RESEARCH SPOTLIGHTS

The current issue of *SIAM Review* features the Research Spotlights article "Random Walks on Simplicial Complexes and the Normalized Hodge 1-Laplacian," coauthored by Michael T. Schaub, Austin R. Benson, Paul Horn, Gabor Lippner, and Ali Jadbabaie. Simplicial complexes (SCs) allow one to model multimode relationships in a way that standard graph models, which hinge on pairwise relationships between nodes, do not. Consider that a k-simplex is a subset of a finite set of vertices of cardinality $k + 1$. Whereas a standard undirected graph consists of nodes and edges (essentially, 0- and 1-simplices), an SC can carry more information: it is a set of simplices with the property that for any element of the set, its subsets are also among the elements in the set. A general SC can therefore contain information about k-simplices for $k > 1$. While SCs have the ability to model more complex relationships, such as biological interactions that occur between sets of molecules, the authors claim that, until now, little has been done to generalize the notion of Laplacian dynamics relative to the underlying graph to SCs.

To this end, the authors therefore propose generalizing the notion of the graph Laplacian by defining an appropriately normalized Hodge Laplacian for the SC and then relating it to a random walk on edges. The normalized Hodge Laplacian under this interpretation can be used to extract information about a simplicial complex in a way that is reminiscent of graph-Laplacian-based analyses for graphs. Interestingly, the correct normalization requires "lifting" to a higher dimensional space.

Although the article involves digesting what may for many readers be several new definitions and concepts, the table of contents, exposition, and illustrated examples aim to make the article accessible to all. The treatment of the topic is comprehensive, including discussion of how to construct SCs as well as associated computational costs. About a third of the main body of the article is devoted to demonstration of their approach on applications. Readers will find promptings for future research projects scattered throughout the article.

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