Formal Abstractions for Packet Scheduling

Mohan, Liu, Foster, Kappé, Kozen

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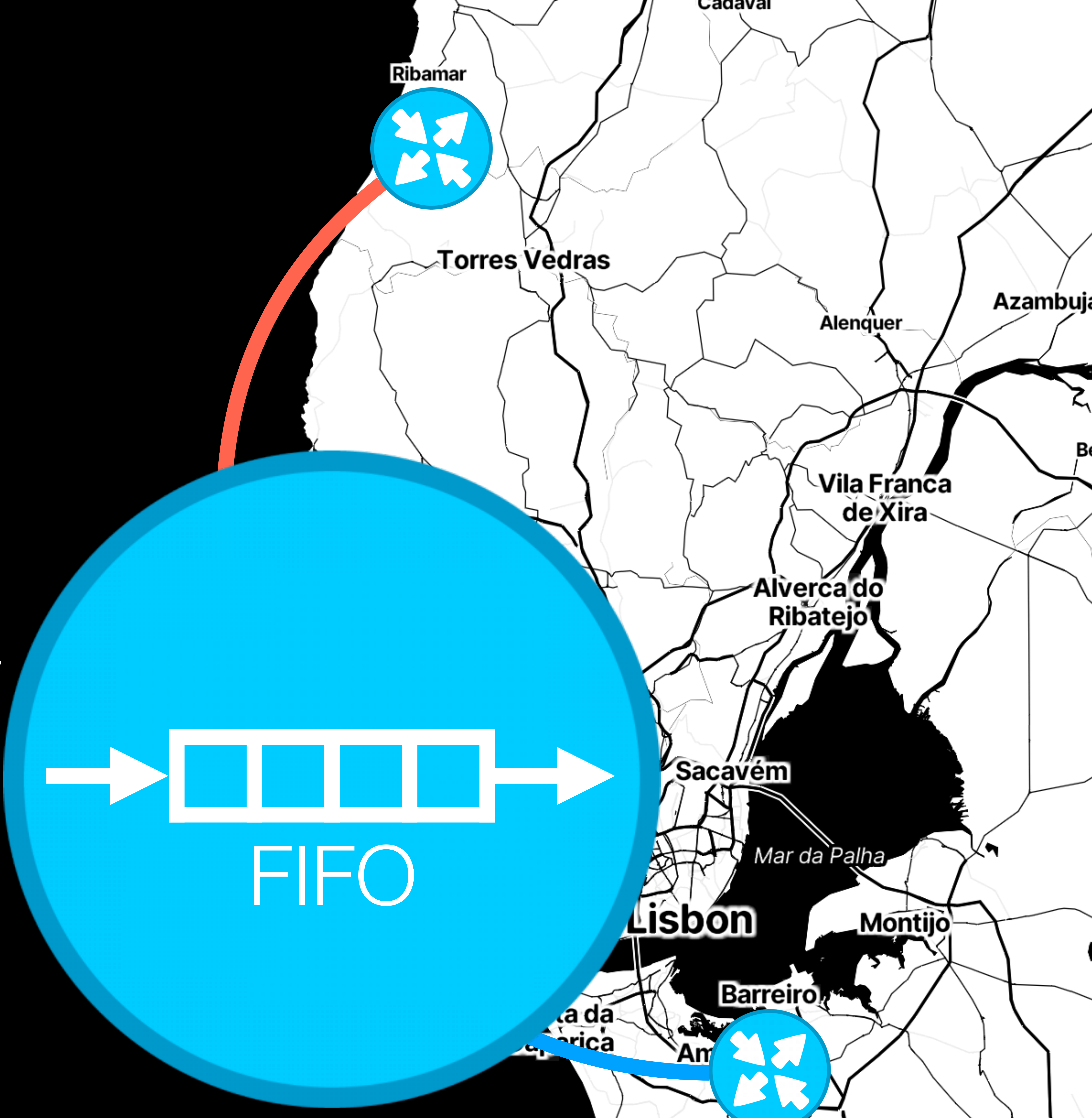

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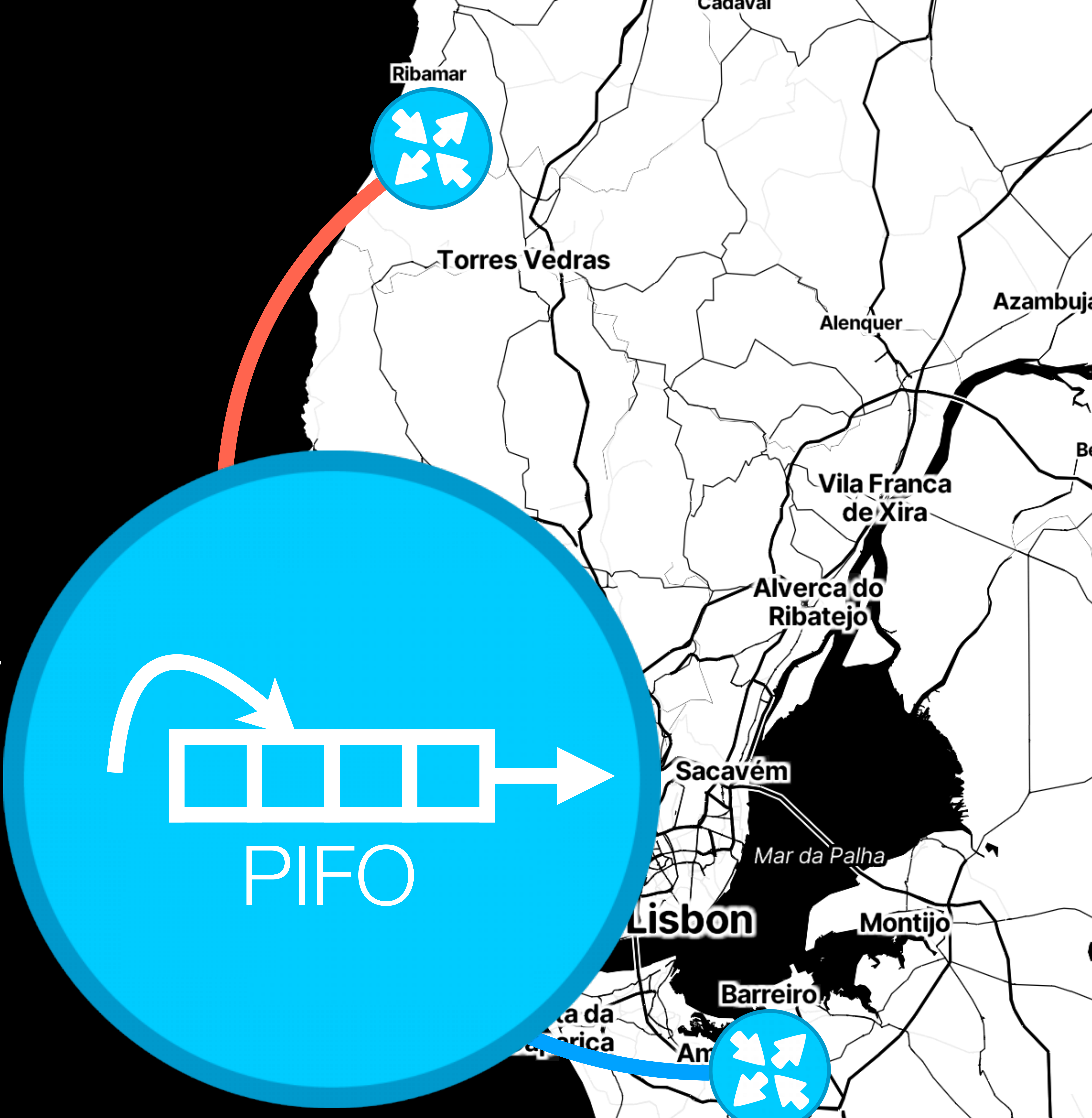

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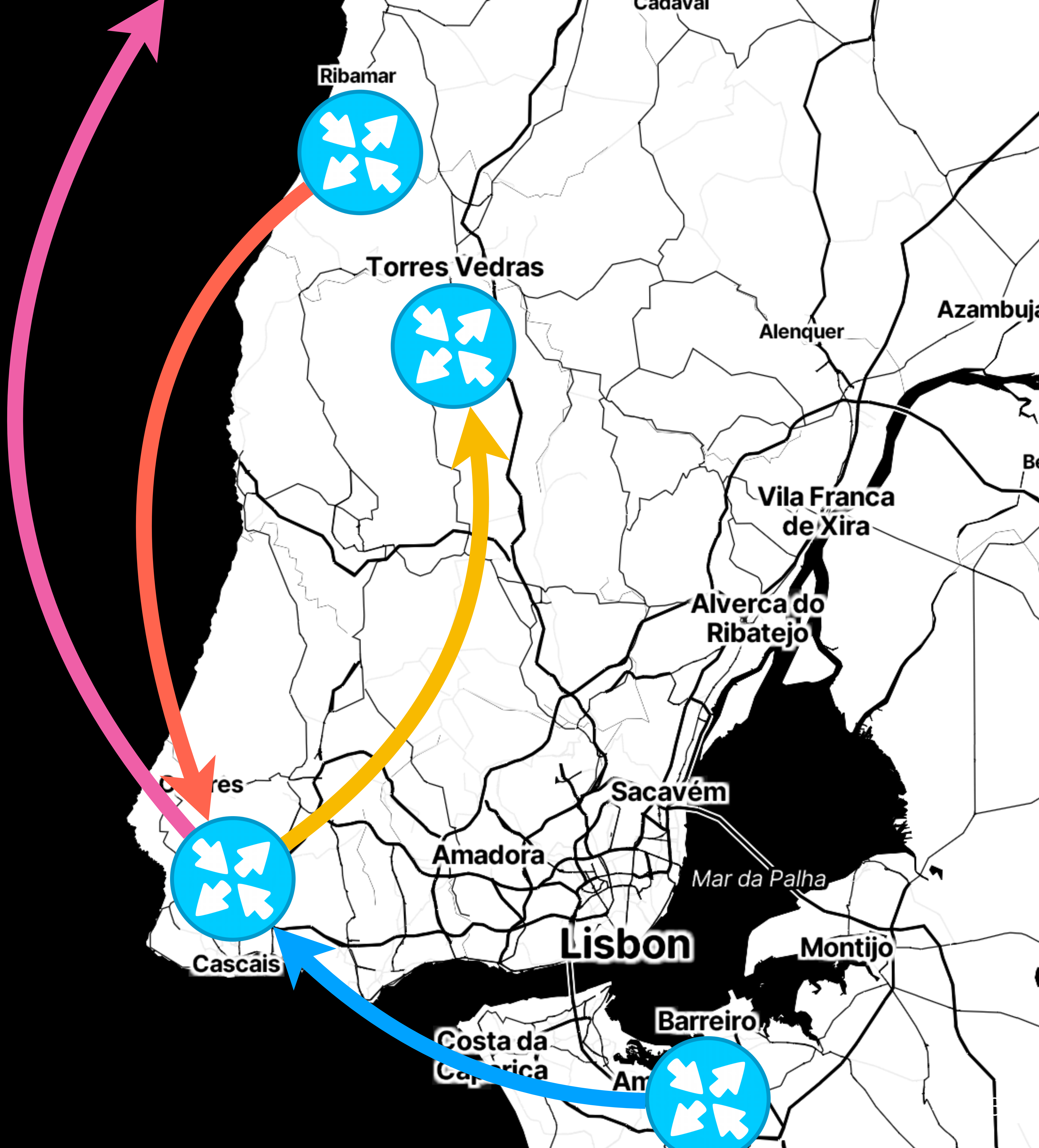

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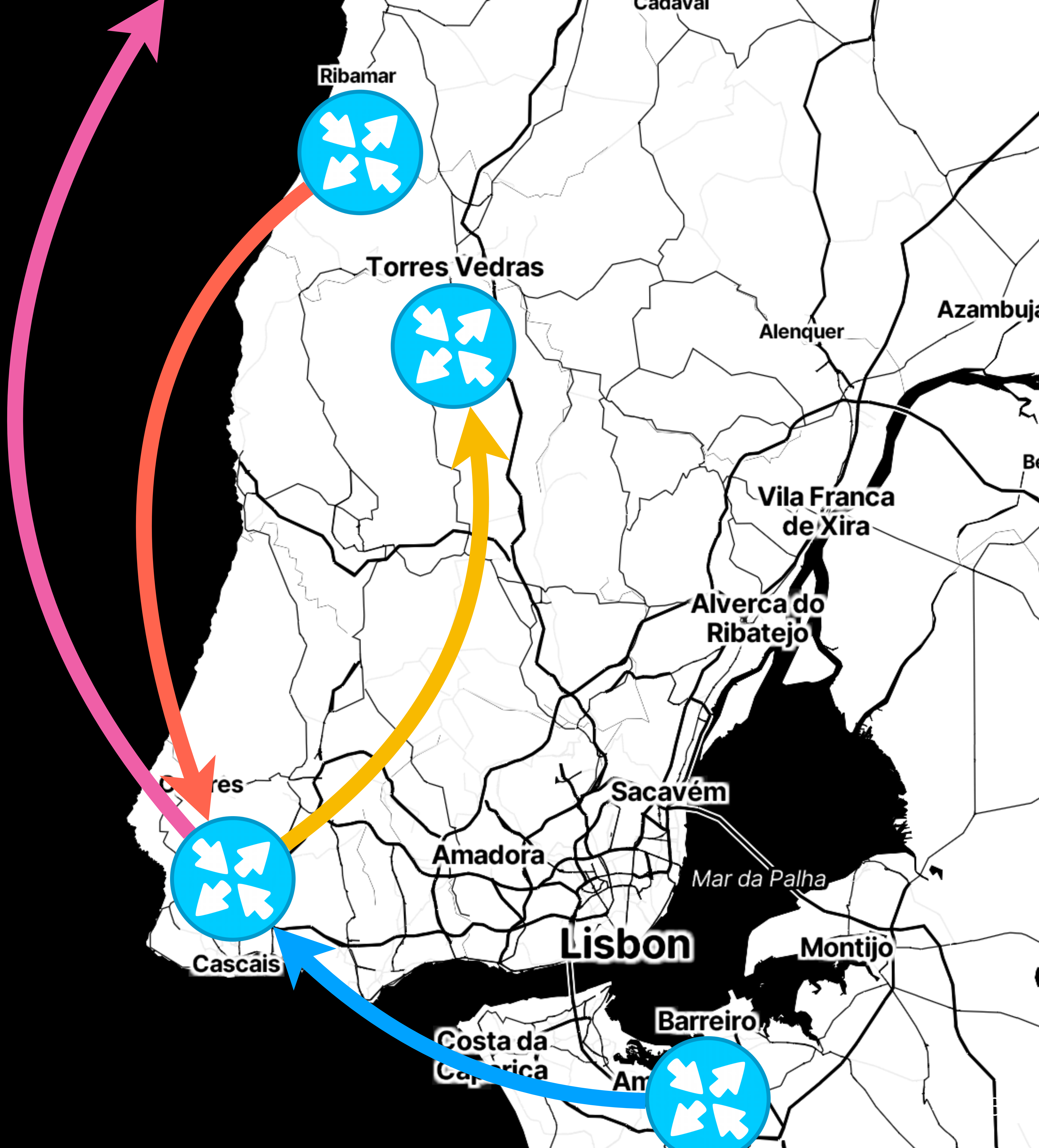

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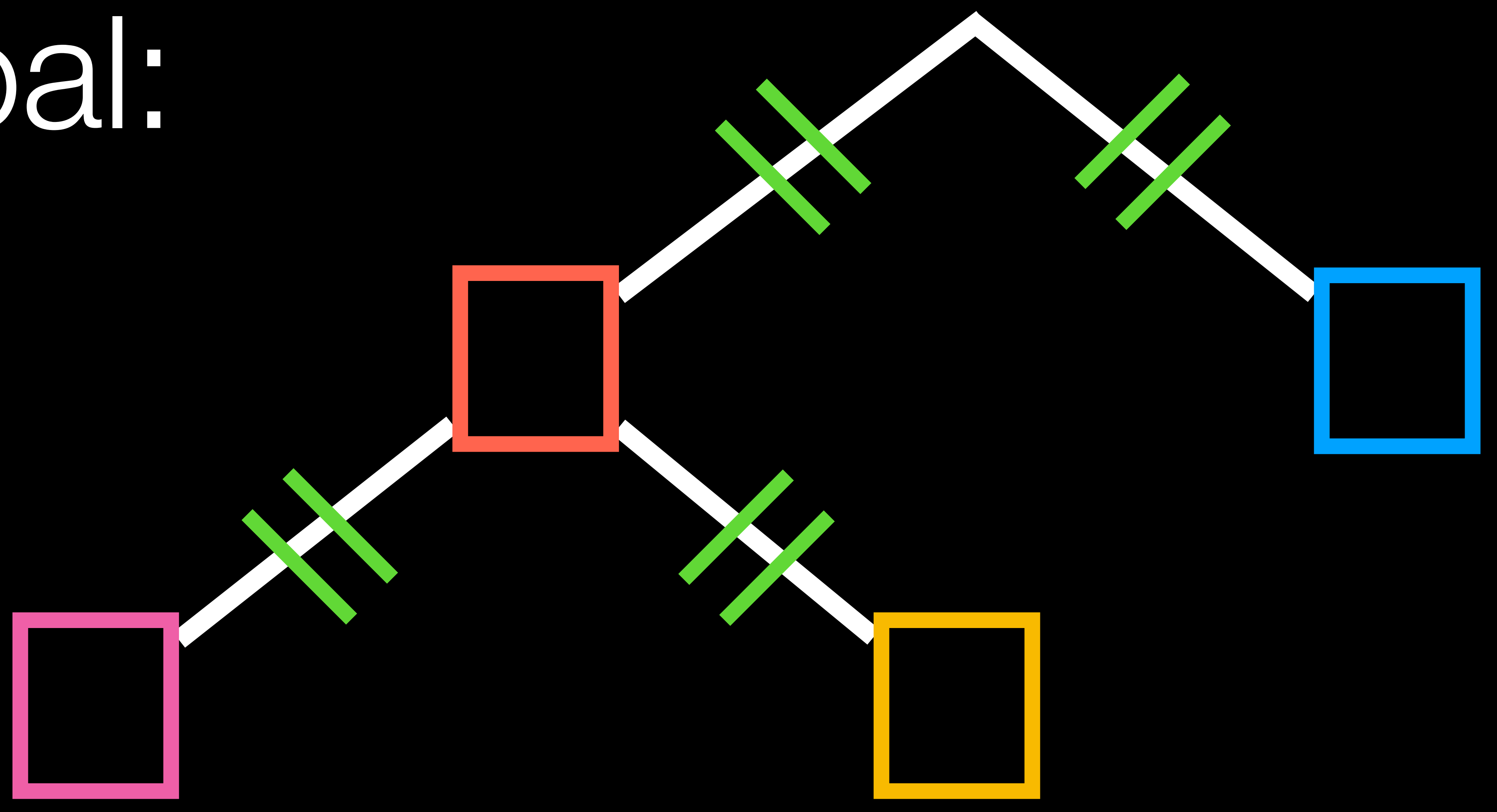

