HiQLab: Simulation of Resonant MEMS

David Bindel

UC Berkeley, CS Division

Overview

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up

- Background, target applications, and grand vision
- Simple “from scratch” examples
- Anchor loss in a disk resonator
- Thermoelastic damping in a beam
- Model reduction of a checkerboard resonator
- Current status and planned work
- Q & A and feedback

Application: RF MEMS

» Overview

Introduction
» Application: RF MEMS
» Damping and Q
» Sources of damping
» What makes it hard?
» Goal for HiQLab

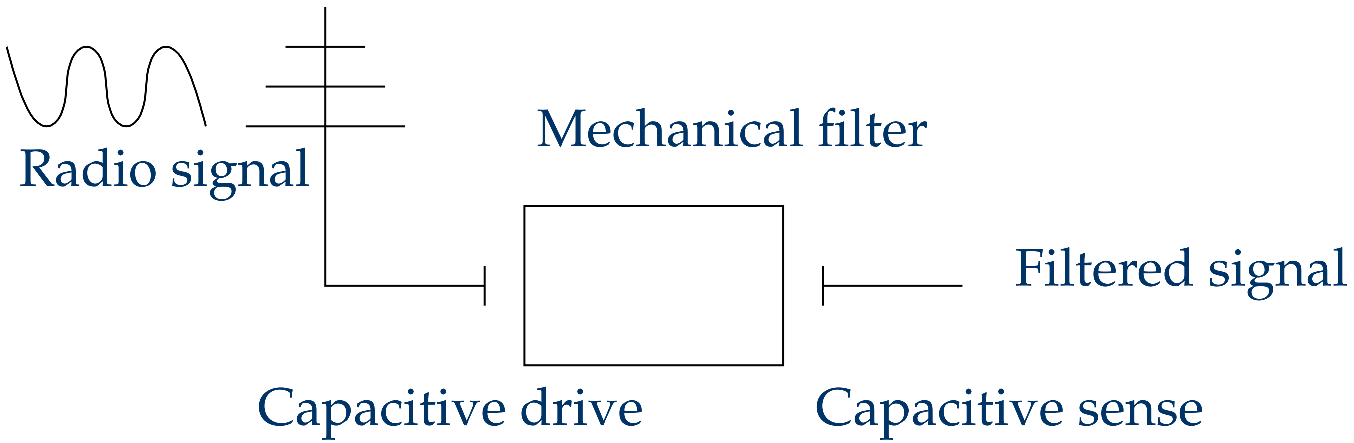
The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up



- Systems of MHz-GHz mechanical resonators
- Useful for
  - Frequency references
  - Filter elements
  - Sensing elements

Damping and Q

» Overview

Introduction
» Application: RF MEMS
» Damping and Q
» Sources of damping
» What makes it hard?
» Goal for HiQLab

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

- Want to minimize damping
  - Electronic filters have too much
  - Understanding of damping in MEMS resonators is lacking
- Characterize damping by Q
  - Non-dimensionalized damping in a one-variable system:

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

  - For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{\Re(\omega)}{2\Im(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

- Goal: Make Q big

Sources of damping

» Overview

Introduction
» Application: RF MEMS
» Damping and Q
» Sources of damping
» What makes it hard?
» Goal for HiQLab

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

- Fluid damping
  - Air is a viscous fluid ($\mathrm{Re} \ll 1$)
  - Can operate in a vacuum
  - Shown not to dominate in many RF designs
- Anchor loss
  - Elastic waves radiate from structure
- Thermoelastic damping
  - Volume changes induce temperature change
  - Diffusion of heat leads to mechanical loss
- Material losses (catch-all)
  - Low intrinsic losses in silicon, diamond, germanium, etc.
  - Greater material losses in metals

» Overview

Introduction
» Application: RF MEMS
» Damping and Q
» Sources of damping
» What makes it hard?
» Goal for HiQLab

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

What makes it hard?

- Physical issues
  - Not clear which loss mechanisms dominate
  - Need physically realistic models
  - Need good material property estimates
- Mathematical issues
  - Operating modes are in the middle of the spectrum
  - Damping makes the problem naturally nonsymmetric
  - ω must be accurately resolved to estimate Q
  - Symmetries and near-symmetries lead to degeneracy
  - Dependence on parameters is critical

» Overview

Introduction
» Application: RF MEMS
» Damping and Q
» Sources of damping
» What makes it hard?
» Goal for HiQLab

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Goal for HiQLab

- Goal is to understand resonant MEMS behavior
  - Initially targeting high-frequency MEMS
  - Particularly care about damping
  - Want to experimentally verify our simulations
- Reason for the simulator
  - Develop physically realistic models
  - Develop fast solvers
  - Help designers

» Overview

Introduction

The Basics

» Software structure
» Software used
» “Hello world”
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

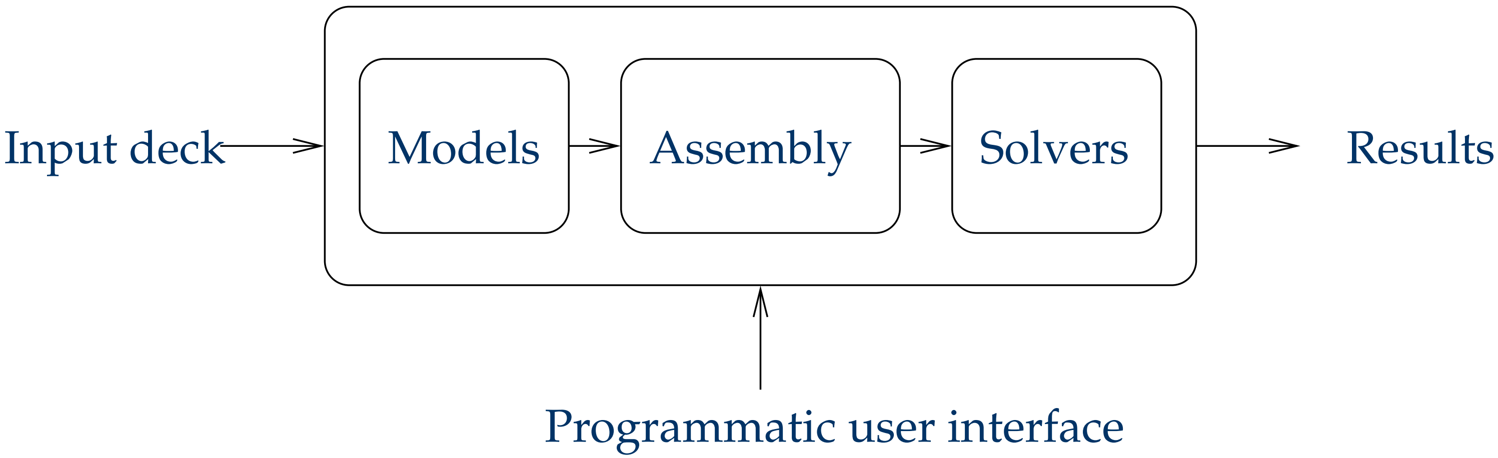
Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Software structure



- Element models (C++)
- Mesh management and system assembly (C++)
- Solvers
  - Standard low-level solver libraries (Fortran, C/C++)
  - High-level solvers provided by MATLAB
  - Specialized algorithms in MATLAB or C++
- Mesh description (Lua)
- User interfaces (MATLAB, Lua)

Software used

» Overview
Introduction
The Basics
» Software structure
» Software used
» “Hello world”
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up

“Lesser artists borrow. Great artists steal.”
– Picasso, Dali, Stravinsky?

- Lua: www.lua.org
  - Evolved from simulator data languages (DEL and SOL)
  - Pascal-like syntax fits on one page; complete language description is 21 pages
  - Fast, freely available, widely used in game design
- MATLAB: www.mathworks.com
  - “The Language of Technical Computing”
  - Good sparse matrix support
  - Star-P: http://www.interactivesupercomputing.com/
- Standard numerical libraries: ARPACK, UMFPACK

» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

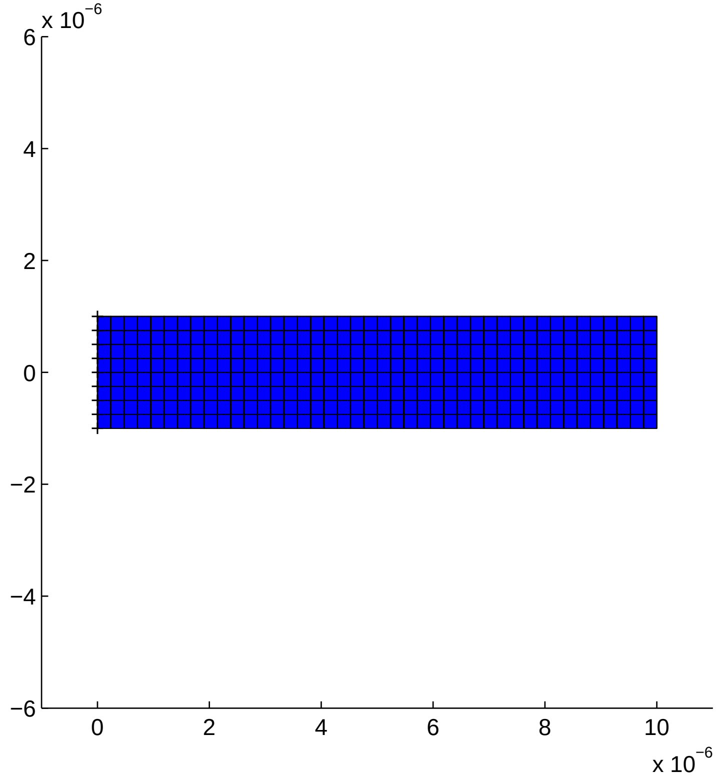
Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

"Hello world"



- Describe a simple cantilever
- Compute and plot the modes of vibration
- Perform a parameter study

» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Describing the mesh

```
require '../common.lua'

l = 10e-6              -- Beam length
w = 2e-6               -- Beam width
dense = 0.5e-6         -- Approximate element size
order = 2              -- Order of elements
nen   = 9              -- Number of element nodes
```

- Common header file defines materials, block generator
- Define symbols for geometry and meshing parameters
- "--" indicates the start of a comment

Describing the mesh

» Overview
Introduction
The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up

```
mesh  = Mesh:new(2,nen,2)
mat   = make_material('silicon2', 'planestrain',
                                 order)
blocks( { 0, l }, { -w/2.0, w/2.0 }, mat )
```

