Structure-preserving model reduction for MEMS modeling

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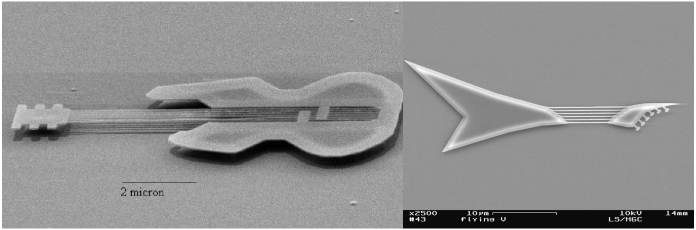
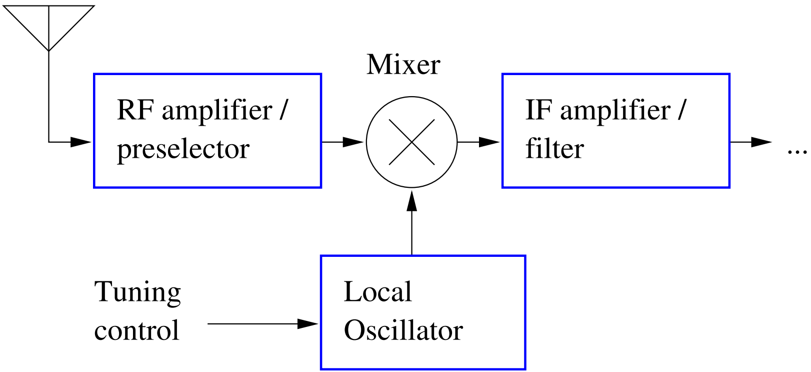
Resonant MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars

The Mechanical Cell Phone



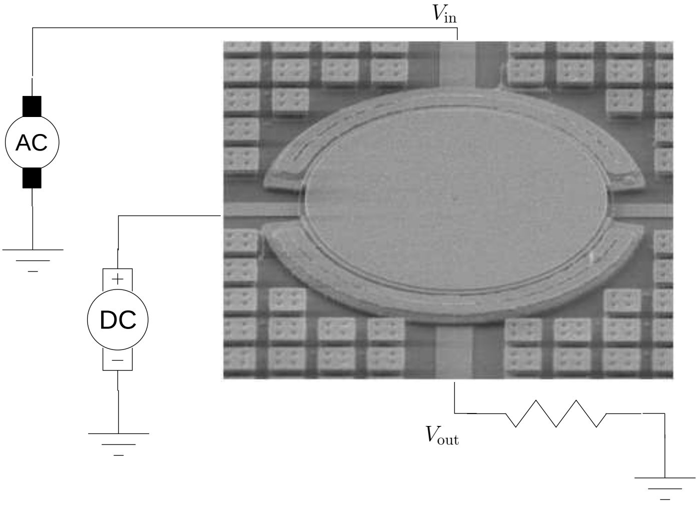
- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

Ultimate Success

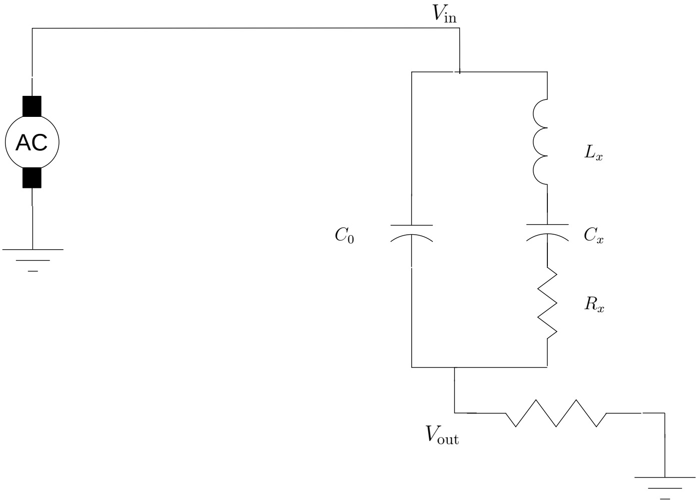
"Calling Dick Tracy!"

