HW 3

Due: Fri, Apr 8

1: Go with the flow The Darcy friction coefficient f for turbulent flow in a pipe is defined in terms of the Colebrook-White equation for large Reynolds number Re (greater than 4000 or so):

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon/D_h}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Here ϵ is the height of the surface roughness and D_h is the diameter of the pipe. For a 10 cm pipe with 0.1 mm surface roughness, find f for Reynolds numbers of 10^4, 10^5, and 10^6. Ideally, you should use a Newton iteration with a good initial guess; it may help to reformulate the problem in terms of a variable other than f.

2: Funky fixed point Suppose we wish to find the fixed point $Ax = f(x)$ by the iteration

$$Ax^{k+1} = f(x^k),$$

where f is Lipschitz (in the two-norm) with constant $M < \sigma_{\min}(A)$. Write an error iteration and analyze it to argue that the fixed point iteration converges.

3: Eine kleine Nacht Kalkül Suppose $g : \mathbb{R} \to \mathbb{R}$ is C^2. Write the gradient and Hessian of

$$f(x) = g(\|x\|^2)$$

in terms of derivatives of g.