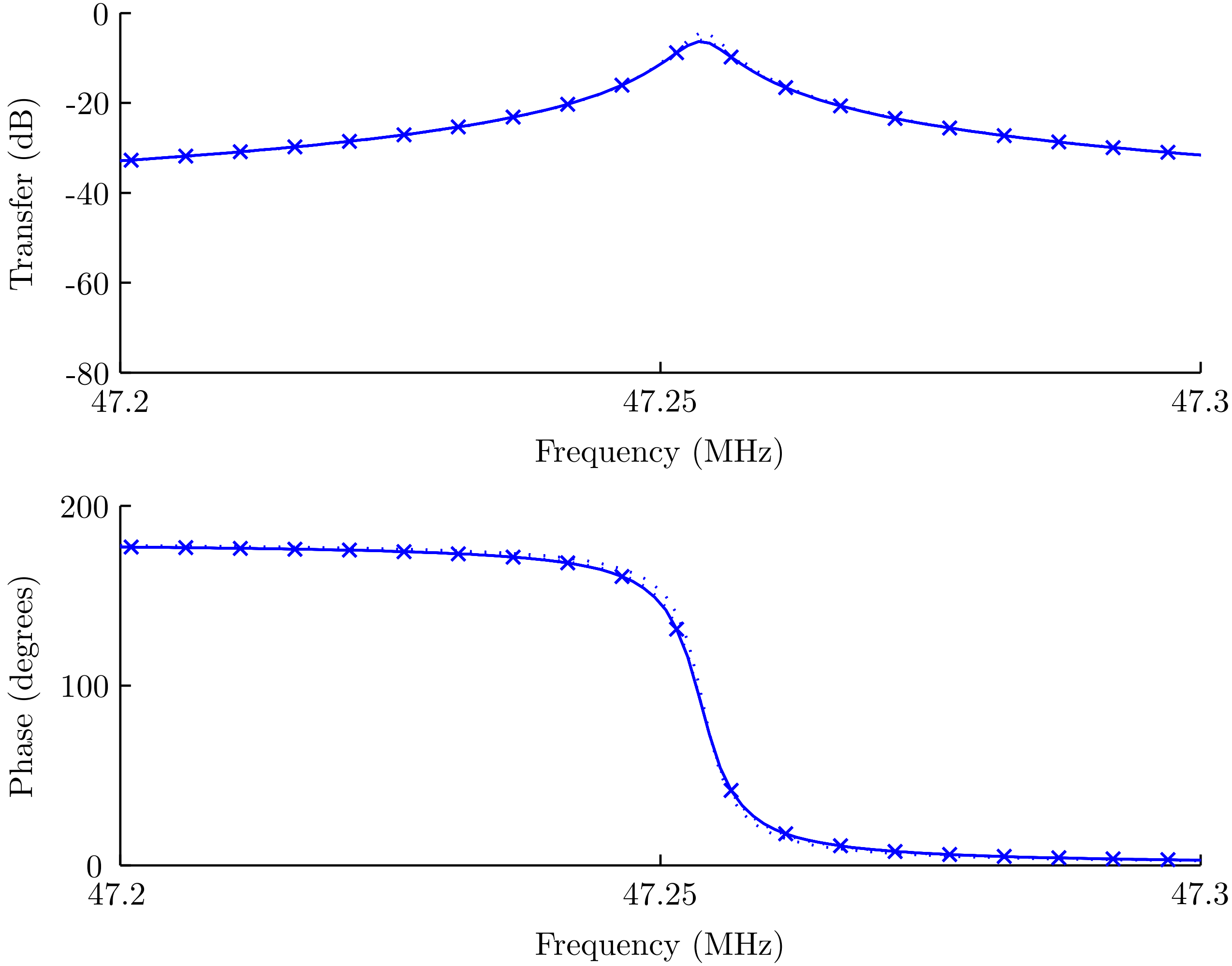
Would like a reduced model which

- Preserves second-order accuracy for converged eigs
- Keeps at least Arnoldi's accuracy otherwise
- Is physically meaningful

Idea:

- Build an Arnoldi basis V
- Double the size: $W = \mathrm{orth}([\mathrm{Re}(V), \mathrm{Im}(V)])$
- Use W as a projection basis
- Resulting system is still a Galerkin approximation with real shape functions for the continuum PML equations