- Define a new mesh (which *must* be called mesh)
  - Number of dimensions = 2
  - Number of element nodes = nen
  - Number of unknowns per node = 2
- Define a polysilicon material in plane strain
- Mesh the region $[0, l] \times [-w/2, w/2]$
  - Element size h is determined by `dense`

Describing BCs

» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

```
mesh:set_bc(function(x,y)
  if x == 0 then return 'uu', 0, 0; end
end)
```

- Define boundary conditions with a function evaluated at every node
- Function returns
  - A string to specify displacement or force BCs
  - Values of boundary displacements or forces
- Examples:
  - Returns nothing – no boundary conditions
  - 'uu', 0, 0 – zero displacement in both x and y
  - ' u', 0 – zero displacement in y only
  - 'f', 1 – unit force in the x direction
  - 'uf', 0, 1 – zero x displacement, unit force in y

Starting in MATLAB

» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

```
>> run ../../init.m
HiQlab 0.1
Copyright    : Regents of the University of California
Build system: i686-pc-linux-gnu
Build date   : Wed Dec 15 12:50:48 PST 2004
Bug reports : dbindel@cs.berkeley.edu
```

- In MATLAB, you *must* run init first
- If you don't see the banner, there is a problem

» Overview

Introduction

The Basics
» Software structure
» Software used
» “Hello world”
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

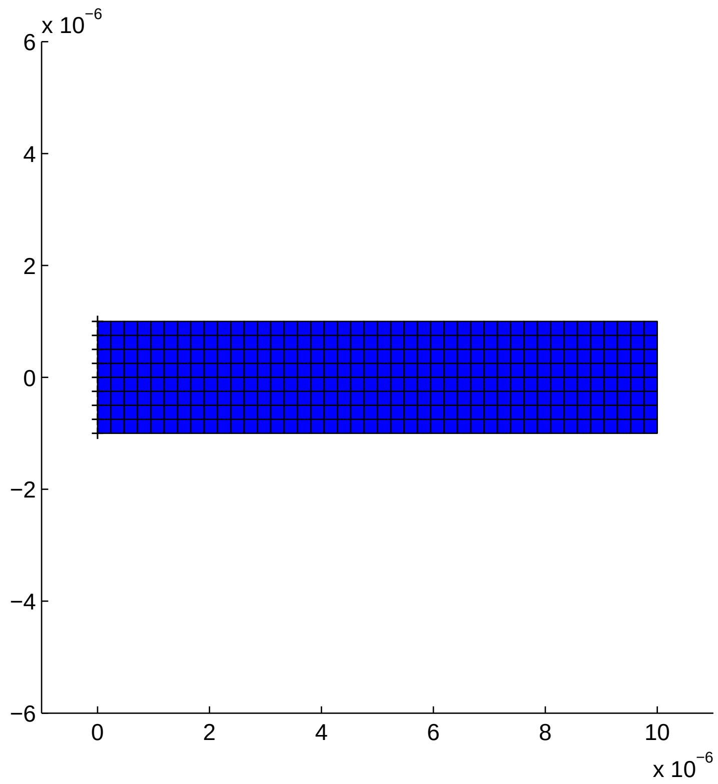
Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Loading and plotting




```
>> mesh = Mesh_load('beammesh.lua');
>> plotmesh(mesh); axis equal
```

Computing modes

» Overview

Introduction

The Basics
» Software structure
» Software used
» “Hello world”
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

$$(K - \omega^2 M)v = 0$$

```
>> [M,K] = Mesh_assemble_mk(mesh);
>> [V,D] = eigs(K,M, 5, 'sm');
>> w = sqrt(diag(D))/2/pi;

w =
   1.0e+08 *
   0.2791 + 0.0000i
   1.4947 - 0.0000i
   2.2295 - 0.0000i
   3.5475 - 0.0000i
   5.8811 - 0.0000i
```

Plotting modes

» Overview

Introduction

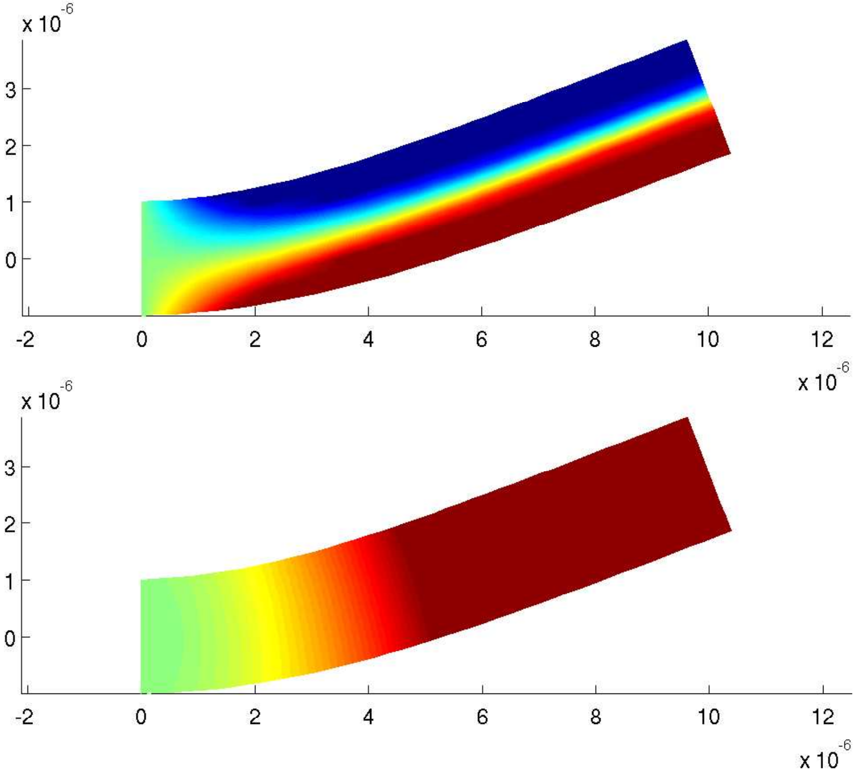
The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up




```
>> opt.axequal = 1;
>> opt.deform = 1;
>> Mesh_scale_u(mesh, V(:,1), 2, 1e-6);
>> plotfield2d(mesh, opt)
```

Parameter sweep

» Overview

Introduction

The Basics
» Software structure
» Software used
» “Hello world”
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Change one line in Lua file:

```
l = l or 10e-6    -- Beam length
```

Now set `l` from MATLAB:

```
l = linspace(10e-6, 20e-6, 11);
for k = 1:length(l)
  param.l = l(k);
  mesh  = Mesh_load('beammesh2.lua', param);
  [M,K] = Mesh_assemble_mk(mesh);
  w(k)  = sqrt(eigs(K,M,1, 'sm'))/2/pi;
  Mesh_delete(mesh);
end
plot(l,real(w)); axis tight
```

Parameter sweep

» Overview

Introduction

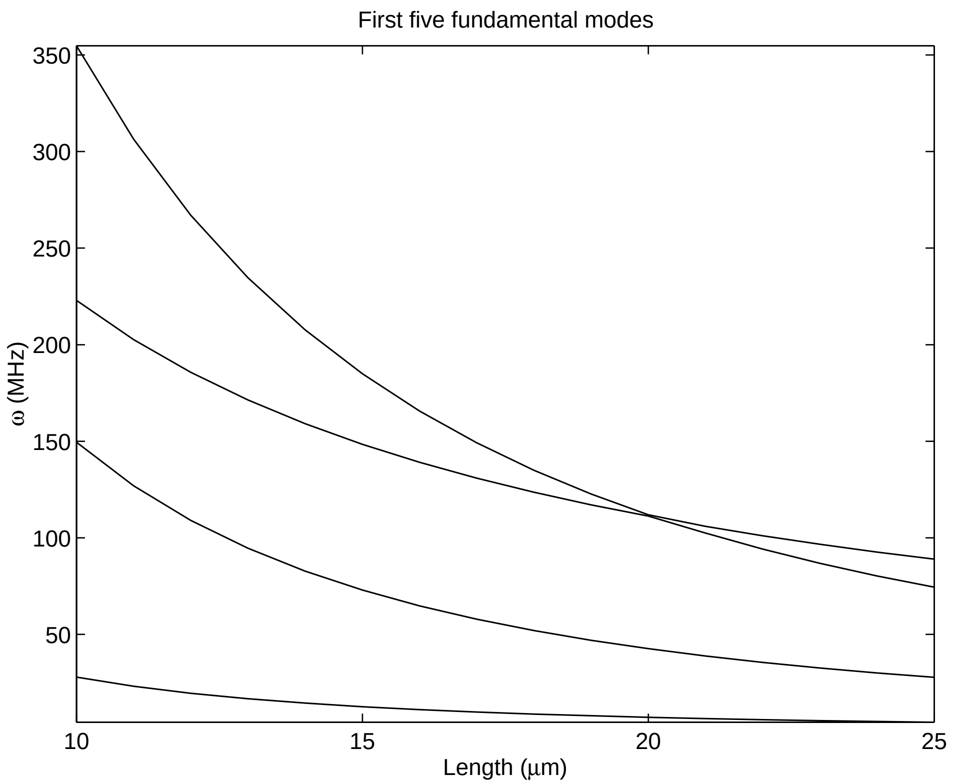
The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up



- Change two lines to plot *several* frequencies
- Notice crossing behavior of third and fourth modes

Running from Lua

» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

```
[tutorial]: hiqlab driver.lua
1       :       27.908006867444 MHz
2       :       149.46698798698 MHz
3       :       222.95156567429 MHz
4       :       354.75135363746 MHz
5       :       588.11440349498 MHz
```

- Can run simulations in standalone code
- Missing some capabilities (esp graphics)
- Less overhead than MATLAB interface

Lua driver file

```
beamf = loadfile 'beammesh.lua'

w0  = 0   -- Frequency estimate
nev = 5   -- Number of eigs
ncv = 10  -- Size of space (~2 nev)
dr  = {}  -- Real parts of eigs
di  = {}  -- Imag parts of eigs

-- Load mesh and compute eigs
beamf()
mesh:initialize()
compute_eigs(mesh, w0, nev, ncv, dr, di);

-- Print eigs
for k = 1,5 do
  print(k, ':', dr[k]/2e6/pi, 'MHz')
end
```


» Overview

Introduction

The Basics
» Software structure
» Software used
» "Hello world"
» Describing the mesh
» Describing the mesh
» Describing BCs
» Starting in MATLAB
» Loading and plotting
» Computing modes
» Plotting modes
» Parameter sweep
» Parameter sweep
» Running from Lua
» Lua driver file

