MATH 118   **FINAL EXAM**   Spring 2002

Last Name: _____   **First Name:** _____

ID Number: _____   **Signature:** _____

Circle your Professor's name:

Arratia   Proskurowski   Saric   Schumitzky   Verona   Von Bremen   Woo

Circle your discussion time:

TuTh 8:00   TuTh 9:00   TuTh 10:00   TuTh 11:00   TuTh 12:00   TuTh 1:00

TuTh 2:00   TuTh 3:00   TuTh 4:00   MW 2:00   MW 3:00   MW 4:00

Directions

You must **show all your work and justify your methods** to obtain full credit. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, e^3, $\sqrt{3}$. Do not use scratch paper. Use the back of the previous page if additional room is needed. **Neither notes nor calculators are allowed**. During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in "silent" mode. Remember, USC considers cheating to be a very serious issue. The exam is worth a total of 200 points.

Problem	Max score	Your score	Problem	Max score	Your score
1	25		5	25	
2	25		6	25	
3	25		7	25	
4	25		8	25	

Total score:

1. At a certain factory, the daily output is $Q(K) = 600\sqrt{K}$ units, where K denotes the capital investment measured in units of $1,000. The current capital investment is $900,000. Use calculus to **estimate** the effect that an additional capital investment of $800 will have on the daily output.

Answer:————————————————————————————————————

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2. Find an equation of the tangent line to the graph of

$$x^2 + 3y^2 + e^y = 2 + 2y \quad \text{at the point} \quad (1, 0).$$

An equation of the tangent line is _____

3. Given that $f'(x) = (x-2)^3 e^{5-x}$ and $f''(x) = -(x-5)(x-2)^2 e^{5-x}$ are the first and the second derivatives of the function $f(x)$, find:

(a) The x-coordinate(s) of all critical points of the function $f(x)$: _____

(b) The interval(s) on which the function $f(x)$ is increasing: _____

(c) The x-coordinate(s) of all the relative extrema of $f(x)$. Specify for each x-coordinate the type of relative extremum:

_____

(d) The interval(s) on which the function $f(x)$ is concave down: _____

(e) The x-coordinate(s) of all the inflection points of the function $f(x)$: _____

Be sure to justify your answers with appropriate derivative tests in the space provided below.

4. Suppose your family owns a rare book whose value t years from now will be

$$V(t) = 200e^{0.6\sqrt{t}} \text{ dollars.}$$

If the prevailing interest rate remains constant at 5% per year compounded continuously, its present value $P(t)$ will be given by:

$$P(t) = 200e^{0.6\sqrt{t}-0.05t}$$

When should you sell the rare book to maximize its present value?

Answer:

5. Evaluate the following integrals:

(a) $\displaystyle \int \left(3 + \frac{1}{2\sqrt{x}} + \frac{1}{x} - 3e^{-x} \right) dx$

Answer: _____

(b) $\displaystyle \int \frac{\ln x}{x^2} dx$

Answer: _____

6. Find the area of the region bounded (enclosed) by the two curves:

$$y = x^2 - x \quad \text{and} \quad y = 2x - 2.$$

The area is: _____

7. A firm produces two products, A and B, and the cost to produce x units of A and y units of B is: $\quad C(x, y) = 1000 + 13x^2 - 2xy + 2y^2 + 67y$.

The firm can sell A for 76 dollars apiece, B for 65 dollars apiece.

(a) Write down the revenue function, $R(x, y)$.

$R(x, y) =$ _____

(b) The profit function is: $\quad P(x, y) = 76x - 2y - 1000 - 13x^2 + 2xy - 2y^2$.

How much of each product should the firm produce, in order to maximize its profit? Be sure to justify your answer with appropriate derivative tests.

Answer: _____

8. Evaluate the double integral $\displaystyle\iint_R 2x^3 e^{yx^2}\, dA,$ where R is the region given by:

$1 \leq x \leq 2$ and $0 \leq y \leq 1$. Choose the order of integration carefully.

Answer: _____