

# Reduced Order Models in Microsystems and RF MEMS

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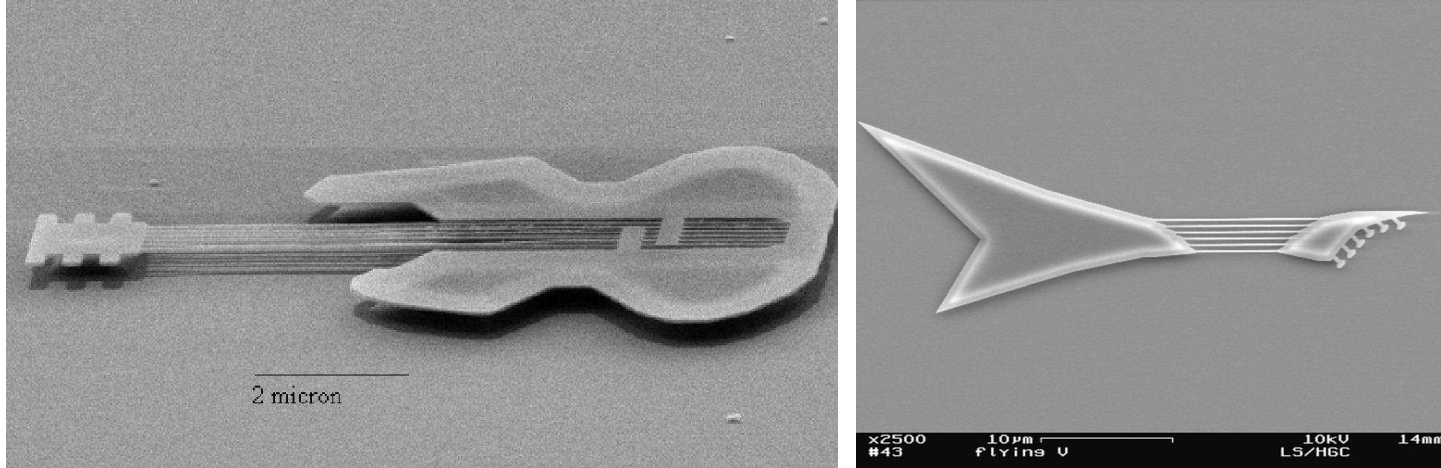
# Collaborators

Faculty	Grad students
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# MEMS Basics

- Micro-electro-mechanical systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - RF devices
- Use IC fabrication technology
- Large surface area / volume ratio
- Still mostly classical (vs. nanosystems)

# RF MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Impact: smaller, lower-power cell phones
  - Replace quartz freq references, filter elements
  - Integrate into CMOS stack
- Other uses:
  - Sensing elements
  - Really high-pitch guitars

# Second-order system equations

Time domain mechanical equations with damping:

$$\begin{aligned}Mu'' + Cu' + Ku &= P\phi \\ y &= V^T u\end{aligned}$$

Linearized:

$$\begin{aligned}\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} u' \\ u \end{bmatrix}' + \begin{bmatrix} C & K \\ -I & 0 \end{bmatrix} \begin{bmatrix} u' \\ u \end{bmatrix} &= \begin{bmatrix} P \\ 0 \end{bmatrix} \phi(t) \\ y &= V^T u\end{aligned}$$

Usual tactic: Model reduction on linearized system.

Goal: Work with second-order system directly.

# Second-order Krylov subspaces

Let  $A, B \in \mathbb{C}^{N \times N}$ ,  $r_0 \in \mathbb{C}^N$ , and define the sequence

$$r_1 = Ar_0,$$

$$r_j = Ar_{j-1} + Br_{j-2} \quad \text{for } j \geq 2$$

Now define a *second-order Krylov subspace*

$$\mathcal{G}_n(A, B; r_0) = \text{span}\{r_0, r_1, r_2, \dots, r_{n-1}\}.$$

Note

$$\begin{bmatrix} r_j \\ r_{j-1} \end{bmatrix} = H^j v, \quad \text{where } H = \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \quad \text{and } v = \begin{bmatrix} r_0 \\ 0 \end{bmatrix}$$

$\implies \mathcal{G}_n(A, B; r_0)$  provides the same info as  $\mathcal{K}_n(H; v)$ !

(T.-J. Su + Craig Jr. '91, cited by White + Ramaswamy '00)

# SOAR-based model reduction

Let  $A = -K^{-1}C$ ,  $B = -K^{-1}M$ ,  $r_0 = -K^{-1}P$ .