Anchor loss

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Disk resonator

» Overview

Introduction

The Basics

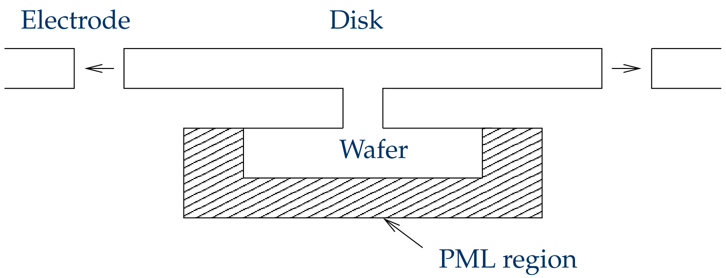
Anchor loss

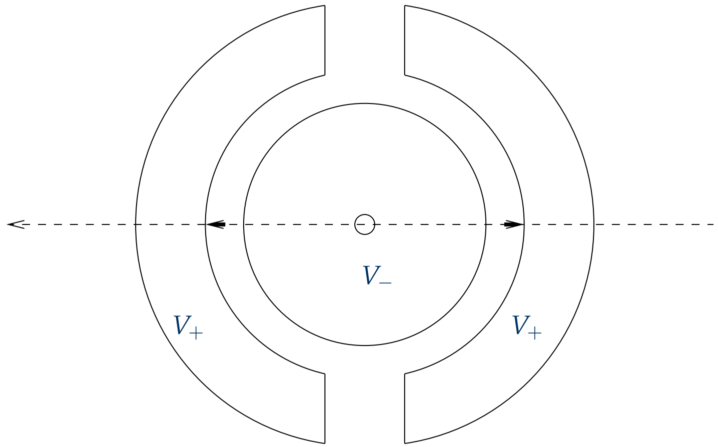
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

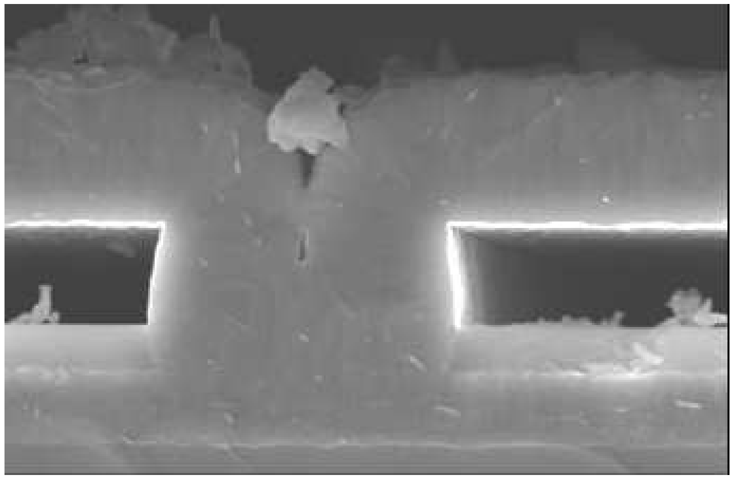
Thermoelastic damping

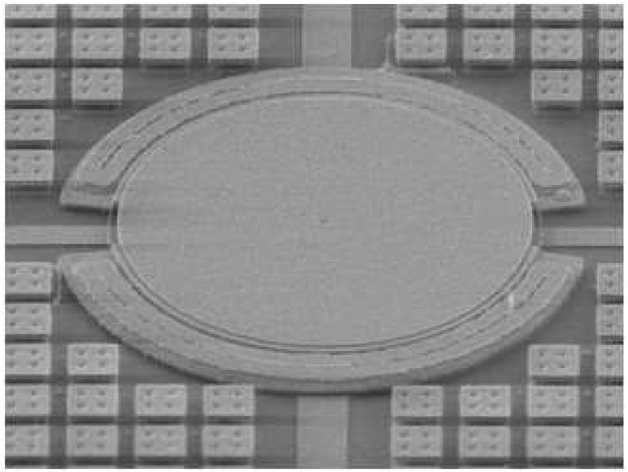
Checkerboard and ROM

Wrap-up









- SiGe disk resonators built by E. Quévy
- Axisymmetric model with bicubic mesh, about 10K nodal points

» Overview

Introduction

The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Substrate model

Goal: Understand energy loss in this resonator

- Dominant loss is elastic radiation from anchor.
- Disk resonator is much smaller than substrate
- Very little energy leaving the post is reflected back
  - Substrate is semi-infinite from disk's perspective
- Possible semi-infinite models
  - Matched asymptotic modes
  - Dirichlet-to-Neumann maps
  - Boundary dampers
  - Perfectly matched layers

Perfectly matched layers

» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up

- Apply a complex coordinate transformation
- Generates a non-physical absorbing layer
- No impedance mismatch between the computational domain and the absorbing layer
- Idea works with general linear wave equations
  - First applied to Maxwell's equations (Berengér 95)
  - Similar idea introduced earlier in quantum mechanics (*exterior complex scaling*, Simon 79)

Scalar wave example

» Overview

Introduction

The Basics

Anchor loss

» Disk resonator

» Substrate model

» Perfectly matched layers

» Scalar wave example

» Defining the PML

» Scalar wave in HiQLab

» Scalar wave BCs

» MATLAB driver

» MATLAB driver

» Disk mesh

» Disk mesh

» Forced response

» Time-averaged energy flux

» Damped modes

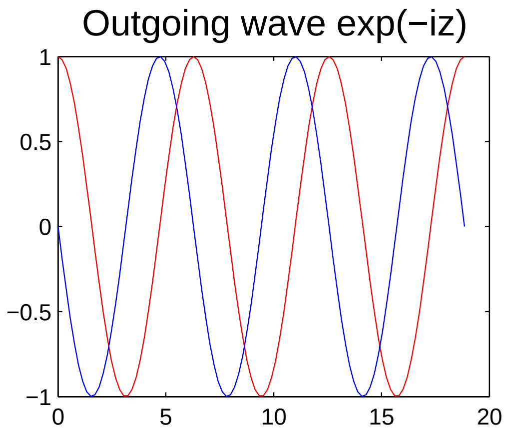
» Q variation

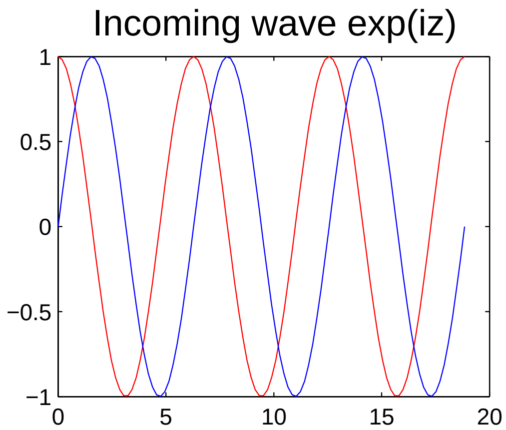
» Truth in advertising

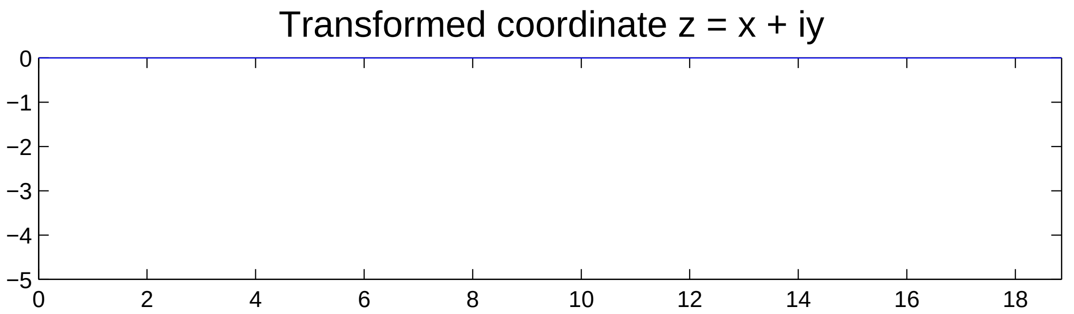
Thermoelastic damping

Checkerboard and ROM

Wrap-up







Scalar wave example

» Overview

Introduction

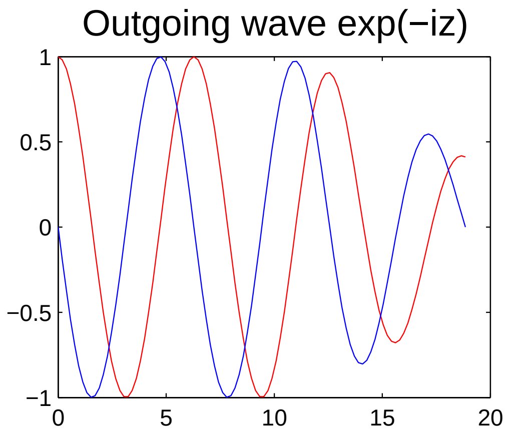
The Basics

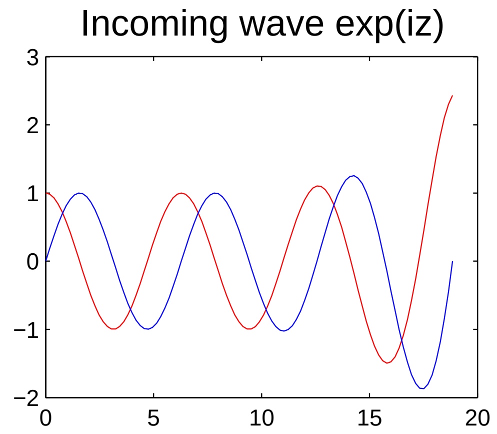
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