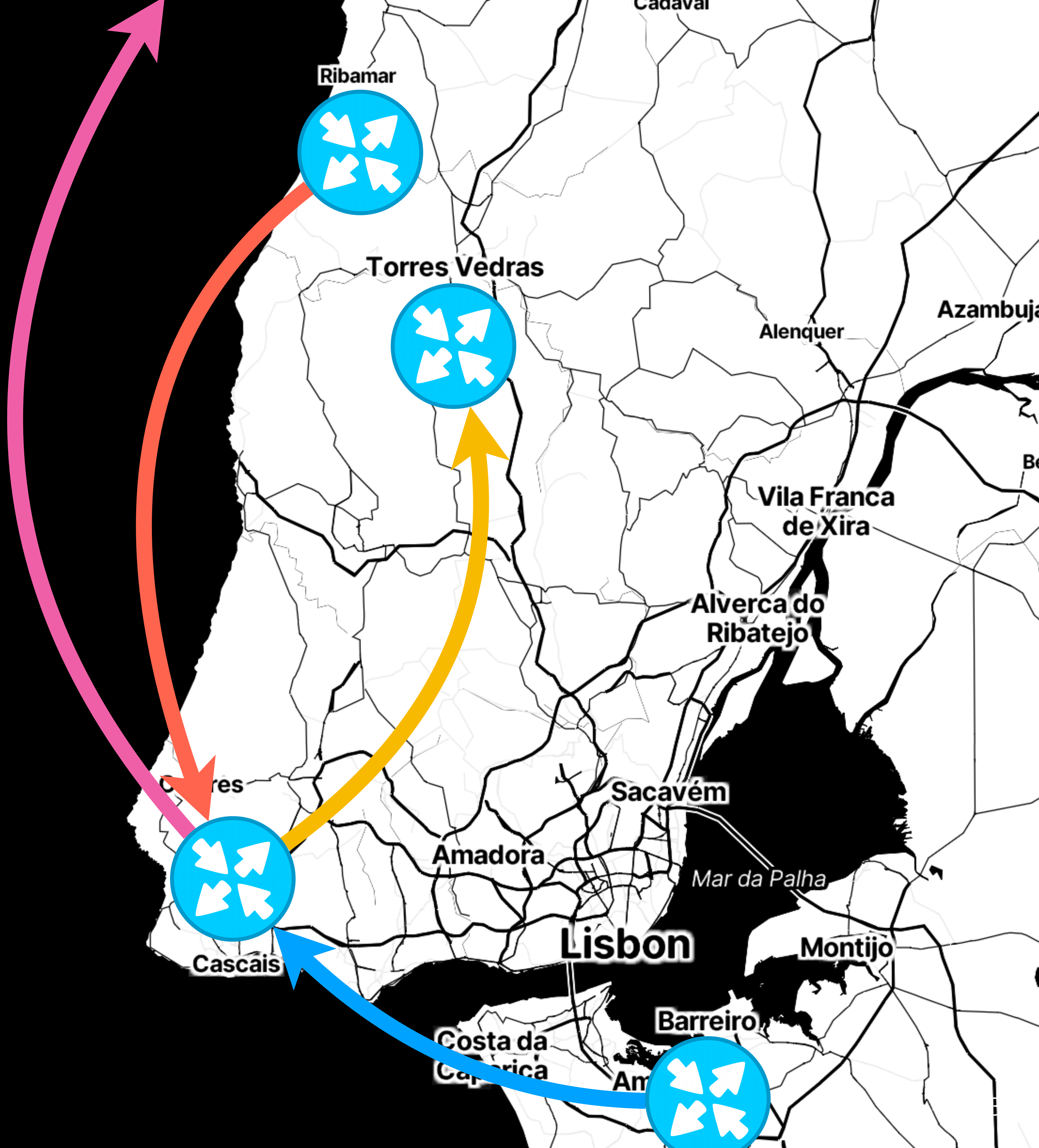

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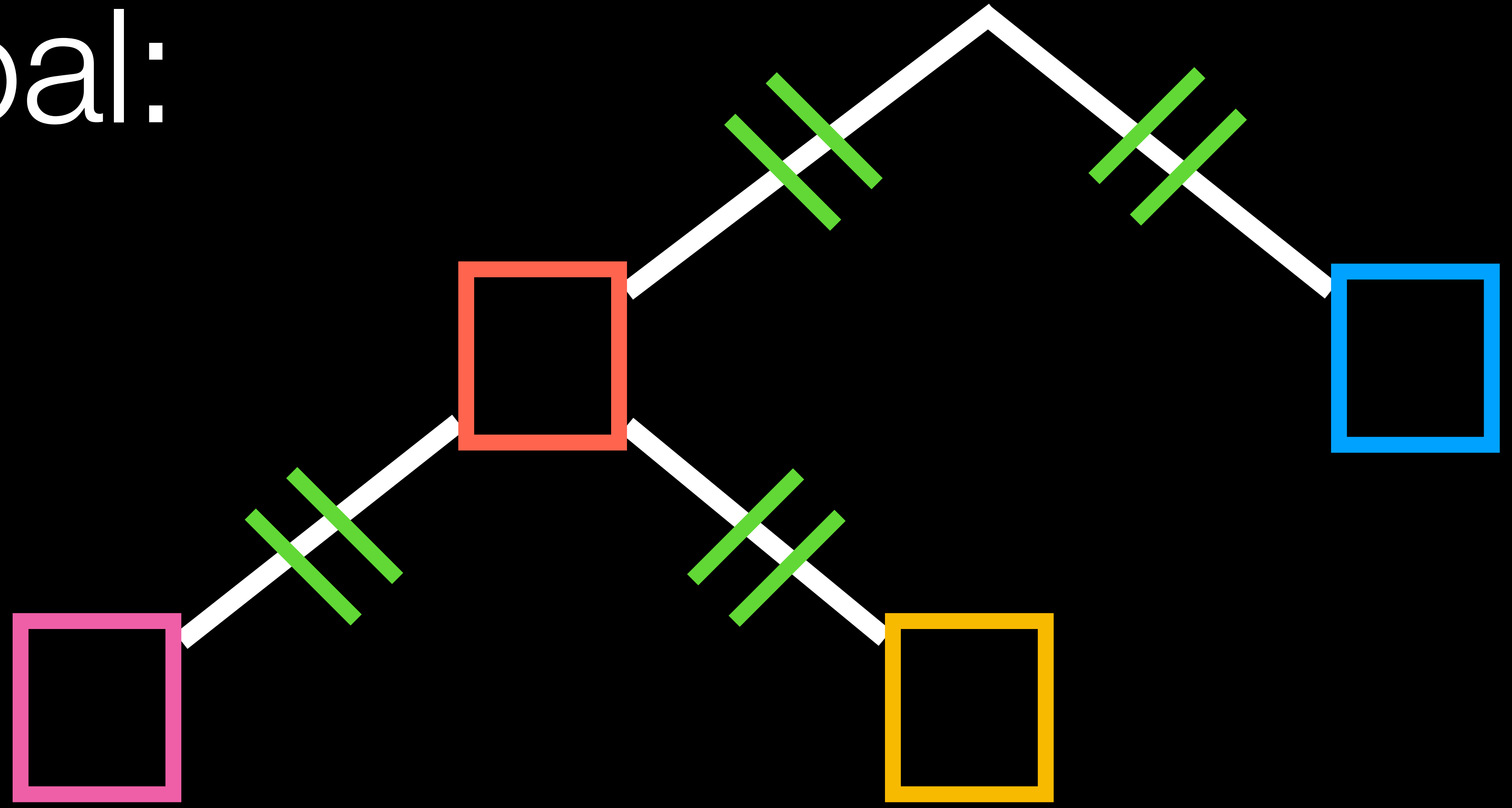
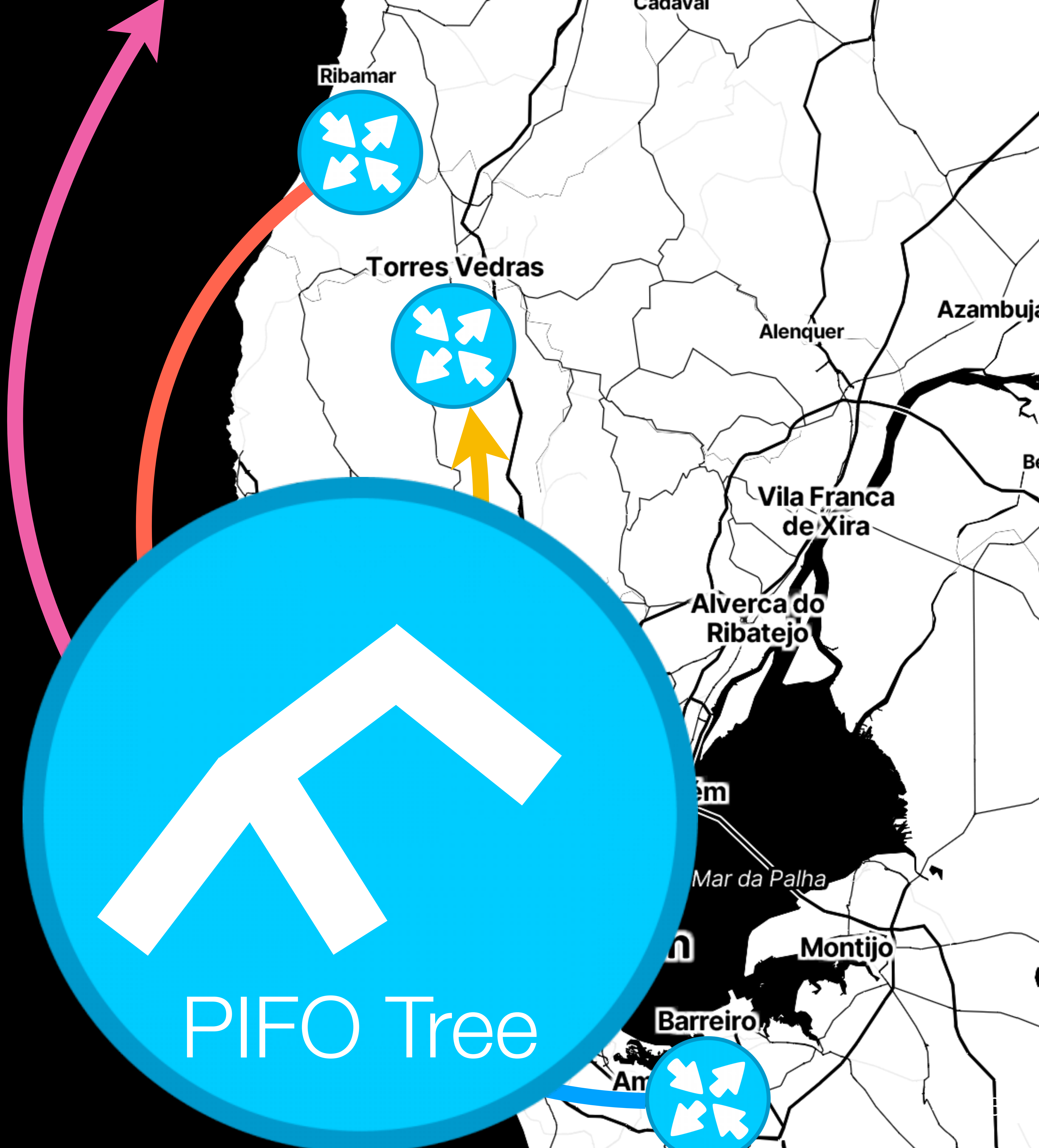
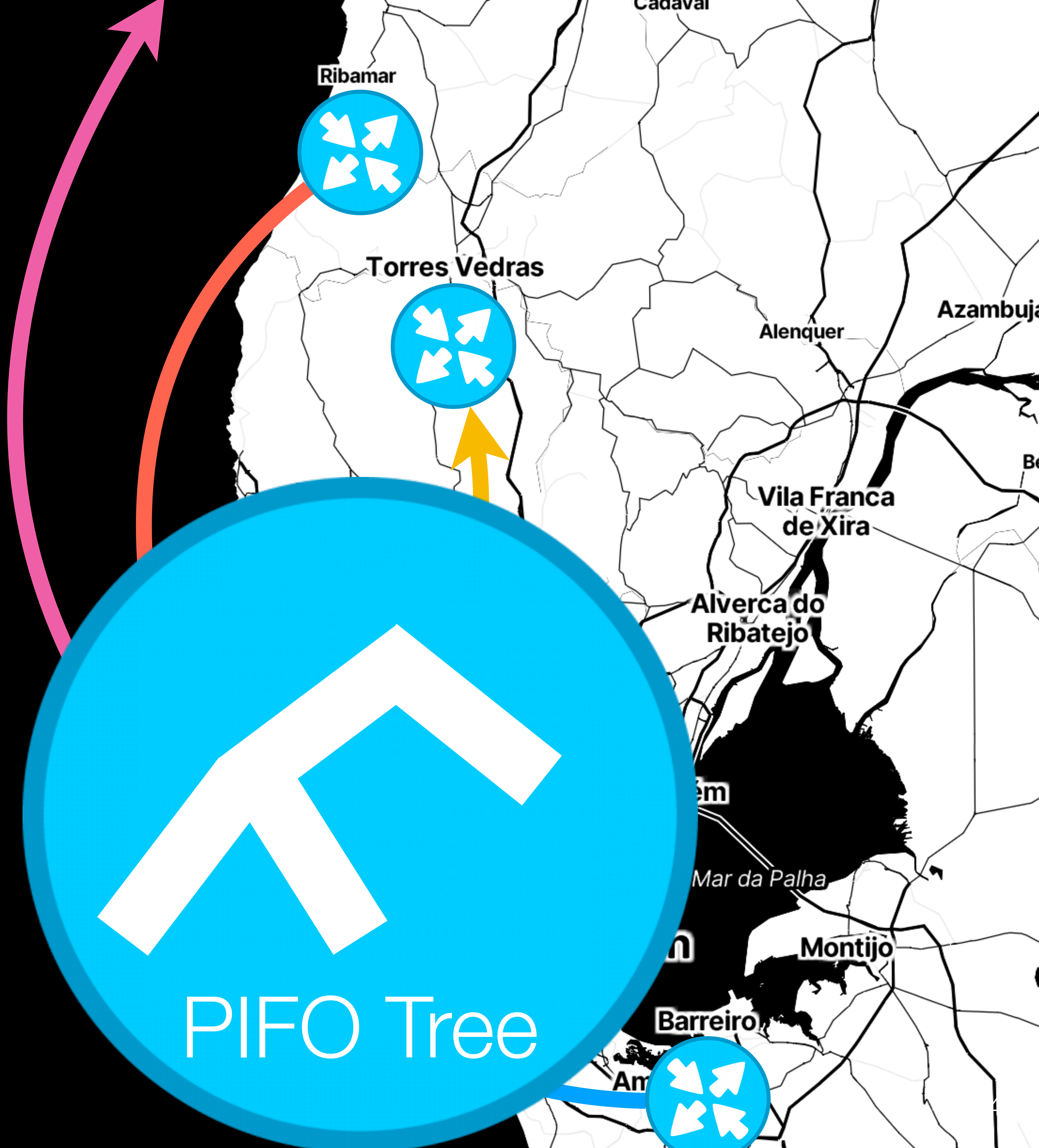




New plan!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Torres Vedras
Alenquer
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Mar da Palha
Montijo
Barreiro
PIFO Tree

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Interleave
small, medium, and large
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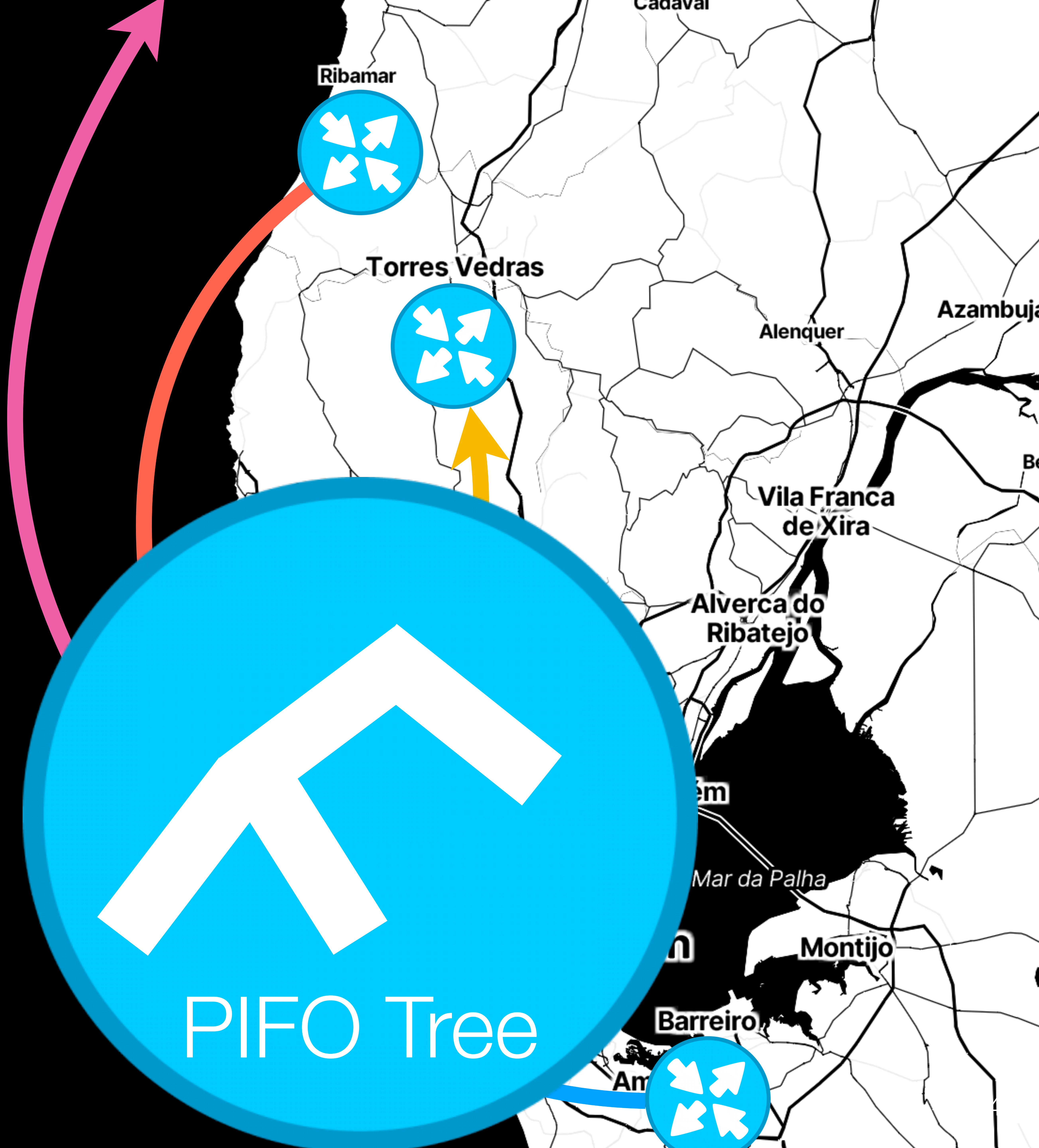

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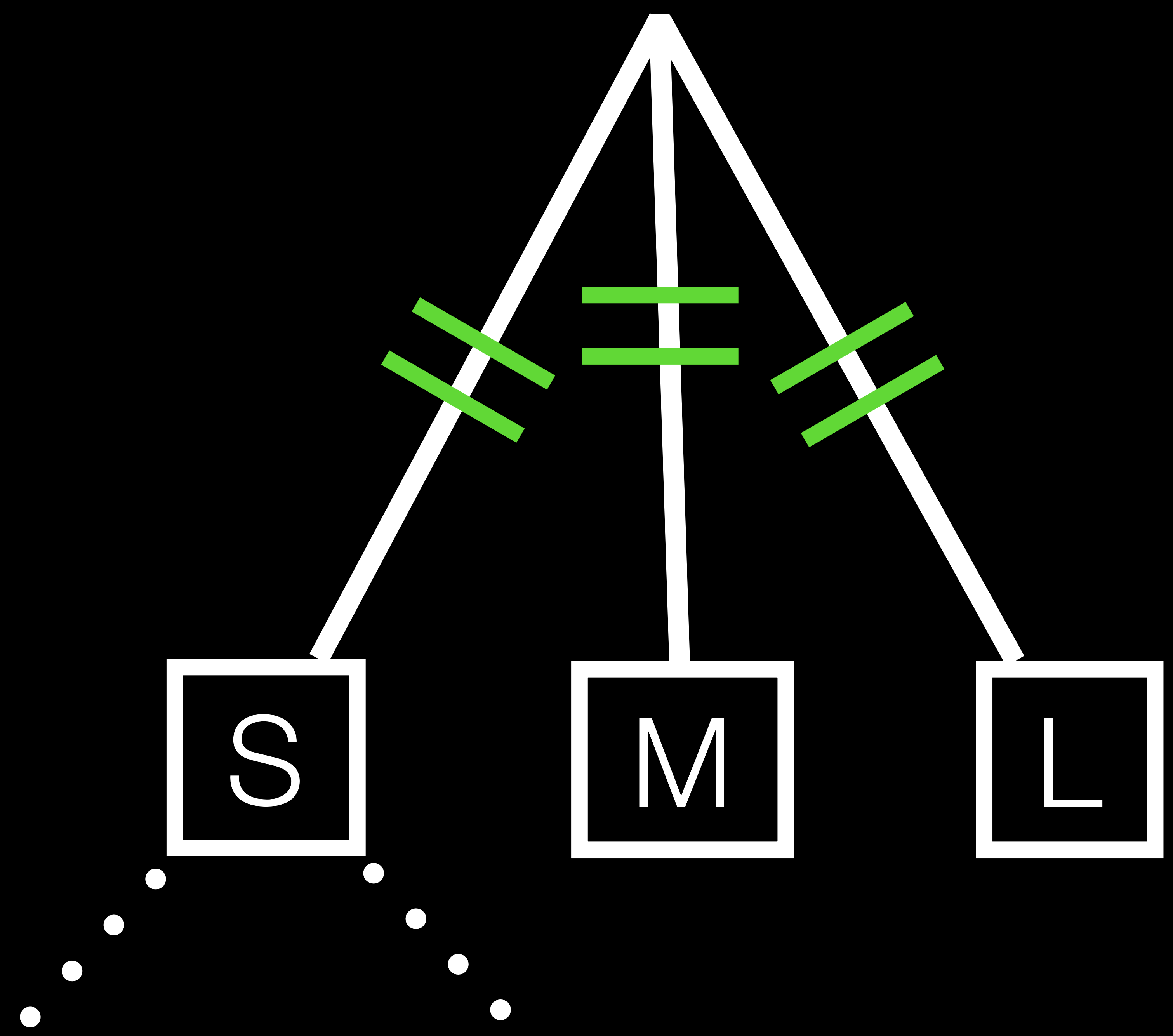

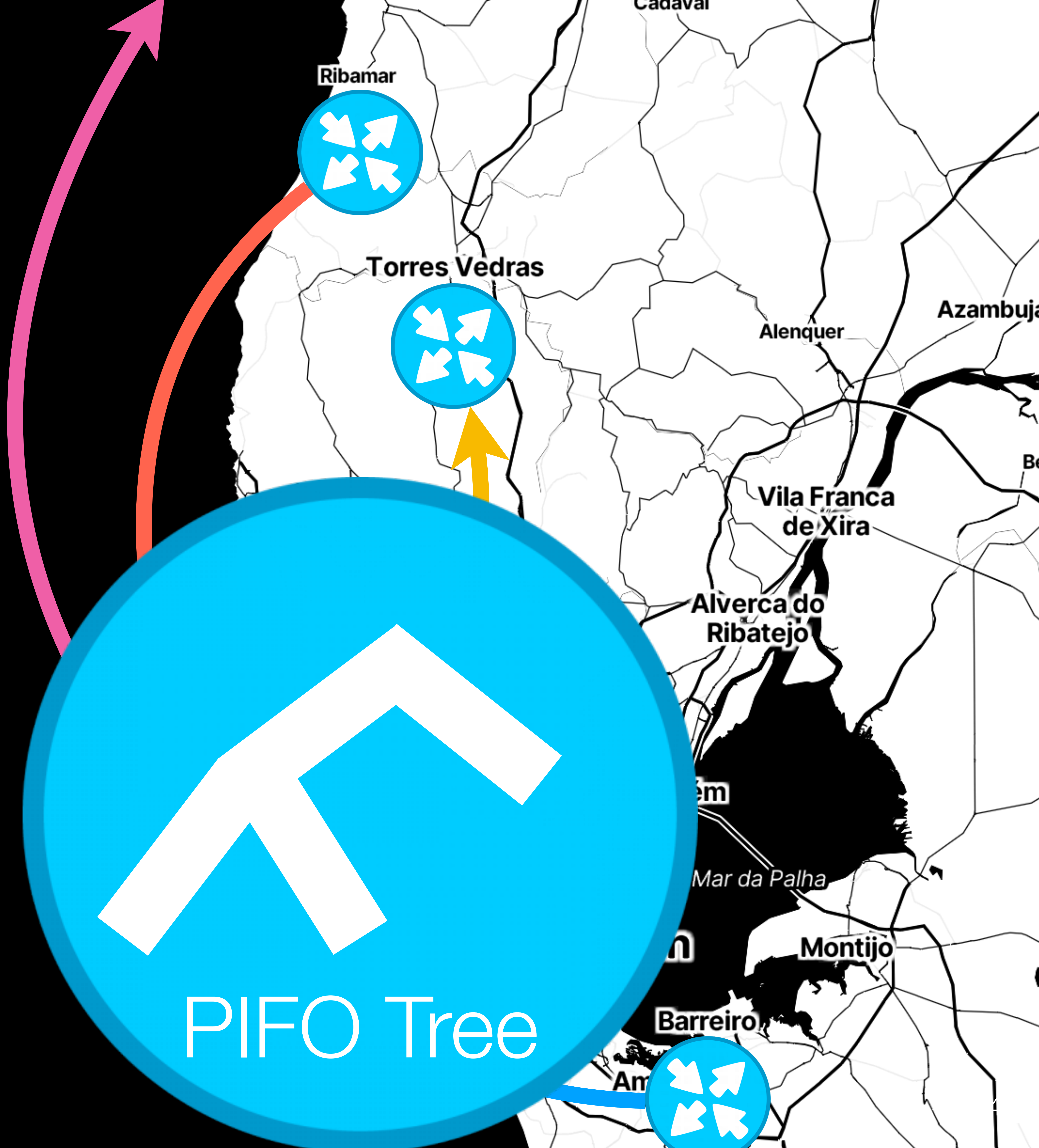

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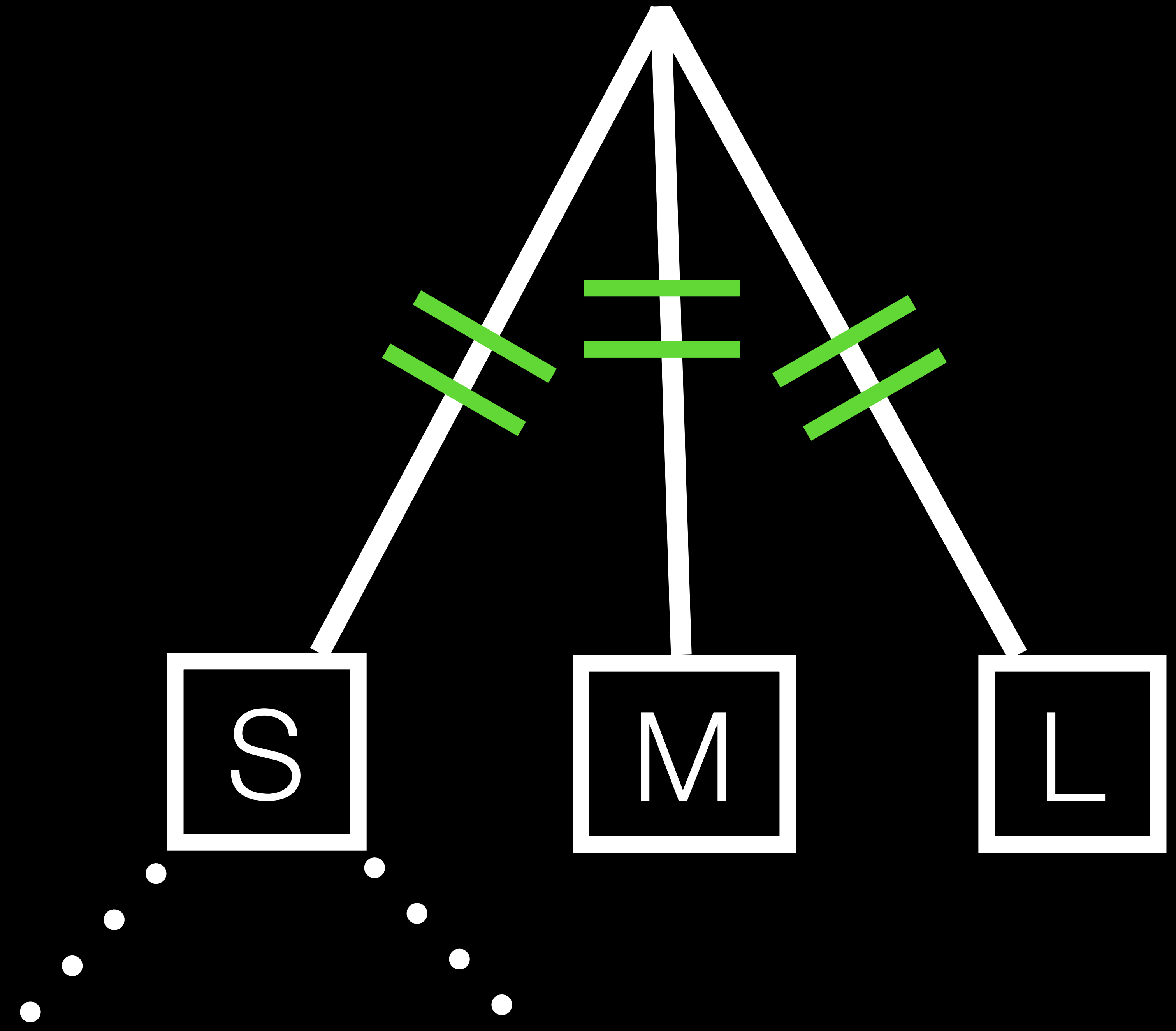

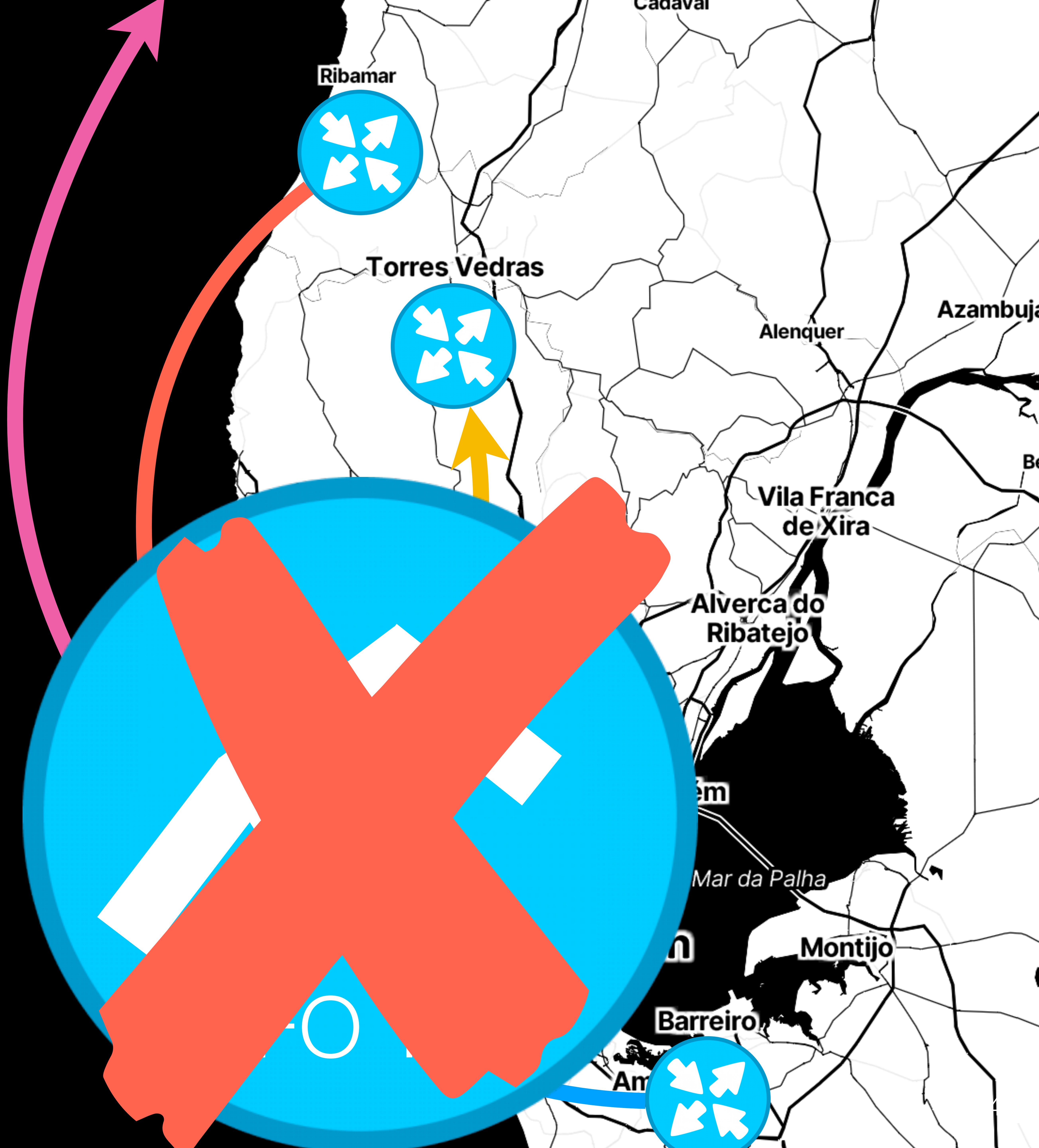

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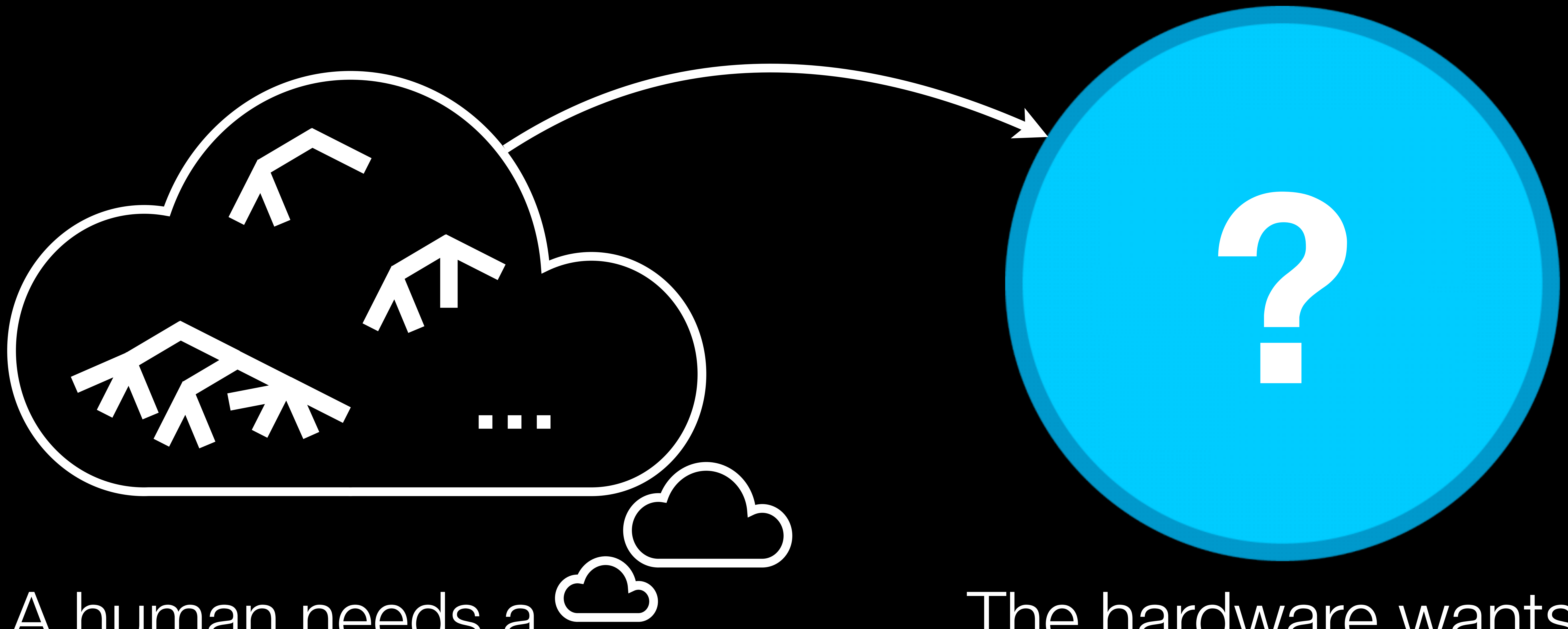

