TEMPERATURE SENSITIVITY OF SOLID-WAVE GYROSCOPES

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ABSTRACT

We analyze the change of angular gain and vibration frequency of solid-wave gyroscopes as a result of geometry perturbations due to thermal expansion. We formulate a temperature sensitivity analysis by assuming a linear dependence of material properties to temperature, and quantify it for common device geometries.

INTRODUCTION

The lumped model of a solid-wave gyroscope involves two degenerate, coupled, damped and driven harmonic oscillators:

$$\ddot{\mathbf{q}} + 2(\gamma\mathbf{I} + A_g\Omega\mathbf{J})\dot{\mathbf{q}} + \omega^2\mathbf{q} = \mathbf{f},\ \mathbf{I} \equiv \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\ \mathbf{J} \equiv \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

where the vectors $\mathbf{q}(t)$ and $\mathbf{f}(t)$ correspond to generalized coordinates and drive forces respectively. ω is the undamped angular vibration frequency and Ω is the rotation rate of the gyro platform. $\gamma = \omega/2Q$ is the damping coefficient and A_g is known as angular gain [1].

As the device temperature T changes, the coefficients of the lumped model drift. Computing $\gamma(T)$, $A_g(T)$ and $\omega(T)$ requires modeling geometric changes of the resonator. Since the damping coefficient $\gamma(T)$ depends on the relevant damping mechanisms, we only compute sensitivity of $A_g(T)$ and $\omega(T)$.

METHOD

Typical solid-wave gyroscope resonators are axisymmetric shells clamped at one of two boundaries and free on the other (Fig. 1). The cross section of the shell's midsurface forms a plane curve on (r, z)-plane. We parametrize this curve by its arclength s and use h, L and d to represent the shell thickness, the length of the midline curve, and the distance from the clamped boundary to the symmetry axis, respectively.

Temperature variations change both material properties and geometry of resonators. The elastic coefficients of conventional materials used in microfabrication change almost linearly within the standard range of operational temperatures: $E(T) = E_0(1 + \alpha_E(T - T_0))$, $\nu(T) = \nu_0(1 + \alpha_\nu(T - T_0))$. Also, due to thermal expansion, the device geometry and material density change slightly: $\rho(T) = \rho_0(1-3\alpha_L(T-T_0))$, where α_L is the thermal coefficient of expansion of the resonator material. We assume the post, attached at $r = d$, expands in radial direction with thermal coefficient α_S corresponding to the substrate material.

First, we discretize the axisymmetric thermal expansion problem: $\mathbf{K}_T\mathbf{u}_T = \mathbf{f}_T$, which is inherently nonlinear; but we only need $\frac{d\mathbf{u}_T}{dT} = \mathbf{K}_T^{-1}\frac{d\mathbf{f}_T}{dT}$ in our analysis. We use this solution to differentiate the resonator shape with respect to temperature and solve for the sensitivities of the lumped

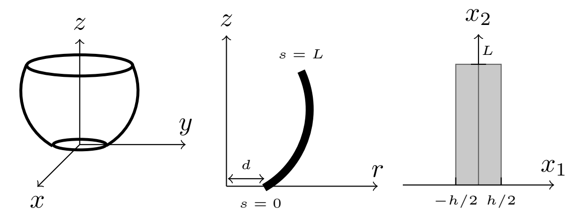

Fig. 1: *Left: Axisymmetric shell. Clamped at the bottom boundary, free at the top boundary. Center: Cross section in (r, z)-plane. Right: Computational mapped domain.*

model coefficients [2]. We non-dimensionalize the equations using units of length $l^* = L$, time $\tau^* = L/\sqrt{E_0/\rho_0}$, mass $m^* = \rho_0 L^3$ and temperature $T^* = \alpha_L^{-1}$.

RESULTS

We investigated temperature sensitivities for three common geometries (Fig. 2). They all have the same midline length and shell thickness. We selected d so that radial extents are the same. Table 1 shows the numerical values used in simulations. In Table 2, we tabulated sensitivities for $m = 2$ and $m = 3$ modes for each resonator.