Form an orthonormal basis  $Q_k$  for  $\mathcal{G}_n(A, B; r_0)$  by a *second-order Arnoldi* (SOAR) iteration (Bai).

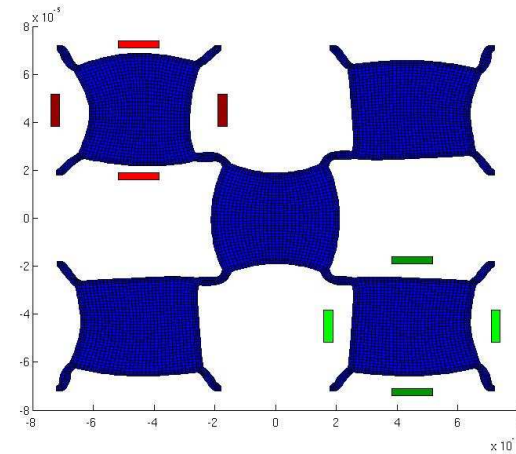
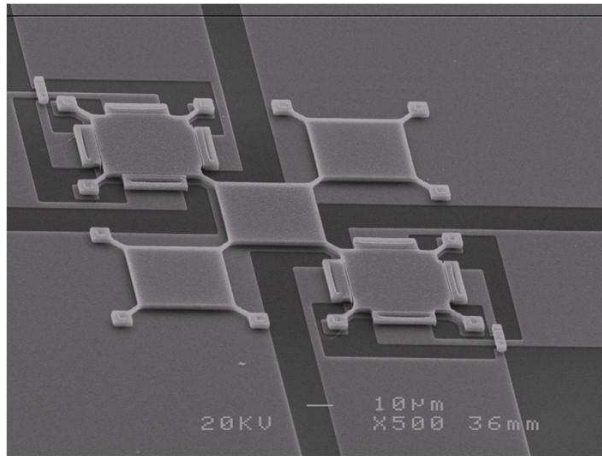
Reduced model:

$$\begin{aligned} M_n u_n'' + C_n u_n' + K_n u_n &= P_n \phi \\ y &= V_n^T u \end{aligned}$$

where  $P_n = Q_n^T P$ ,  $V_n = Q_n^T V$ ,  $M_n = Q_n^T M Q_n, \dots$

Basic structure (and properties) are preserved.

# Example: Checkerboard resonator



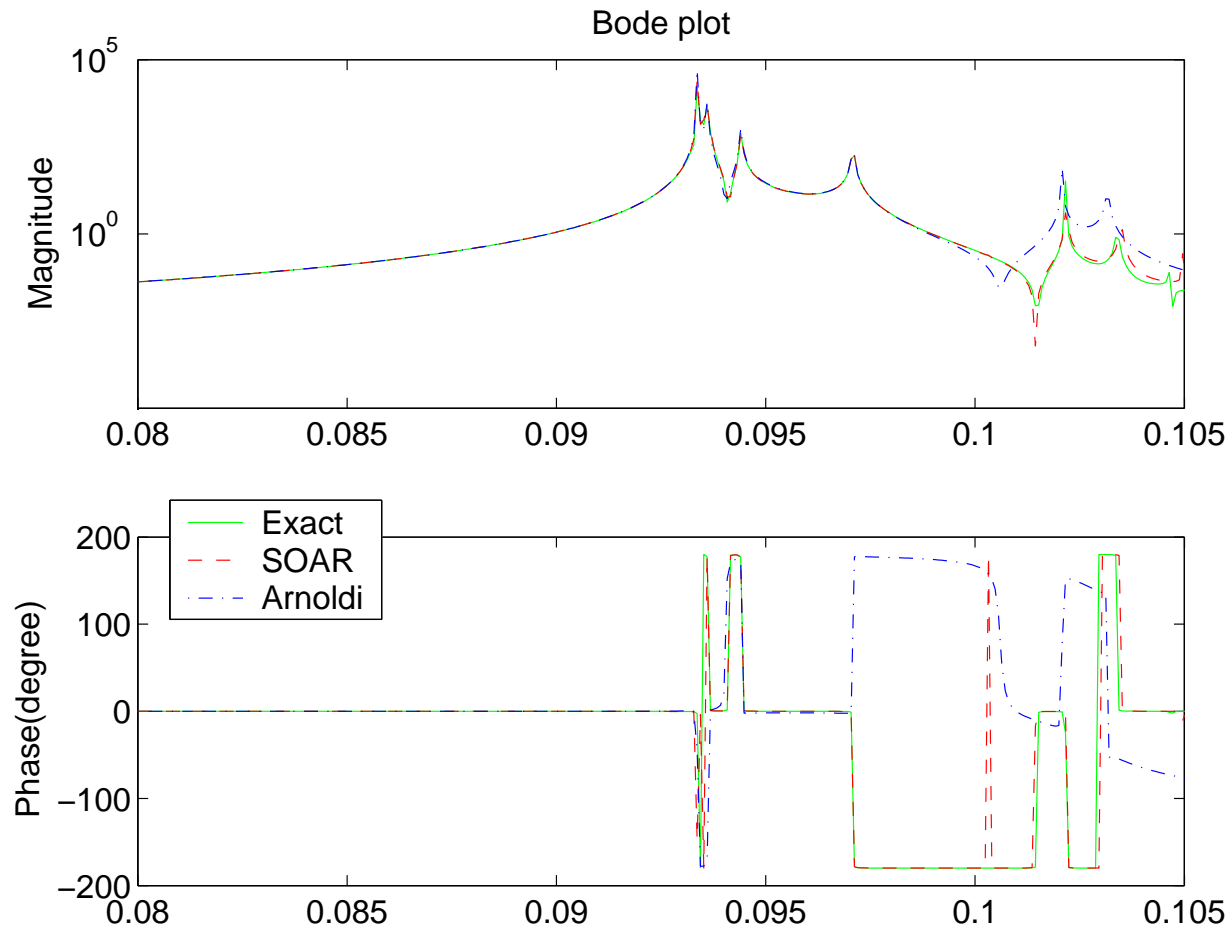
(Design and layout by S. Bhave)