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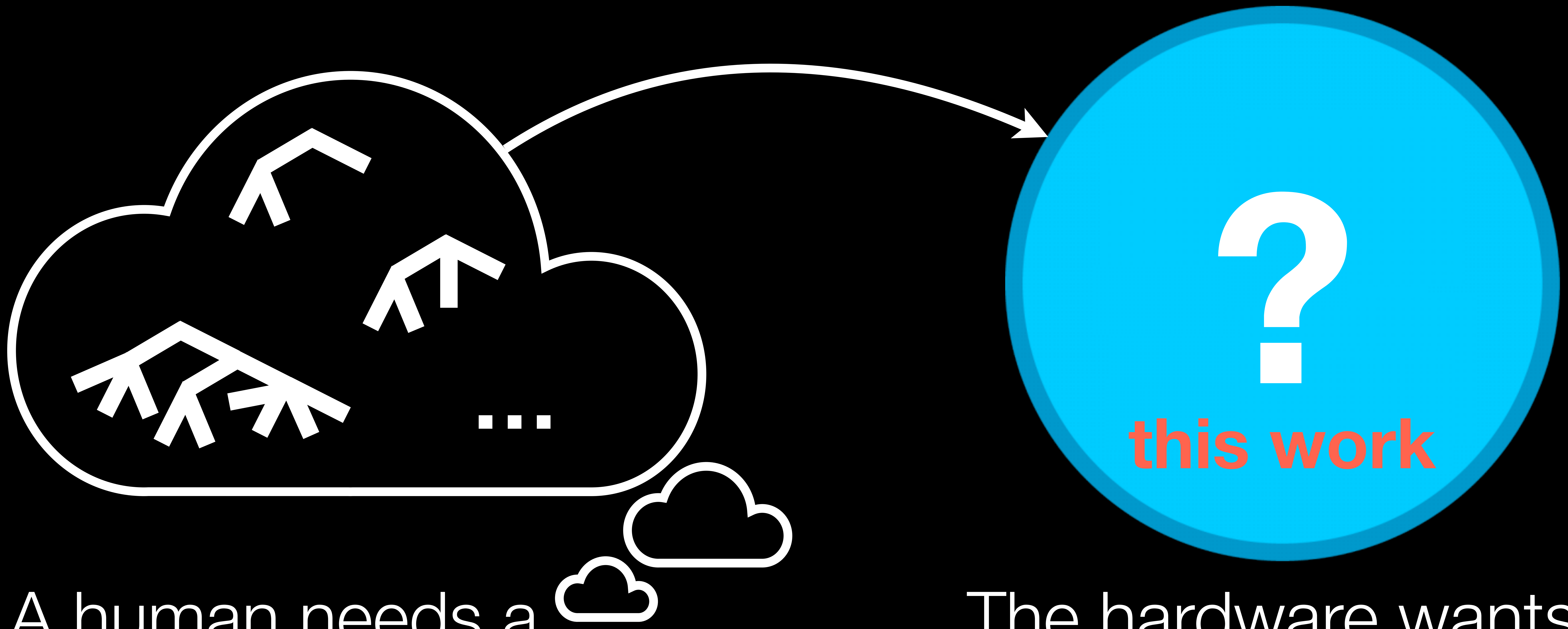

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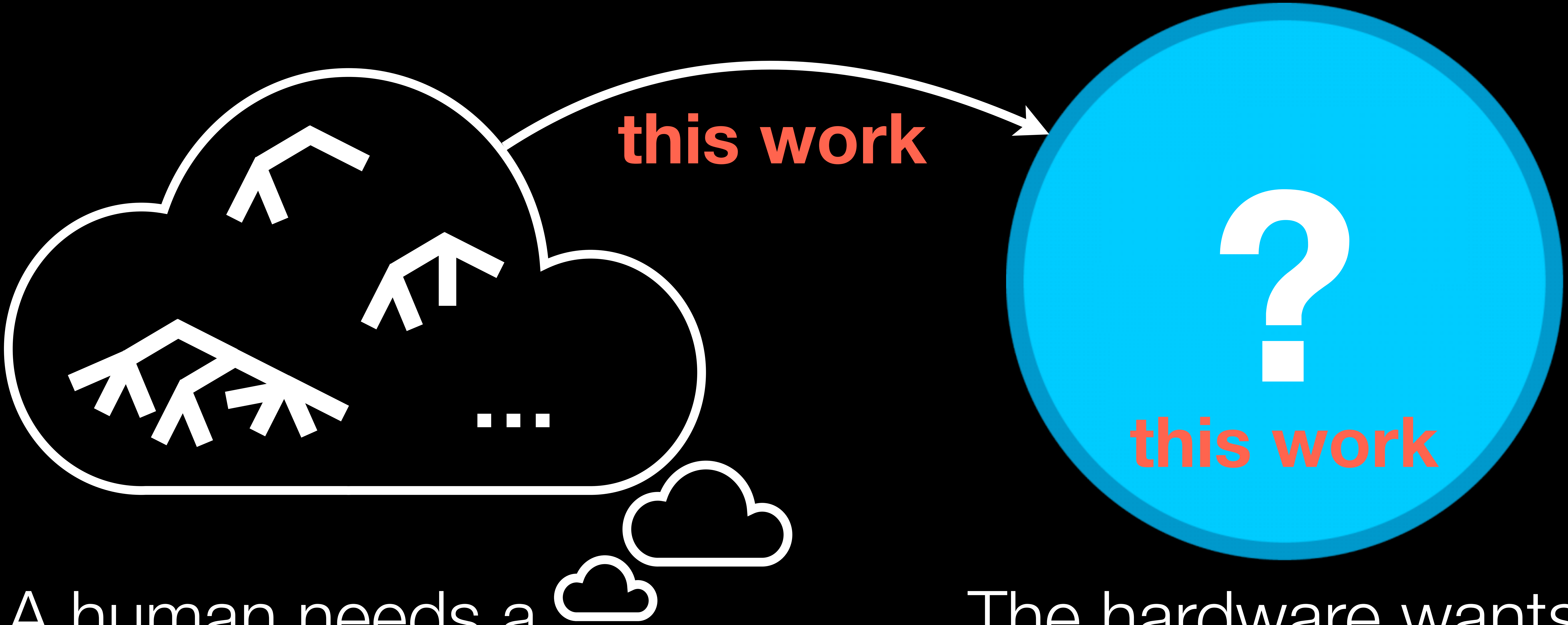

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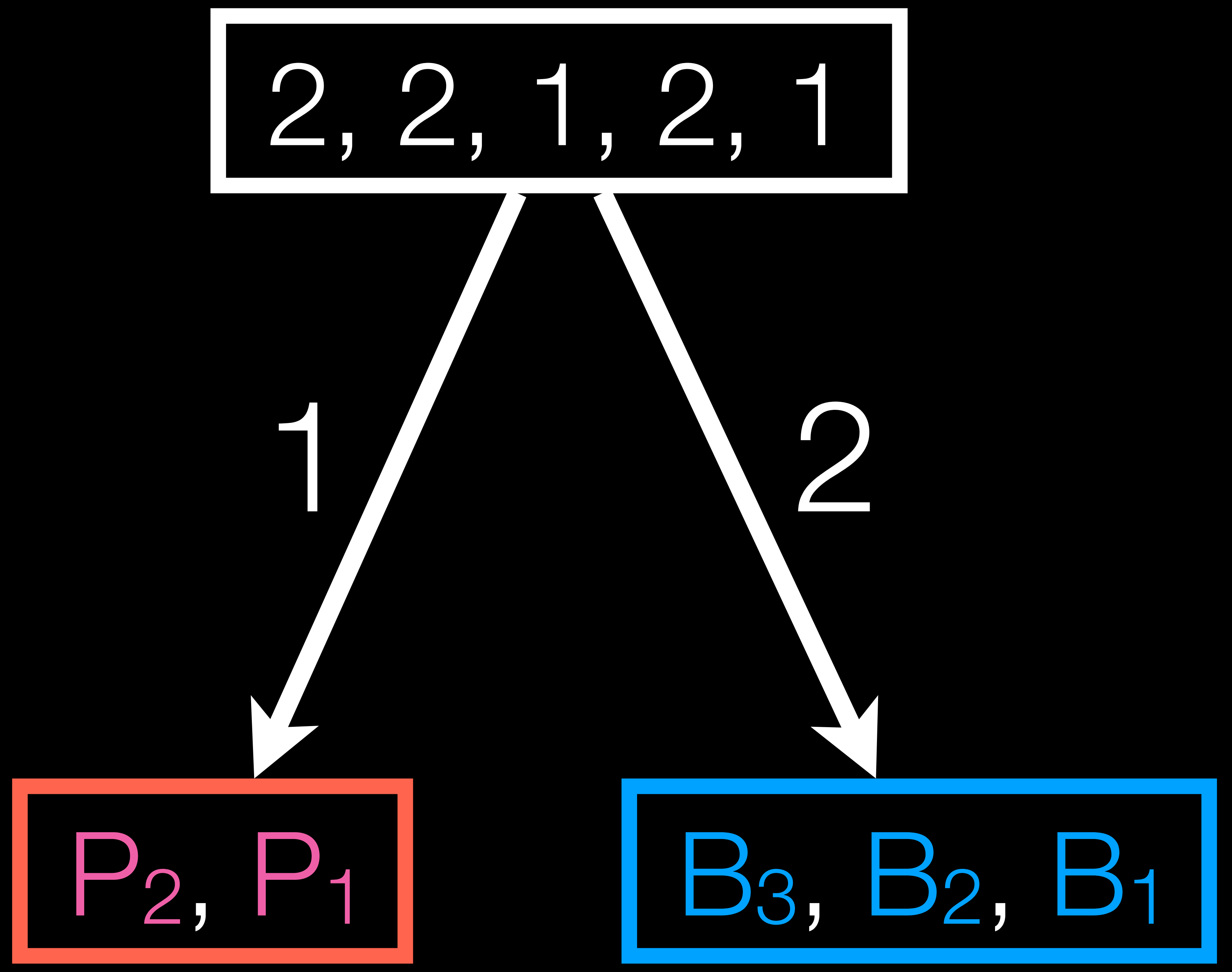

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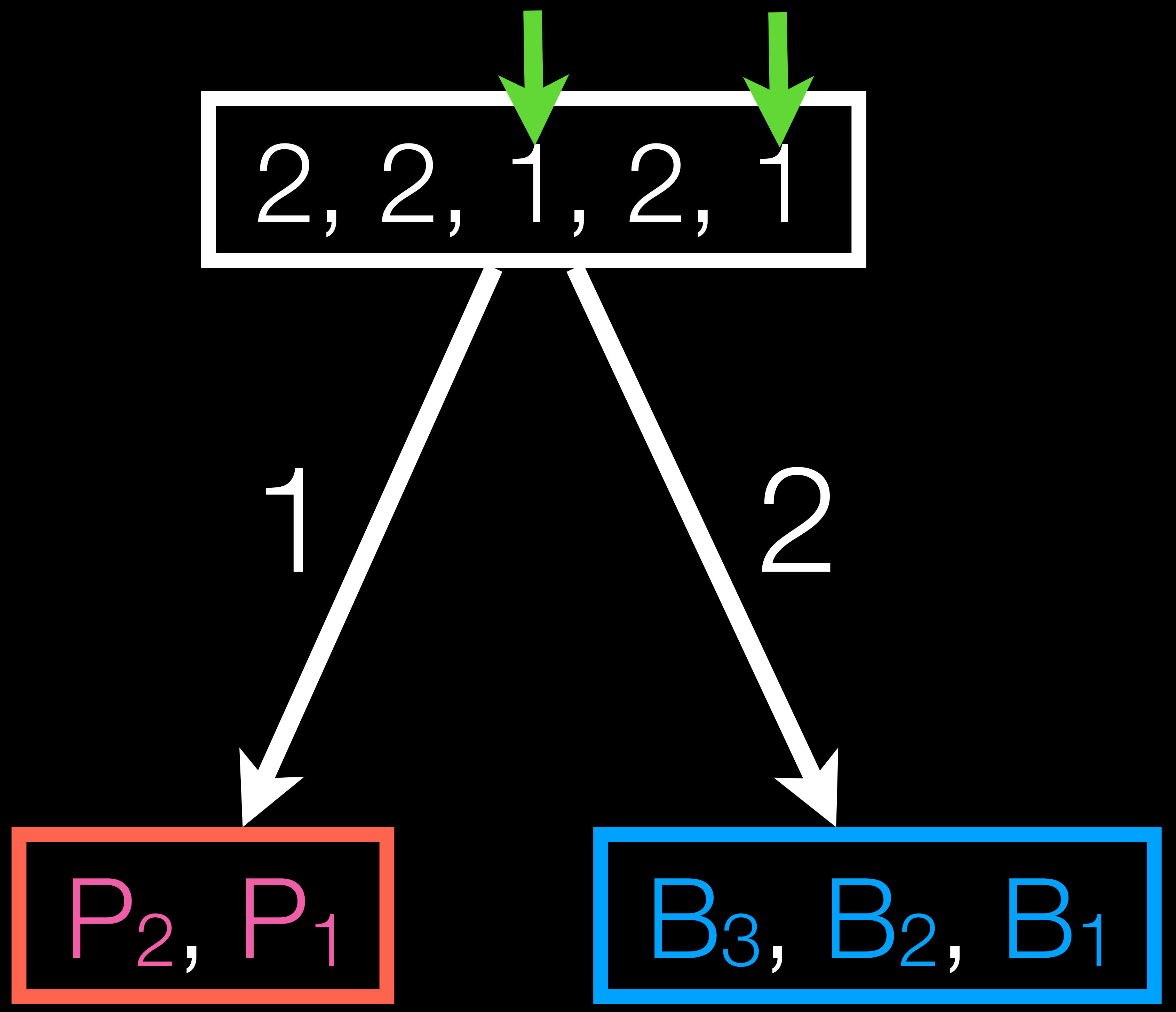

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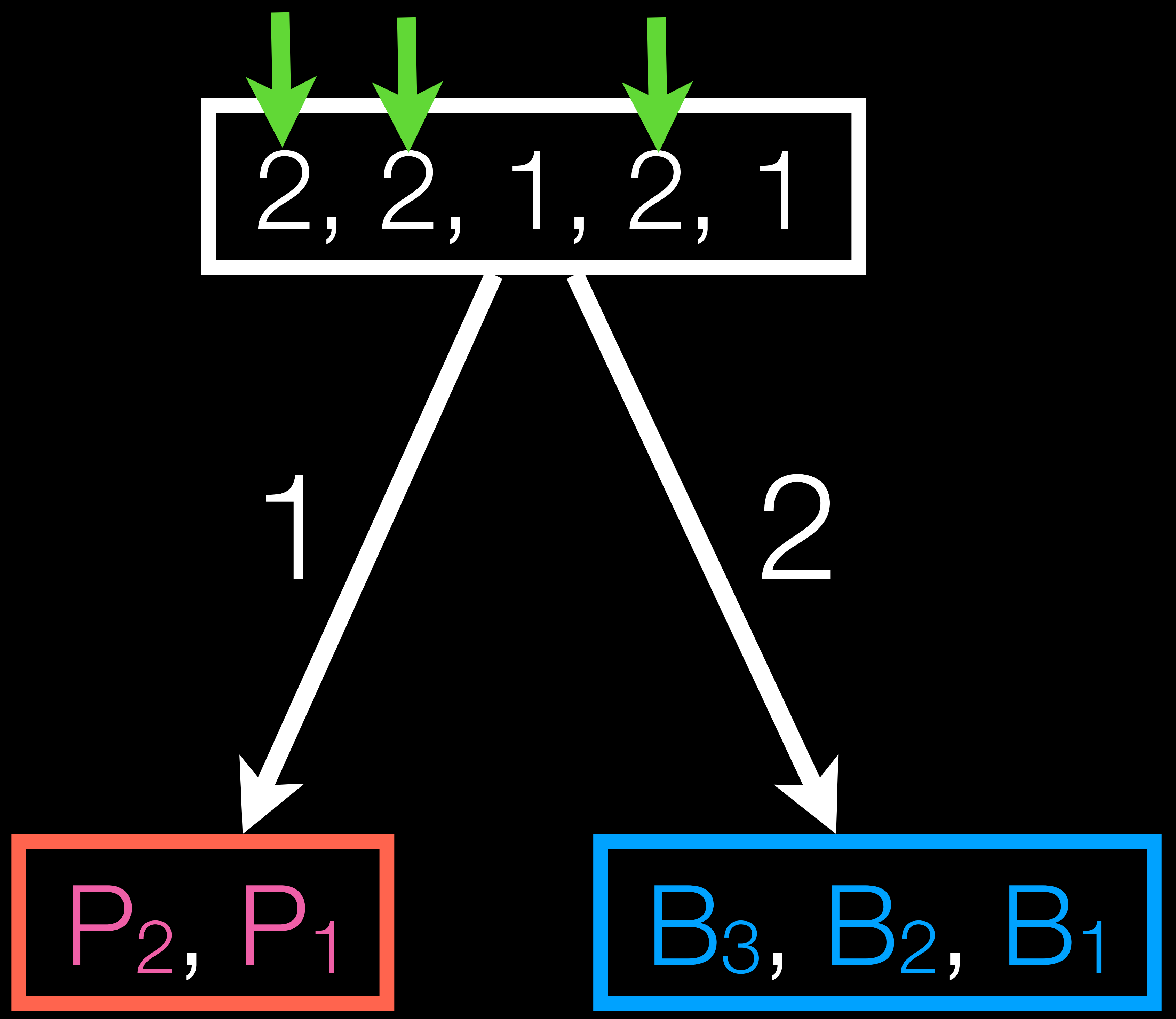

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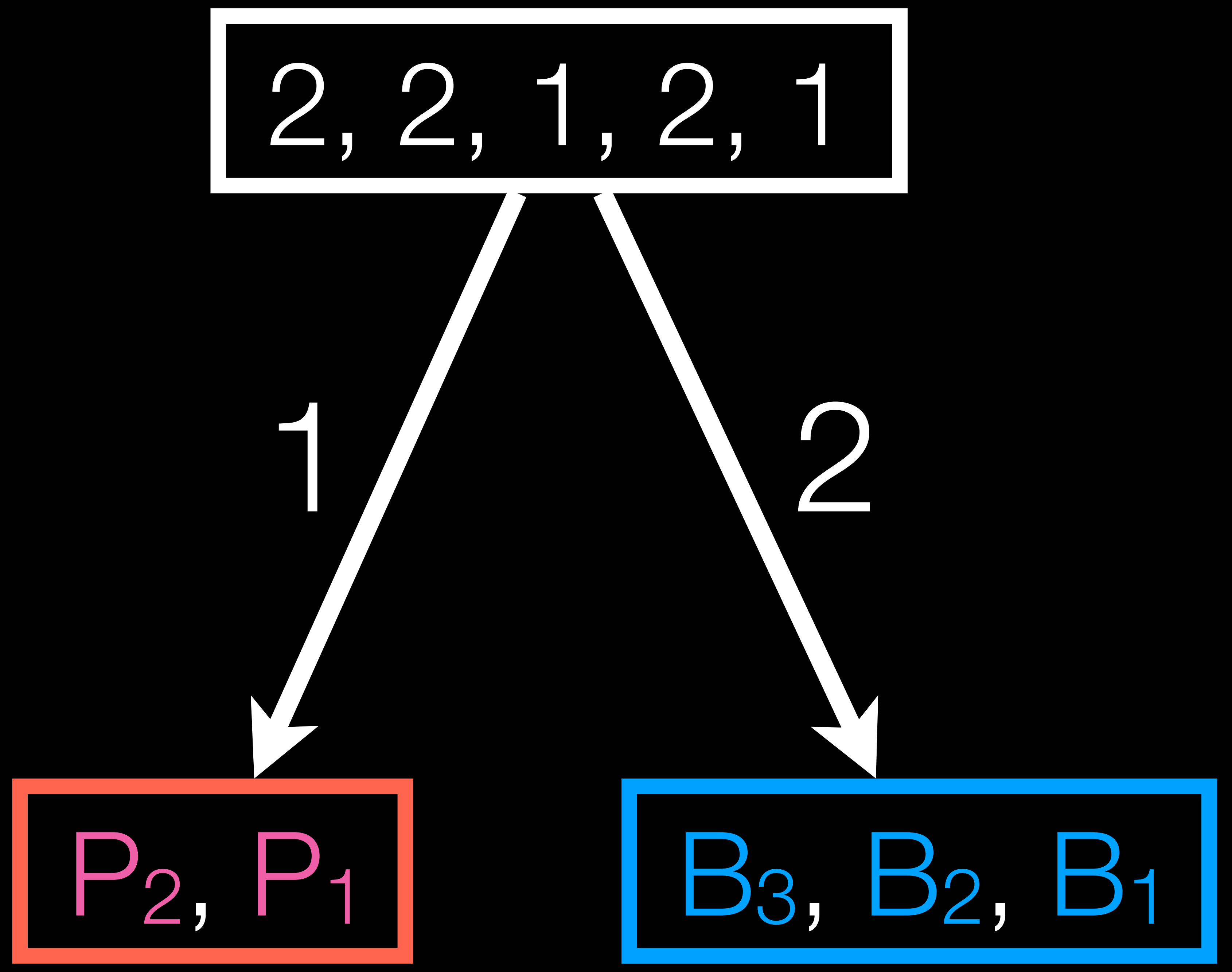

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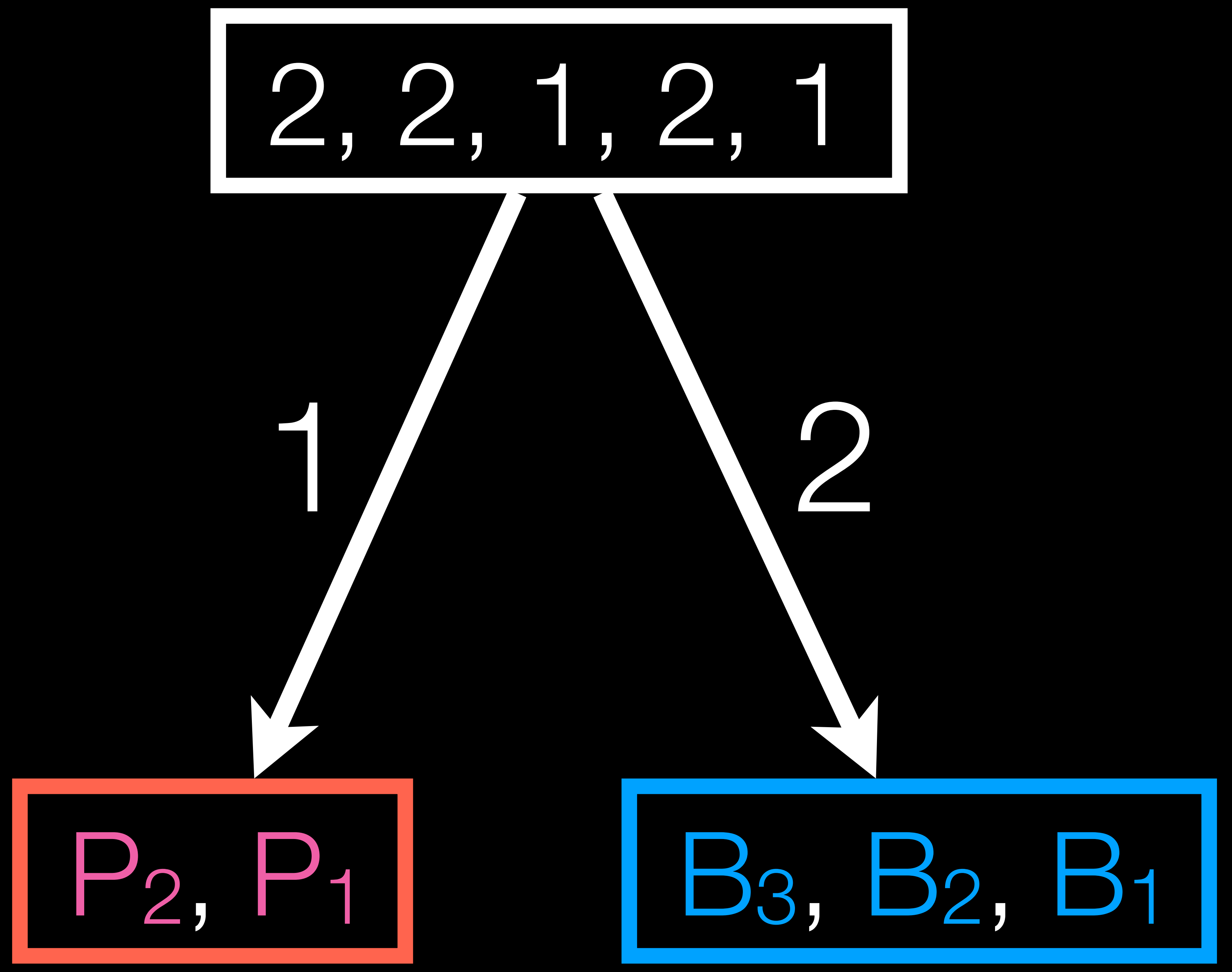

