Global Stochastic Optimization of Stellarator Coils

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Bigger Picture

Simons Collaboration: “Hidden Symmetries and Fusion Energy”

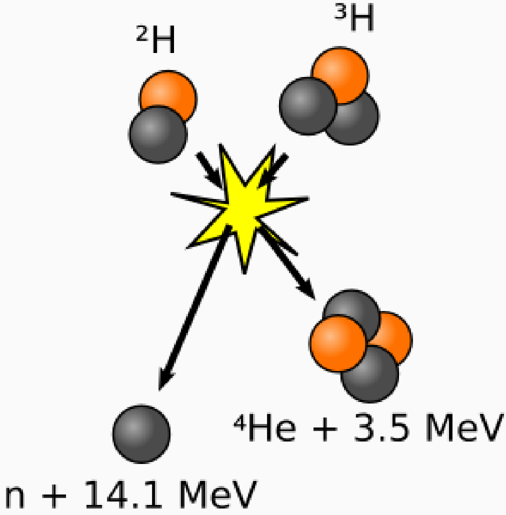
`https://hiddensymmetries.princeton.edu/`

Princeton, NYU, Maryland, IPP Greifswald, Warwick, CU Boulder, UW Madison, EPFL, ANU, UT Austin, U Arizona.

- Phase 1: Sep 2017-Aug 2022
- Phase 2: Sep 2022-Aug 2025

See MS392: Computational Challenges in Plasma Confinement (Fri 11:30-1:10 in Auditorium)

D-T fusion



Lawson criterion

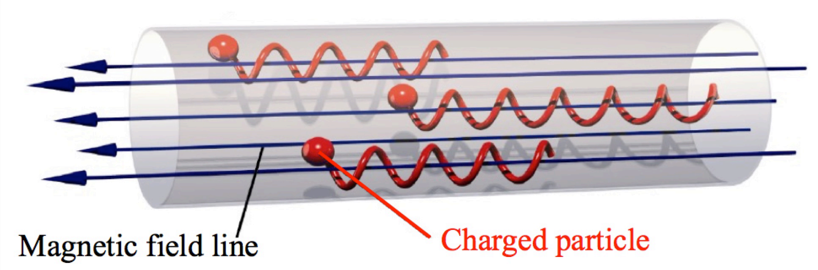
Figure of merit: $nT\tau_E$ where

- n is number density
- T is temperature
- τ_E is energy confinement time

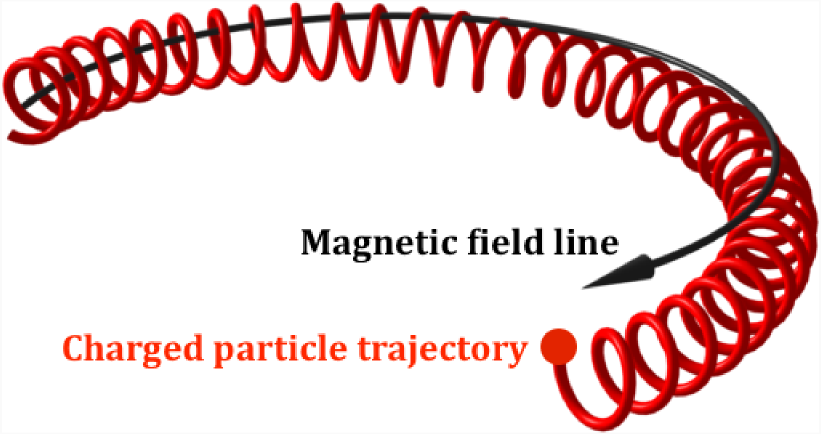
Min value required at $T = 14$ keV (about 162×10^6 K) is

