Math 118 **Sample Final Exam** **Spring 2001**

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Students should expect problems from any of the required sections of the textbook. This sample exam provides an idea of the level of complexity and approximate length of the common final exam.

Instructions: You must **show your work** to receive full credit. Simplify your answers. You need not evaluate expressions such as $\sqrt{2}$, $\ln 3$, e^5. Any fraction should be written in lowest terms or as a decimal. **No notes; no graphing calculators**. Remember, USC considers cheating to be a very serious issue.

1. (a) [5 points] Find the value of the constant C that makes the function $f(x)$ continuous on $(-\infty,\infty)$:

$$f(x) = \begin{cases} \frac{x^2-1}{x+1}, & \text{if } x < -1; \\ Cx^2, & \text{if } x \geq -1. \end{cases}$$

   (b) [5 points] Is the above (resulting) continuous function $f(x)$ differentiable at $x = -1$? Justify your answer.

2. [10 points] Use the definition of the derivative to find $\frac{d}{dx}x^{-1/2}$. Then find an equation of the tangent line to the graph of the function $y = x^{-1/2}$ at $x = 1$.

3. [10 points] Assume $z = \ln\sqrt{y^2+1}$, $y = \frac{x+1}{x-1}$. Use the chain rule to find $\frac{dz}{dx}$ as a function of x only. Simplify your answer.

4. [10 points] Determine where the function $g(x) = (x^2-4)^2$ is increasing and decreasing, and where its graph is concave up and down. Find the relative extrema and inflection points. Draw the graph.

5. [5 points] The total cost of producing a certain commodity is the sum of the setup cost plus the operating cost. The setup cost is 10 dollars per machine. A manufacturing firm receives an order for 1000 units of the commodity. Each machine can produce 100 units per hour. Once the machines have been set up, they can be overseen by a single supervisor, so the operating cost is 20 dollars per hour for any number of machines. Find the number of machines that should be used to keep the total cost as low as possible.

6. [10 points] A manufacturer can produce VCRs at a cost of \$125 apiece and estimates that if they are sold for x dollars apiece, consumers will buy approximately $1{,}000e^{-0.02x}$ each week.

   (a) Express the profit P as a function of x.

   (b) At what price should the manufacturer sell the VCRs to maximize profit?

7. [5 points] Find $\frac{dy}{dx}$ if $y = (x+3)^{2x+1}$. Hint: Use logarithmic differentiation.

8. [8 points] Find the function whose tangent has slope $\frac{1}{x}\ln x$ for each value of $x > 0$ and whose graph passes through the point $(e,3)$.

9. [7 points] Use integration by parts to find the following integral

$$\int \frac{x}{\sqrt{2x+1}} dx.$$

10. [5 points] Find the area of the region bounded by the curve $y = \frac{2}{x}$ and the line $2y + x = 5$.

11. [10 points] Three farmers want to dig a well between them in such a way that the sum of the squares of distances from the well to their houses is minimal. If the coordinates of their houses are (1,1), (2,−1), and (3,−2) on a rectangular map grid, where units are in miles, find the coordinates (a,b) of the well.

12. (a) [5 points] Evaluate $\int_0^3 \int_0^2 x^2 e^{-xy} dx dy$.

    (b) [5 points] Find the volume of the solid bounded above by the surface $f(x,y) = xy + 1$ and below by the rectangular region $R = \{(x,y) \mid 1 \le x \le 2, 1 \le y \le 3\}$.

Short Answers

1. (a) −2; (b) no.
2. $\frac{-1}{2x^{3/2}}$; $y = -\frac{1}{2}x + \frac{3}{2}$.
3. $\frac{x+1}{(1-x)(x^2+1)}$.
4. increasing on (−2,0) ∪ (2,∞); decreasing on (−∞,−2) ∪ (0,2); concave up on $(-\infty,-\frac{2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}},\infty)$; concave down on $(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}})$; local maximum at (0,16); local minima at (−2,0) and (2,0); inflection points at $(-\frac{2}{\sqrt{3}},\frac{64}{9})$ and $(\frac{2}{\sqrt{3}},\frac{64}{9})$.
5. 4 machines.
6. (a) $P(x) = (x-125)(1000e^{-0.02x})$; (b) \$175.
7. $[2(x+3)\ln(x+3) + 2x + 1](x+3)^{2x}$.
8. $y = \frac{1}{2}(\ln x)^2 + \frac{5}{2}$.
9. $x\sqrt{2x+1} - \frac{1}{3}(2x+1)^{3/2} + C$.
10. $\frac{15}{4} - 4\ln 2$.
11. $(2,-\frac{2}{3})$.
12. (a) $\frac{7}{9}e^{-6} + \frac{17}{9}$; (b) 8.