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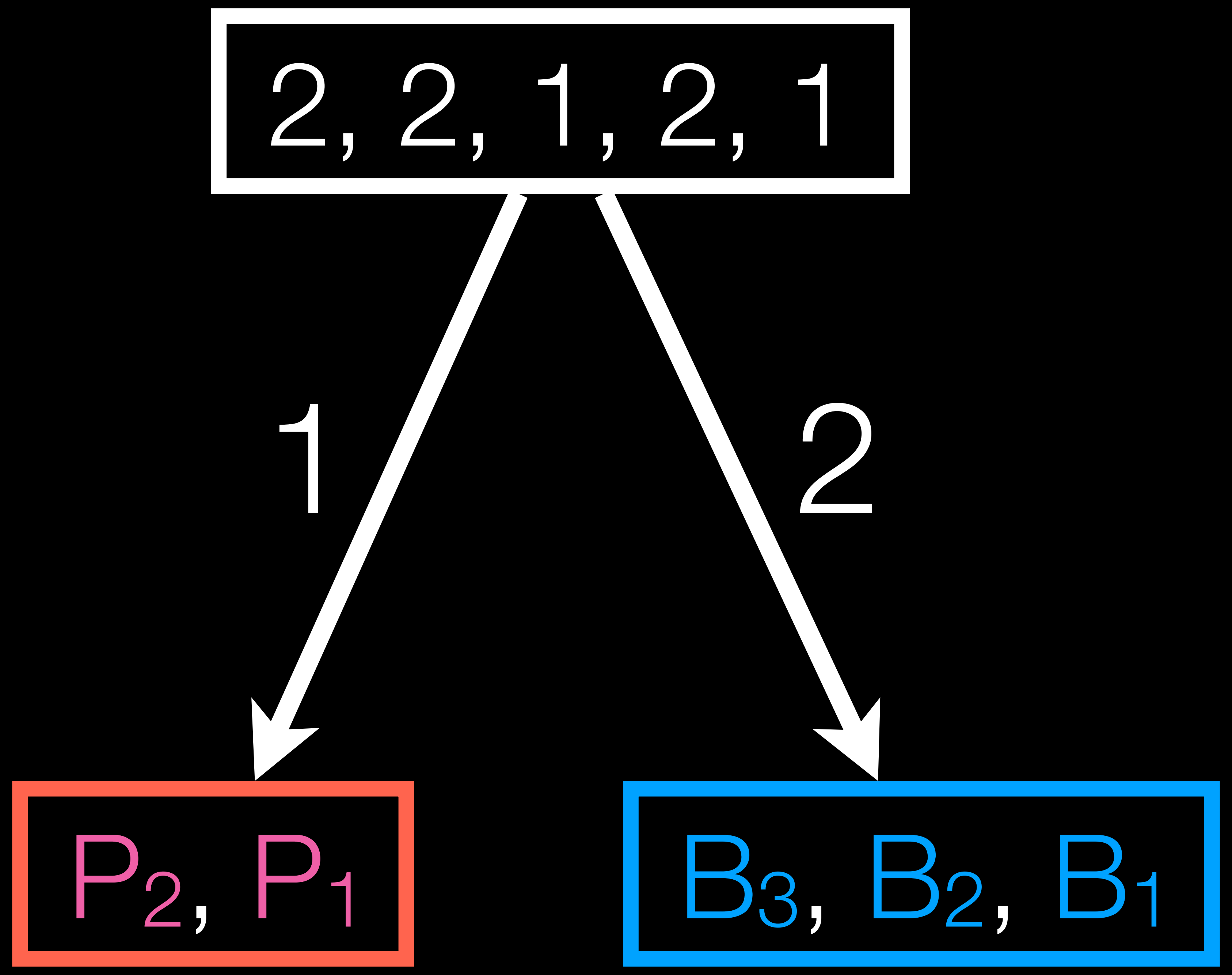

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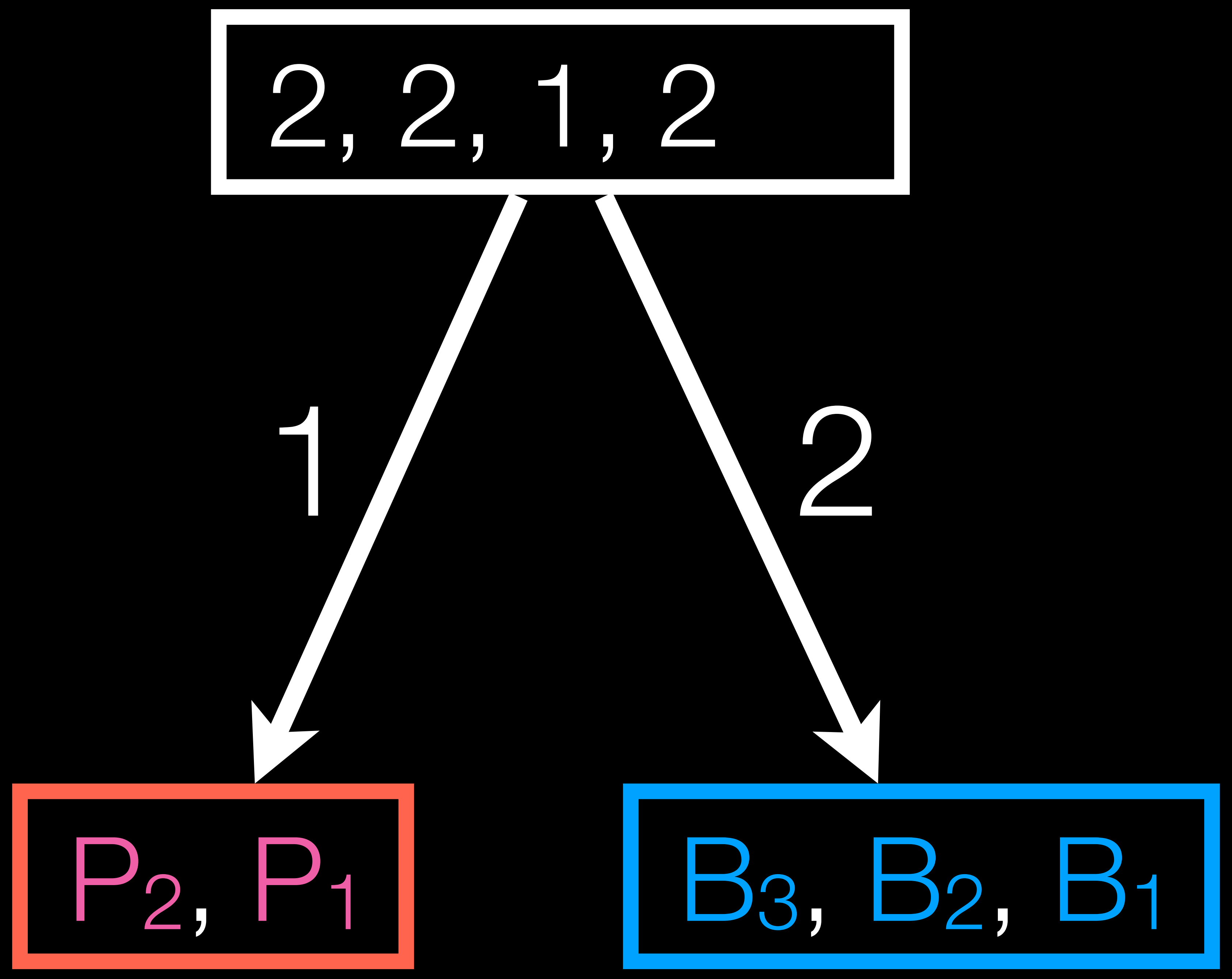

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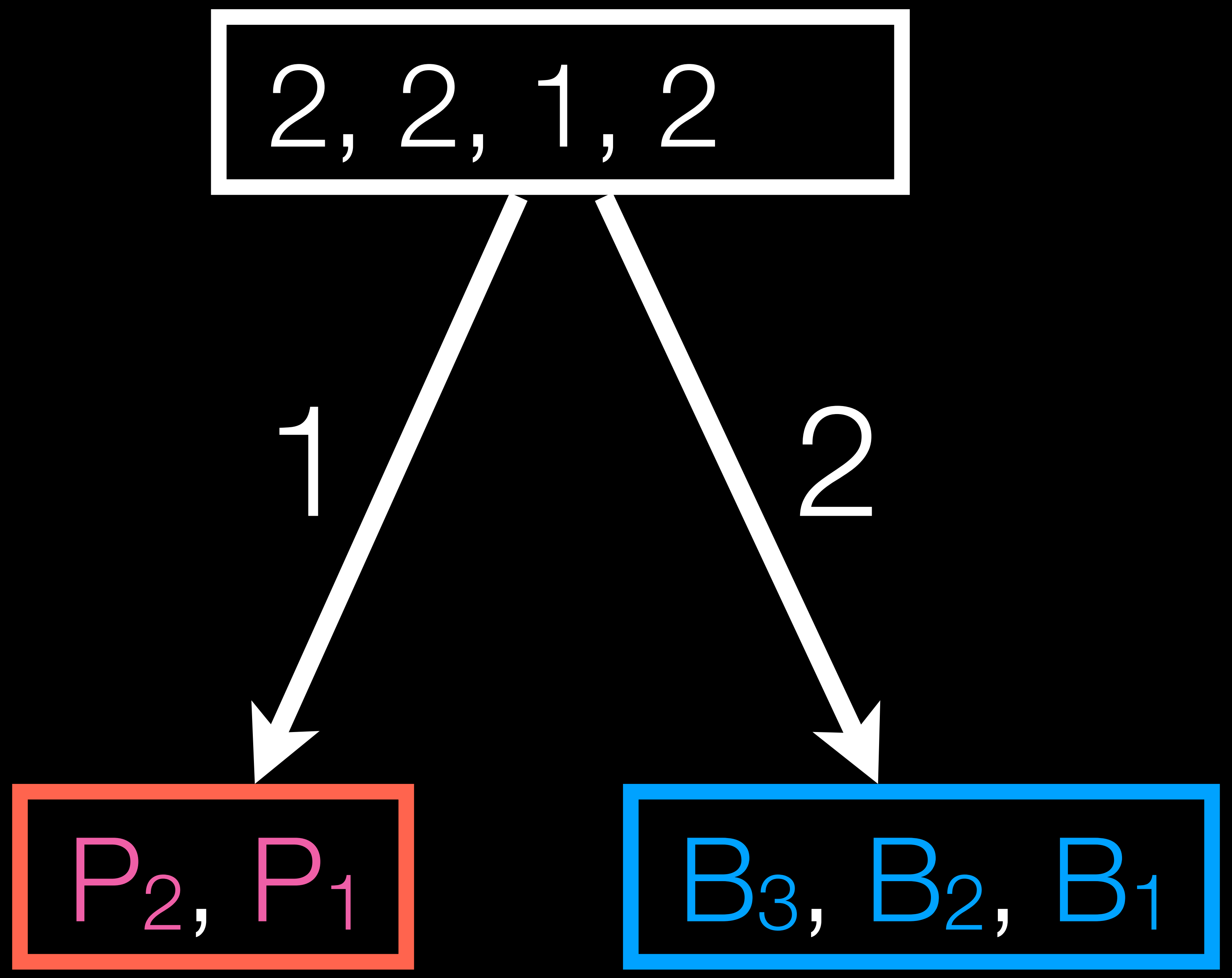

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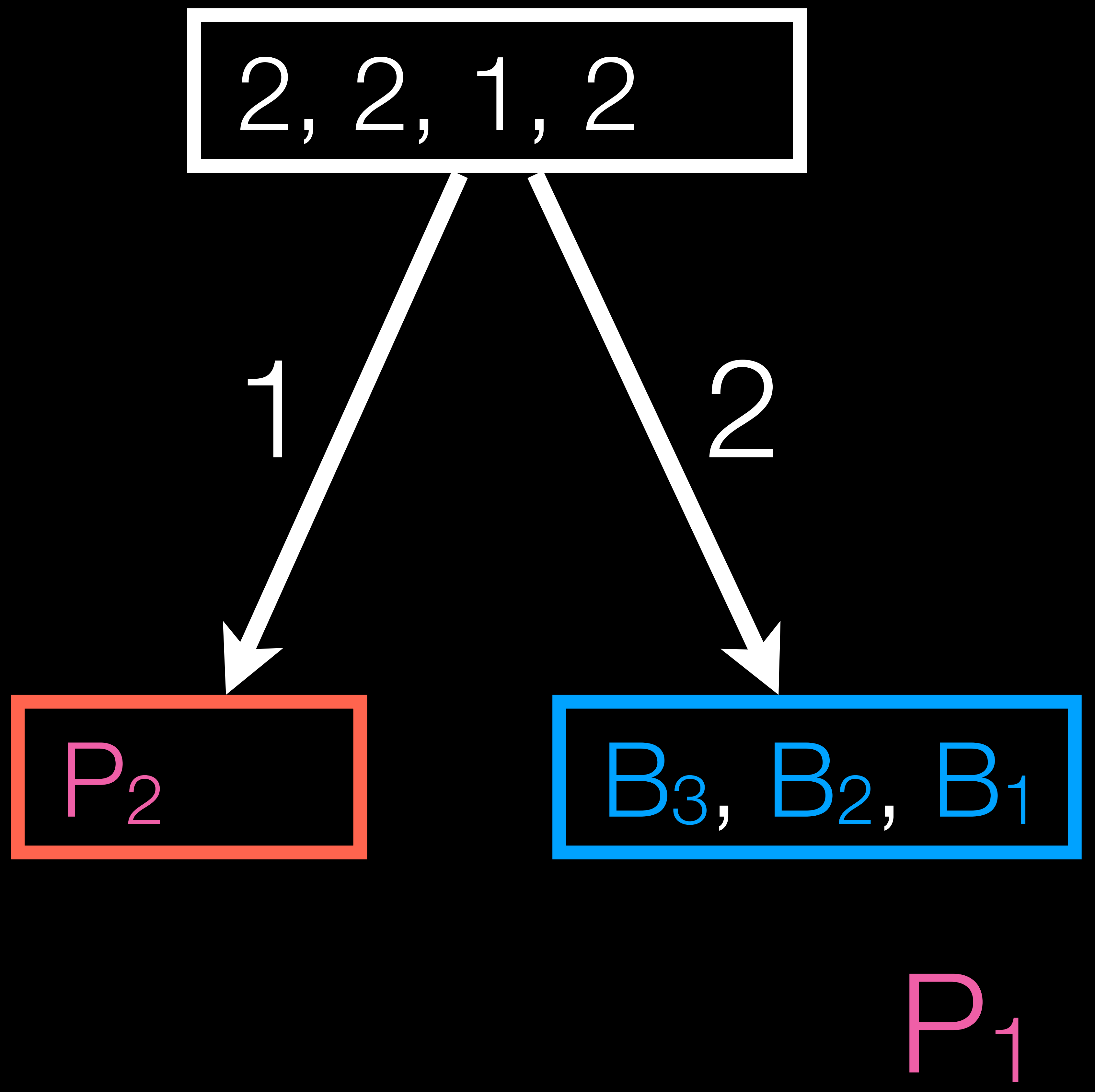

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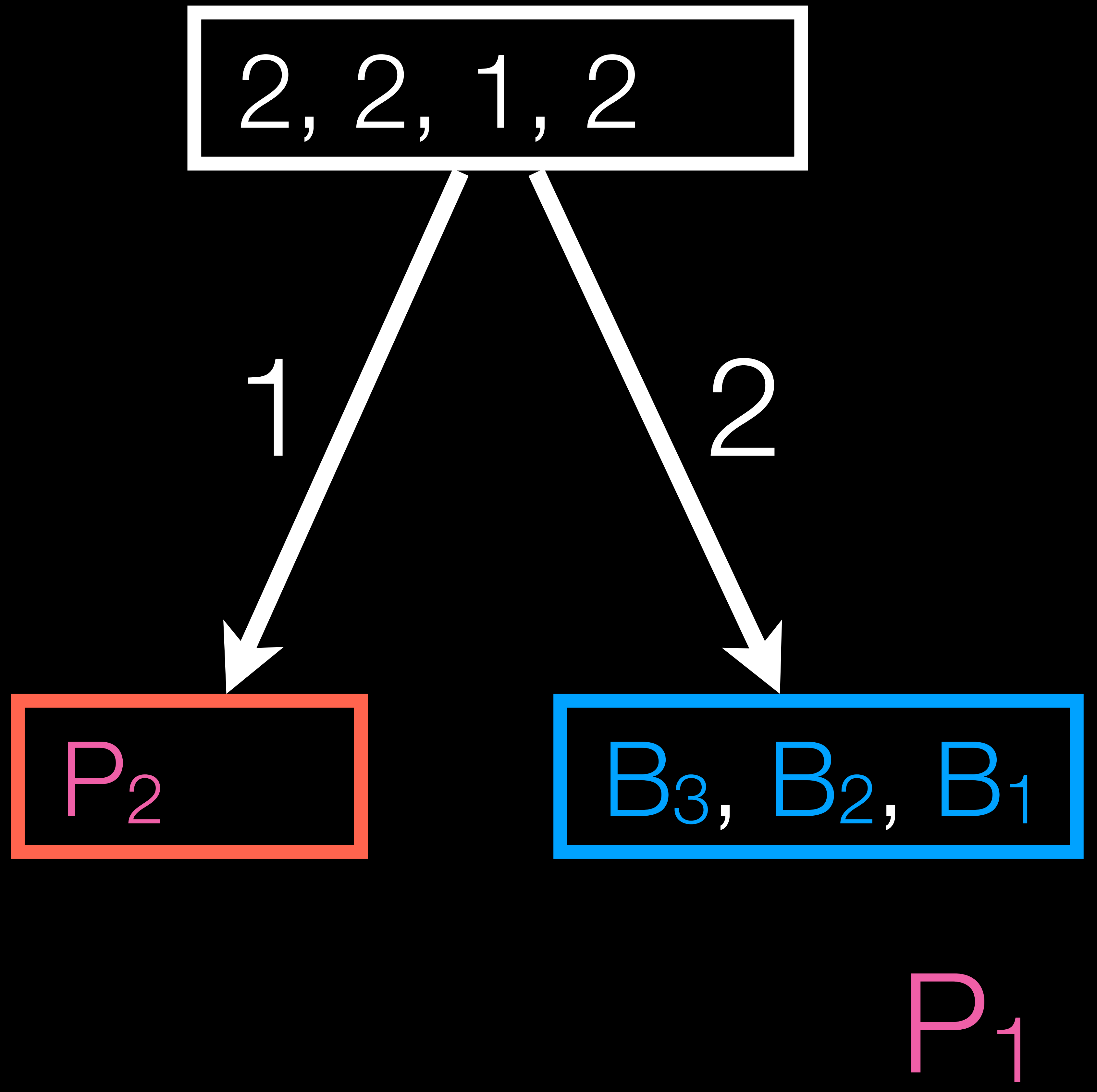

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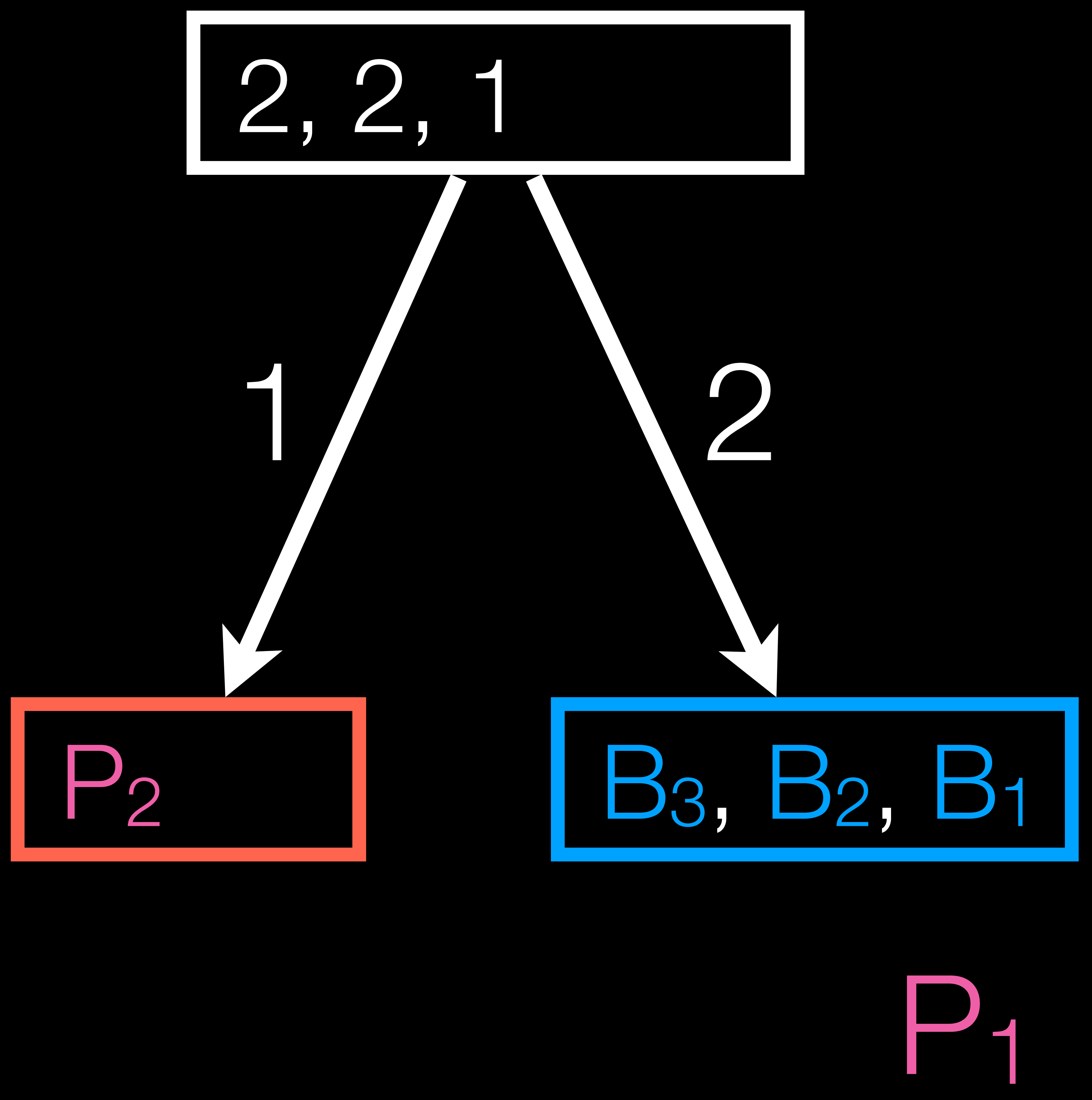

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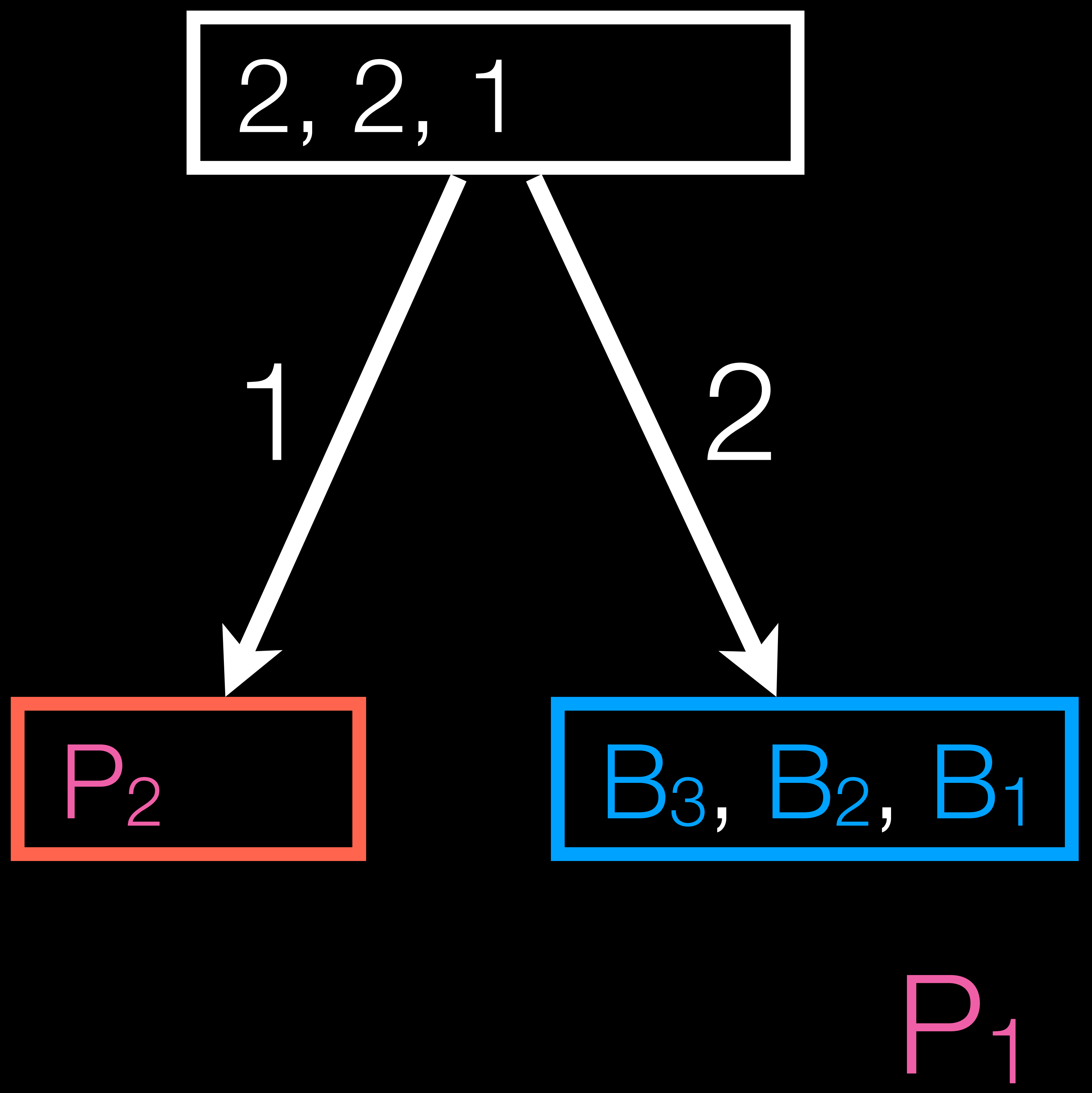