Since all thermal coefficients are small, we can write temperature sensitivity of angular gain or frequency as a sum of all effects: $y = C_L\alpha_L + C_S\alpha_S + C_E\alpha_E + C_\nu\alpha_\nu$, where y is either $(1/A_g)dA_g/dT$ or $(1/\omega)d\omega/dT$, and C's are constants for a particular resonator. By setting all α's to zero except one, we can compute the contribution of each thermal effect (Table 3, 4).

Design $\mathcal{C}$ is particularly sensitive to Poisson's ratio, and so is more sensitive to temperature variations than $\mathcal{H}$ and $\mathcal{T}$. The effects of substrate thermal expansion matter just as much to thermal sensitivity in the other designs, highlighting the importance of selection of the post material and radius. For all devices and modes, angular gain is independent of Young's modulus. Frequency sensitivity depends strongly on thermal variations of Young's modulus, much more than on any geometric effects; hence, new geometries are unlikely to improve frequency sensitivity. In contrast, by proper choice of the post radius, we can achieve temperature insensitive angular gain (Fig. 3). We can also formulate basic shape optimization problems as in Fig. 4, where the cost function is defined proportional to $(\frac{1}{A_g}\frac{dA_g}{dT})^2$.

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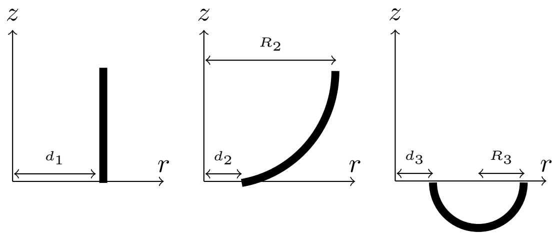

Fig. 2: *Left: Cylindrical shell $\mathcal{C}$. Center: Truncated hemispherical shell $\mathcal{H}$. Right: Half-toroidal shell $\mathcal{T}$.*

Table 1: *Material properties and geometry of resonators*

Parameter	Symbol	Value
Young's modulus	E_0	$80\,GPa$
Poisson's ratio	ν_0	0.26
Material density	ϕ_0	$2200\,kg/m^3$
Thermal expansion coefficient of resonator	α_L	$1.4 \times 10^{-6}\,K^{-1}$
Thermal expansion coefficient of substrate	α_S	$2.6 \times 10^{-6}\,K^{-1}$
Thermal coefficient of E	α_E	$2.5 \times 10^{-4}\,K^{-1}$
Thermal coefficient of ν	α_ν	$3.0 \times 10^{-4}\,K^{-1}$
Thickness to length ratio	h/L	0.01
Post radius to length ratio (cylindrical)	d_1/L	$12/5\pi$
Post radius to length ratio (spherical)	d_2/L	$12\sin(\pi/12)/5\pi$
Post radius to length ratio (toroidal)	d_3/L	$2/5\pi$
Radius of hemisphere to length ratio	R_2/L	$12/5\pi$
Minor radius of torus to length ratio	R_3/L	$1/\pi$

Table 2: *Temperature sensitvities*

$\mathcal{D}$	m	A_g	$\frac{1}{A_g}\frac{dA_g}{dT}$ $(\frac{ppb}{K})$	ω $(\frac{1}{L}\sqrt{\frac{E_0}{\rho_0}})$	$\frac{1}{\omega}\frac{d\omega}{dT}$ $(\frac{ppm}{K})$
$\mathcal{C}$	2	0.744	−613	0.345	102.2
	3	0.577	−1694	0.214	108.5
$\mathcal{H}$	2	0.554	−305	0.028	108.2
	3	0.487	−126	0.061	101.5
$\mathcal{T}$	2	0.304	−576	0.040	121.6
	3	0.357	−220	0.091	110.5

Table 3: *Thermal effect components of angular gain sensitivity*

$\mathcal{D}$	m	$\frac{1}{A_g}\frac{dA_g}{dT}$ $(\frac{ppb}{K})$	$C_L\alpha_L$	$C_S\alpha_S$	$C_E\alpha_E$	$C_\nu\alpha_\nu$
$\mathcal{C}$	2	−613	56	−313	0	−356
	3	−1694	251	−199	0	−1747
$\mathcal{H}$	2	−305	−105	−269	0	70
	3	−126	0	−145	0	18
$\mathcal{T}$	2	−576	−4	−441	0	−131
	3	−220	0	−231	0	11