$$nT\tau_E \geq 3.5 \times 10^{28} \text{ K s/m}^3$$

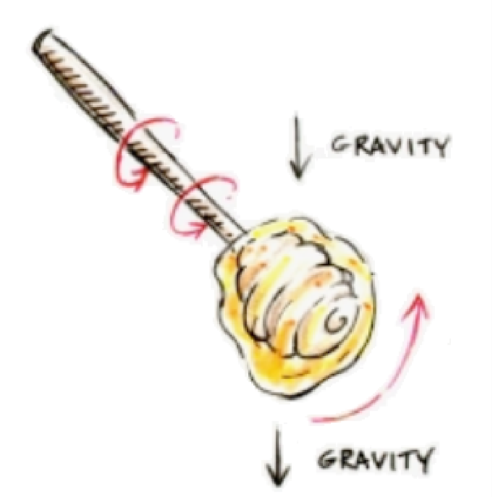
Magnetic confinement basics



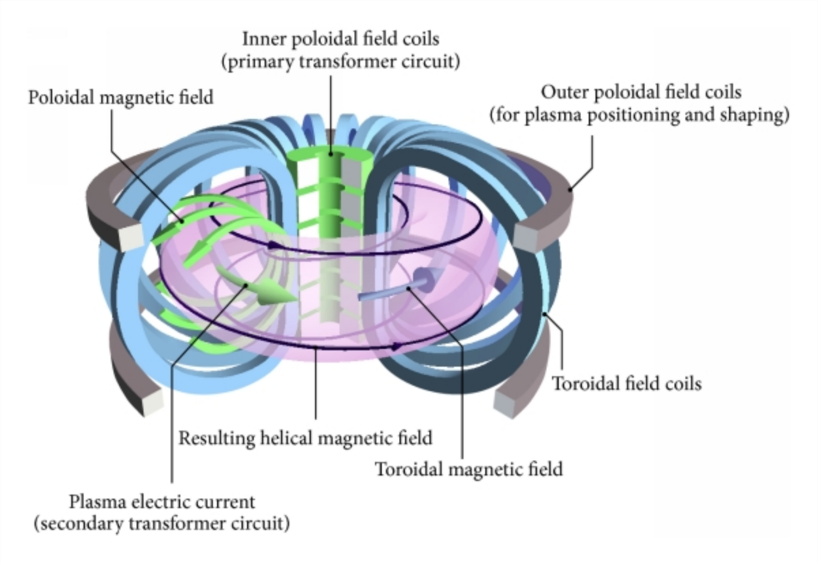
Magnetic confinement basics



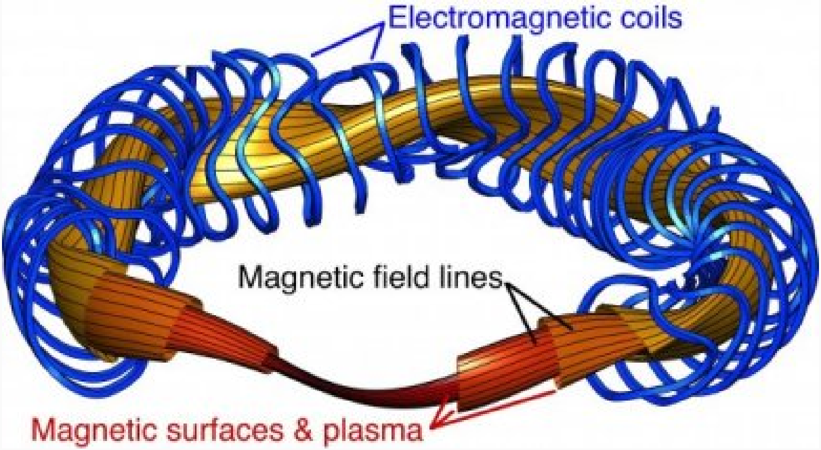
Magnetic confinement basics



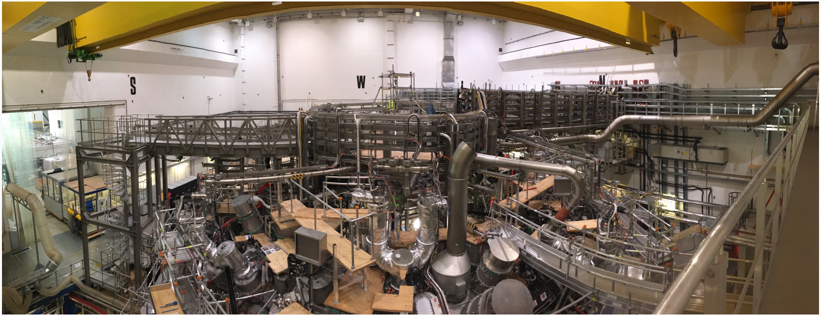
The big name: Tokamaks



Stellarator Concept



Wendelstein 7-X Machine



Operating since 2015-12-10;
plasma discharges lasting several min.

What Makes a Good Stellarator?

