HW for 2018-06-12

(due: 2018-06-19)

1: Trial and error Using your favorite language and plotting system, produce a log-log plot of $\|(A + hE)^{-1} - (A^{-1} - hA^{-1}EA^{-1})\|$ versus h for small h and for

$$A = \begin{bmatrix} 6 & 3 \\ 1 & 3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

What is the asymptotic slope on the log-log plot, and why?

2: Sliding to the simplex In many data analysis problems, we want a vector $x \in \mathbb{R}^n$ that represents a probability distribution; that is, $x \geq 0$ elementwise and $\sum_i x_i = 1$. Given a target vector u,

1. Write the KKT conditions for the constrained problem that finds the closest probability distribution vector:

$$\text{minimize } \|x - u\|^2 \text{ s.t. } \sum_i x_i = 1 \text{ and each } x_i \geq 0$$

2. Argue that for some λ, the solution satisfies $x_i = \max(u_i + \lambda, 0)$.