Example: Disk resonator response



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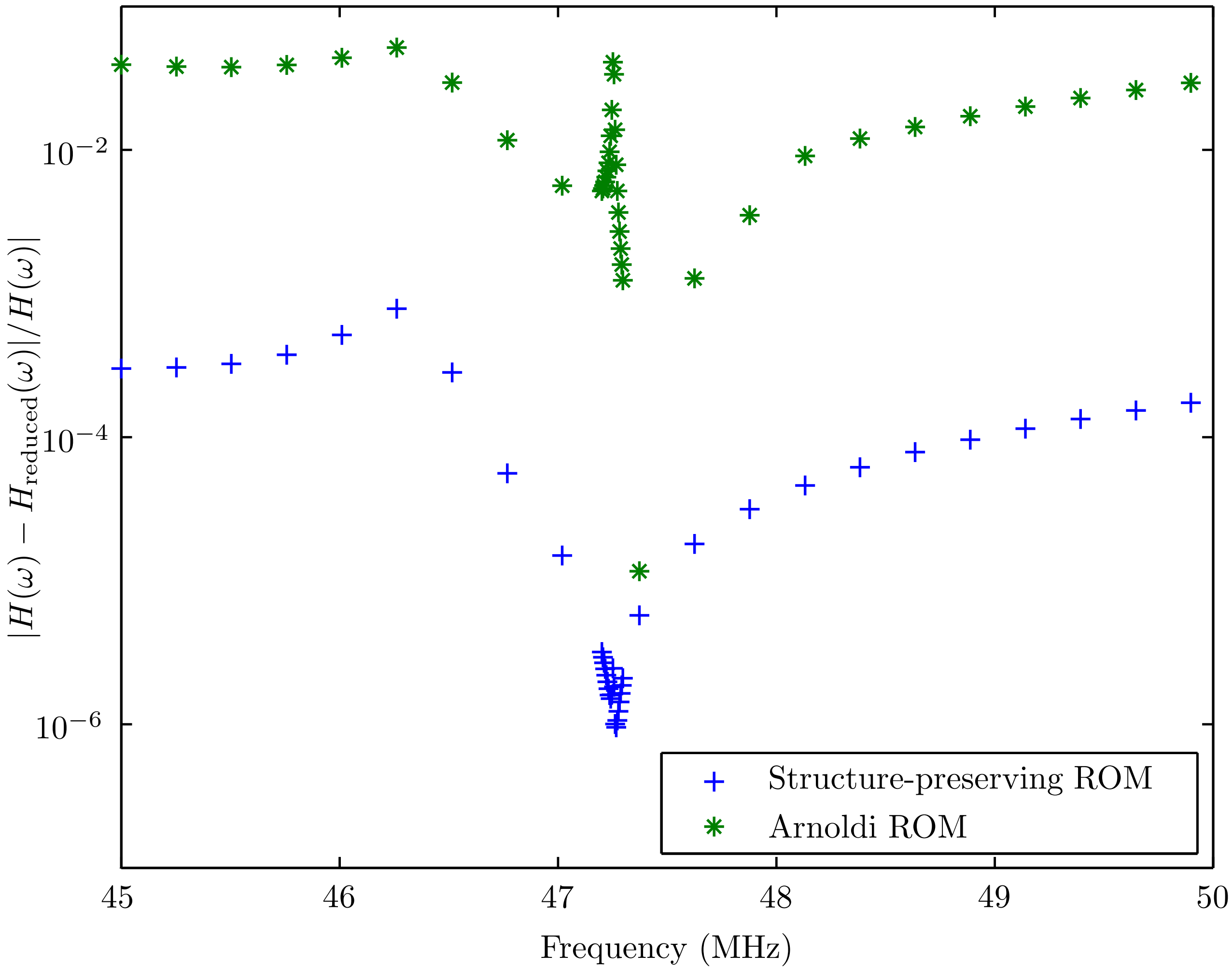


Thermoelastic damping (TED)

u is displacement, $T = T_0 + \theta$ is temperature

$$\begin{aligned} \sigma &= C\epsilon \\ \rho u_{tt} &= \nabla \cdot \sigma \\ \rho c_v \theta_t &= \nabla^2 \theta \end{aligned}$$

- Second-order mechanical + first-order thermal equation

Thermoelastic damping (TED)

u is displacement, $T = T_0 + \theta$ is temperature

$$\begin{aligned} \sigma &= C\epsilon - \beta\theta 1 \\ \rho u_{tt} &= \nabla \cdot \sigma \\ \rho c_v \theta_t &= \nabla^2 \theta - \beta T_0 \operatorname{tr}(\epsilon_t) \end{aligned}$$

- Second-order mechanical + first-order thermal equation
- Temperature change causes stress (thermal expansion)
- Volumetric strain rate causes thermal fluctuations

Thermoelastic damping (TED)