- Old dream: a Dick Trace watch phone!
- New dream: long battery life for smart phones

Example Resonant System



Example Resonant System



The Designer's Dream

Ideally, would like

- Simple models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up
- Backed by error analysis

We aren't there yet.

The Hero of the Hour

Major theme: use problem structure for better models

- Algebraic
  - Structure of ODEs (e.g. second-order structure)
  - Structure of matrices (e.g. complex symmetry)
- Analytic
  - Perturbations of physics (thermoelastic damping)
  - Perturbations of geometry (near axisymmetry)
  - Perturbations of boundary conditions (clamping)
- Geometric
  - Simplified models: planar motion, axisymmetry, ...
  - Substructures

SOAR and ODE structure

Damped second-order system:

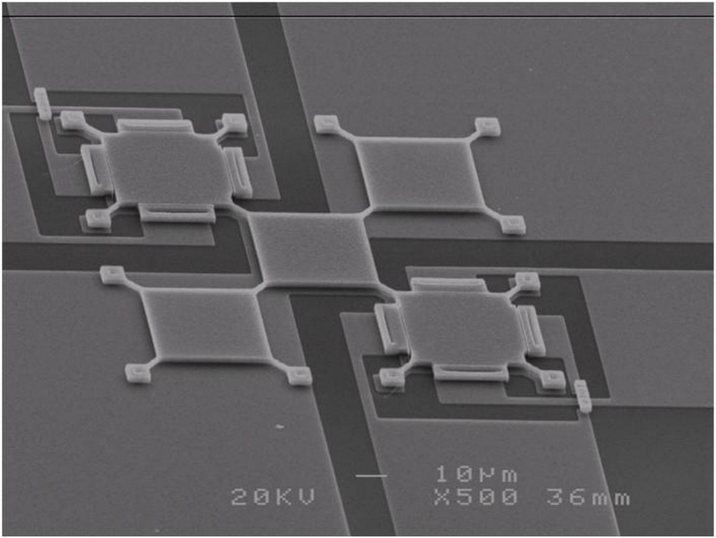
$$\begin{aligned} Mu'' + Cu' + Ku &= P\phi \\ y &= V^T u. \end{aligned}$$

Projection basis Q_n with Second Order ARnoldi (SOAR):

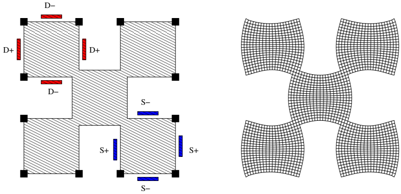
$$\begin{aligned} M_n u_n'' + C_n u_n' + K_n u_n &= P_n \phi \\ y &= V_n^T u \end{aligned}$$