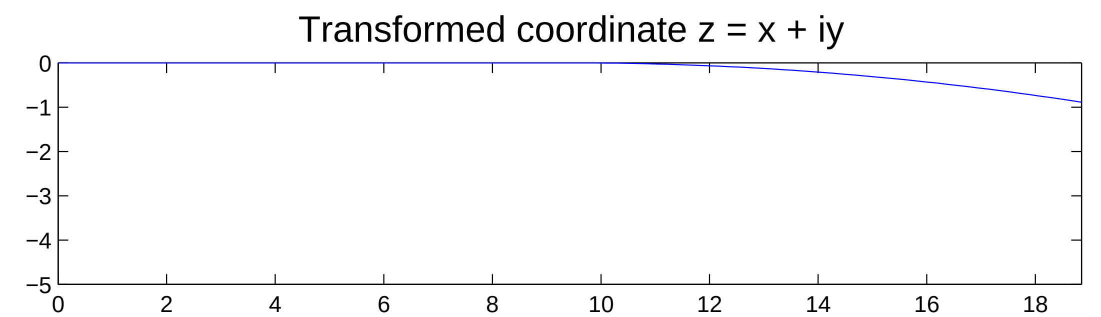
Thermoelastic damping

Checkerboard and ROM

Wrap-up







Scalar wave example

» Overview

Introduction

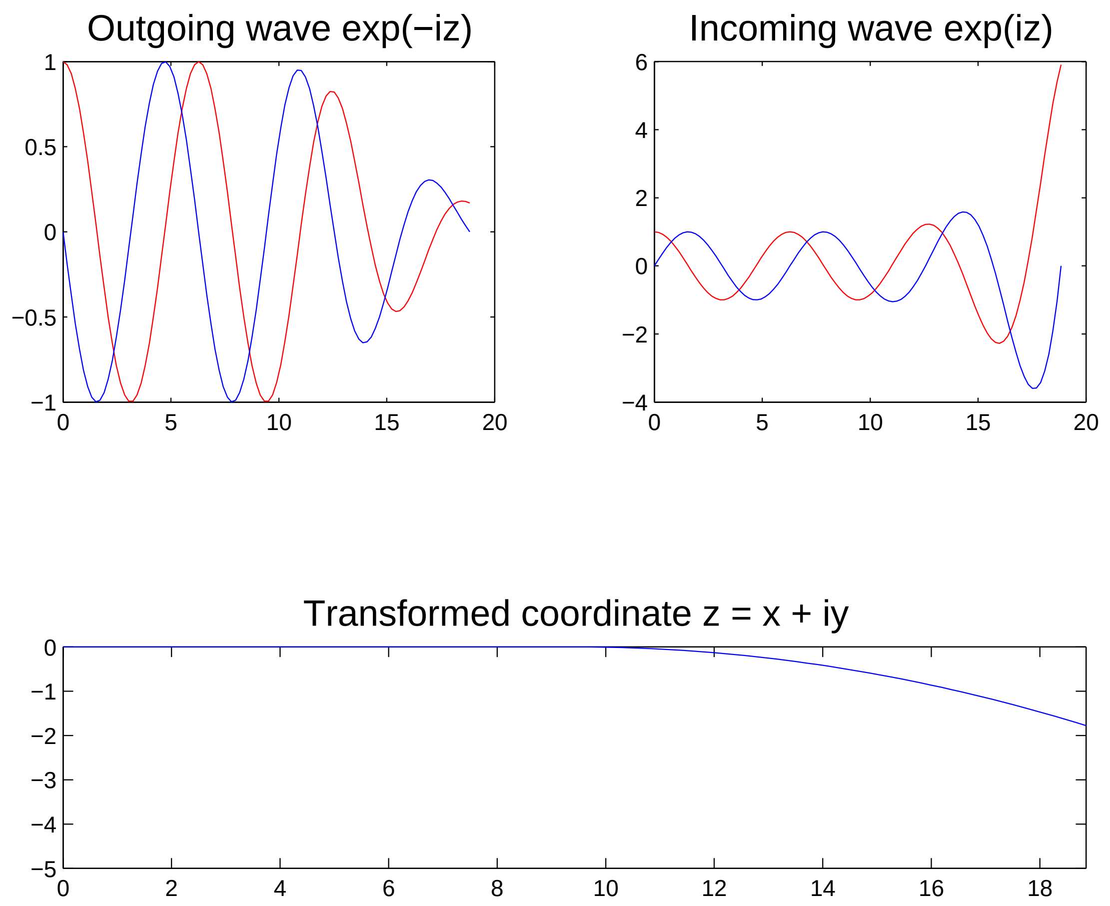
The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up



Scalar wave example

» Overview

Introduction

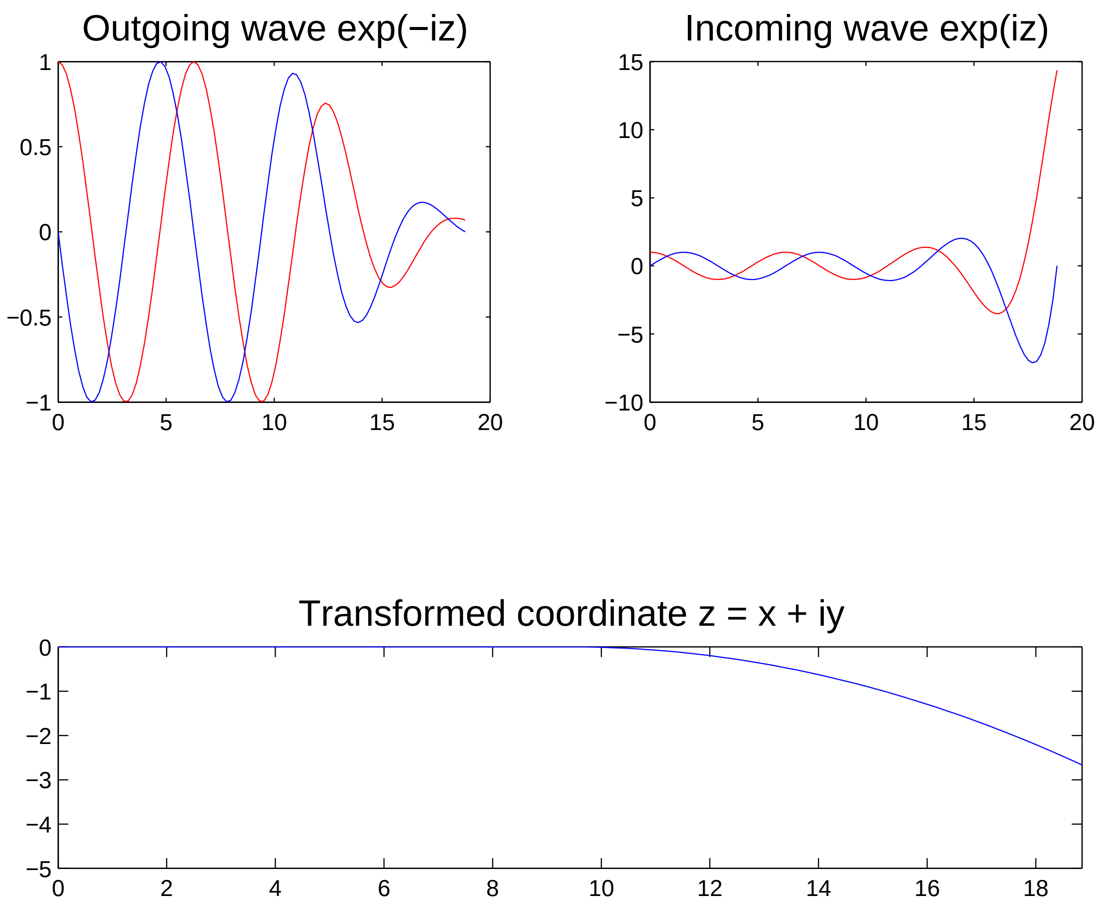
The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