Math 118 **More Practice Problems for the Final Exam** **Spring 2001**

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1. Differentiate the given function and simplify your answer

   (a) $f(x) = (3x^4 - 5x^2 + 7)^6$

   (b) $g(y) = \frac{3y+1}{\sqrt{1-5y}}$

   (c) $h(x) = e^{-1/(2x)}$

2. Evaluate the following integrals:

   (a) $\int 3x\sqrt{x^2+8}dx$

   (b) $\int \frac{2t\ln(t^2+1)}{t^2+1}dt$

   (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

3. Evaluate the integral $\int \frac{x}{\sqrt{x+2}}dx$

   (a) by using integration by parts,

   (b) by making a substitution.

4. Let $f(x) = \frac{\ln x}{x^2}$.

   (a) Find the slope of the tangent line to the curve $f(x) = \frac{\ln x}{x^2}$ at the point $x = e$ and write the equation of the tangent line to $f(x) = \frac{\ln x}{x^2}$ at the point $x = e$.

   (b) For what value of x is the tangent line to the curve $f(x) = \frac{\ln x}{x^2}$ horizontal?

   (c) Find the absolute maximum and the absolute minimum of the function f on the interval $[1,e]$.

5. A manufacturer can produce VCRs at a cost of \$125 apiece and estimates that if they are sold for x dollars apiece, consumers will buy approximately $1{,}000e^{-0.02x}$ each week.

   (a) Express the profit P as a function of x.

   (b) Determine where the function $P(x)$ is increasing and decreasing, and where the graph is concave up and down. Find any relative extrema and inflection points. Sketch the graph of $P(x)$.

6. For $f(x,y) = ye^{x^2+3x} + \ln y$, find $f_x(\ln 2, 1)$ and $f_y(\ln 2, 1)$.

7. For the function $f(x,y) = x^3 + y^2 + 6xy - 6x - 7y$, find all of the critical points and classify each critical point as a local maximum, a local minimum, or a saddle point.

8. (a) Evaluate $\int_{e^2}^{e} \frac{2}{x\ln x}dx$.

   (b) Evaluate $\int_1^2\int_1^3 xy\ln\frac{x}{y}dxdy$.

Short Answers

1. (a) $6(3x^4-5x^2+7)^5(12x^3-10)$; (b) $\frac{11-15y}{2(1-5y)^{3/2}}$; (c) $\frac{e^{-1/(2x)}}{2x^2}$
2. (a) $(x^2+8)^{3/2}+C$; (b) $\frac{1}{2}(\ln(t^2+1))^2+C$; (c) $2e^{\sqrt{x}}+C$
3. (a) $2x\sqrt{x+2}-\frac{4}{3}(x+2)^{3/2}+C$; (b) $\frac{2}{3}(x+2)^{3/2}-4(x+2)^{1/2}+C$
4. (a) $-\frac{1}{e^3}$, $y=-\frac{1}{e^3}x+\frac{2}{e^2}$; (b) $\sqrt{e}$; (c) absolute maximum value $=\frac{1}{2e}$, absolute minimum value $=0$.
5. (a) $P(x)=(x-125)(1000e^{-0.02x})$; (b) increasing on (0,175); decreasing on (175,∞); concave up on (225,∞); concave down on (0,225); relative maximum at $(175,50000e^{-3.5})=(175,1509.87)$; inflection point at $(225,100000e^{-4.5})=(225,1110.90)$.
6. $8(2\ln 2+3)\cdot 2^{\ln 2}$; $8\cdot 2^{\ln 2}+1$
7. $(1,\frac{1}{2})$ is a saddle point; $(5,-\frac{23}{2})$ is a local minimum.
8. (a) $-2\ln 2$; (b) $\frac{27}{4}\ln 3-8\ln 2$.