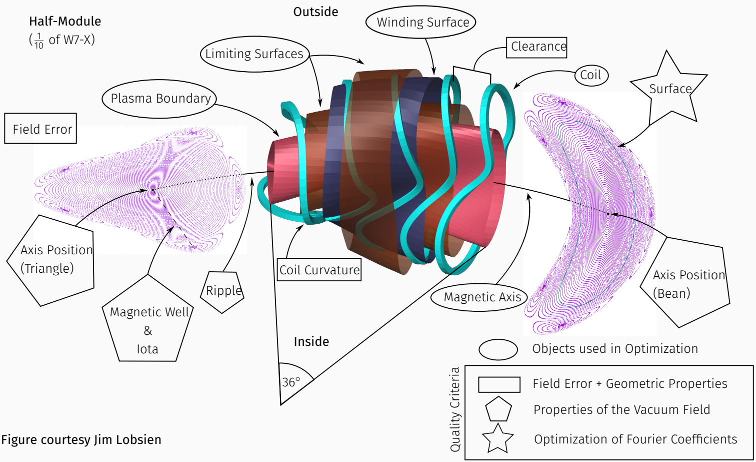


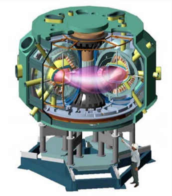
Figure courtesy Jim Lobsien

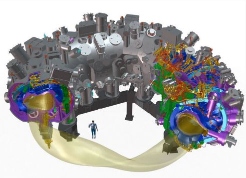
Optimization Under Uncertainty

Low construction tolerances:

- NCSX: 0.08%
- Wendelstein 7-X: 0.1% – 0.17%

Higher tolerances as coil opt goal!

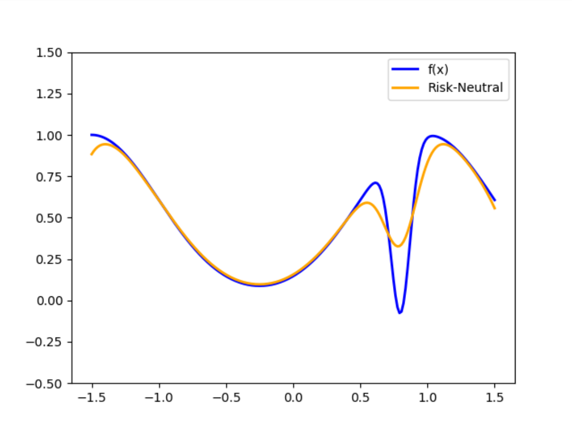




Also want tolerance to

- Changes to control parameters
- Uncertainty in physics or model

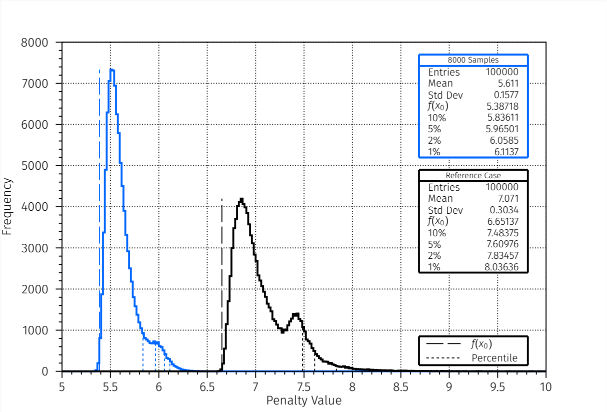
Risk-neutral OUU



Want efficient OUU in ~ 200 dimensions

$$\min_{x \in \Omega} \mathbb{E}_U[f(x - U)]$$

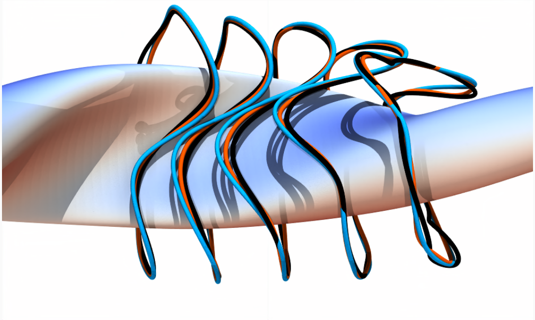
(Recent) Prior: Monte Carlo Approach



Robustness & mean perf greatly improved (w/ $\sim 10^8$ evals)

J.-F. Lobsien, M. Drevlak, T. Kruger, S. Lazerson, C. Zhu, T. S. Pedersen, *Improved performance of stellarator coil design optimization*, Journal of Plasma Physics, 2020.