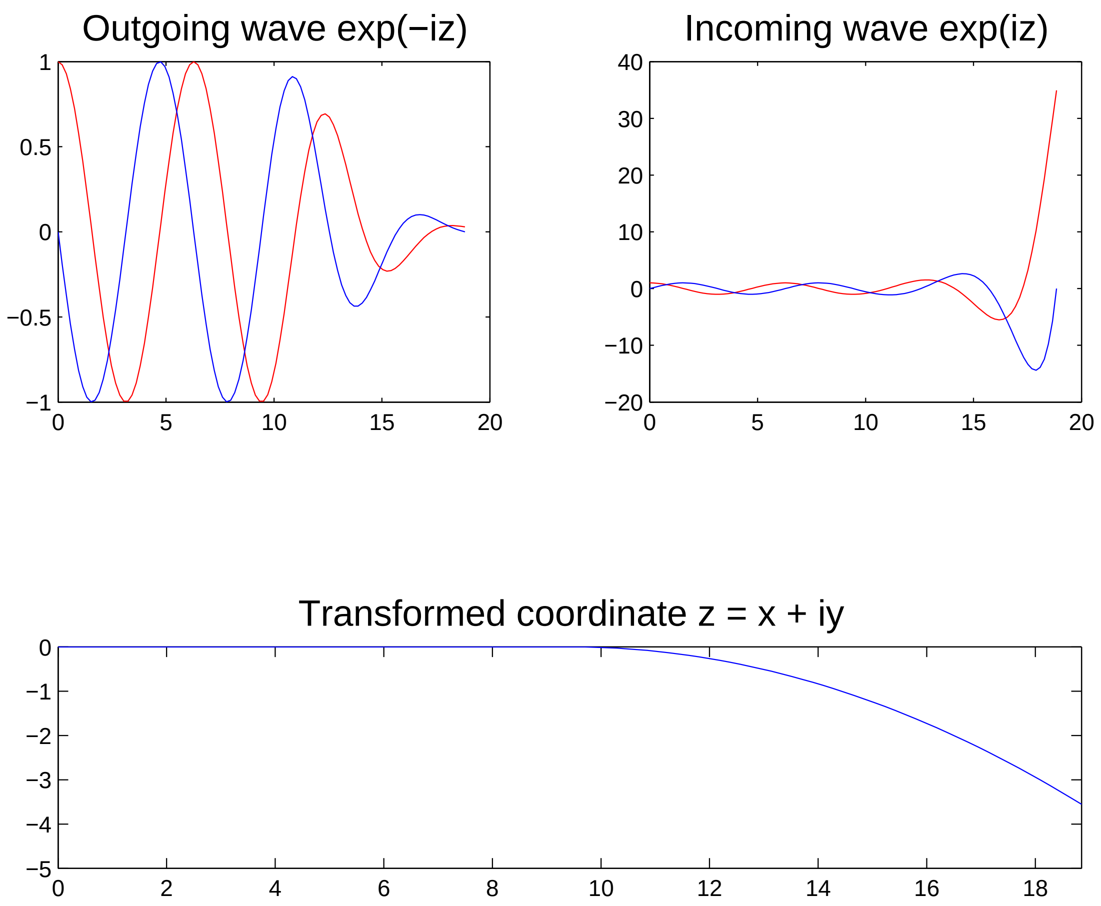
Checkerboard and ROM

Wrap-up



» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up

Scalar wave example



Scalar wave example

» Overview

Introduction

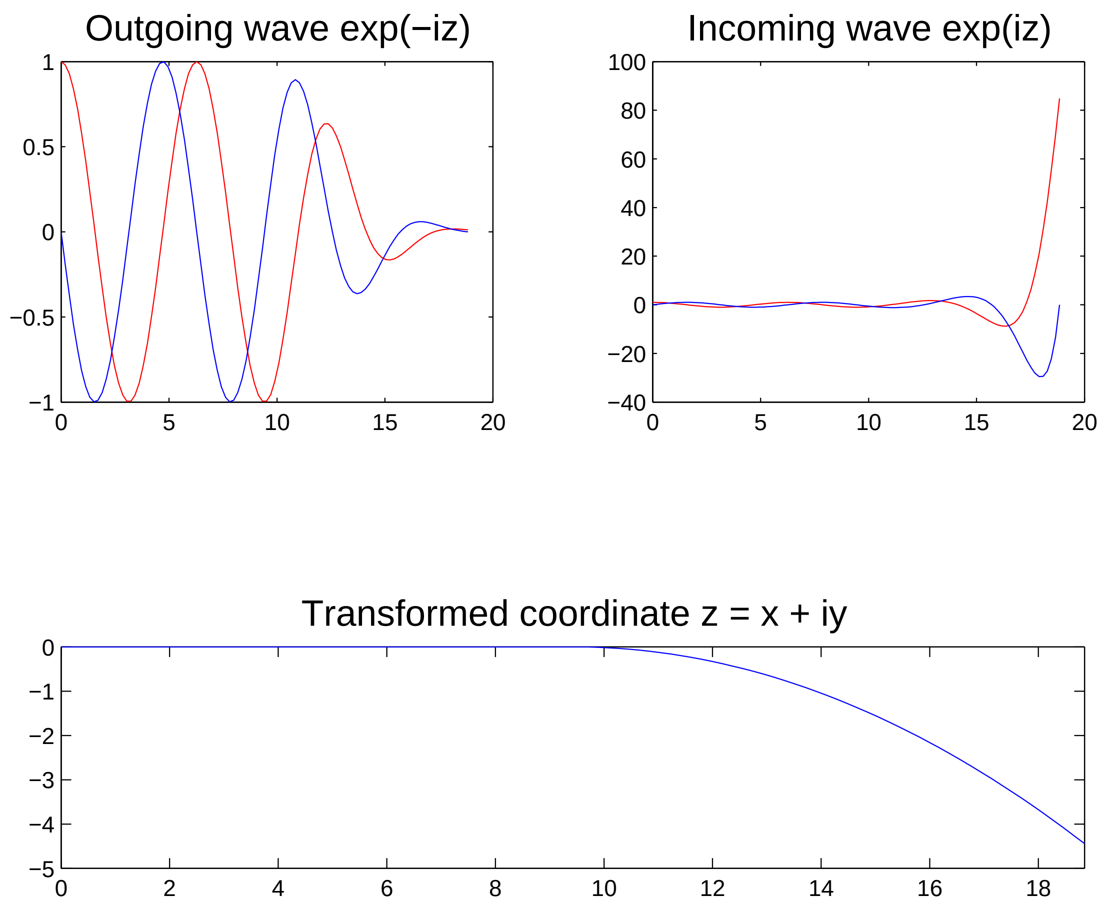
The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up



Clamp solution at transformed end to isolate outgoing wave.

Defining the PML

» Overview

Introduction

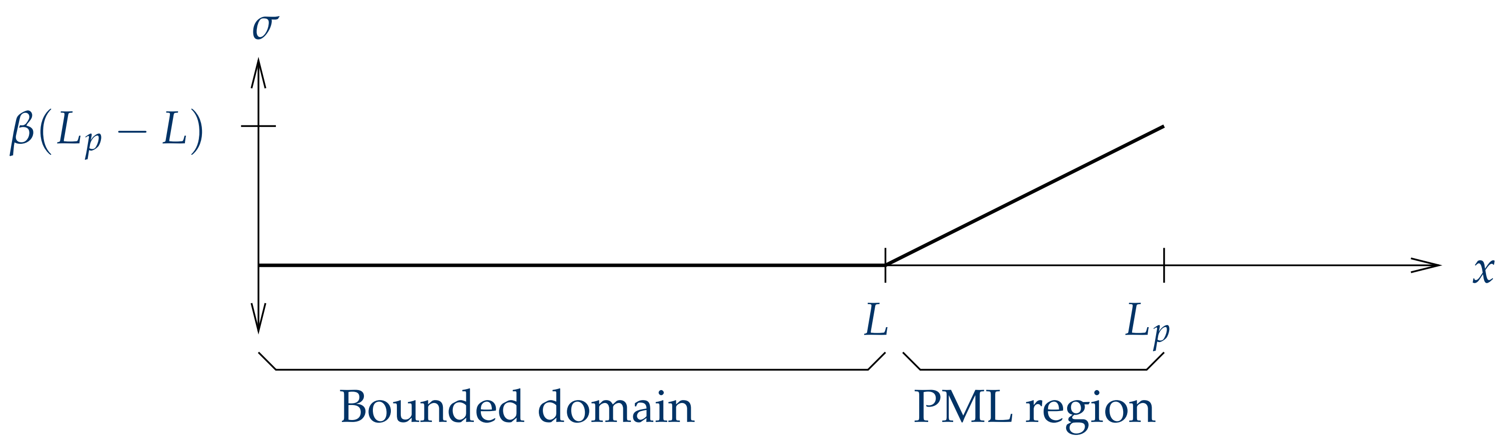
The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up



- Only need $dz/dx = 1 - i\sigma(x)$
- Usually choose σ to be piecewise linear
  - $\sigma = 0$ on ordinary domain
  - $\sigma > 0$ in PML region
- In higher dim, transform each x_i independently

Scalar wave in HiQLab

» Overview

Introduction

The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up


```
require '../common.lua'

Ne1   = Ne1   or 10  -- Elements in first region
Ne2   = Ne2   or 10  -- Elements in second region
order = order or 3   -- Polynomial order
nen   = order+1;     -- Number of element nodes

mesh = Mesh:new(1,nen,1);
D    = mesh:own( PMLScalar1d:new(1, 1, nen) );
mesh:add_block(-Ne1, Ne2, order*(Ne1+Ne2)+1,
               D, order);
```

- Mesh $[-N_1, N_2]$ with cubic elements
- Define a material for a scalar wave with PML

Scalar wave BCs

» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up

```
epw   = epw   or 10  -- Elements per wave
dpw   = dpw   or 1   -- Damping per wave

D:set_stretch(function(x)
   return max(x*dpw/epw,0)
end)

mesh:set_bc(function(x)
   if x == -Ne1 then return 'u', 1 end
end)
```

- $\sigma(x) = 0$ on $[-N_1, 0]$
- $\sigma(x)$ varies linearly from 0 to 1 on $[0, N_2]$
  - σ_{max} small only for demonstration
  - A more reasonable σ_{max} would be $10 - 40$
- Displacement BCs at left end generate waves

MATLAB driver

» Overview

Introduction

The Basics

Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

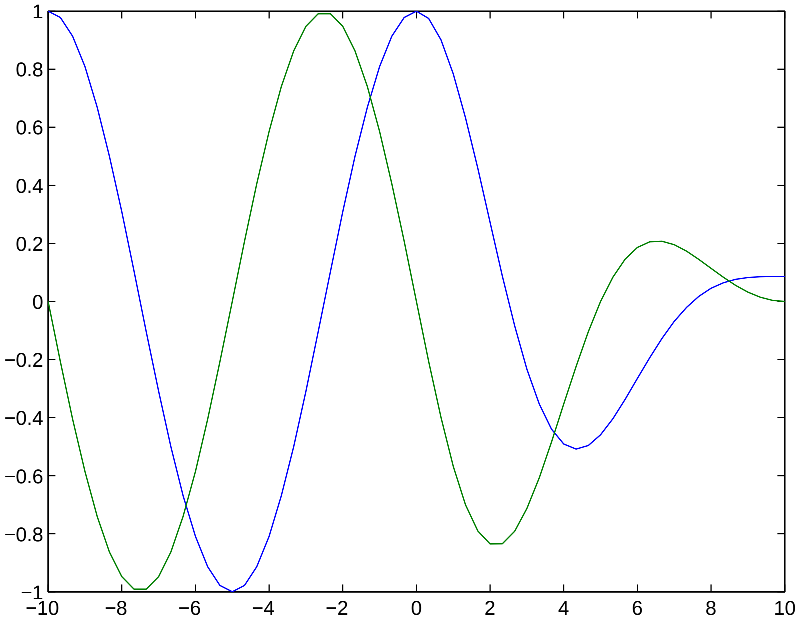
Wrap-up

```
mesh  = Mesh_load('pml1d.lua');
k = 2*pi/10;
Mesh_make_harmonic(mesh, k);
[M,K] = Mesh_assemble_mk(mesh);
F     = Mesh_assemble_R(mesh);
u     = -(K-k^2*M)\F;
Mesh_set_u(mesh, u);
```

- `Mesh_make_harmonic` sets $v = i\omega u$ and $a = -\omega^2 u$
- F is the forcing vector from BCs
- u is the time-harmonic response

MATLAB driver

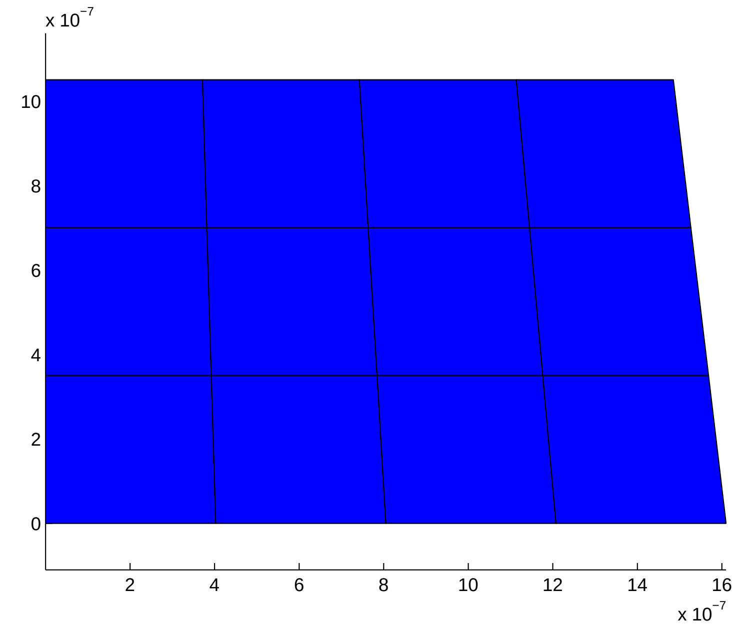
» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up




```
x = Mesh_get_x(mesh);
u = Mesh_get_disp(mesh);
plot(x, real(u), x, imag(u));
Mesh_delete(mesh);
```

» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up

Disk mesh




```
mesh:add_block_shape(npostx, ndisky, pelt, order,
  {0,       0,   0,      hpost,
   rpost2, 0,   rpost, hpost})
```

Mesh of mapped blocks. Arguments specify block size, type of element, and (x, y) coordinates of each corner.