where $P_n = Q_n^T P$, $V_n = Q_n^T V$, $M_n = Q_n^T M Q_n, \ldots$

Checkerboard Resonator

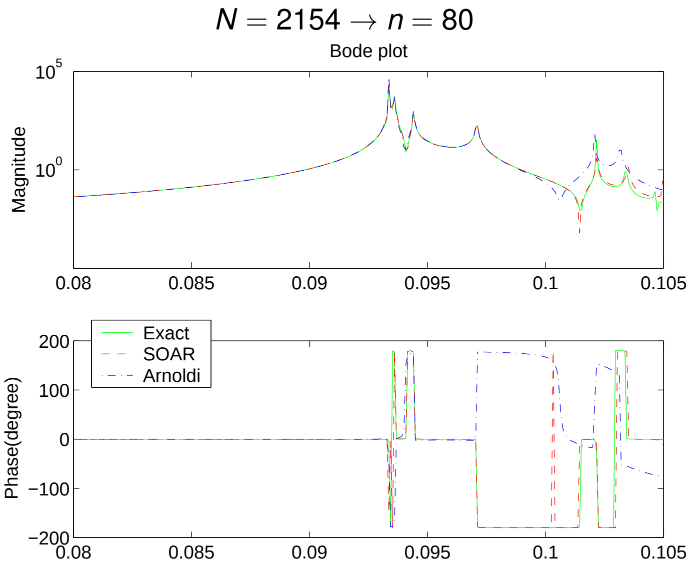


Checkerboard Resonator

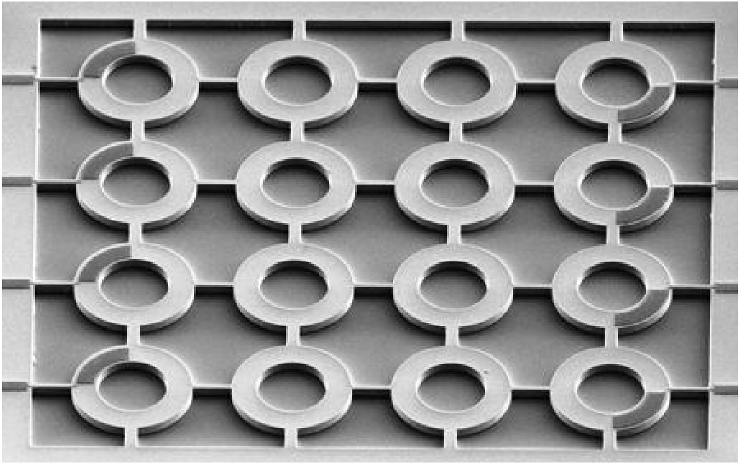


- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Performance of SOAR vs Arnoldi



Aside: Next generation



Complex Symmetry

Model with radiation damping (PML) gives complex problem:

$$(K - \omega^2 M)u = f, \text{ where } K = K^T, M = M^T$$

Forced solution u is a stationary point of

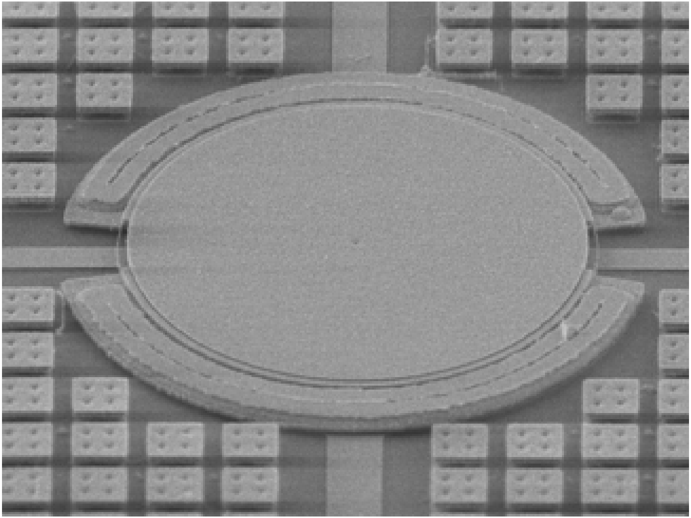
$$I(u) = \frac{1}{2}u^T(K - \omega^2 M)u - u^T f.$$

Eigenvalues of (K, M) are stationary points of

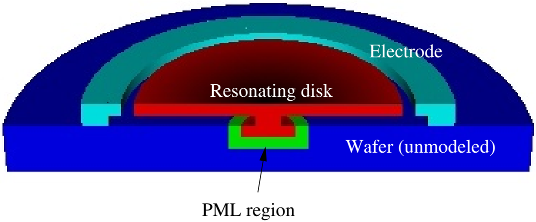
$$\rho(u) = \frac{u^T K u}{u^T M u}$$

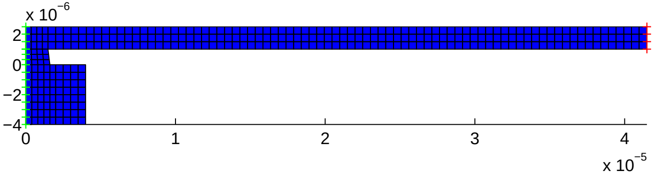
First-order accurate vectors $\implies$
second-order accurate eigenvalues.