- Array of coupled resonators
- Anchored at corners
- Excited at **northwest**
- Sensed at **southeast**
- Surfaces move a few nm

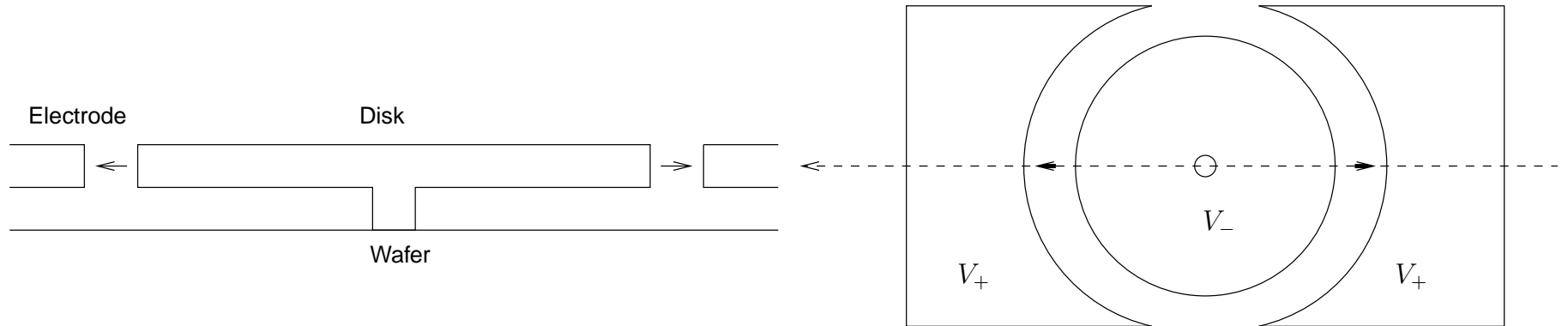


# Performance of SOAR vs Arnoldi

$$N = 2154 \rightarrow n = 80$$



# Example: Disk resonator anchor loss



Goal: Understand energy lost through radiation from post.

- Treat substrate as a half-space
  - Use a complex-valued change of coordinates to build an absorbing *Perfectly Matched Layer* (PML) (Basu and Chopra '03)
  - $M$  and  $K$  are *complex* symmetric
- Need a fine mesh to resolve quality of resonant peaks

# PML ROM: Preserving structure

Weak form of the PML equation (frequency domain) is

$$s^2 \int_{\Omega} \rho w \cdot u J d\Omega + \int_{\Omega} \tilde{\epsilon}(w) : \mathbf{C} \tilde{\epsilon}(u) J d\Omega = \int_{\Gamma} w \cdot p \tilde{n} J d\Gamma$$

Finite element discretization (Bubnov-Galerkin) yields complex-symmetric mass and stiffness.