» Overview

Introduction

The Basics

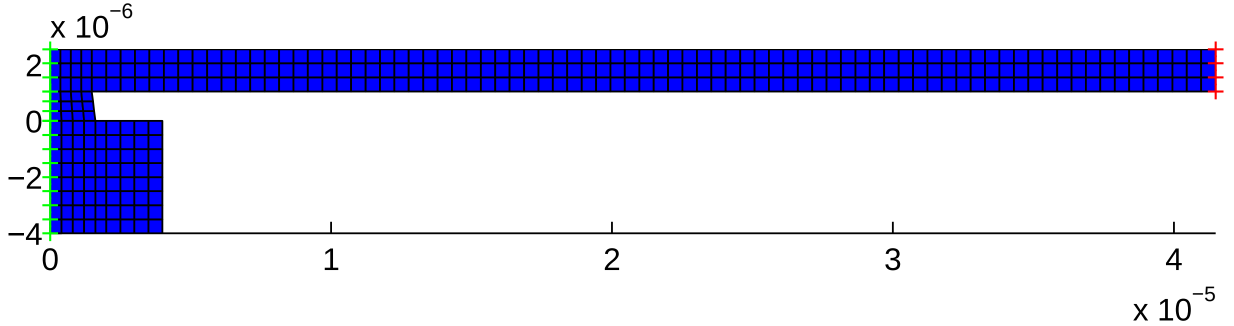
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up

Disk mesh

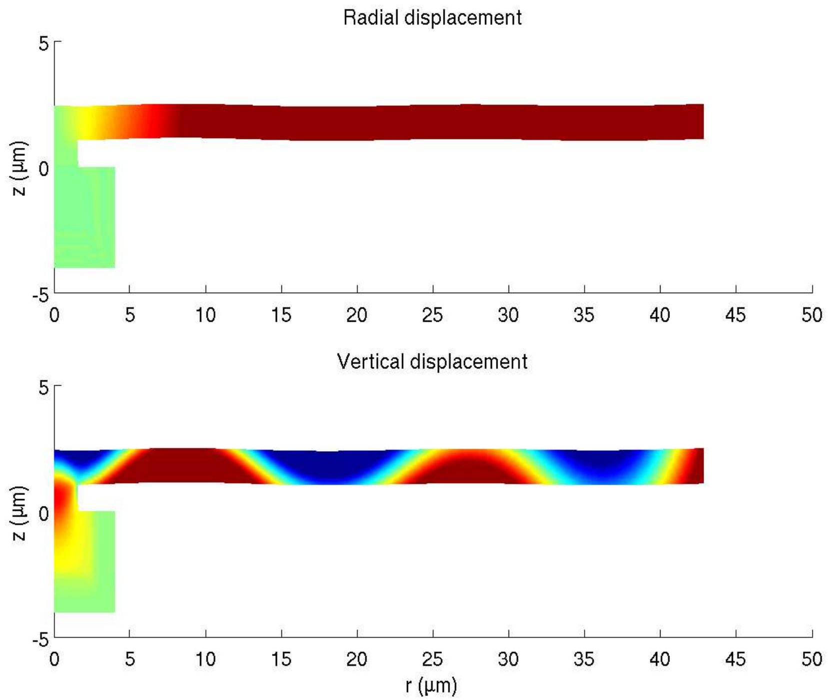



```
mesh:tie(dense/100)
```

Mapped blocks are tied together to form disk mesh.
Usually use higher mesh density than shown.

Forced response

» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up




```
mesh  = Mesh_load('diskmesh.lua', param);
[M,K] = Mesh_assemble_mk(mesh);
F     = Mesh_assemble_R(mesh);
u     = -(K - wforce^2*M) \ F;
Mesh_scale_u(mesh, u, 1, 1e-6);
plotcycle2d(mesh,1,plotopt);
```

Time-averaged energy flux

» Overview

Introduction

The Basics

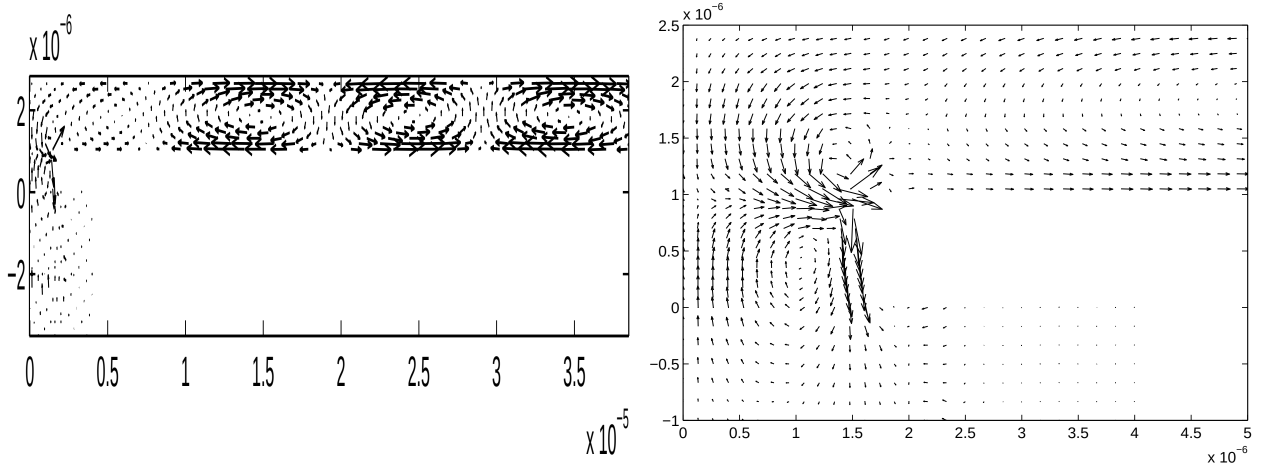
Anchor loss

» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up

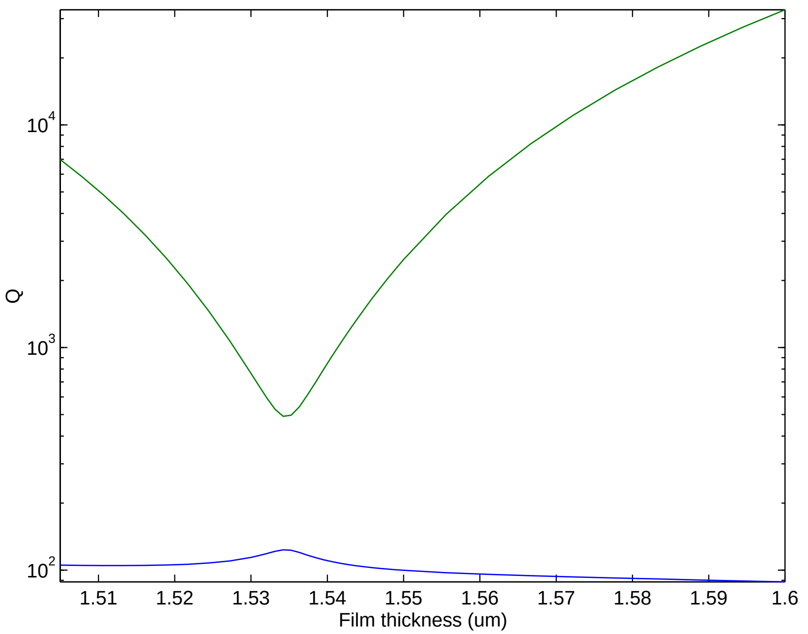



```
Mesh_make_harmonic(mesh, wforce);
p = Mesh_get_x(mesh);
E = Mesh_mean_power(mesh);
quiver(p(1,:), p(2,:), E(1,:), E(2,:));
```

- Compute $\bar{E} = T^{-1} \int_0^T \sigma(t) \cdot v(t)\, dt$
- Energy flows are strongest at surfaces
- Stress singularity at corner sprays energy

Damped modes

» Overview
Introduction
The Basics
Anchor loss
» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising
Thermoelastic damping
Checkerboard and ROM
Wrap-up



In a loop, compute

```
param.hdisk = t(k);
mesh = Mesh_load('diskmesh.lua', param);
[M,K] = Mesh_assemble_mk(mesh);
[VV,w,Q] = pml_mode(M,K,w0,2);
Mesh_delete(mesh);
```

Q variation

» Overview

Introduction

The Basics

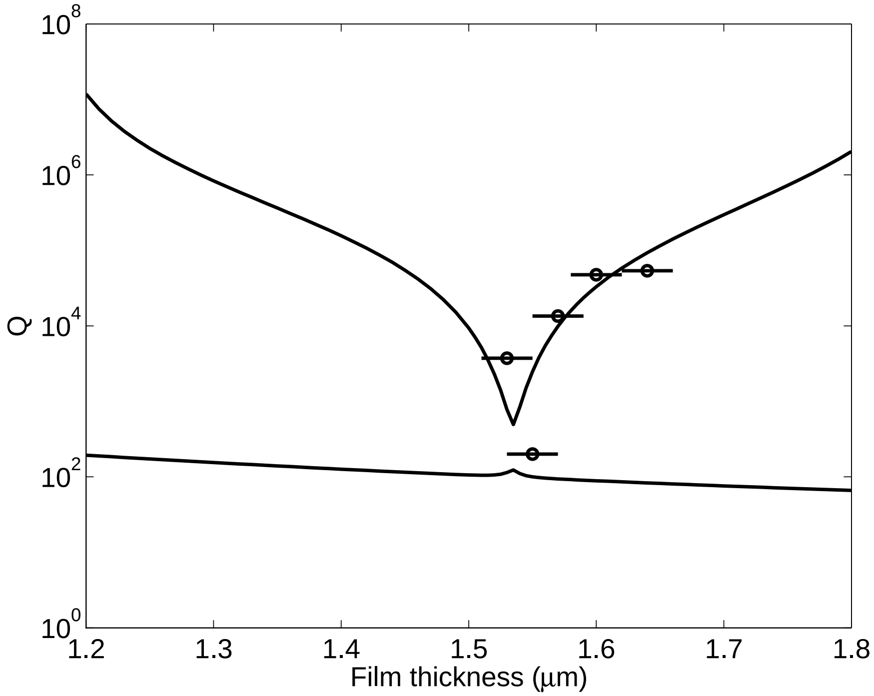
Anchor loss

» Disk resonator
» Substrate model
» Perfectly matched layers
» Scalar wave example
» Defining the PML
» Scalar wave in HiQLab
» Scalar wave BCs
» MATLAB driver
» MATLAB driver
» Disk mesh
» Disk mesh
» Forced response
» Time-averaged energy flux
» Damped modes
» Q variation
» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up