Table 4: *Thermal effect components of frequency sensitivity*

$\mathcal{D}$	m	$\frac{1}{\omega}\frac{d\omega}{dT}$ $(\frac{ppm}{K})$	$C_L\alpha_L$	$C_S\alpha_S$	$C_E\alpha_E$	$C_\nu\alpha_\nu$
$\mathcal{C}$	2	102.16	−4.90	0.67	125	−18.6
	3	108.48	−7.96	1.05	125	−9.5
$\mathcal{H}$	2	108.25	−6.73	0.37	125	−10.3
	3	101.55	0.64	−1.06	125	−23.0
$\mathcal{T}$	2	121.63	0.64	−0.28	125	−3.7
	3	110.52	0.70	−0.60	125	−14.6

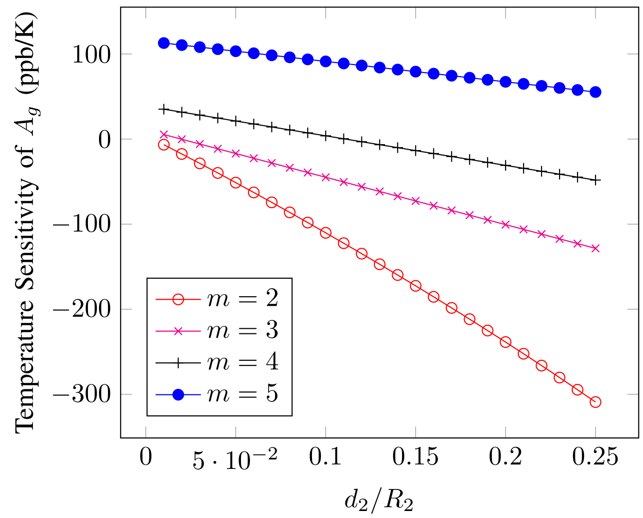

Fig. 3: *Temperature sensitivities of angular gain of truncated hemispherical shell as a function of post radius to shell radius ratio. Smaller posts are desirable for $m = 2, 3$ modes. It seems possible to design $m = 4$ with A_g insensitive to temperature.*

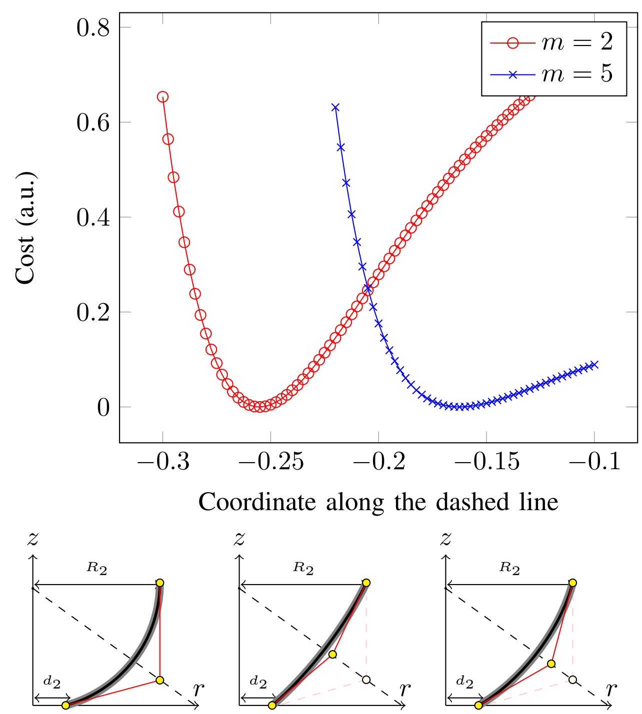

Fig. 4: *The simplest shape optimization problem. Bottom left: Three control points representing truncated hemispherical shell. The middle point is free to move along the dashed line while the other two are fixed. Bottom middle and right: Optimal shape and new position of the control point which minimizes temperature sensitivity of angular gain for modes $m = 2$ and $m = 5$ respectively. Top: Local minimum of the cost function for $m = 2$ and $m = 5$ by sweeping the position of control point on the dashed line.*