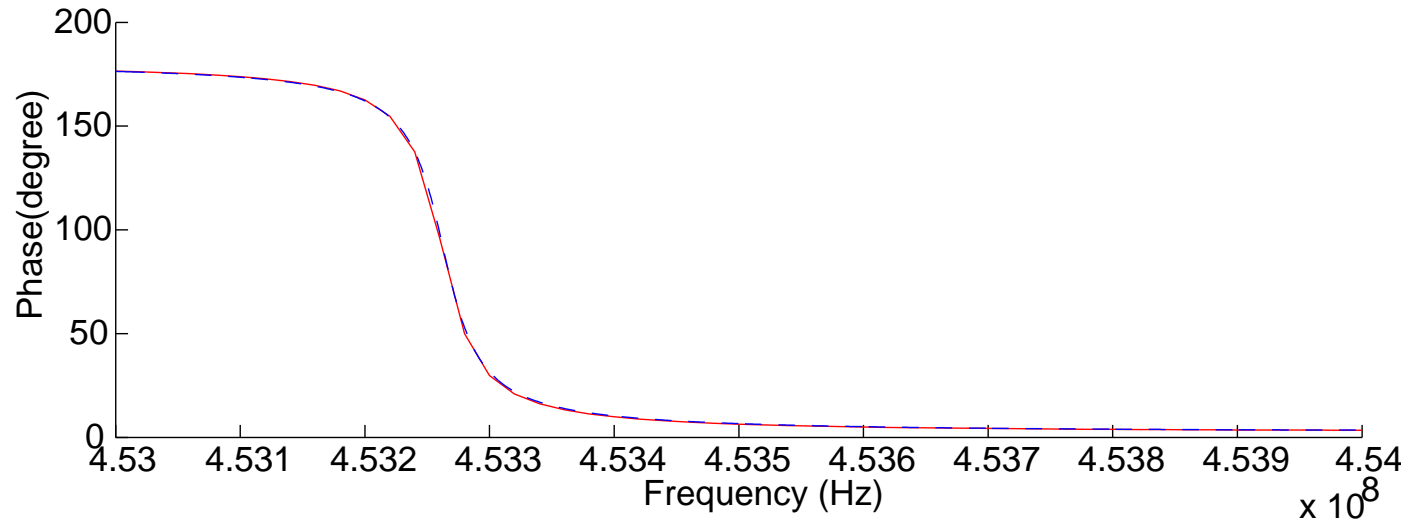
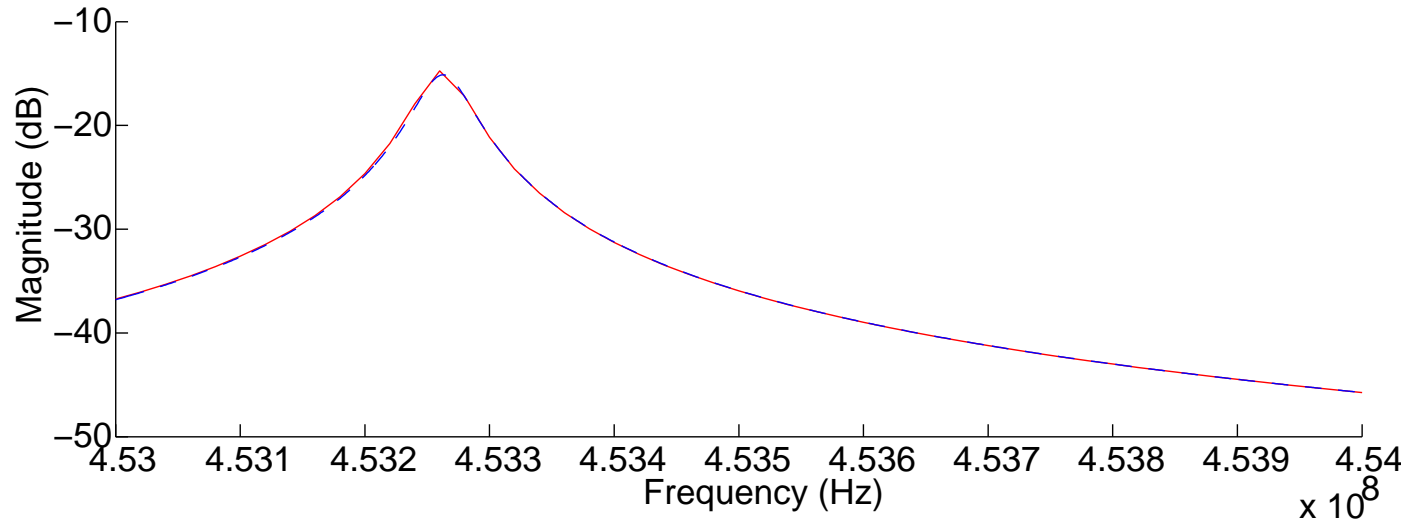
Reduction procedure:

- Run Arnoldi on  $(s_0^2 M + K)^{-1}$  to get basis  $\hat{Q}_n$
- $Q_n = \text{orth}([\text{Re}(\hat{Q}_n), \text{Im}(\hat{Q}_n)])$
- $M_n = Q_n^T M Q_n$  and  $K_n = Q_n^T K Q_n$  are symmetric

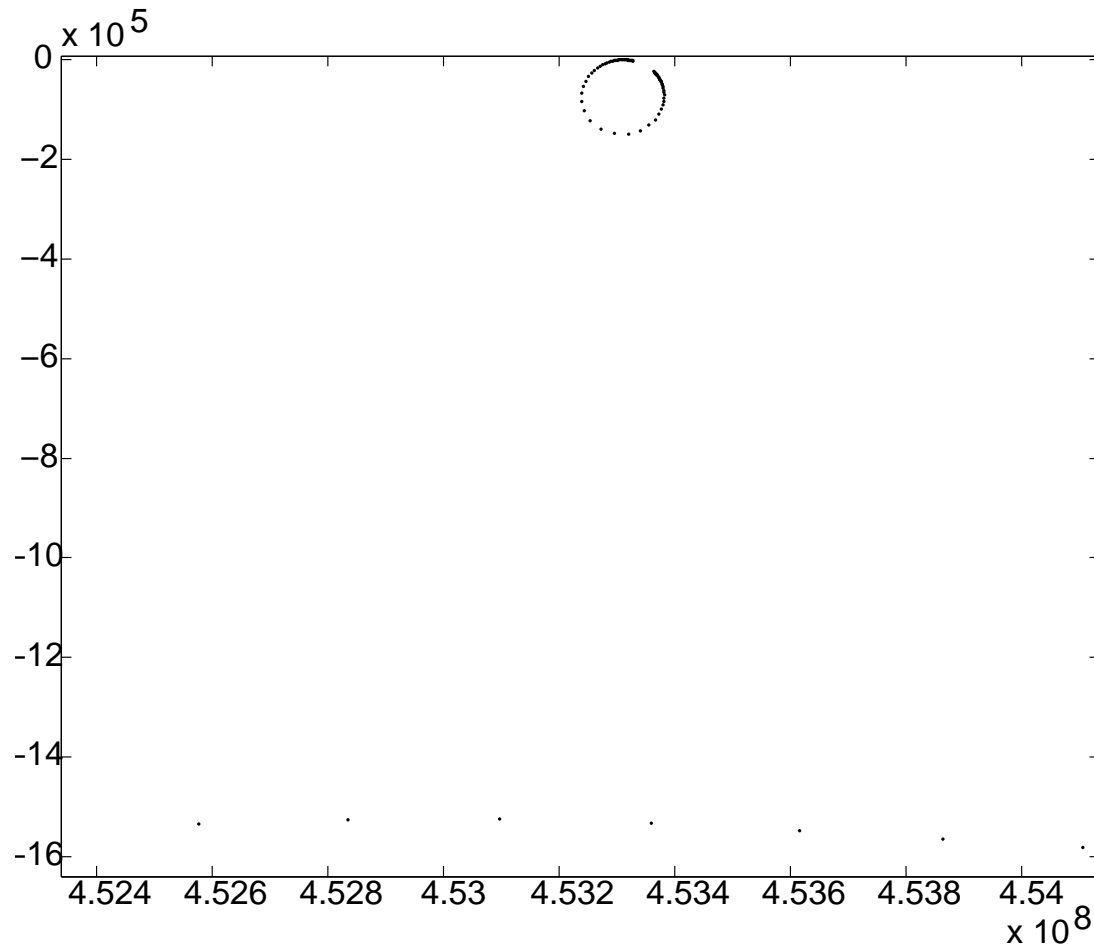
ROM is a Bubnov-Galerkin projection of original equation.

# Anchor loss

$$N = 57475 \rightarrow n = 3$$



# Why $n = 3$ ?



- Behavior in this region is dominated by two poles
- Mode “mixing” dramatically affects predicted resonance!
- Impact: Eliminate most anchor loss for disk resonator?

# Ongoing and future work

Continued development of predictive CAD for RF MEMS

Parameter-dependent model reduction

Structure-preserving model reduction for thermoelasticity:

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}'' + \begin{bmatrix} 0 & 0 \\ D_{tu} & D_{tt} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}' + \begin{bmatrix} K_{uu} & K_{ut} \\ 0 & K_{tt} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix} \phi(t)$$

Integration into the SUGAR framework for MEMS simulation