- Surprising variation in Q as film thickness changes
- Confirmed by experiment on a set of 40 μm disks
- Effect comes from interaction of radial and bending modes

Truth in advertising

» Overview

Introduction

The Basics

Anchor loss

» Disk resonator

» Substrate model

» Perfectly matched layers

» Scalar wave example

» Defining the PML

» Scalar wave in HiQLab

» Scalar wave BCs

» MATLAB driver

» MATLAB driver

» Disk mesh

» Disk mesh

» Forced response

» Time-averaged energy flux

» Damped modes

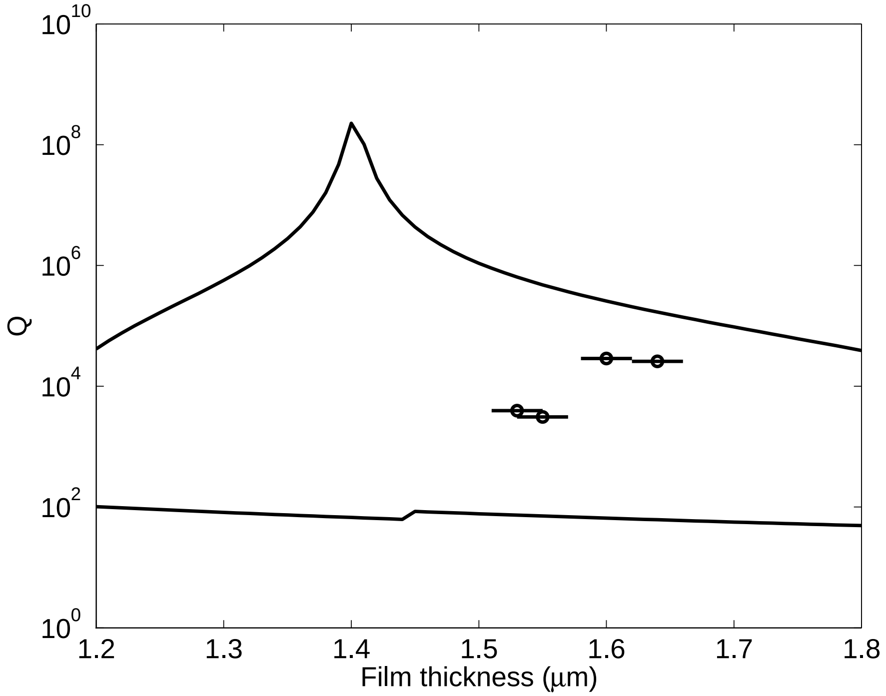
» Q variation

» Truth in advertising

Thermoelastic damping

Checkerboard and ROM

Wrap-up



Data from a set of $30\mu m$ radius disks.

Thermoelastic damping (TED)

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
» Thermoelastic damping (TED)
» Nondimensionalization
» Scaling analysis
» Clamped SCS beam
» Clamped SCS vibration
Checkerboard and ROM
Wrap-up

u is displacement and $T = T_0 + \theta$ is temperature

$$\begin{aligned} \sigma &= C\epsilon - \beta\theta 1 \\ \rho u_{tt} &= \nabla \cdot \sigma \\ \rho c_v \theta_t &= \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \, \mathbf{tr}(\epsilon_t) \end{aligned}$$

- Volumetric strain rate drives energy transfer from mechanical to thermal domain
  - Irreversible diffusion $\implies$ mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators
- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
» Thermoelastic damping (TED)
» Nondimensionalization
» Scaling analysis
» Clamped SCS beam
» Clamped SCS vibration
Checkerboard and ROM
Wrap-up

Nondimensionalization

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ u_{tt} &= \nabla \cdot \sigma \\ \theta_t &= \eta \nabla^2 \theta - \operatorname{tr}(\epsilon_t) \end{aligned}$$

$$\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}$$

$$\begin{aligned} \text{Length} &\sim L \\ \text{Time} &\sim L/c, \text{ where } c = \sqrt{E/\rho} \\ \text{Temperature} &\sim T_0 \frac{\beta}{\rho c_v} \end{aligned}$$

Scaling analysis

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
» Thermoelastic damping (TED)
» Nondimensionalization
» Scaling analysis
» Clamped SCS beam
» Clamped SCS vibration
Checkerboard and ROM
Wrap-up

$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
u_{tt} &= \nabla \cdot \sigma \\
\theta_t &= \eta \nabla^2 \theta - \mathrm{tr}(\epsilon_t)
\end{aligned}
$$

$$
\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}
$$

- Micron-scale poly-Si devices: ξ and η are $\sim 10^{-4}$.
- Small η leads to thermal boundary layers
- Can linearize about $\xi = 0$ for a fast solver
- *Must* note scaling behavior to get accurate eigenvalues

Clamped SCS beam

» Overview

Introduction

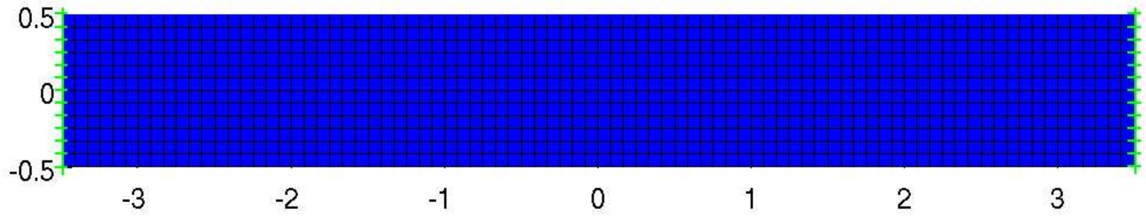
The Basics

Anchor loss

Thermoelastic damping
» Thermoelastic damping (TED)
» Nondimensionalization
» Scaling analysis
» Clamped SCS beam
» Clamped SCS vibration

Checkerboard and ROM

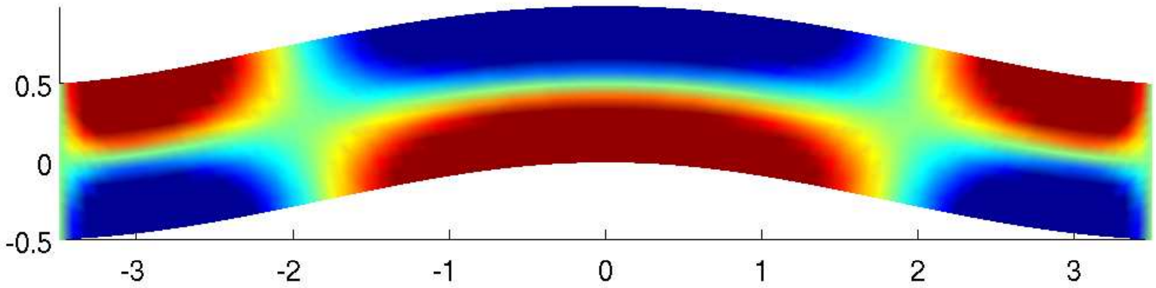
Wrap-up



- Mesh similar to before, but clamped at both ends
- Use full thermoelastic element (single-crystal Si)
- Given length scale and material properties, automatically compute other characteristic scales
- Compute with dimensionless model

Clamped SCS vibration

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
» Thermoelastic damping (TED)
» Nondimensionalization
» Scaling analysis
» Clamped SCS beam
» Clamped SCS vibration
Checkerboard and ROM
Wrap-up



$Q = 8700$ at 87.7 MHz

```
tedopt.type = 'pert';
tedopt.cT   = cT;
[V,w,Q]     = tedmode(mesh,w0,1,tedopt);
```

- Compute thermoelastic modes using `tedmode`
- Two methods: full solve and perturbation-based
  - Full solve (default) is more expensive
  - Perturbation method only works for ξ small

Checkerboard resonator

» Overview

Introduction

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

» Checkerboard resonator

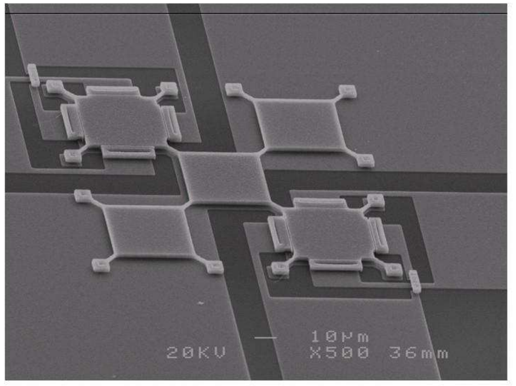
» Checkerboard mesh

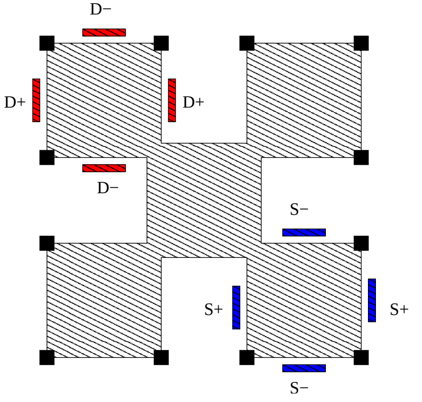
» Model reduction

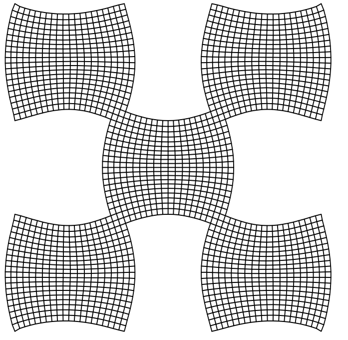
» Bode plot

» Interactive visualization

Wrap-up







- Array of loosely coupled resonators
- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Checkerboard mesh

