Math 118 **Final Exam** **Spring, 2001**

Y. Bahturin, I. Han, F. Lin, E. Verona, Z. Vorel (in alphabetical order)

1. Consider the function

$$f(t) = \begin{cases} t^2 + 6, & \text{if } t < 5; \\ -8t + C, & \text{if } 5 \le t. \end{cases}$$

   (a) [5 points] Find the value of the constant C that makes the function $f(t)$ continuous on $(-\infty, \infty)$.

   (b) [5 points] Graph the function $f(t)$.

   (c) [5 points] Is the (resulting) continuous function $f(t)$ differentiable at all points in the interval $-\infty < t < \infty$? Explain.

2. [20 points] Use the definition to find the derivative of $f(x) = \dfrac{1}{3x^2}$. Then find an equation of the tangent line to the graph of f at $x = 1$.

3. (a) [12 points] Calculate the third derivative of $f(x) = \dfrac{1}{3x+2}$.

   (b) [3 points] Based on the pattern in the first three derivatives of the function in part (a), find an expression for the fiftieth derivative, $f^{(50)}(x)$.

4. Let $f'(x) = x(x-3)^2$ be the first derivative of the function $f(x)$.

   (a) [5 points] Find the interval(s) on which $f(x)$ is increasing.

   (b) [7 points] Find the x-coordinate(s) of the relative extrema, if any, of $f(x)$. Identify each extremum as a maximum or a minimum.

   (c) [8 points] Find the interval(s) on which the graph of $f(x)$ is concave down.

5. [20 points] A manufacturer can produce perfume bottles at a cost of \$8 apiece and estimates that if they are sold for x dollars apiece, consumers will buy approximately $2000e^{(2-0.1x)}$ perfume bottles each month. The profit is given by

$$P(x) = (x-8) \cdot 2000e^{(2-0.1x)} \quad .$$

   At what price should the manufacturer sell the perfume bottles to maximize profit? Be sure to explain why this corresponds to a maximum and not to a minimum.

6. [15 points] Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (\sqrt{x})^x$.

7. [15 points] Evaluate the definite integral:

$$\int_0^2 \frac{6x^3 - 3x}{\sqrt{x^4 - x^2 + 4}} dx$$

8. [20 points] Find the function whose tangent has slope $1 + \ln x$ for each value of $x > 0$ and whose graph passes through the point (1,−2).

9. (a) [5 points] Graph the curves $y = \dfrac{1}{2x}$ and $8y + x - 5 = 0$.

(b) [5 points] Find the x-coordinates of the intersection points of the curves in part (a).

(c) [10 points] Find the area of the region bounded by the curves in part (a).

10. [20 points] Three towns want to build a shopping mall to serve their inhabitants, in such a place that the sum of the squares of the distances from the mall to the centers of the respective towns is minimal. If the coordinates of the centers of the towns on the rectangular map grid are (5,1), (−3,2), and (1,−5), where the units are in miles, find the coordinates of the mall.

11. [20 points] Evaluate $\int_{-1}^{2} \int_{0}^{1} 3t^2 e^{2st} \, ds \, dt$.