Our Approach: fast TuRBO-ADAM



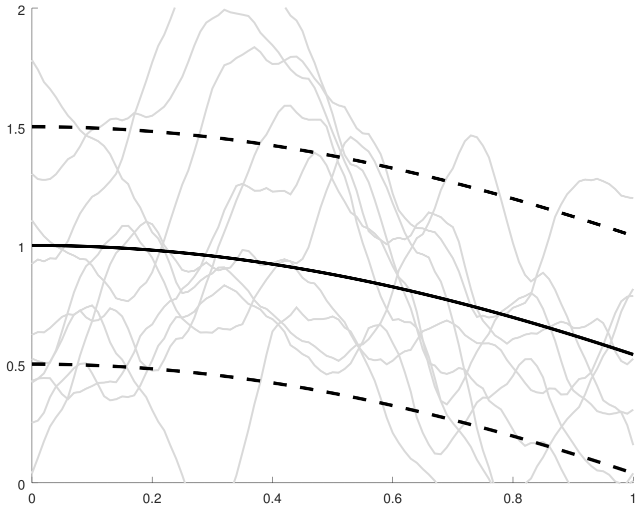
Black: ref; red: TuRBO-ADAM 10mm; blue: TuRBO-ADAM 20mm.

Evaluate objective with FOCUS from PPPL.

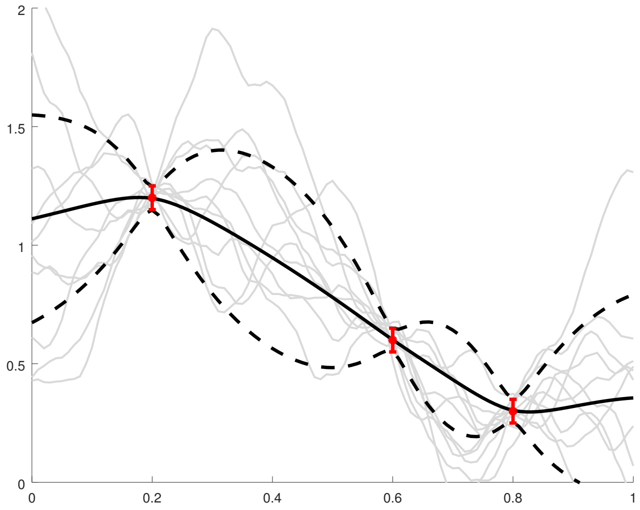
- Global search with modified TuRBO
- Local refinement with ADAM with control variate

Costs about 0.01% the evaluation budget.

Gaussian Processes (GPs)



Being Bayesian



Bayesian Optimization (BO)

Typical GP-based BO:

- Evaluate f on initial sample in Ω
- Condition a GP on sample data
- Until budget exhausted
  - Optimize *acquistion function* $\alpha(x)$ over Ω
    (e.g. $\alpha_{\mathrm{EI}}(x) = E\left[[f(x_{\mathrm{best}}) - f(x)]_+\right]$ where x_{best} is best so far)
  - Evaluate at selected point
  - Update the GP model

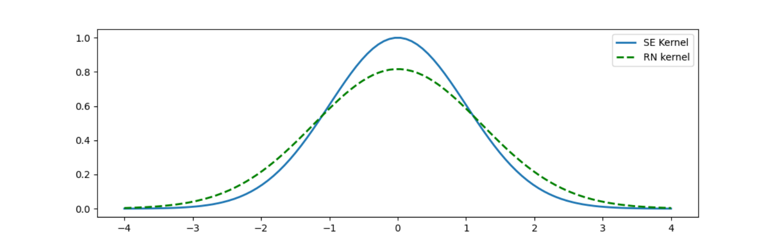
TuRBO: Trust-Region BO

For high-d: combine local BO with multi-start strategy

- Rough global sampling at M points
- Local GP models and trust-region around each point
- Thompson sampling to choose which local model (and trust region) to refine next

(Eriksson, Pearce, Gardner, Turner, Poloczek, 2019)

TuRBO + OUU



- TuRBO builds GP models for $f(x)$ (nominal objective)
- Simple transform from GP for $f(x)$ to GP for $E_U[f(x+U)]$ (Beland and Nair, 2017)

Problem: TuRBO explores a lot — want more refinement

Adam + Control Variates

- Regular Adam: stochastic gradient algorithm with "adaptive momentum" for step size control. Use directions

$$g(x) = \nabla f(x + U)$$

  for a random draw U (can also do mini-batch).

- Variance reduction with control variates (Wang, Chen, Smola, Xing, 2013)

$$\begin{aligned} g(x) &= \nabla f(x + U) + \alpha(\hat{g}(x) - E[\hat{g}(x)]) \\ \hat{g}(x) &= \nabla f(x) + HU. \end{aligned}$$

- True Hessian not avail, so set H to be an approximate Hessian (BFGS approximation via gradients from Adam).

The Longer Version

S. Glas, M. Padidar, A. Kellison, and D. Bindel, “Global Stochastic Optimization of Stellarator Coil Configurations,” *Journal of Plasma Physics* 88(2), 2022.

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