Surrogate-Based Optimization of Stellarators

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Simons Collaboration: "Hidden Symmetries and Fusion Energy"

https://hiddensymmetries.princeton.edu/

Princeton, NYU, Maryland, IPP Greifswald, Warwick, CU Boulder, UW Madison, EPFL, ANU, UT Austin, U Arizona.

Cornell group:

- Silke Glas (Simons postdoc)
- Misha Padidar (CAM PhD student)
- Ariel Kellison (CS PhD student)
- Nick Parrilla (predoc student)
- Paco Rilloraza (ugrad student)

with involvement from many others.

Magnetic confinement basics



Magnetic confinement basics



Magnetic confinement basics



- Particles confined to magnetic surfaces (invariant tori).
- Drift cancels over the full trajectory.

(V. I. Arnold, Small denominators and problems of stability of motion in classical and celestial mechanics, Russ. Math. Surv., 1963.)

The big name: Tokamaks



Stellarator Concept



Wendelstein 7-X Machine



Operating since 2015-12-10; plasma discharges lasting up to 30 min.

Optimization Under Uncertainty

Low construction tolerances:

- NCSX: 0.08%
- Wendelstein 7-X: 0.1% 0.17%

Higher tolerances as coil opt goal!

Also want tolerance to

- Changes to control parameters
- Uncertainty in physics or model



Risk-neutral OUU



Want efficient OUU in \sim 200 dimensions

 $\min_{x\in\Omega}\mathbb{E}_U[f(x-U)]$

(Recent) Prior: Monte Carlo Approach



Robustness & mean perf greatly improved (w/ ~ 10⁸ evals) J.-F. Lobsien, M. Drevlak, T. Kruger, S. Lazerson, C. Zhu, T. S. Pedersen, Improved performance of stellarator coil design optimization, Journal of Plasma Physics, 2020.

Our Approach: fast TuRBO-ADAM



Black: ref; red: TuRBO-ADAM 10mm; blue: TuRBO-ADAM 20mm.

Evaluate objective with FOCUS from PPPL.

- Global search with modified TuRBO
- Local refinement with ADAM with control variate

Costs about 0.01% the evaluation budget.

Combine two ideas:

- TuRBO: Trust-Region Bayesian Optimization (Eriksson, Pearce, Gardner, Turner, Poloczek, 2019)
- BO under uncertainty (Beland and Nair, 2017)

TuRBO idea



- Do a rough global sampling at M points.
- Local Gaussian process models of *f* near each point.
- Thompson sampling to choose which local model (and trust region) to refine next.

OUU adaptation



- TuRBO builds GP models for *f*(*x*) (nominal objective)
- Simple transform from GP for f(x) to GP for $E_U[f(x + U)]$

ADAM + control variates

• Regular ADAM: stochastic gradient algorithm based on

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g(x) = \nabla f(x+U)
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for a random draw U (can also do mini-batch).

• Variance reduction with control variates (Wang, Chen, Smola, Xing, 2013)

$$g(x) = \nabla f(x + U) + \alpha(\hat{g}(x) - E[\hat{g}(x)])$$
$$\hat{g}(x) = \nabla f(x) + HU.$$

• True Hessian not avail, so set *H* to be an approximate Hessian (BFGS approximation via gradients from ADAM).

And more!



https://hiddensymmetries.princeton.edu/ (Look at 2019 annual meeting for more talks!)