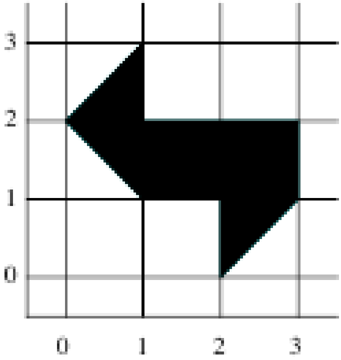
Spectral densities and social networks

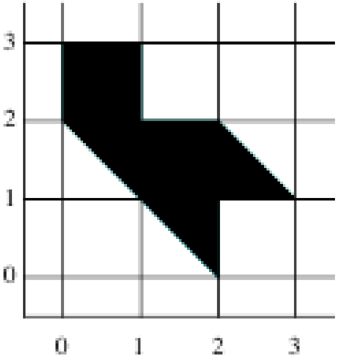
David Bindel

Department of Computer Science
Cornell University

NYCAM 4, 9 Nov 2013

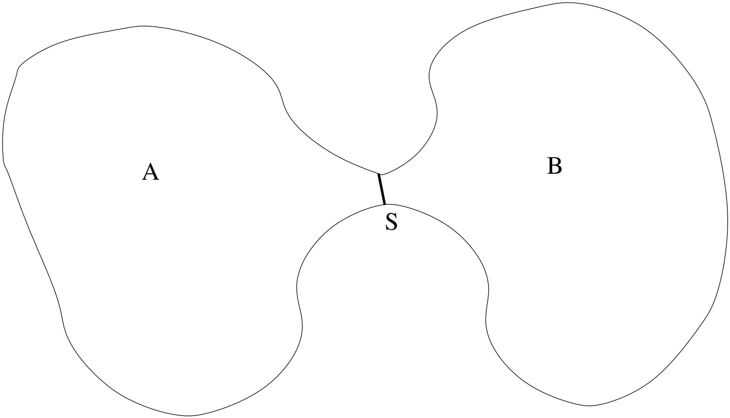
Can One Hear the Shape of a Drum?





$$-\nabla^2 u = \lambda u \text{ on } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$

What Do You Hear?



Size of bottlenecks (Cheeger inequality)

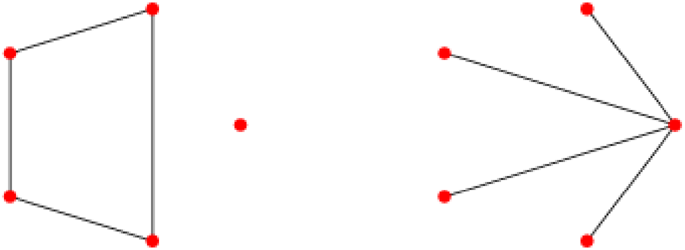
$$h \leq 2\sqrt{\lambda_2}$$

What Do You Hear?

Volume (Weyl law)

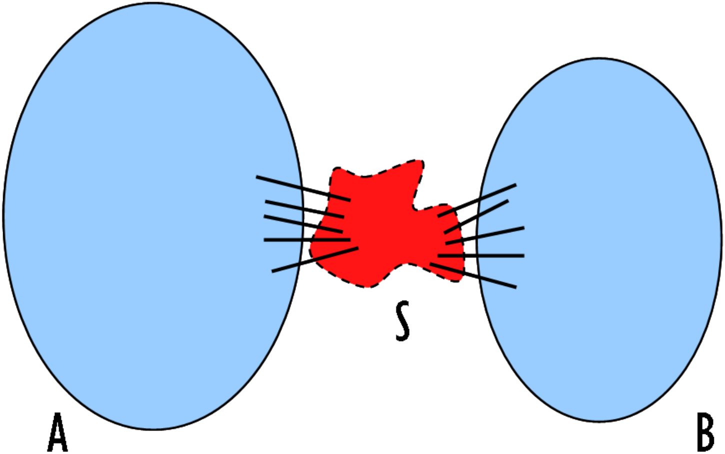
$$\lim_{x\to\infty} \frac{N(x)}{x^{d/2}} = (2\pi)^{-d}\omega_d \,\mathrm{vol}(\Omega), \quad N(x) = \{\text{\# eigenvalues } \leq x\}$$

Can One Hear the Shape of a Graph?



From eigenvalues of adjacency, Laplacian, normalized Laplacian?

What Do You Hear?



Size of separators (Cheeger inequality)

What Do You Hear?

What information hides in the eigenvalue distribution?

1. Discretizations of Laplacian: something like Weyl's law
2. Sparse random graphs: Wigner semicircular distribution
3. "Real" networks: less well understood

Exploring Spectral Densities

Kernel polynomial method (see Weisse, Reviews of Modern Physics)