Non-dimensionalized equation:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ u_{tt} &= \nabla \cdot \sigma \\ \theta_t &= \eta\nabla^2\theta - \mathrm{tr}(\epsilon_t) \end{aligned}$$

- Typical MEMS scales: ξ and η small
- Perturbation about $\xi = 0$ is effective

Perturbation computation

Discrete time-harmonic equations:

$$\begin{aligned} -\omega^2 M_{uu} u + K_{uu} u + K_{ut} \theta &= 0 \\ i\omega D_{tt} \theta + K_{tt} \theta + i\omega D_{tu} u &= 0 \end{aligned}$$

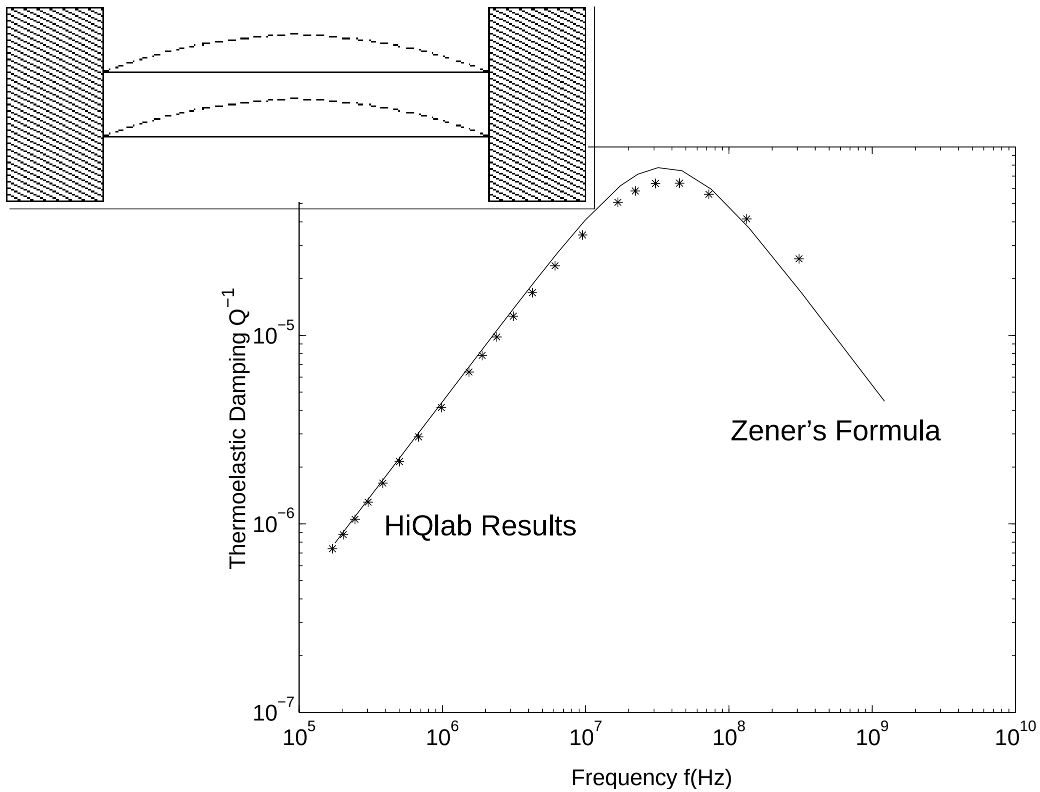
Approximate ω by perturbation about $K_{u\theta} = 0$:

$$\begin{aligned} -\omega_0^2 M_{uu} u_0 + K_{uu} u_0 &= 0 \\ i\omega_0 D_{\theta\theta} \theta_0 + K_{\theta\theta} \theta_0 + i\omega_0 D_{tu} u_0 &= 0 \end{aligned}$$

Choose $v : v^T u_0 \neq 0$ and compute

$$\begin{bmatrix} (-\omega_0^2 M_{uu} + K_{uu}) & -2\omega_0 M_{uu} u_0 \\ v^T & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta\omega \end{bmatrix} = \begin{bmatrix} -K_{u\theta} \theta_0 \\ 0 \end{bmatrix}$$

Comparison to Zener's model



- Good match to Zener's approximation for TED in beams
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi

Conclusions

- MEMS resonator simulations give interesting problems
- Damped resonators $\Longrightarrow$ nonlinear eigenproblems
  - Introduce auxiliary variables to get exact or approximate linear problem
  - There's still useful structure in non-Hermitian problems!
- References:
  - Bindel and Govindjee. "Elastic PMLs for Resonator Anchor Loss Simulation." (IJNME, to appear)
  - HiQLab home page: `www.cs.berkeley.edu/~dbindel/hiqlab/`