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
» Checkerboard resonator
» Checkerboard mesh
» Model reduction
» Bode plot
» Interactive visualization
Wrap-up

- A more complex tied mesh example
- Hierarchical: “checker” and “beam” functions
- Can vary
  - Dimensions
  - Number of checkers
  - Linking strategy (corner-coupled or beam-coupled)
  - Drive and sense positions
- Define functions for drive and sense
  - Lua functions look like those passed to `set_bc`

```
function drive_checker(x,y)
  ... return string and values ...
end
```

  - MATLAB functions construct corresponding vectors

```
L = Mesh_get_sense_u(mesh, 'sense_checker');
B = Mesh_get_sense_f(mesh, 'drive_checker');
```

Model reduction

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
» Checkerboard resonator
» Checkerboard mesh
» Model reduction
» Bode plot
» Interactive visualization
Wrap-up

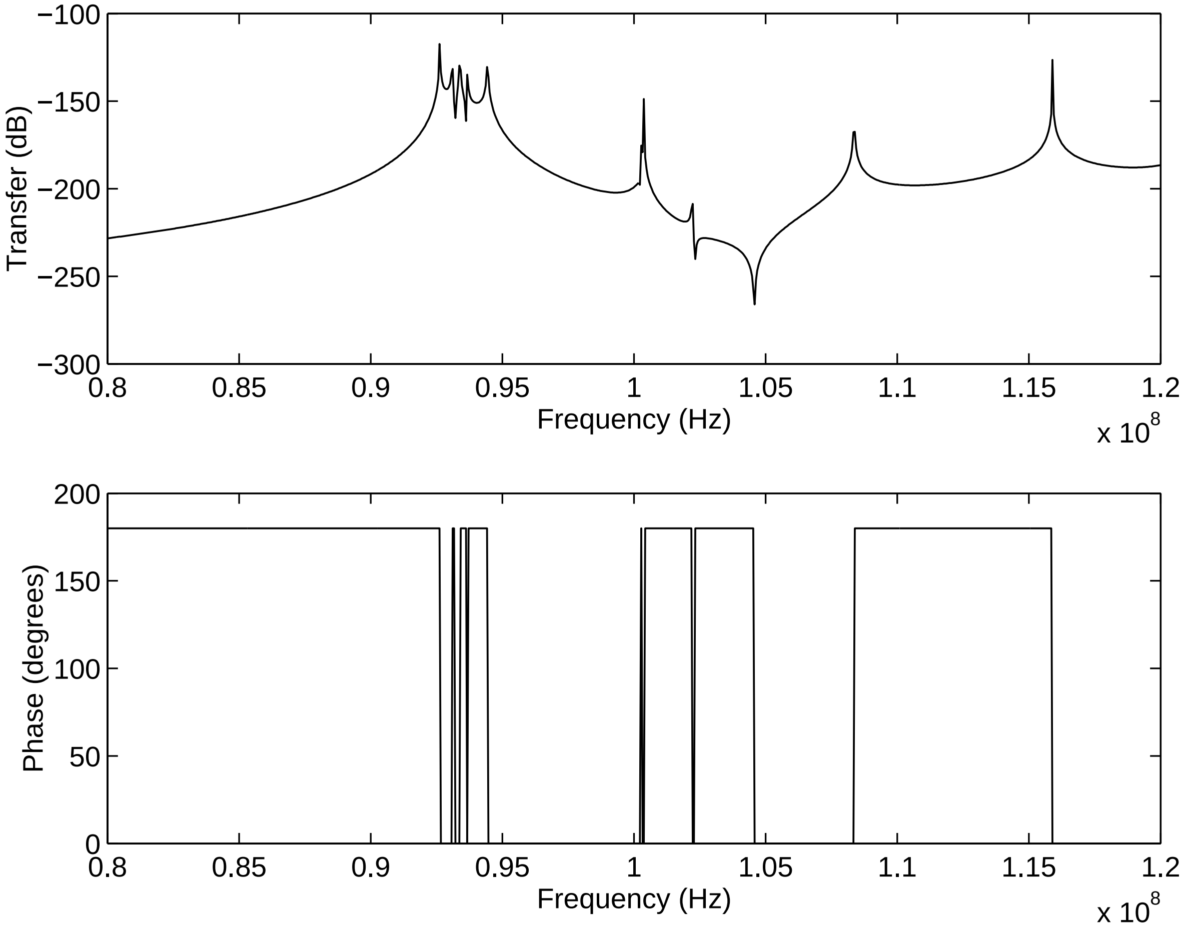
Approximate frequency response near ω_0 with a k-dimensional reduced model.

$$H(\omega) = L^T(K - \omega^2 M)B \approx L_k^T(K_k - \omega^2 M_k)B_k$$

```
freq = linspace(wmin, wmax, n);
[Mk,Kk,Lk,Bk,Vk] = rom_arnoldi(M,K,L,B, k, w0);
H = compute_bode_mech(freq, Mk,Kk,Lk,Bk);
plot_bode(freq, H);
```

Bode plot

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
» Checkerboard resonator
» Checkerboard mesh
» Model reduction
» Bode plot
» Interactive visualization
Wrap-up



Interactive visualization

» Overview

Introduction

The Basics

Anchor loss

Thermoelastic damping

Checkerboard and ROM

» Checkerboard resonator

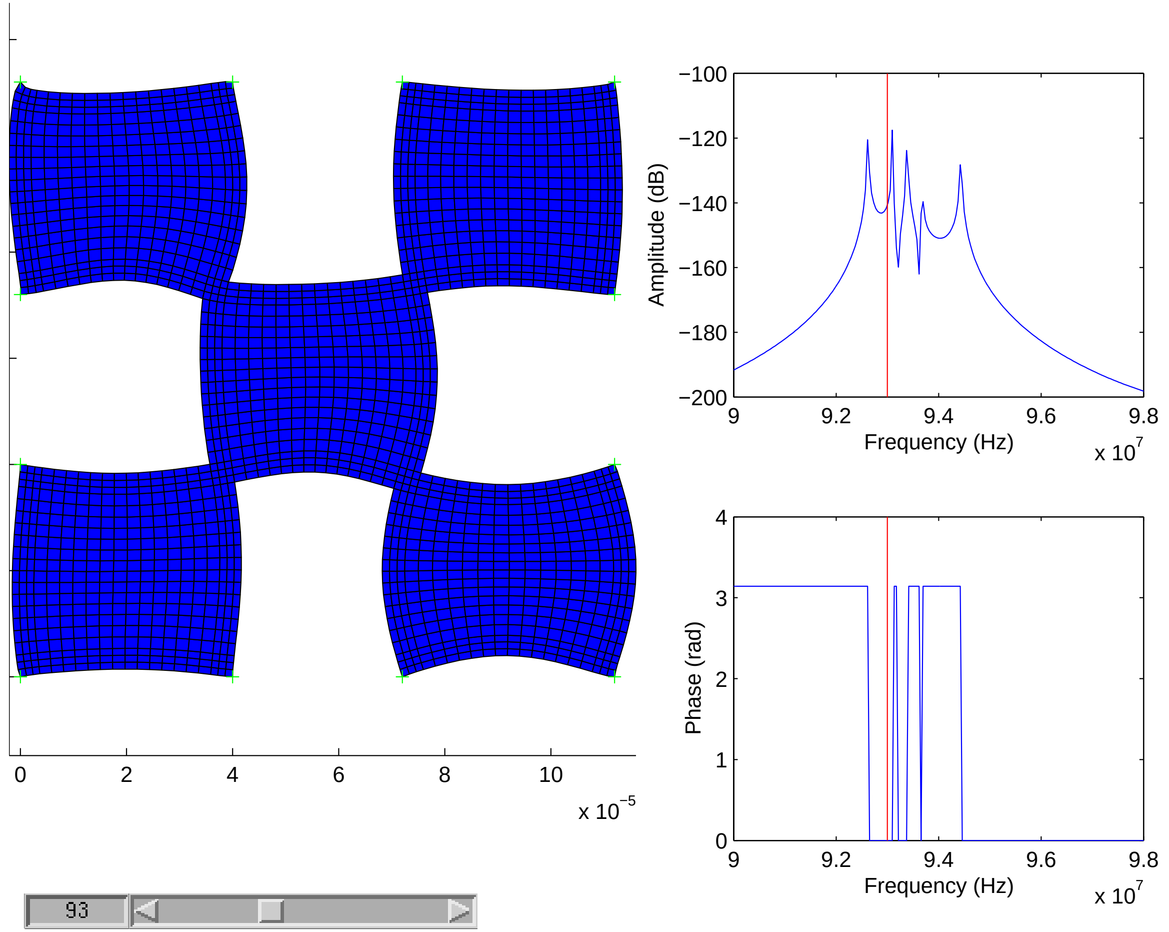
» Checkerboard mesh

» Model reduction

» Bode plot

» Interactive visualization

Wrap-up



» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up
» Current status
» Planned work
» Conclusions

Current status

- Alpha version available with MATLAB and Lua front-ends
  - Already allows some interesting analyses
  - Ongoing verification work (E. Quévy)
- Pre/post-processing:
  - Language for parameterized mesh description
  - Mainly use tied meshes of mapped blocks
  - MATLAB graphics, some ability to export to OpenDX
- Elements:
  - Isotropic and anisotropic scalar, elastic, TED elements
  - Linear, quadratic, and cubic shapes supported
  - All elements allow PML mapping
- Analyses:
  - Modal analysis: standard, PML, and TED
  - Forced response and energy flux computation
  - Model reduction and fast Bode plots
  - Parameter sweeps on any of the above

Planned work

- Short term
  - 3D calculations
  - Intrinsic material loss models
  - Piezo models?
  - Studies of grain boundary effects in TED
  - Continued comparisons to disk resonators
  - Comparisons to flextural resonator data
  - User documentation
  - Finding a new name!
- Long term
  - Computer optimization of bandpass filter designs
  - Continued simulation/verification of actual devices

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up
» Current status
» Planned work
» Conclusions

Conclusions

- HiQLab page: www.cs.berkeley.edu/∼dbindel/hiqlab
- Questions or comments?

» Overview
Introduction
The Basics
Anchor loss
Thermoelastic damping
Checkerboard and ROM
Wrap-up
» Current status
» Planned work
» Conclusions