HW 3

Due in class on Monday, Feb 28

1: Everybody make some noise! The following MATLAB program generates a vector of observations corrupted by Gaussian noise:

```
n = 10000;
alpha = 1;
beta = 2;
sig   = 0.1;
xmax = 1;
x = xmax*rand(n,1);
z = sig^2*randn(n,1)
y = alpha*x + beta + z;
```

For $m = 3, 4, \ldots, n$, compute the least-squares fit of $\{(x_i, y_i)\}_{i=1}^m$. Do this for $x_{\max} = 1$, $x_{\max} = 10$, and $x_{\max} = 100$. For each value of $x_{\max}$, make a semi-logarithmic plot of the *relative* error in the vector (α, β). On the same plot, using the fact that you know the noise vector z and can compute the residual in each case, show the relative error *bound* computed in terms of the condition number of A (which can be computed via `cond`) and the angle θ (which you can access via the residual and the noise). You may assume that the backward error in the matrix due to roundoff is negligible, though you should briefly explain why. Submit your scripts and plots with your work.

2: The continuous connection. Set up and solve a system of normal equations to find a, b, c that minimize

$$\phi(a, b, c) = \int_0^\pi (ax^2 + bx + c - \sin(x))^2 \, dx.$$

Using `linspace`, set up equally-spaced meshes of $n = 11, 21, \ldots, 1001$ points on $[0, \pi]$, and for each mesh, solve a discrete least squares system to minimize

$$\phi_n(a, b, c) = \sum_{j=0}^{n-1} (ax_j^2 + bx_j + c - \sin(x_j))^2.$$

On a log-log plot, show the convergence of a_n, b_n, c_n to the continuous minimum a, b, c as a function of $h = \pi/(n+1)$.