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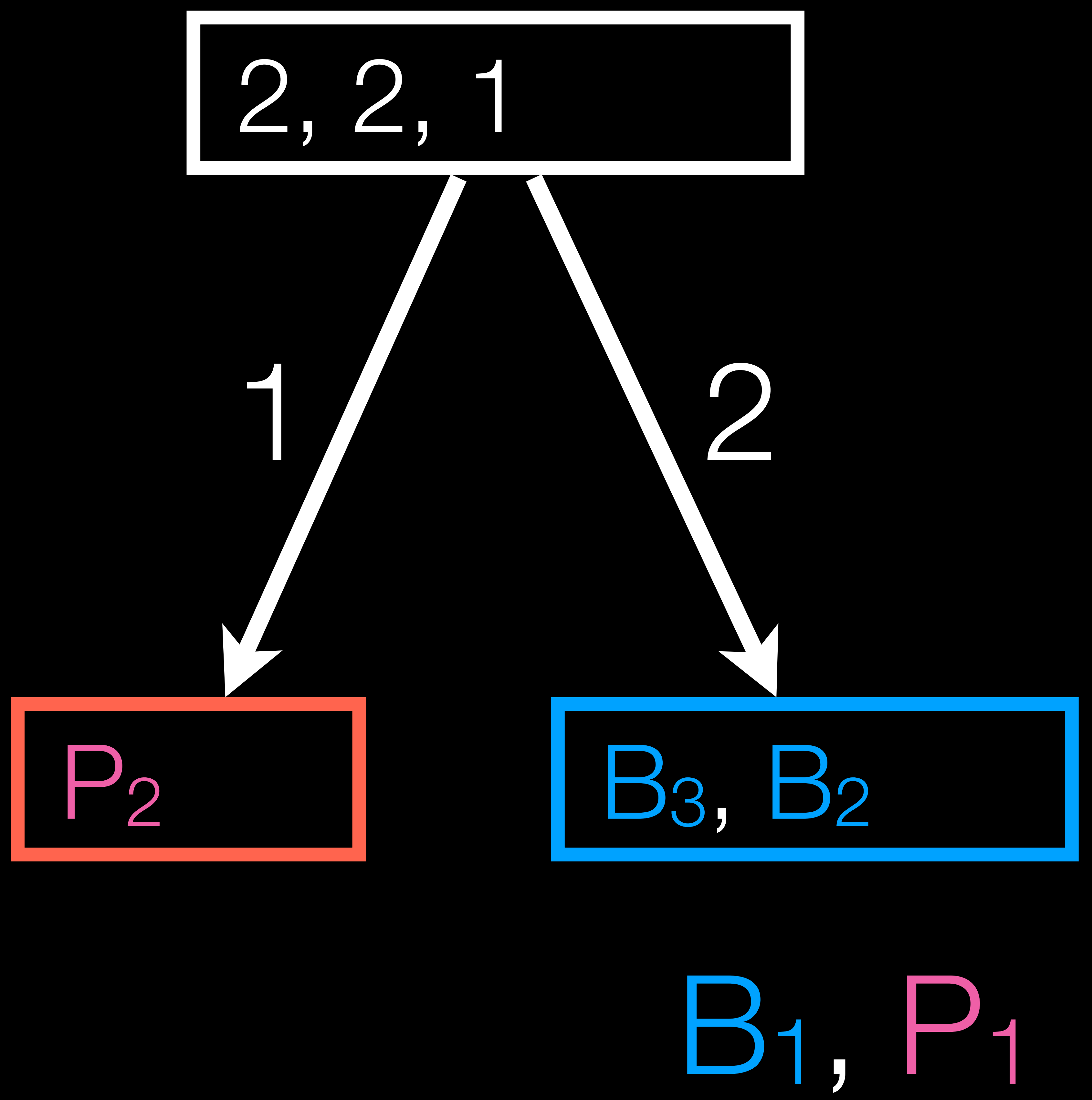

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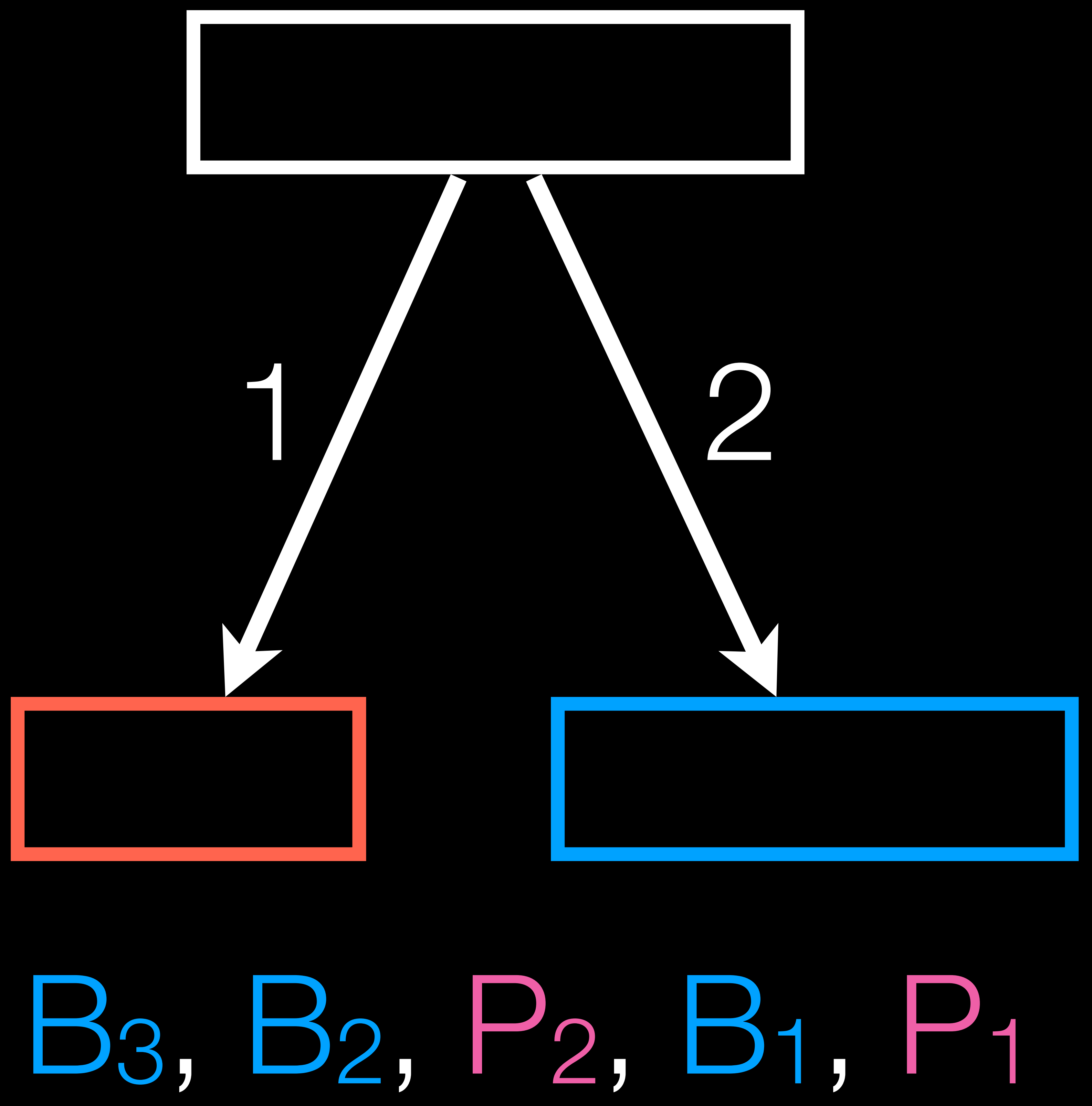

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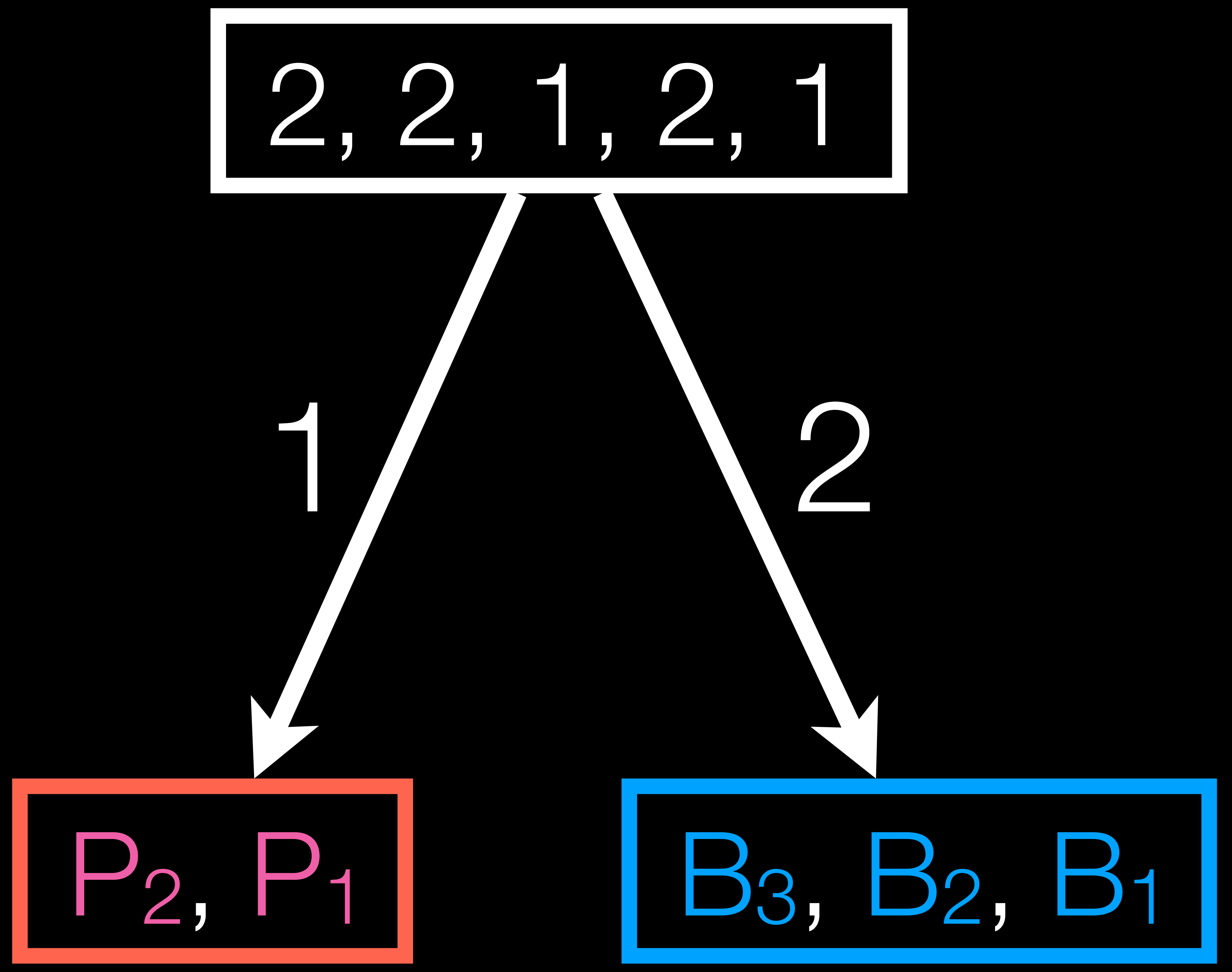

B_3, B_2, P_2, B_1, P_1

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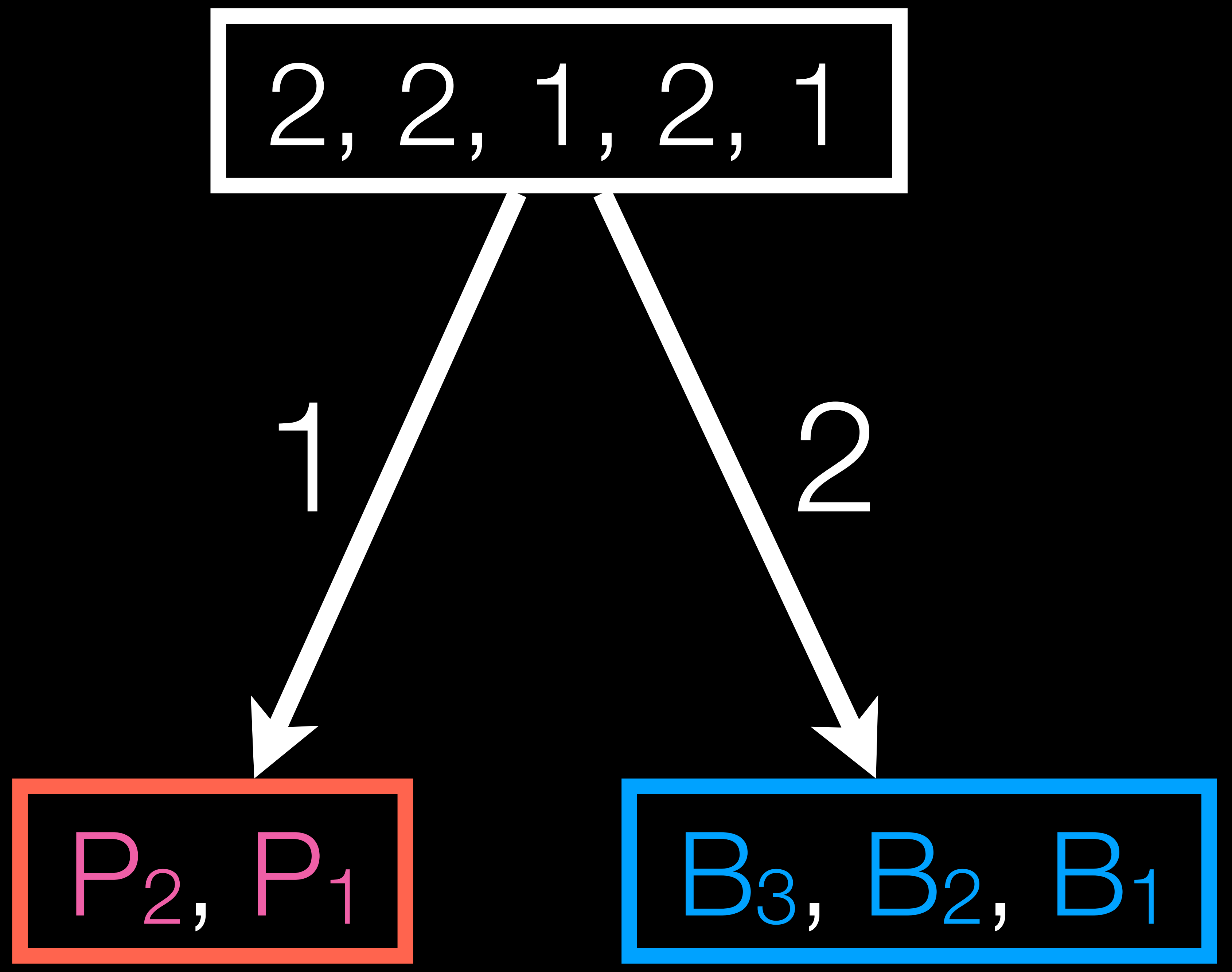

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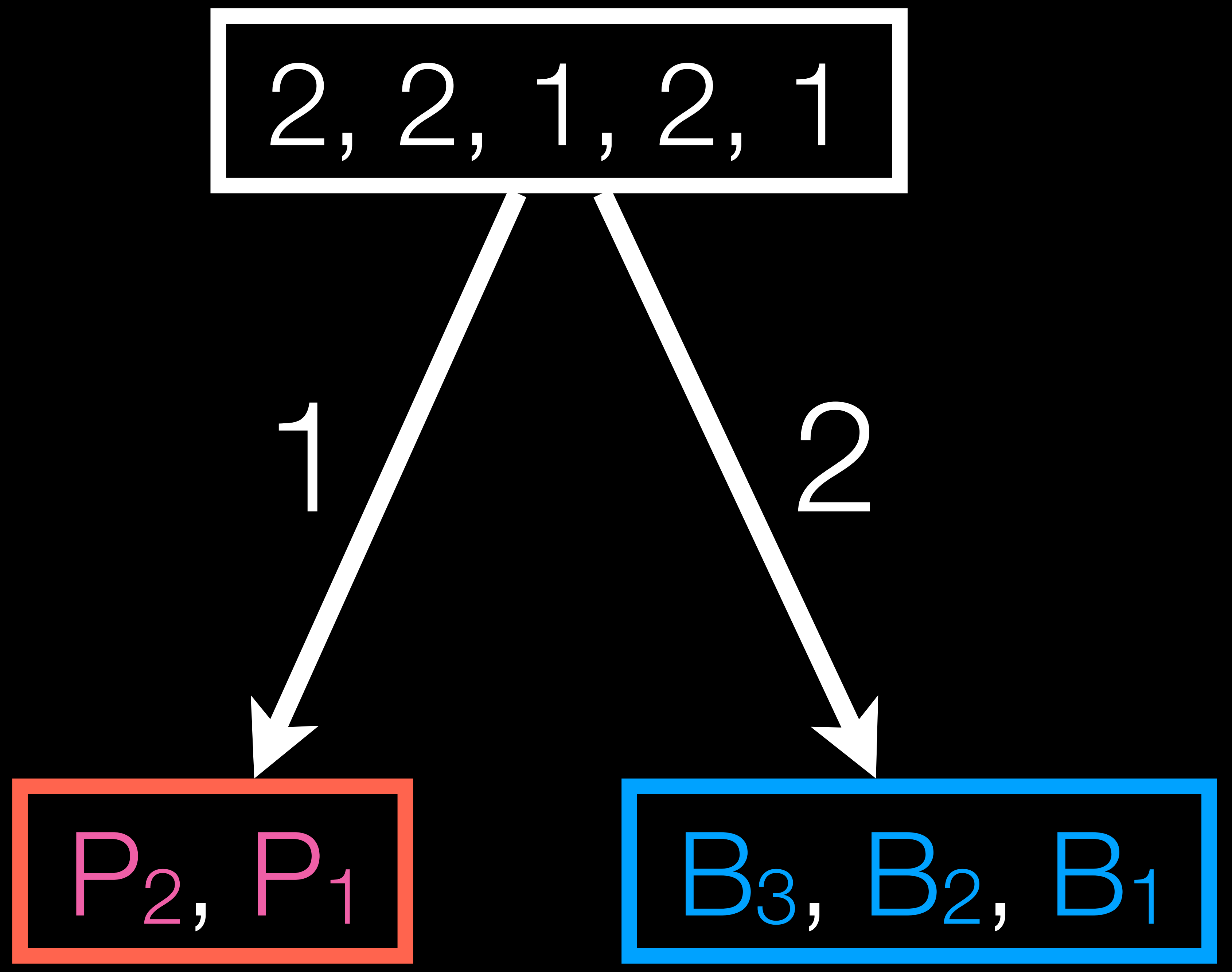

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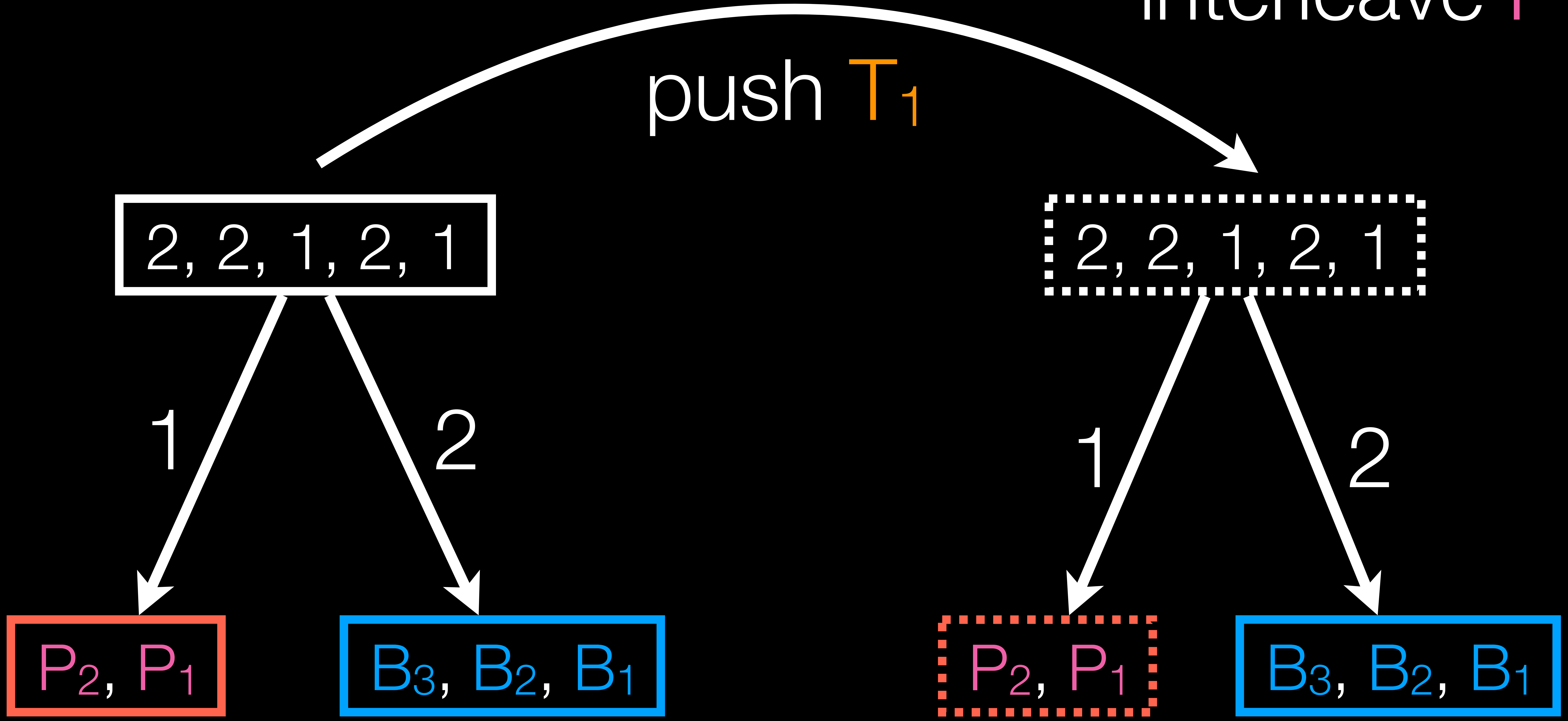

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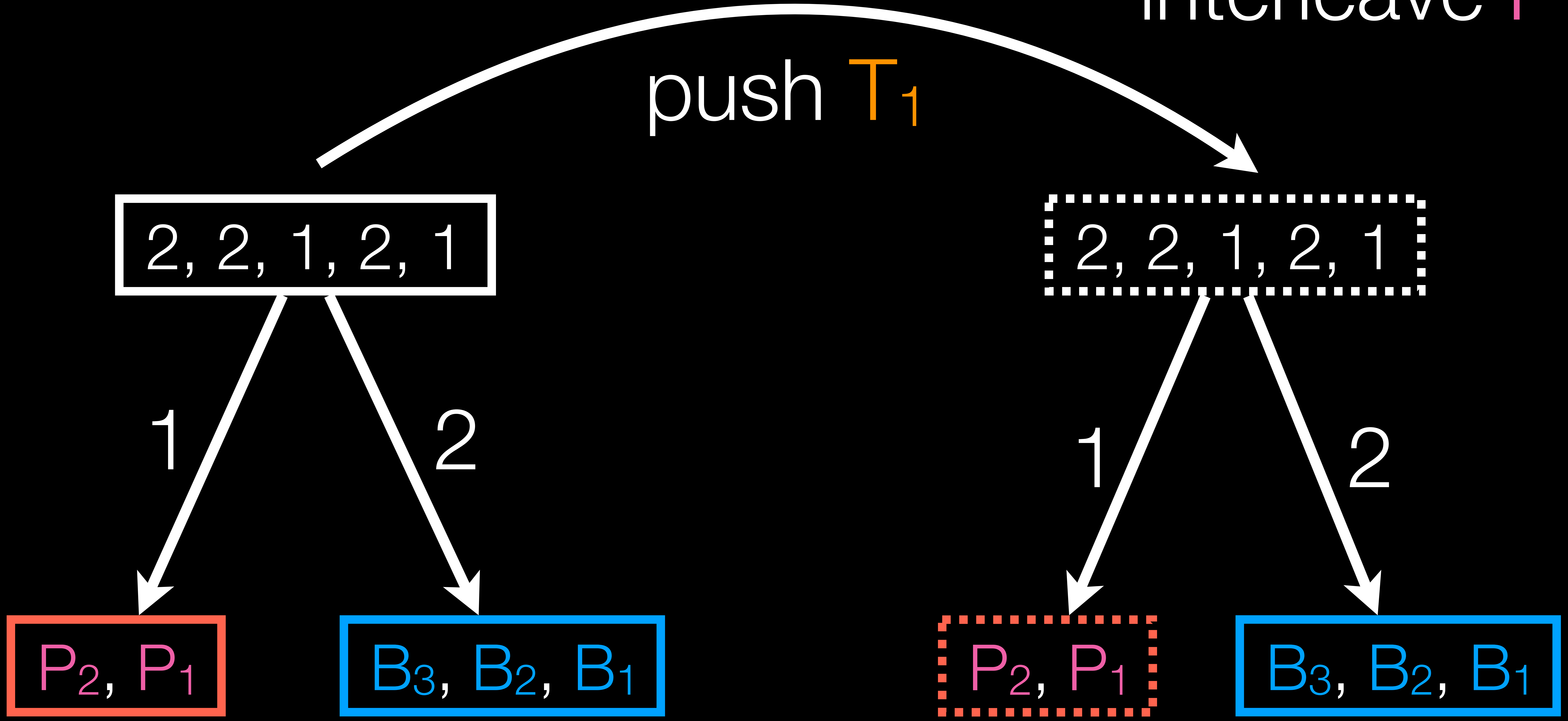

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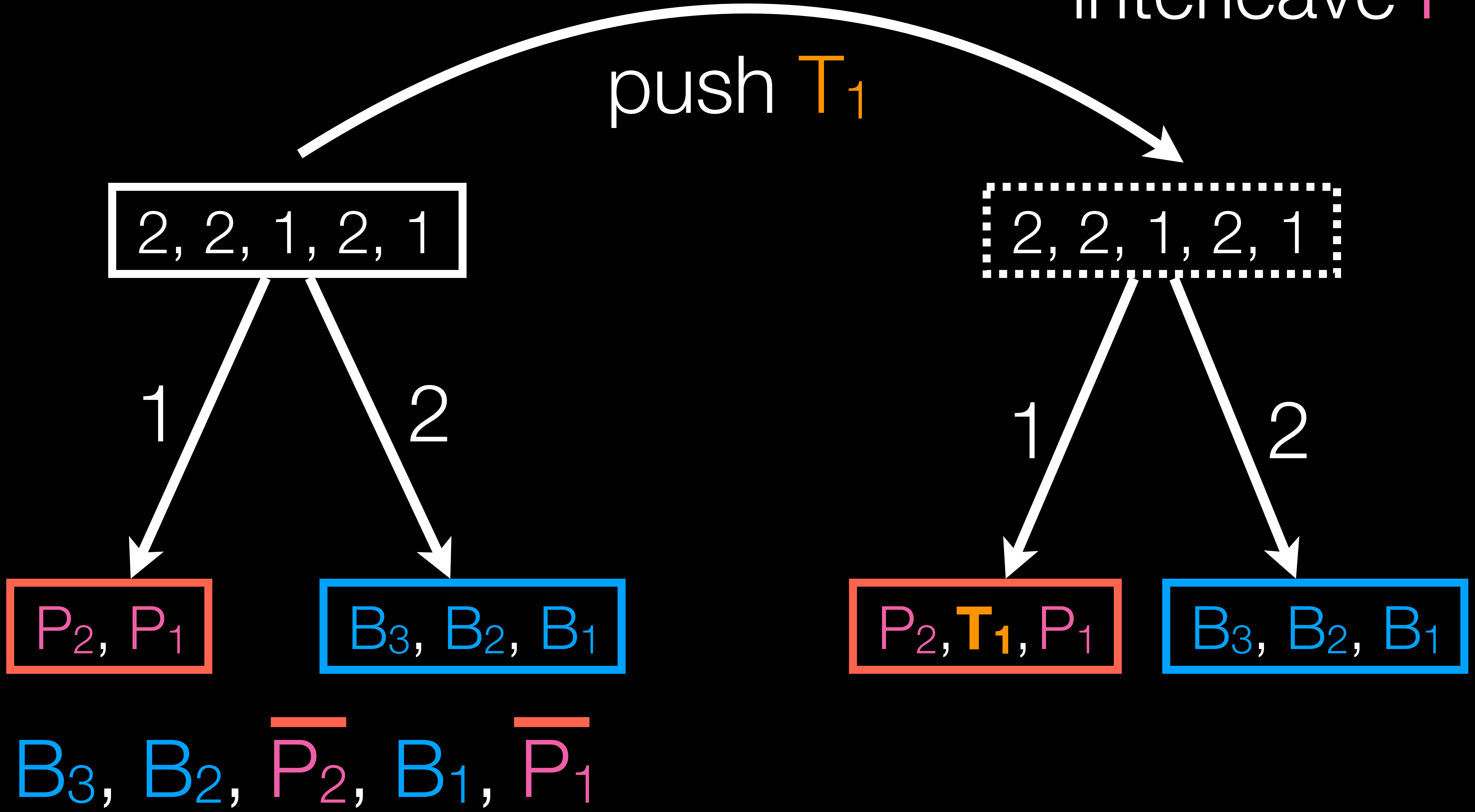