Disk Resonator Simulations



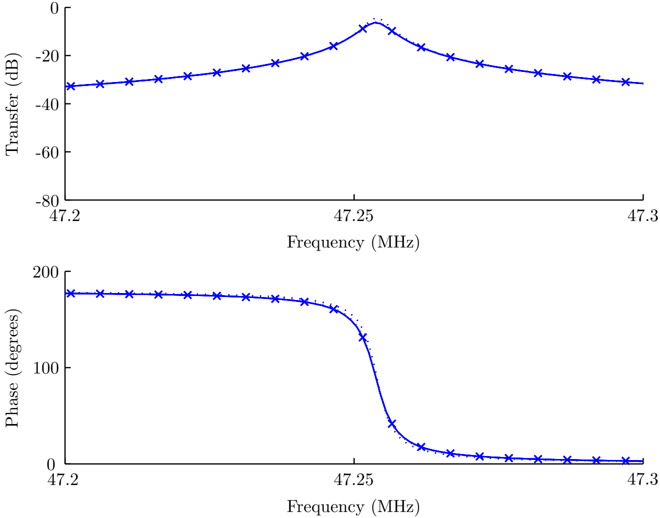
Disk Resonator Mesh





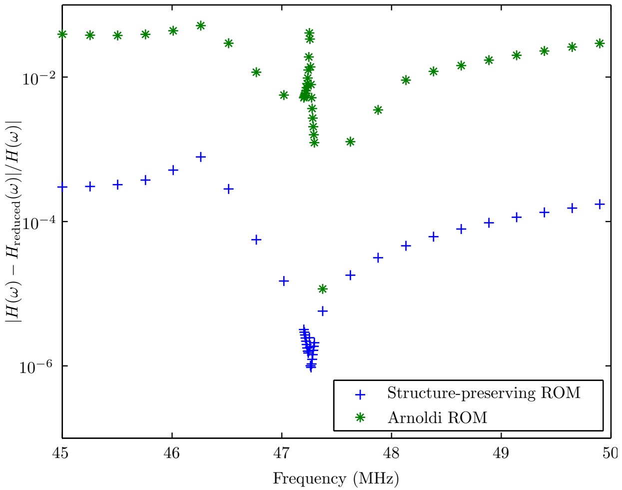
- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Symmetric ROM Accuracy



Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

Symmetric ROM Accuracy



Preserve structure $\Longrightarrow$
get twice the correct digits

Perturbative Structure

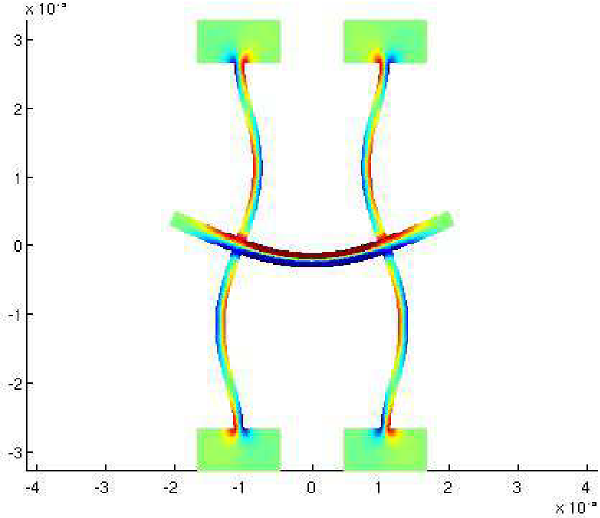
Dimensionless continuum equations for thermoelastic damping:

$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
\ddot{u} &= \nabla \cdot \sigma \\
\dot{\theta} &= \eta\nabla^2\theta - \mathrm{tr}(\dot{\epsilon})
\end{aligned}
$$