- Think of spectral distribution as a generalized function

$$\int_{-1}^{1} \mu(x) f(x)\, dx = \frac{1}{N} \sum_{k=1}^{N} f(\lambda_k)$$

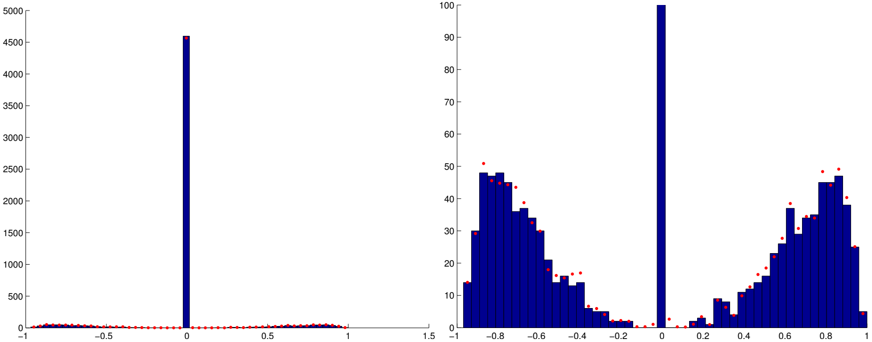
- Write $f(x) = \sum_{j=1}^{\infty} c_j T_j(x)$ and $\mu(x) = \sum_{j=1}^{\infty} d_j \phi_j(x)$, where $\int_{-1}^{1} \phi_j(x) T_k(x)\, dx = \delta_{jk}$
- Estimate $d_j = \frac{1}{N} \operatorname{tr}(T_j(A)) = \frac{1}{N} E[z^T T_j(A) z]$, z a random probe vector
- Truncate series for $\mu(x)$ and filter (avoid Gibbs)

Much cheaper than computing all eigenvalues!

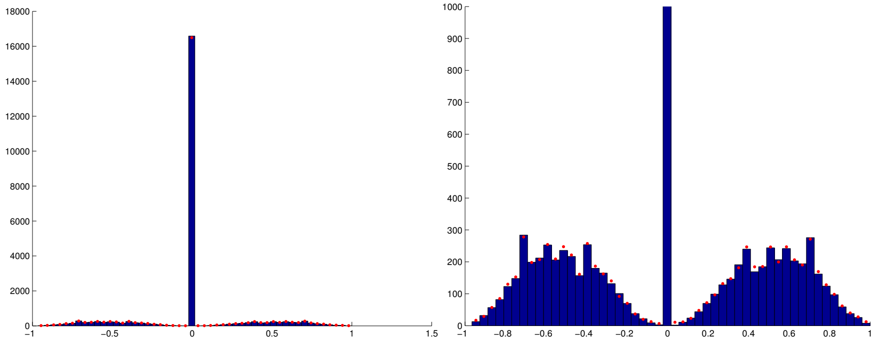
Exploring Spectral Densities

- Consider spectrum of normalized Laplacian (random walk matrix)
- Approximate via KPM and compare to full eigencomputation
- Joint work with David Gleich

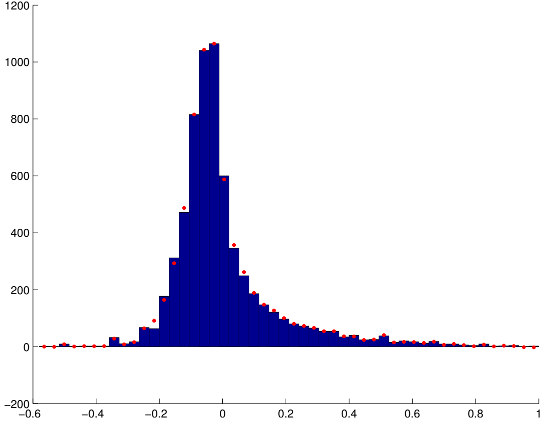
Erdos


5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
–1
–0.5
0
0.5
1
1.5
100
90
80
70
60
50
40
30
20
10
0
–1
–0.8
–0.6
–0.4
–0.2
0
0.2
0.4
0.6
0.8
1

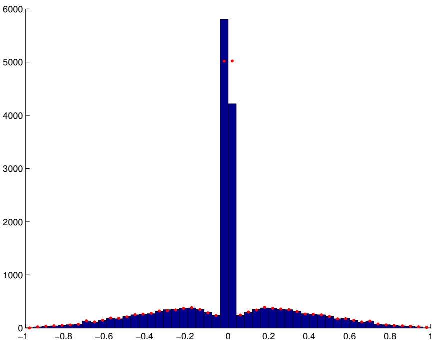
Internet topology



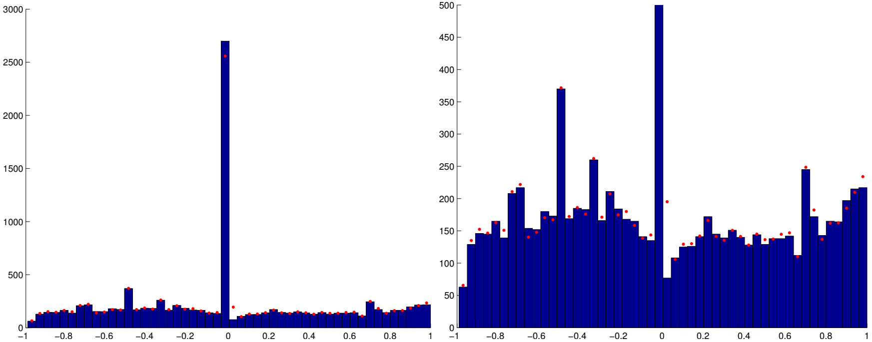
Marvel characters



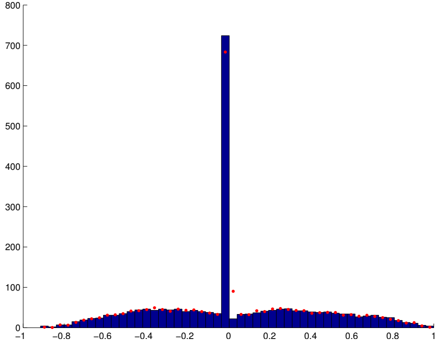
Marvel comics



PGP


3000
2500
2000
1500
1000
500
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
500
450
400
350
300
250
200
150
100
50
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

Yeast


800
700
600
500
400
300
200
100
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
100
90
80
70
60
50
40
30
20
10
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

What Do You Hear?