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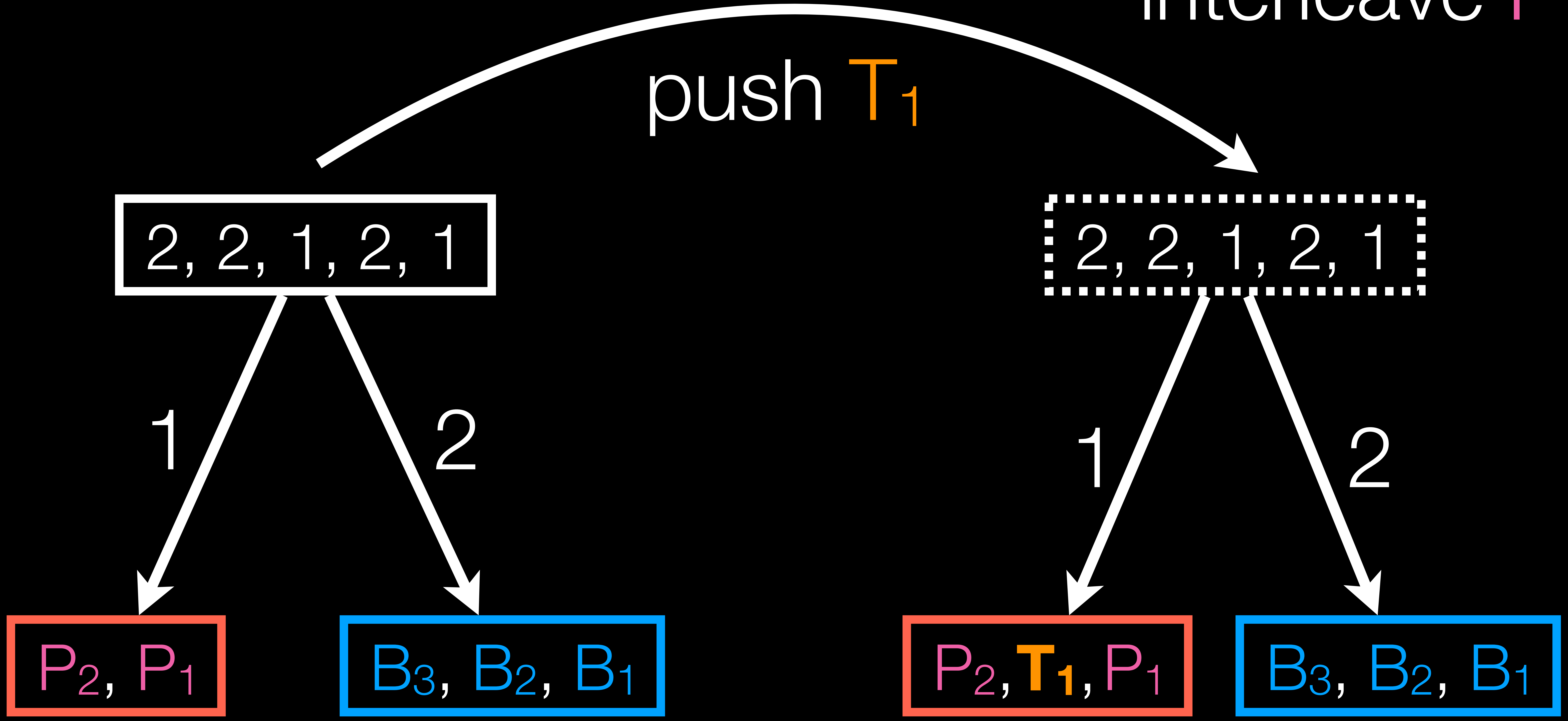

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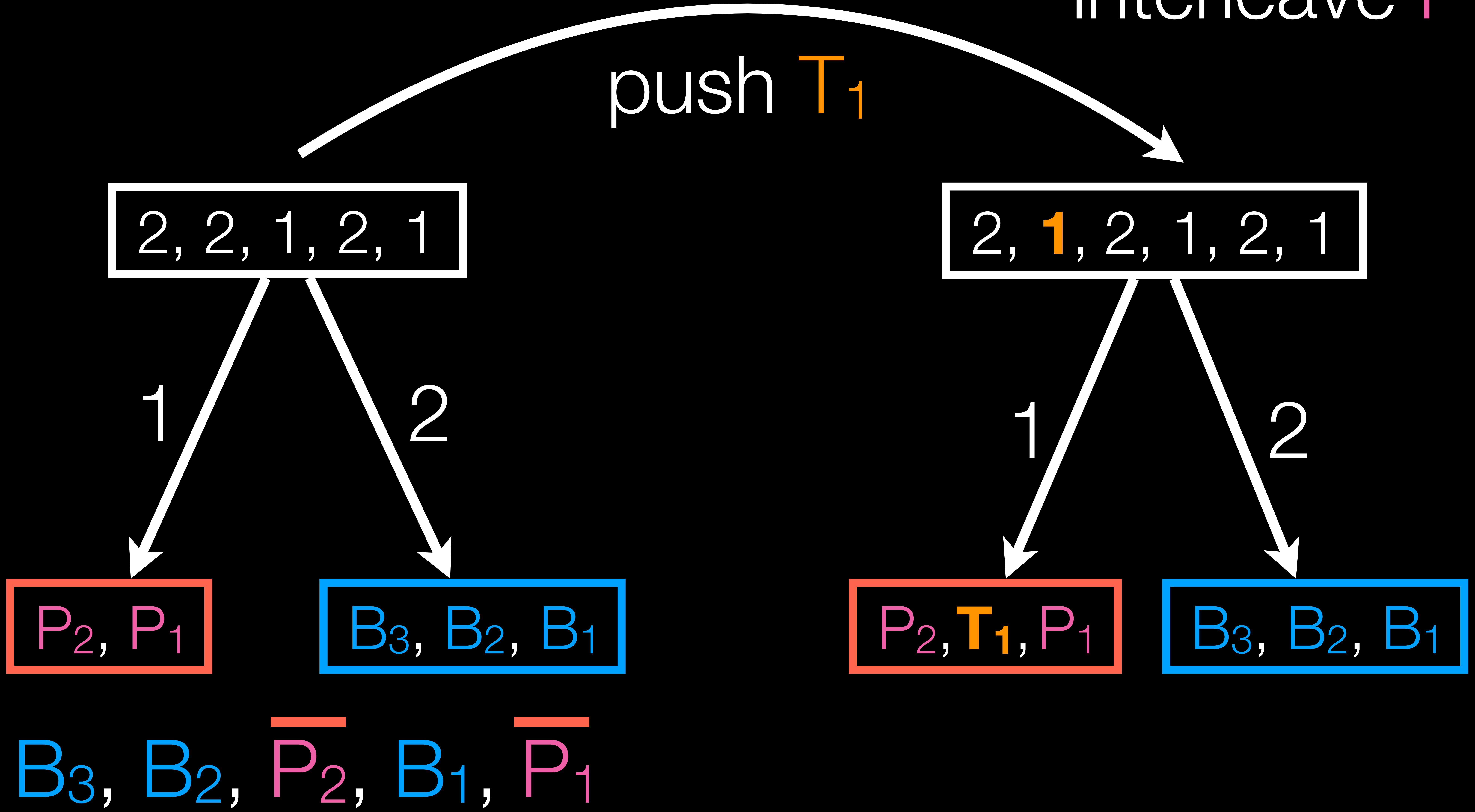

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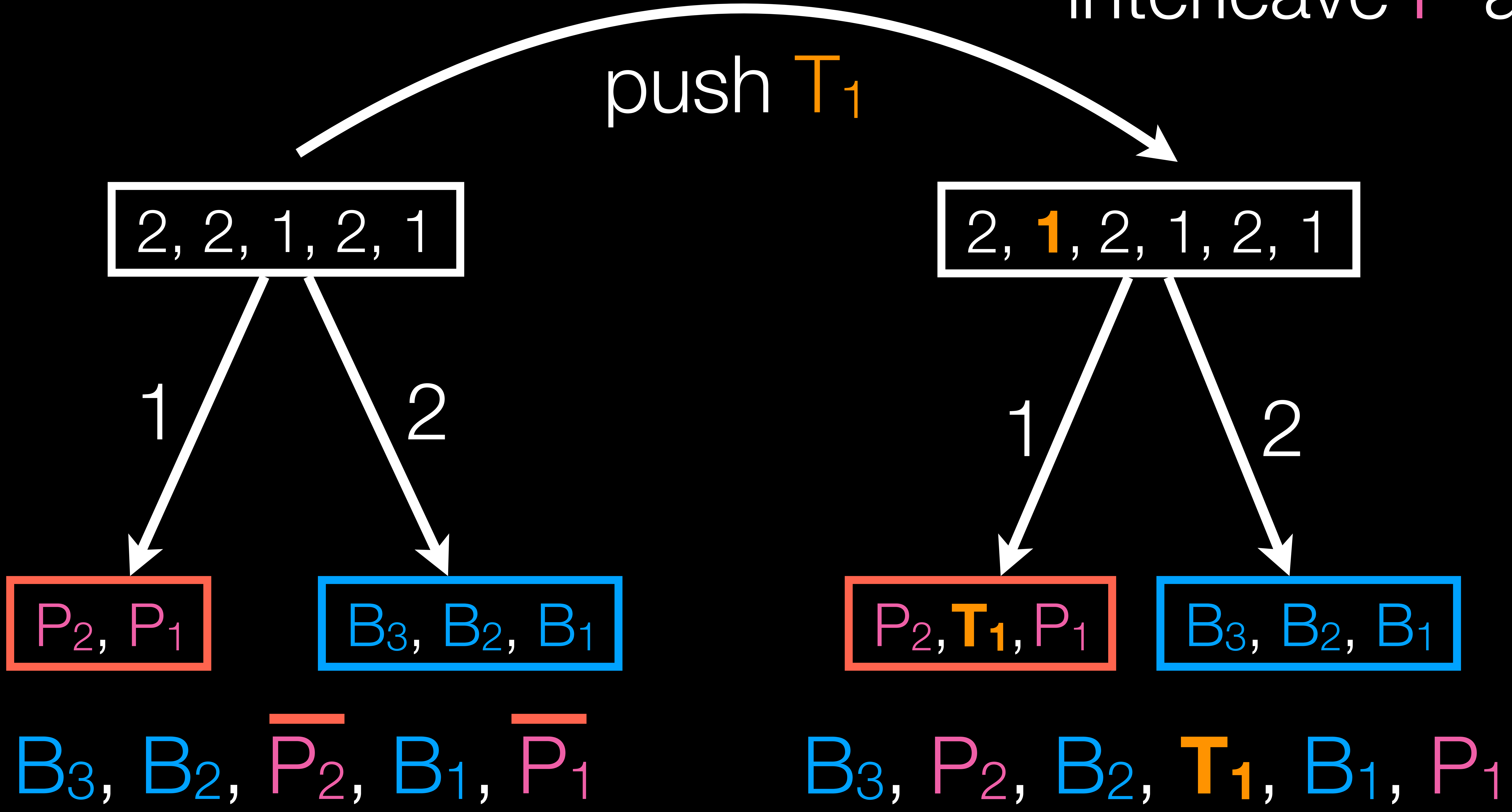

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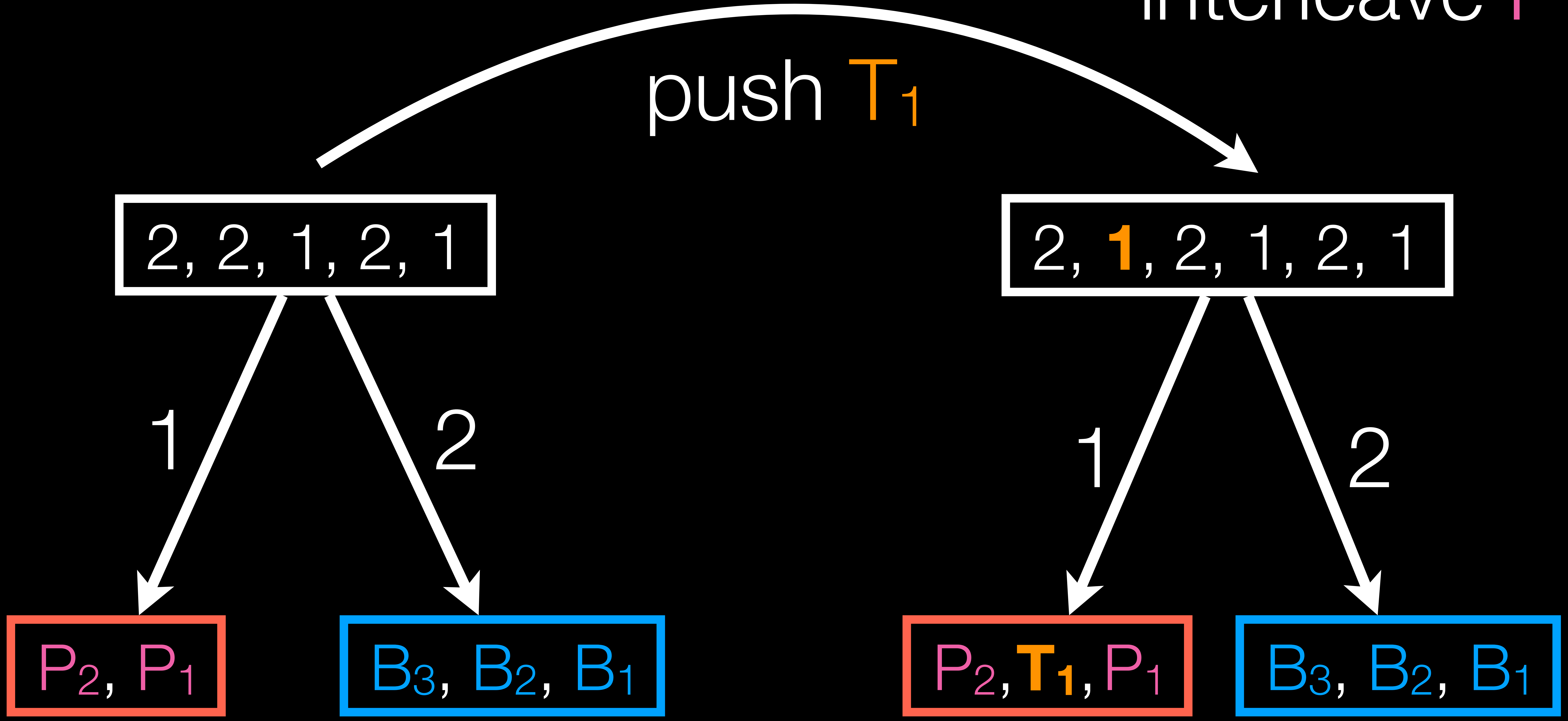

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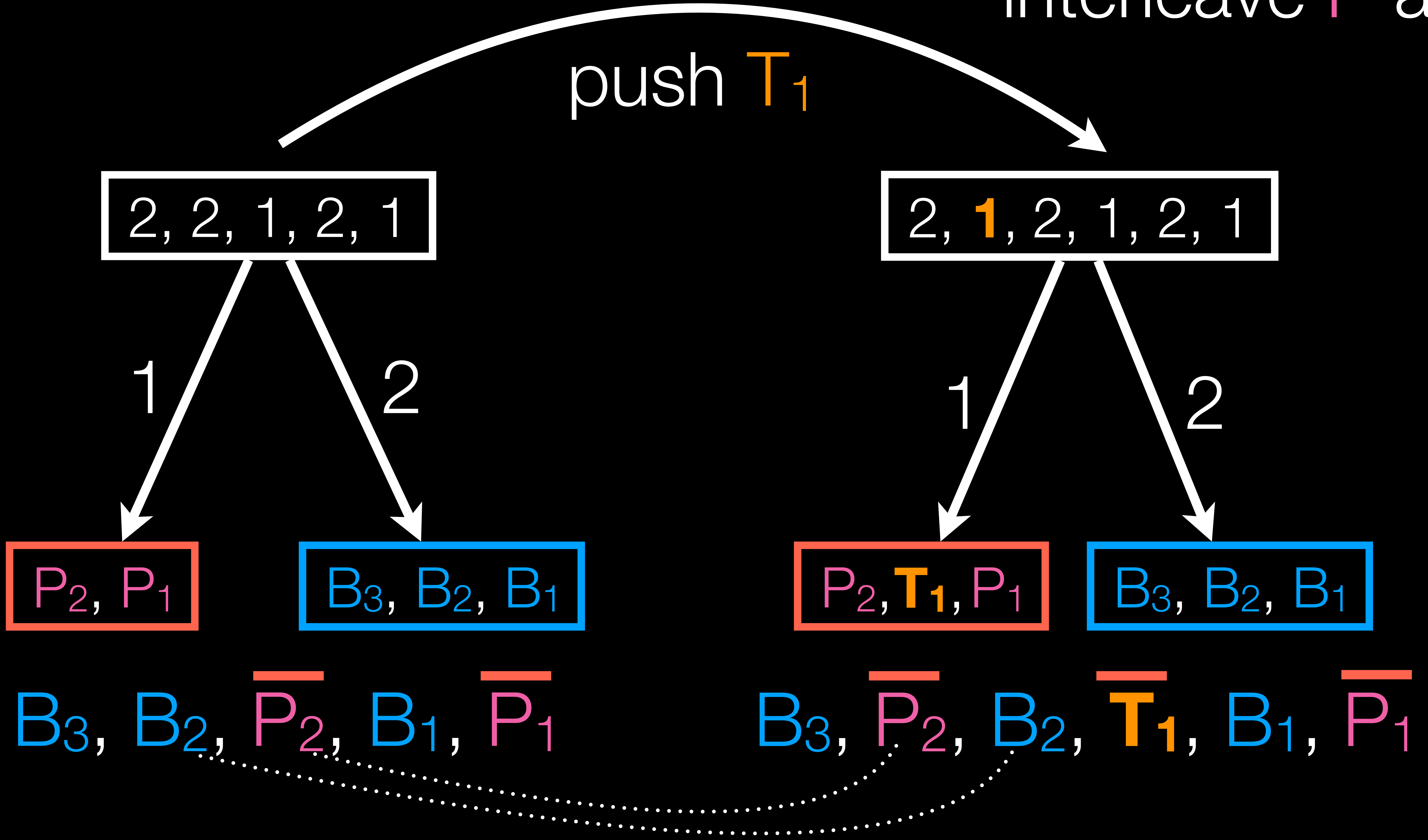

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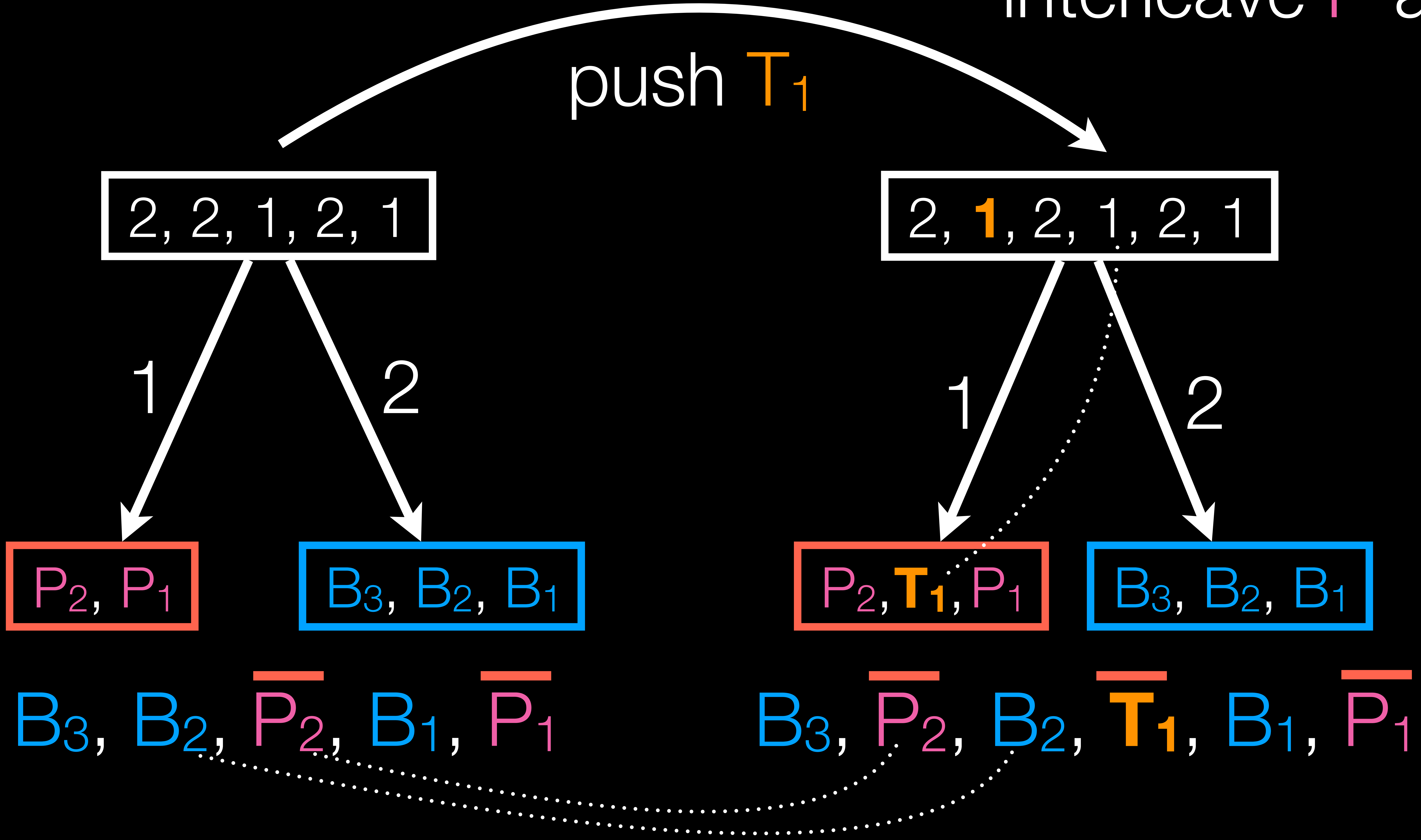

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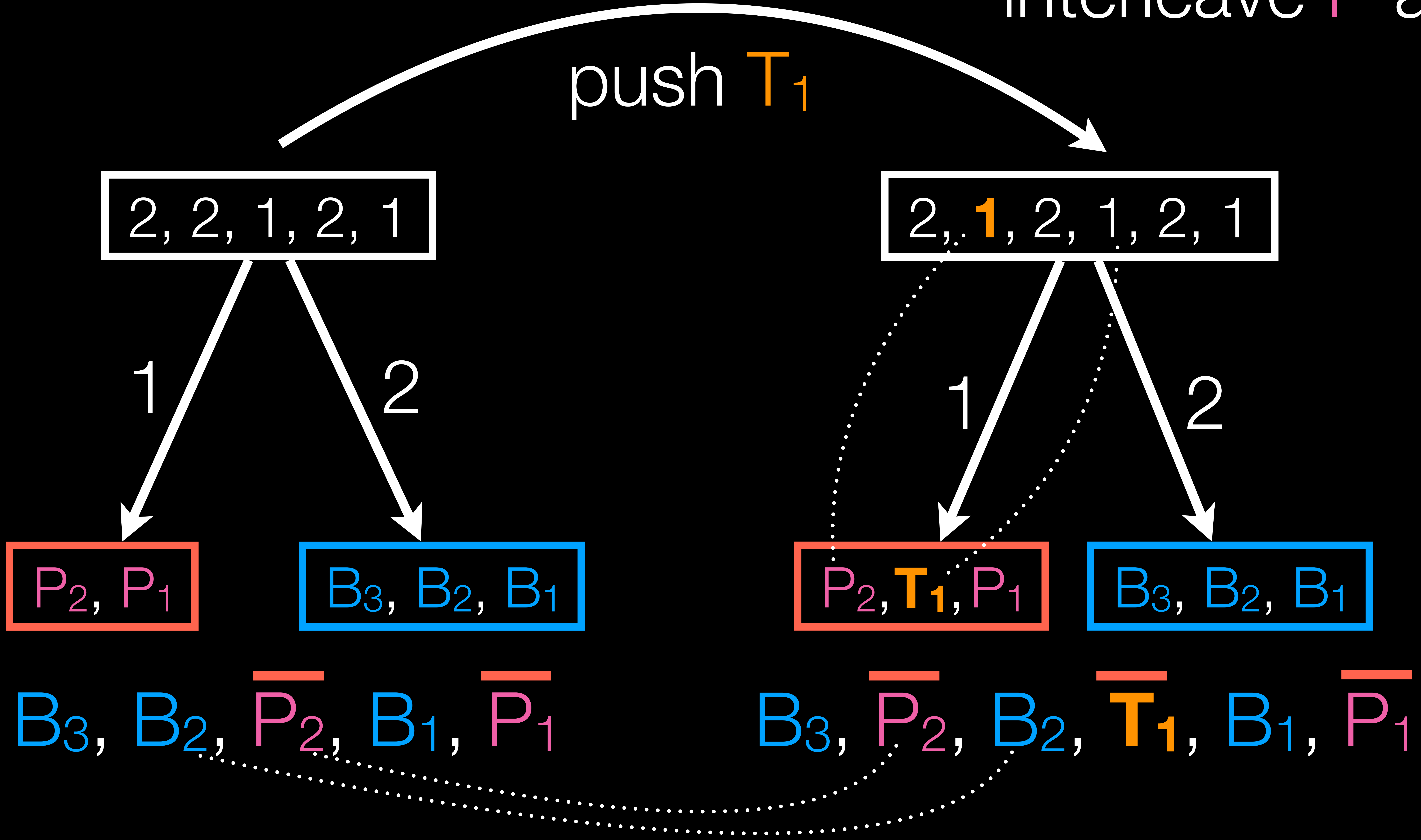

