Last Name: _____   **First Name:** _____

ID Number: _____   **Signature:** _____

Circle your Professor's name:

Haydn   Saric   Schumitzky   Verona   Von Bremen   Vorel   Woo

Circle your discussion time:

TuTh 8:00   TuTh 9:00   TuTh 10:00   TuTh 11:00   TuTh 12:00

TuTh 2:00   TuTh 3:00   TuTh 4:00   MW 2:00   MW 3:00   MW 4:00

Directions

You must **show all your work and justify your methods** to obtain full credit. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, e^3, $\sqrt{3}$. Do not use scratch paper. Use the back of the previous page if additional room is needed. **Neither notes nor calculators are allowed**. During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in "silent" mode. Remember, USC considers cheating to be a very serious issue. The exam is worth a total of 200 points. Each problem is worth 20 points.

1. Use the definition of the derivative to find $f'(x)$, where $f(x) = \sqrt{x^2 + 1}$.

$f'(x) =$ _____

**2.** (a) Use implicit differentiation to find the slope of the tangent line to the curve
$$y + \sqrt{x}\, y^4 = 3 \quad \text{at the point} \quad (4, 1).$$

The slope of the tangent line at $(4, 1)$ is _____

(b) Find an equation of the tangent line to the curve $y + \sqrt{x}\, y^4 = 3$ at the point $(4, 1)$.

An equation of the tangent line is _____

3. A rectangle is positioned with its base on the x-axis and inside the region bounded by the graph of $f(x) = 27 - x^2$, as shown in the accompanying figure. Use calculus to find the dimensions of the rectangle of maximum area. Justify why your answer is indeed a maximum.
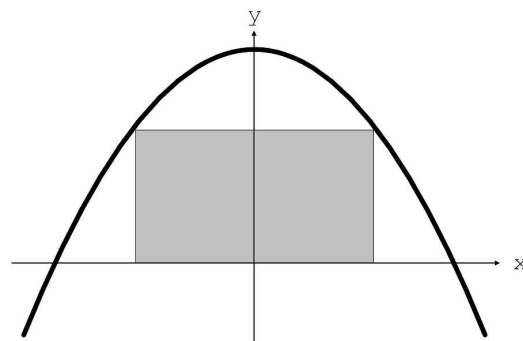
The dimensions of the rectangle of maximum area are: _____

The rectangle has maximum area because: _____

_____

_____

4. Let $f'(x) = -2xe^{-x^2}$ be the derivative of the function $f(x)$.

(a) Find the x-coordinate(s) of all critical points of the function $f(x)$ and classify each critical point as a relative maximum, a relative minimum, or neither. Explain yor answer.

x-coordinate(s) and type of extremum: _____

Because: _____

_____

(b) Find the x-coordinate(s) of all the inflection points of the function $f(x)$. Justify why they are indeed inflection points.

The inflection points have x-coordinate(s): _____

They are indeed inflection points because: _____

_____

_____

5. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (x^2 + 3)^{\ln x}$, $x > 0$. Write your answer in terms of x alone.

$\dfrac{dy}{dx} =$ _____

6. Find the function whose tangent has slope $\dfrac{\ln(3x)}{x}$ for each value of $x > 0$ and whose graph passes through the point $(e, 2)$.

The function is _____

7. Evaluate the integral $\displaystyle \int \frac{1-x}{\sqrt{x^2-2x+5}}\, dx$

Answer: _____

8. Evaluate the definite integral $\displaystyle \int_1^e \left(\frac{x}{2} + x\ln x\right) dx$

Answer: _____

9. The profit obtained by producing x units of product A and y units of product B is given by the function

$$p(x,y) = 8x + 10y - \frac{1}{100}(x^2 + xy + y^2) + 500.$$

Find the values of x and y that produce the maximum profit. Justify that your answer is indeed a maximum.

$x =$_____ $y =$_____

Justification: _____

_____

_____

10. Evaluate the double integral $\displaystyle \iint_R \frac{1}{x^3} e^{y/x}\, dA,$ where R is the region given by:

$1 \le x \le 2$ and $0 \le y \le 1$. Choose the order of integration carefully.

Answer: _____