HW 4

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Contrary conditioning Given a scalar $C > 1$, find $A \in \mathbb{R}^{2\times 2}$ for which all eigenvalues are one but $\kappa_2(A) \geq C$.

2: Interesting identity Suppose $X, Y \in \mathbb{R}^{n\times k}$. Show that if $\lambda \neq 0$ is an eigenvalue of XY^T, then

$$\begin{bmatrix} -\lambda I & X \\ Y^T & -I \end{bmatrix}$$

is singular. Via this formulation, show λ is also an eigenvalue of $Y^T X$.

3: Vector recovery Suppose $T \in \mathbb{R}^{n\times n}$ is upper triangular and $\lambda = t_{ii}$ is a simple eigenvalue of T. Give a code to compute a column eigenvector v in $O(n^2)$ time.

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function [v] = uptri_eigenvec(T,i)
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4: Somewhat symmetric Suppose $A = H + E$ where $H = H^*$. Argue that if $\lambda = \alpha + \beta\mathrm{i}$ is an eigenvalue of A, then $|\beta| \leq n\|E\|_2$.

5: Real rotations Suppose $A \in \mathbb{R}^{n\times n}$ has a complex conjugate pair of eigenvalues $\mu \exp(\pm \mathrm{i}\theta) = \alpha \pm \beta\mathrm{i}$ and corresponding eigenvectures $u \pm v\mathrm{i}$, with μ larger than the magnitude of any other eigenvalue. Show that power iteration from a random starting vector gives the sequence

$$v_k \approx u \cos(k\theta + \gamma) - v \sin(k\theta + \gamma)$$

for large k.

6: Double-shift iteration Suppose $A \in \mathbb{R}^{n\times n}$ has a complex conjugate pair of eigenvalues near $\alpha + \beta\mathrm{i}$. Without resorting to complex arithmetic, give a two-dimensional variant of Rayleigh quotient iteration that gives a rapidly-convergent estimate for the invariant subspace associated with the pair of eigenvalues. You may wish to build from the codes in the notes first.

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function [V,L] = rqi2d(A, a, b, rtol)
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