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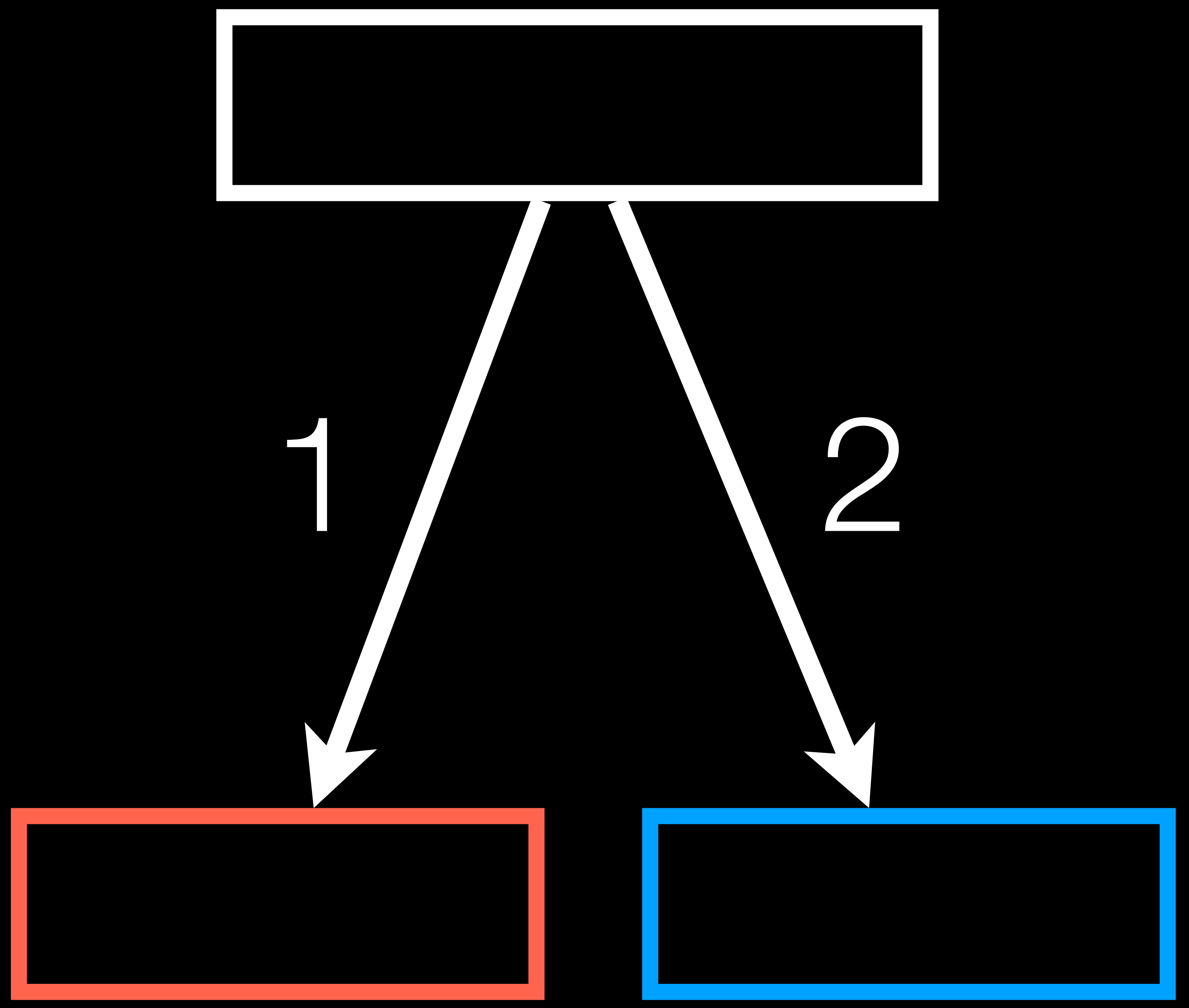

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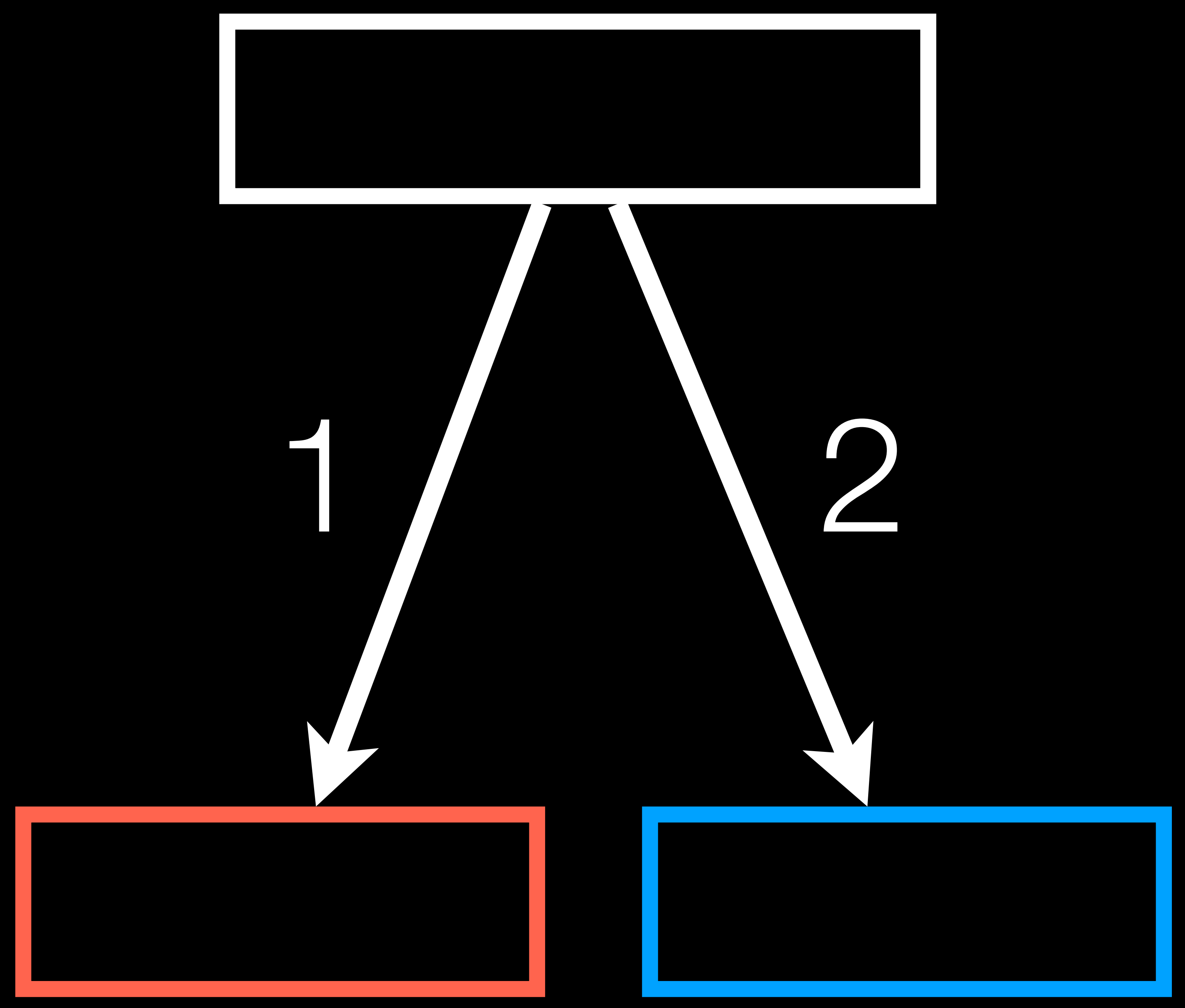
A PIFO tree manifests a *programming language.*



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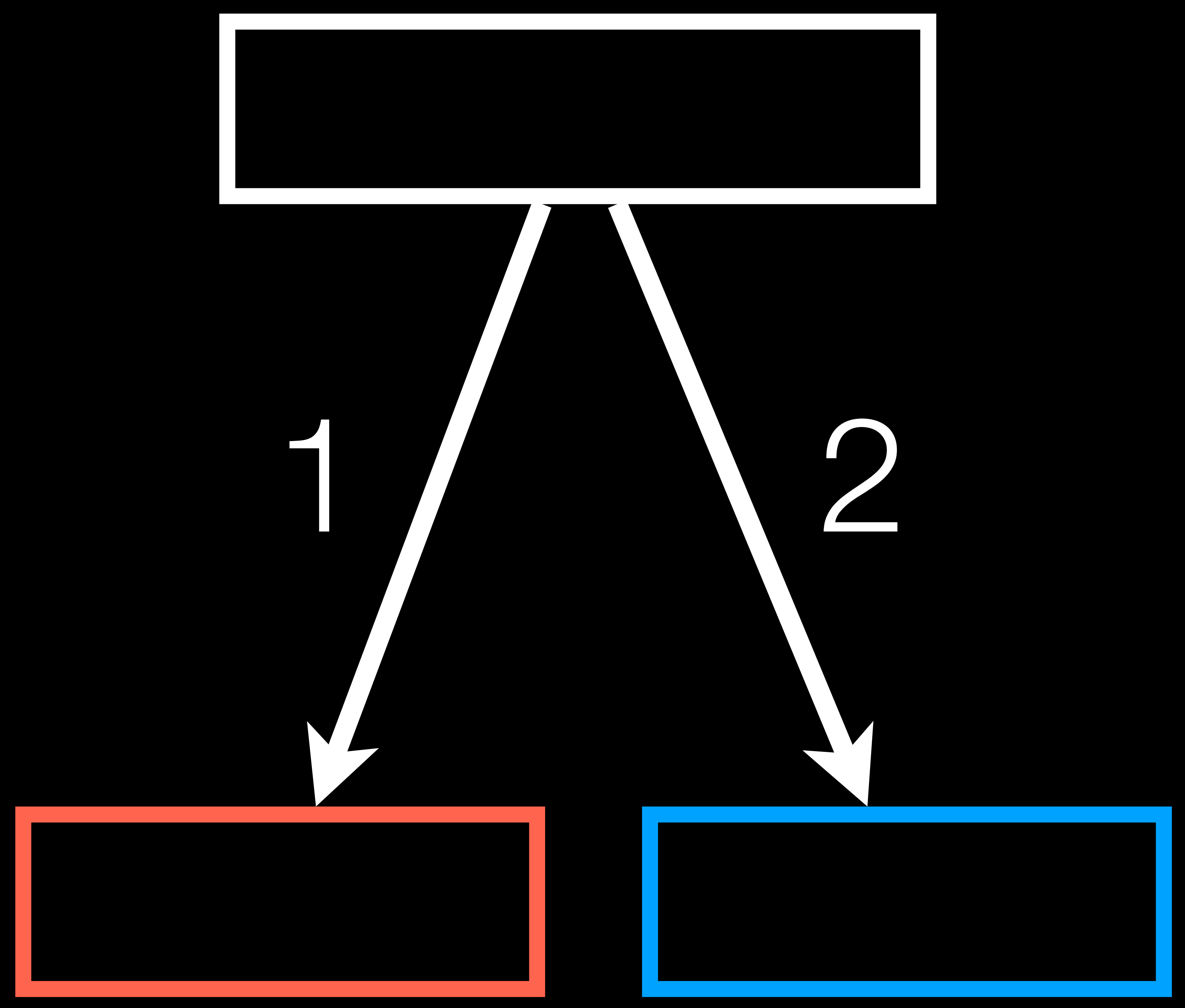
A program is precisely a *scheduling algorithm.*



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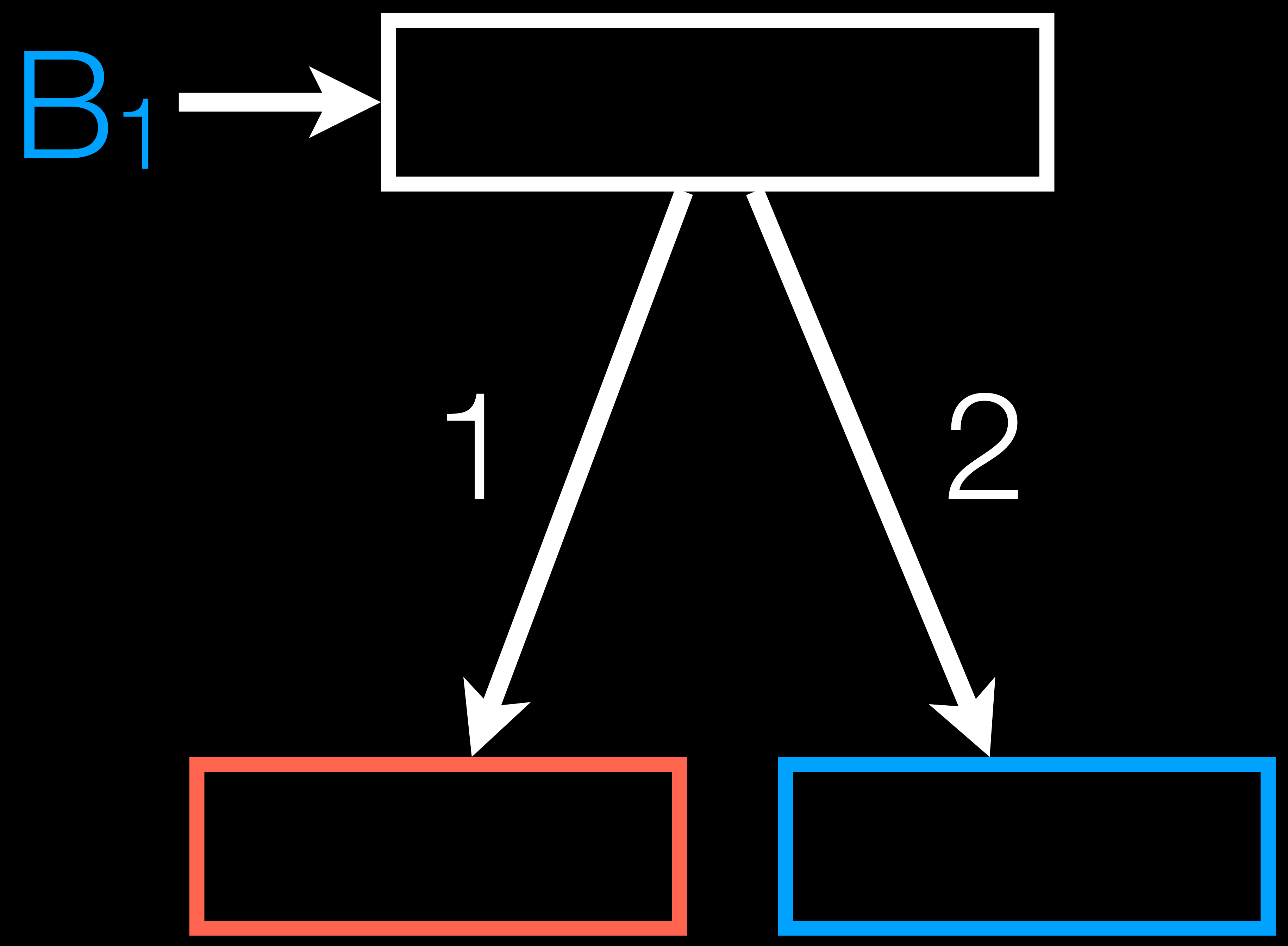

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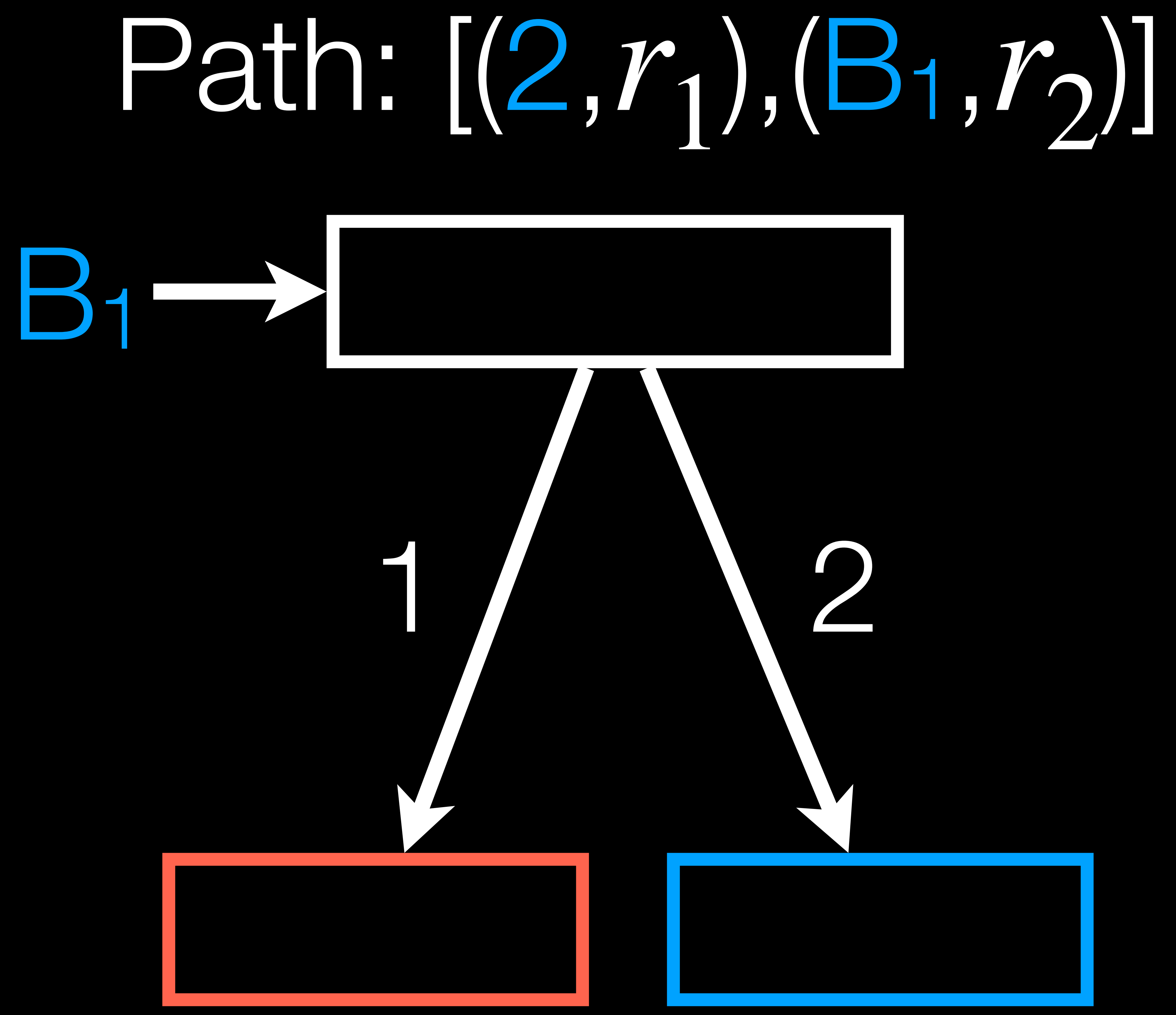

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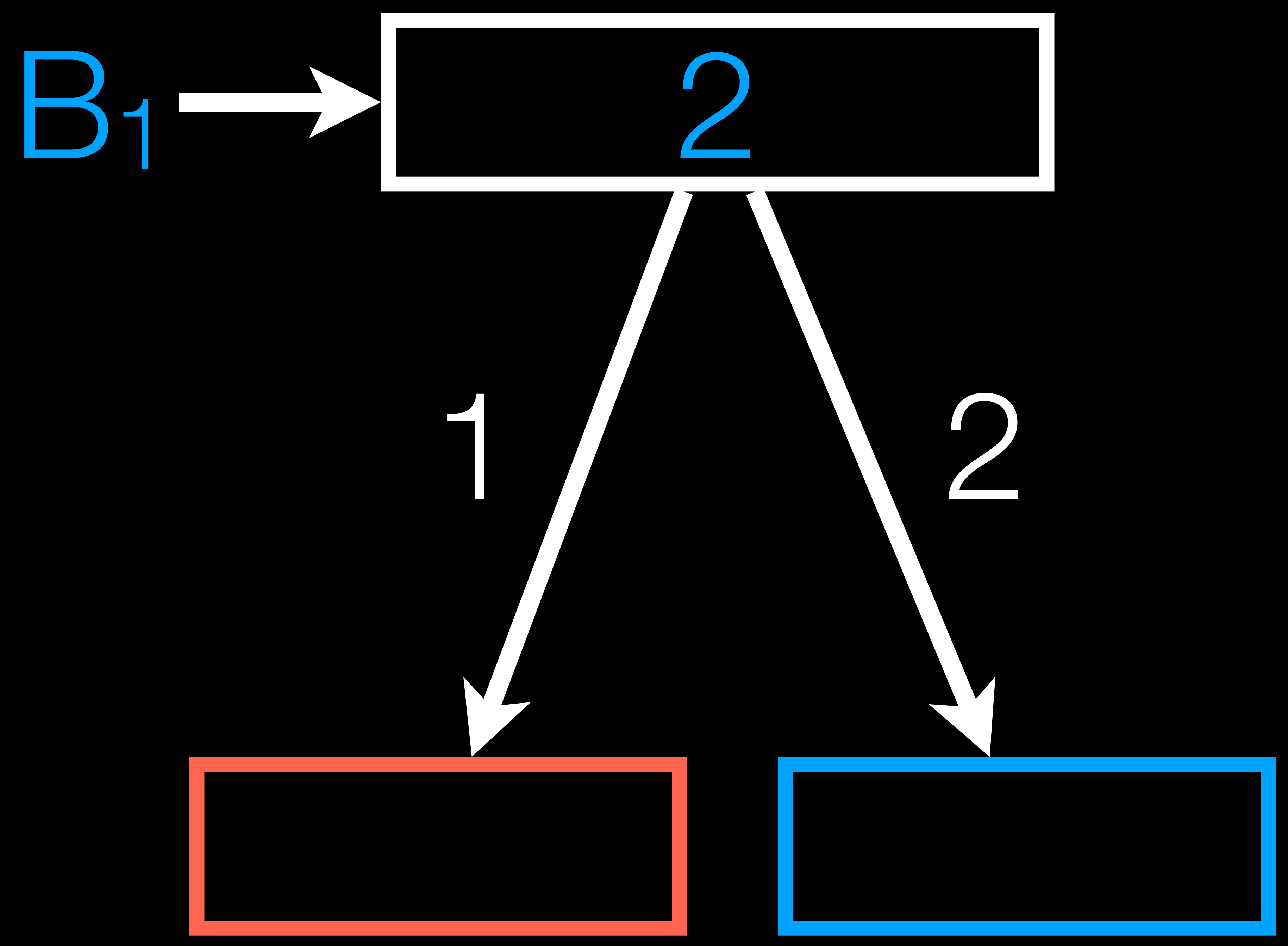

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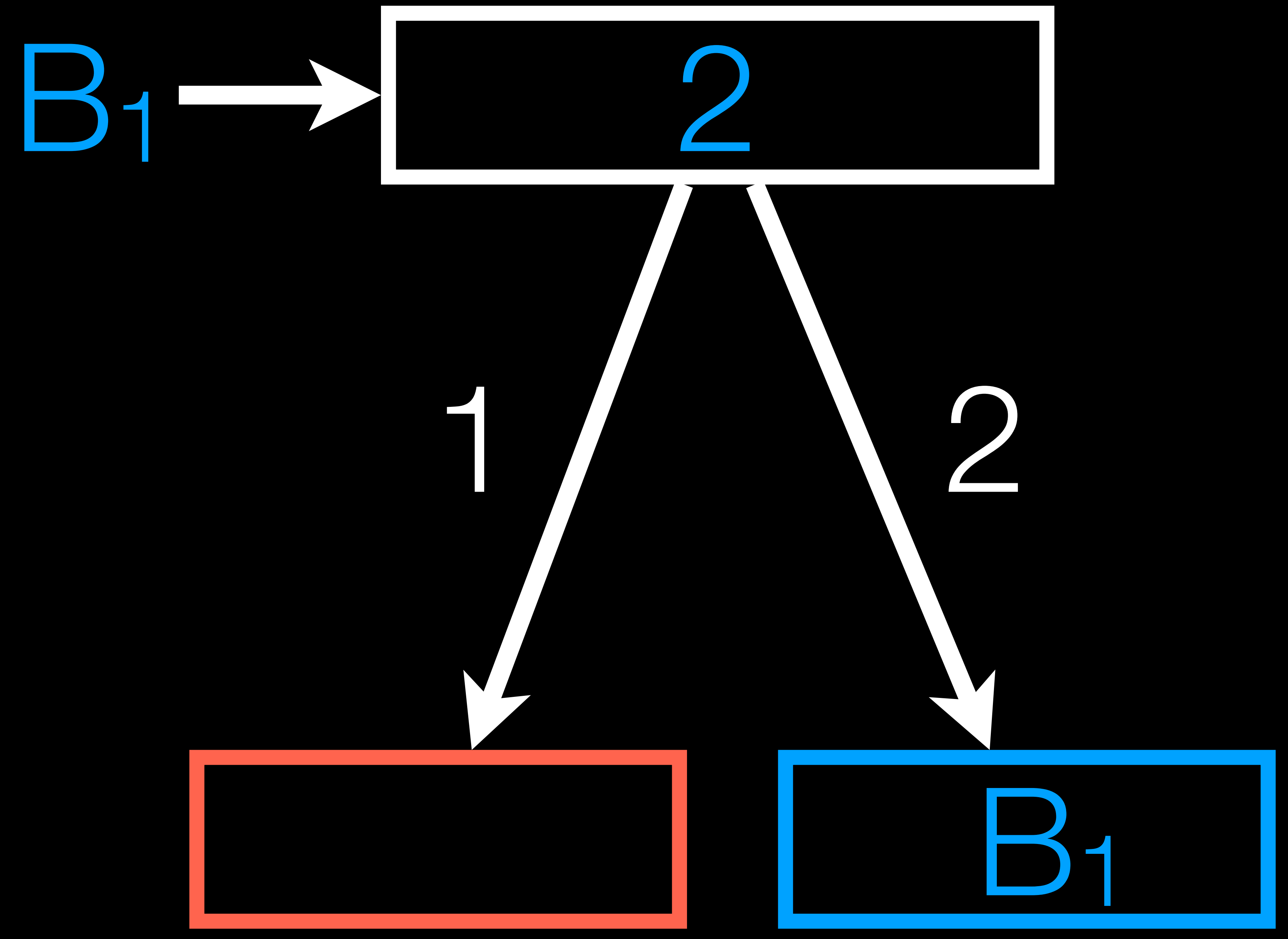

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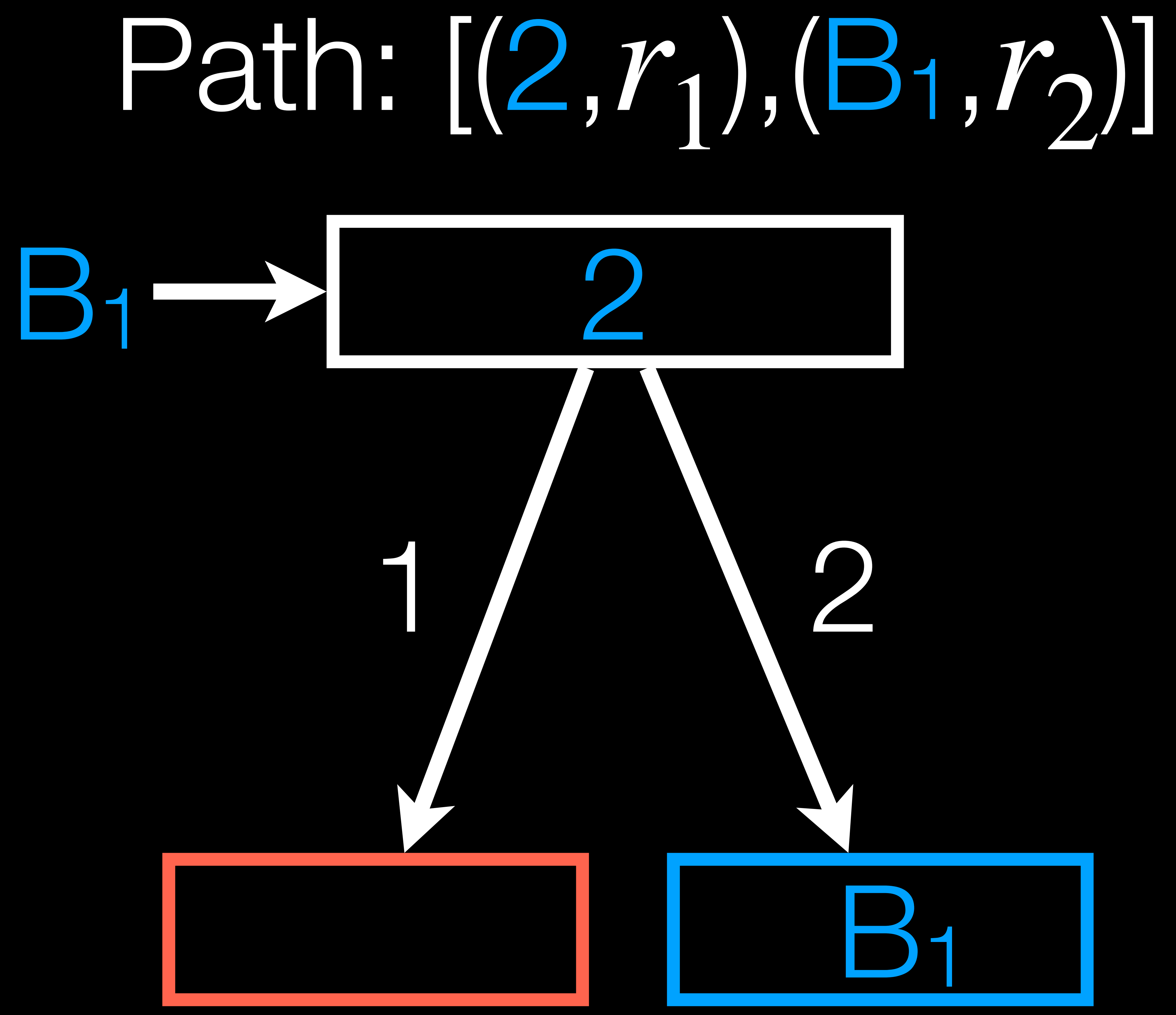
tree shape ⟷ language expressivity



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Compare expressivity of languages?