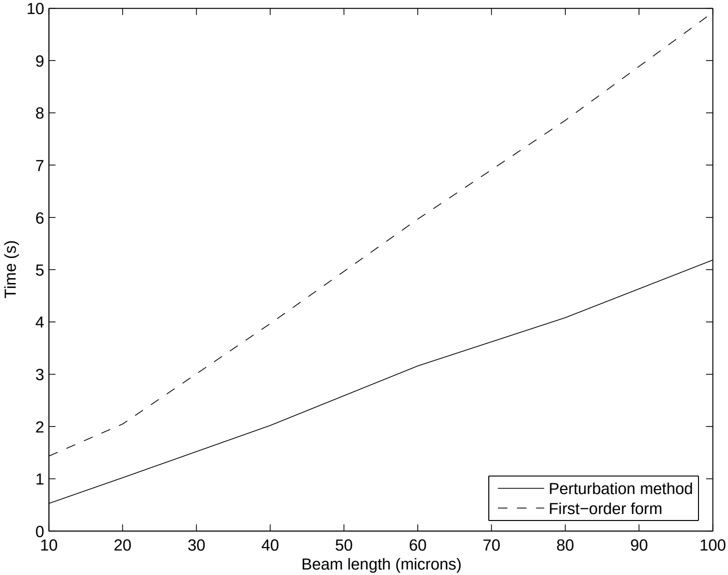
Dimensionless coupling ξ and heat diffusivity η are 10^{-4} $\implies$ perturbation method (about $\xi = 0$).

Large, non-self-adjoint, first-order coupled problem $\rightarrow$ Smaller, self-adjoint, mechanical eigenproblem + symmetric linear solve.

Thermoelastic Damping Example



Performance for Beam Example



Aside: Effect of Nondimensionalization

100 μm beam example, first-order form.

Before nondimensionalization

- Time: 180 s
- $\text{nnz}(L) = 11\text{M}$

After nondimensionalization

- Time: 10 s
- $\text{nnz}(L) = 380\text{K}$

Semi-Analytical Model Reduction

We work with hand-build model reduction all the time!

- Circuit elements: Maxwell equation + field assumptions
- Beam theory: Elasticity + kinematic assumptions
- Axisymmetry: 3D problem + kinematic assumption

Idea: Provide *global shapes*

- User defines shapes through a callback
- Mesh serves defines a quadrature rule
- Reduced equations fit known abstractions

Global Shape Functions

Normally:

$$u(X) = \sum_j N_j(X)\hat{u}_j$$

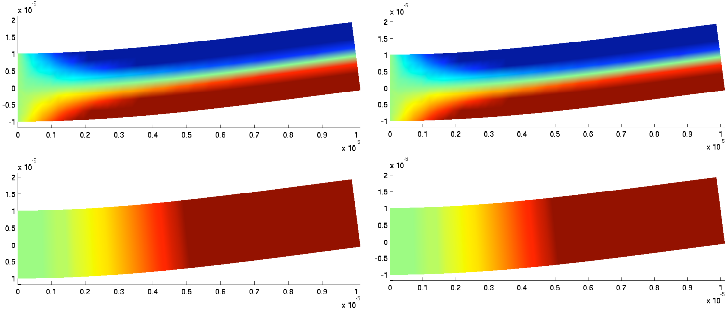
Global shape functions:

$$\hat{u} = \hat{u}^l + G(\hat{u}^g)$$

Then constrain values of some components of $\hat{u}^l$, $\hat{u}^g$.

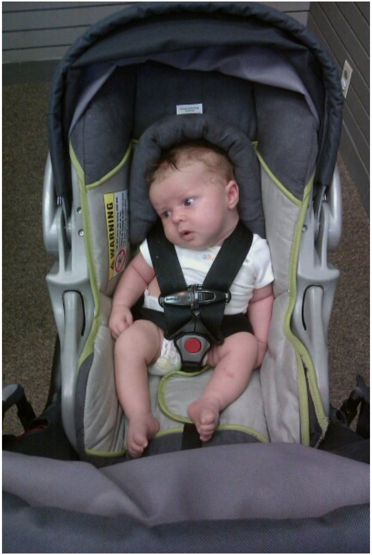
"Hello, World!"

Which mode shape comes from the reduced model (3 dof)?



(Left: 28 MHz; Right: 31 MHz)

The latest



http://www.cs.cornell.edu/~bindel