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Compare expressivity of languages?
Compare expressivity of *trees*?

Which leads to some very PL-ey questions:

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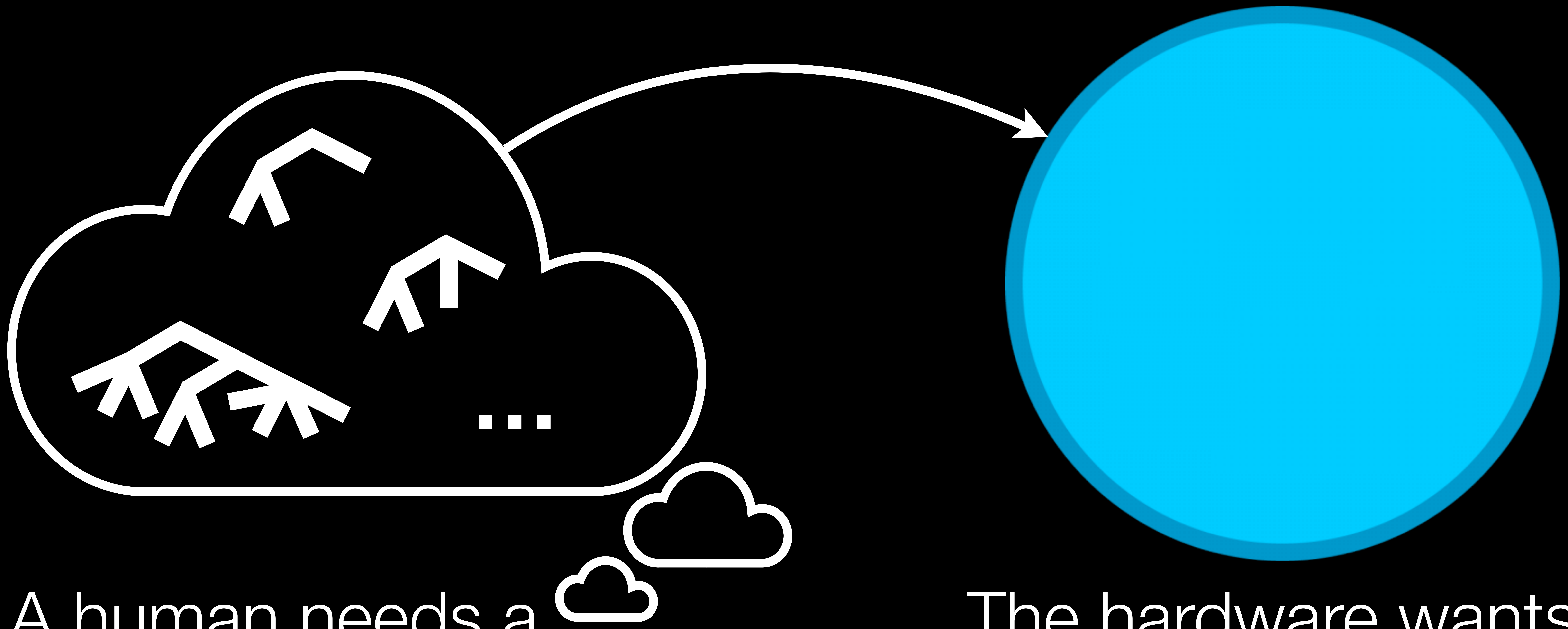
Compare expressivity of languages?
Compare expressivity of *trees*?

Compile a program so it runs against a new tree?

~~No general way to deploy our gadget.~~



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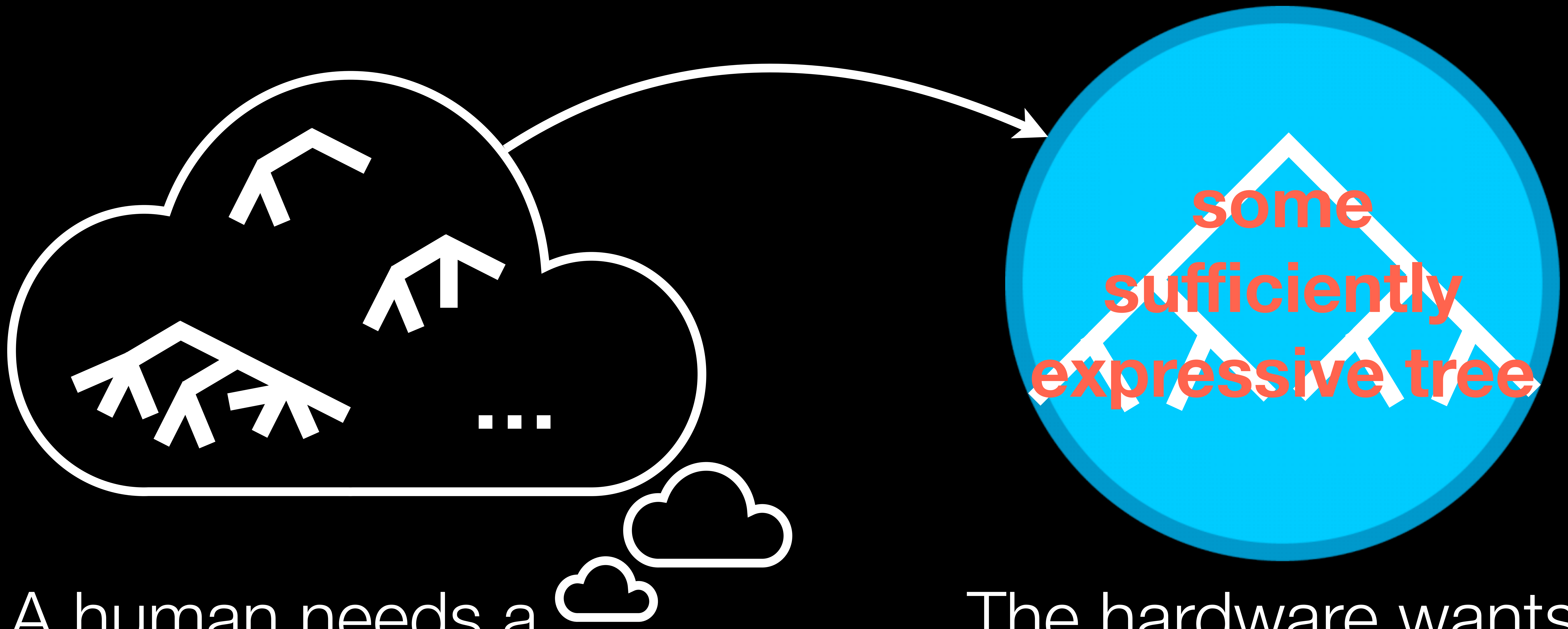

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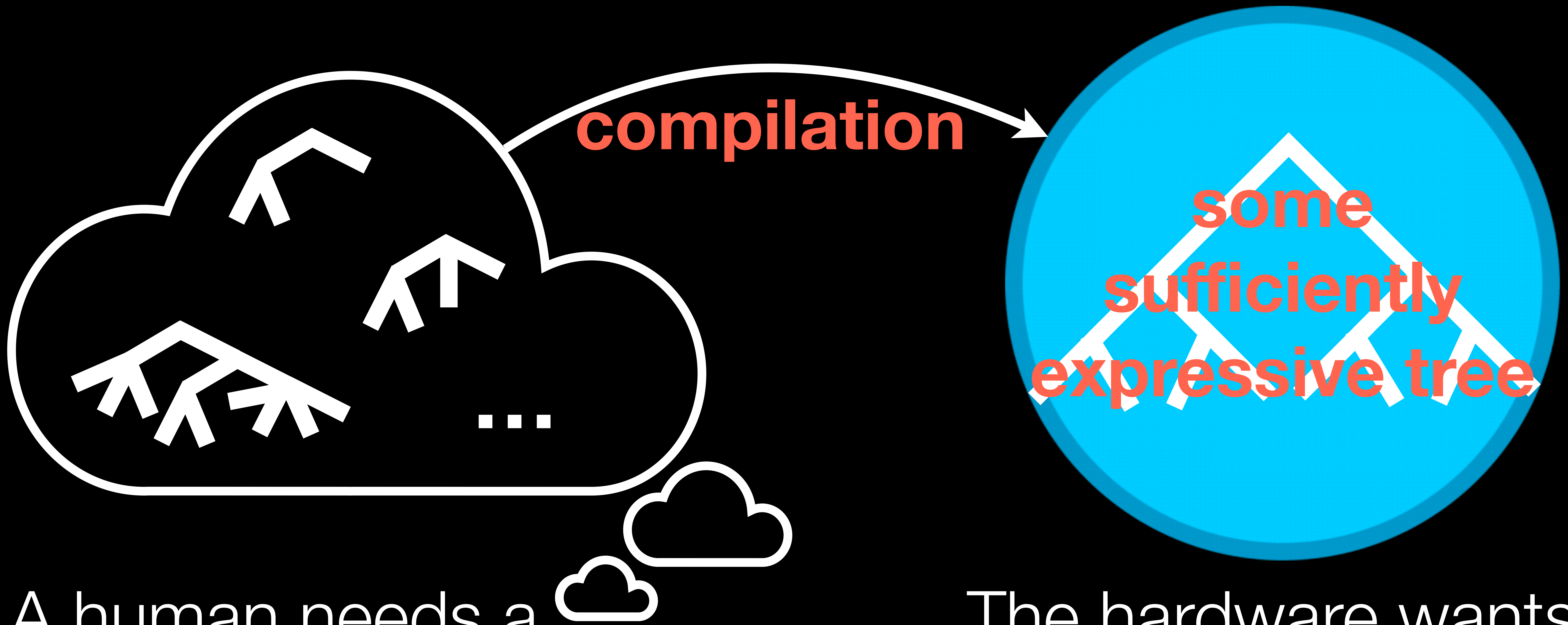

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Contributions

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Formal model of PIFO trees

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General theorems of expressiveness w.r.t. tree shape

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General theorems of expressiveness w.r.t. tree shape

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Contributions

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Compiler

Simulator

Expressivity of trees

Trees with more leaves are more expressive.

Taller trees are more expressive.

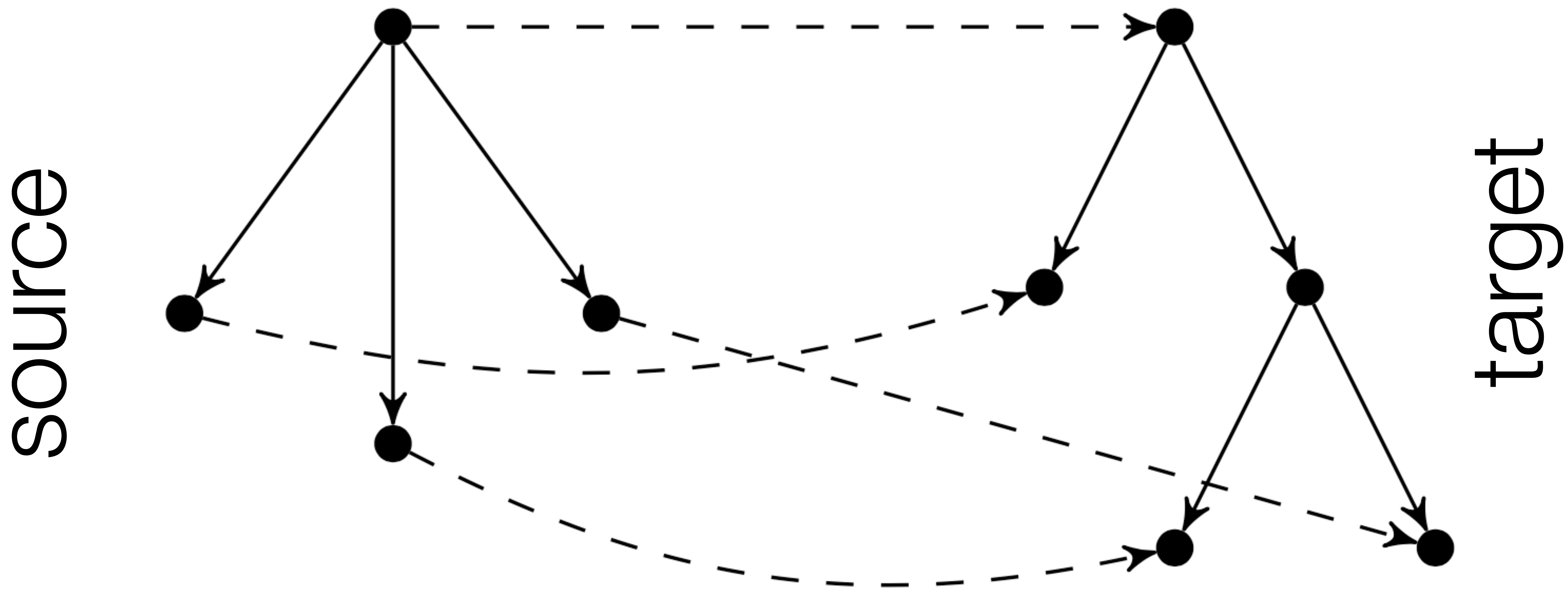
Expressivity of trees

Trees with more leaves are more expressive.
Taller trees are more expressive.

Captured elegantly by:

Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.

Expressivity of trees



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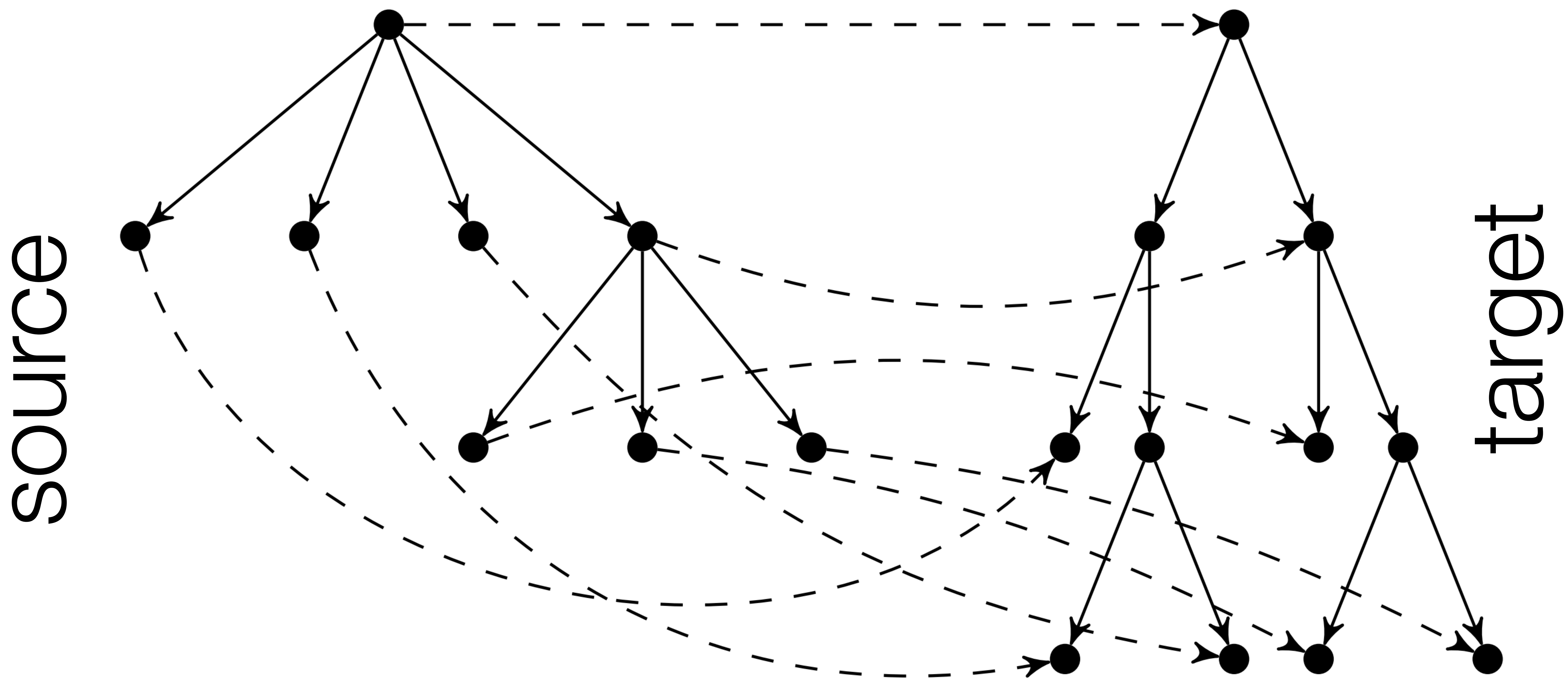

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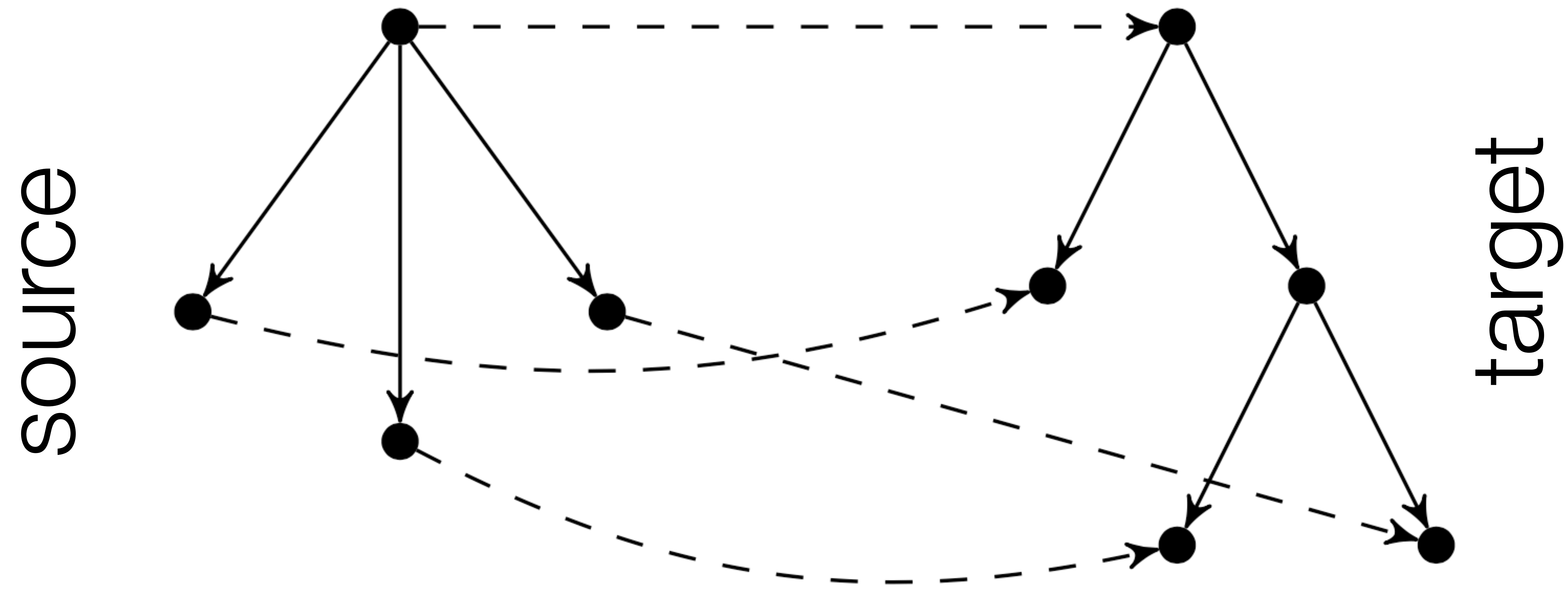

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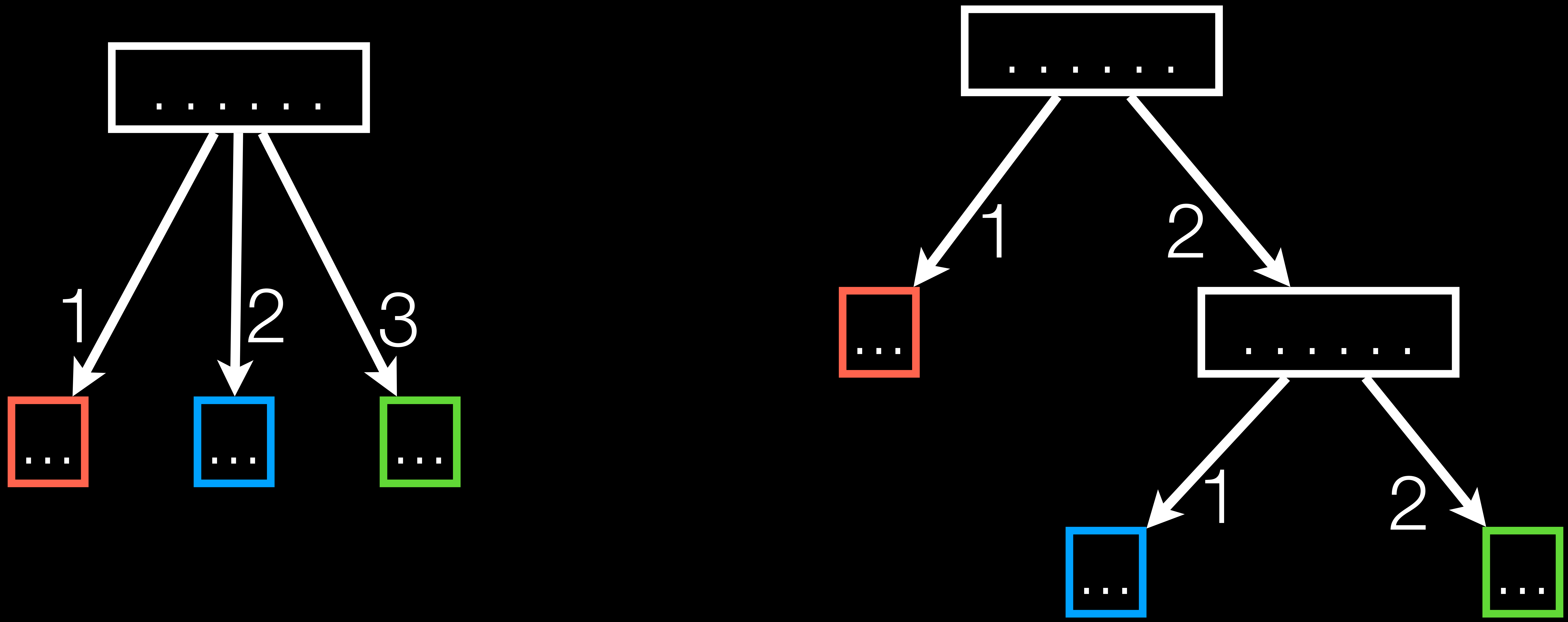
Compiling programs



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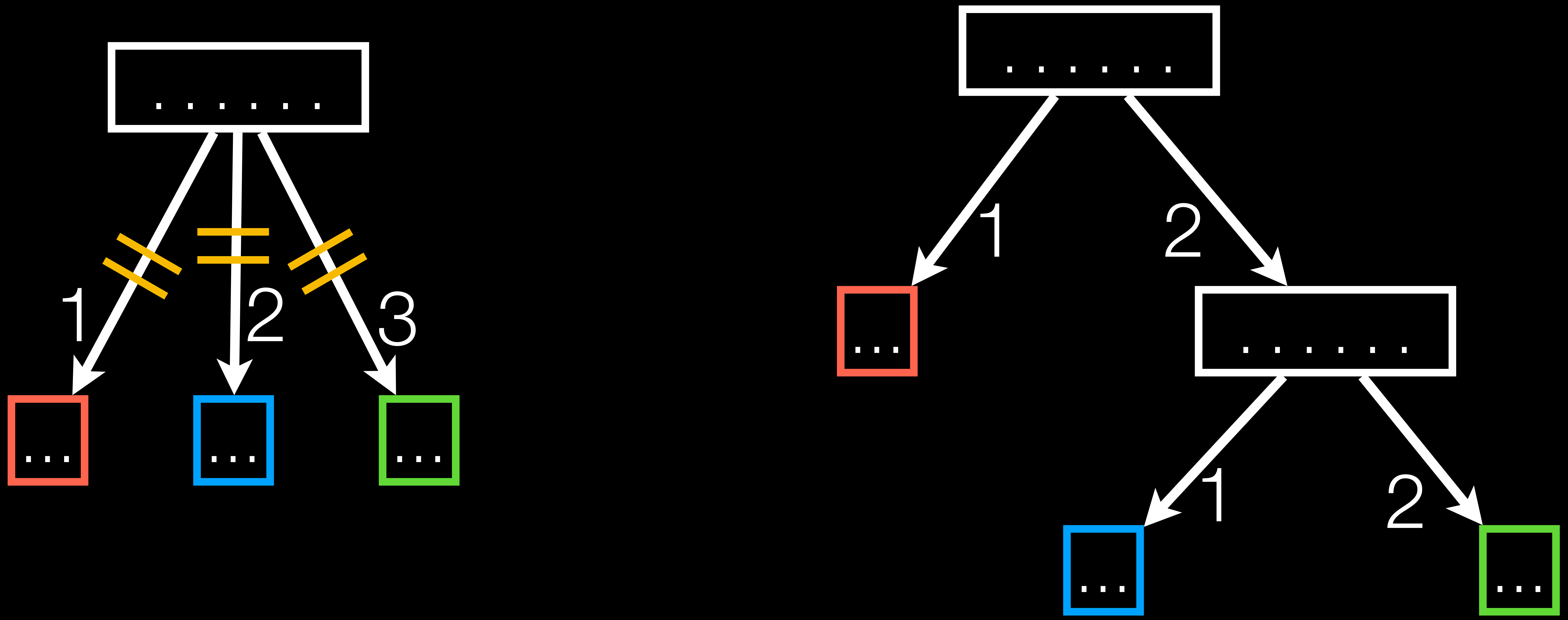

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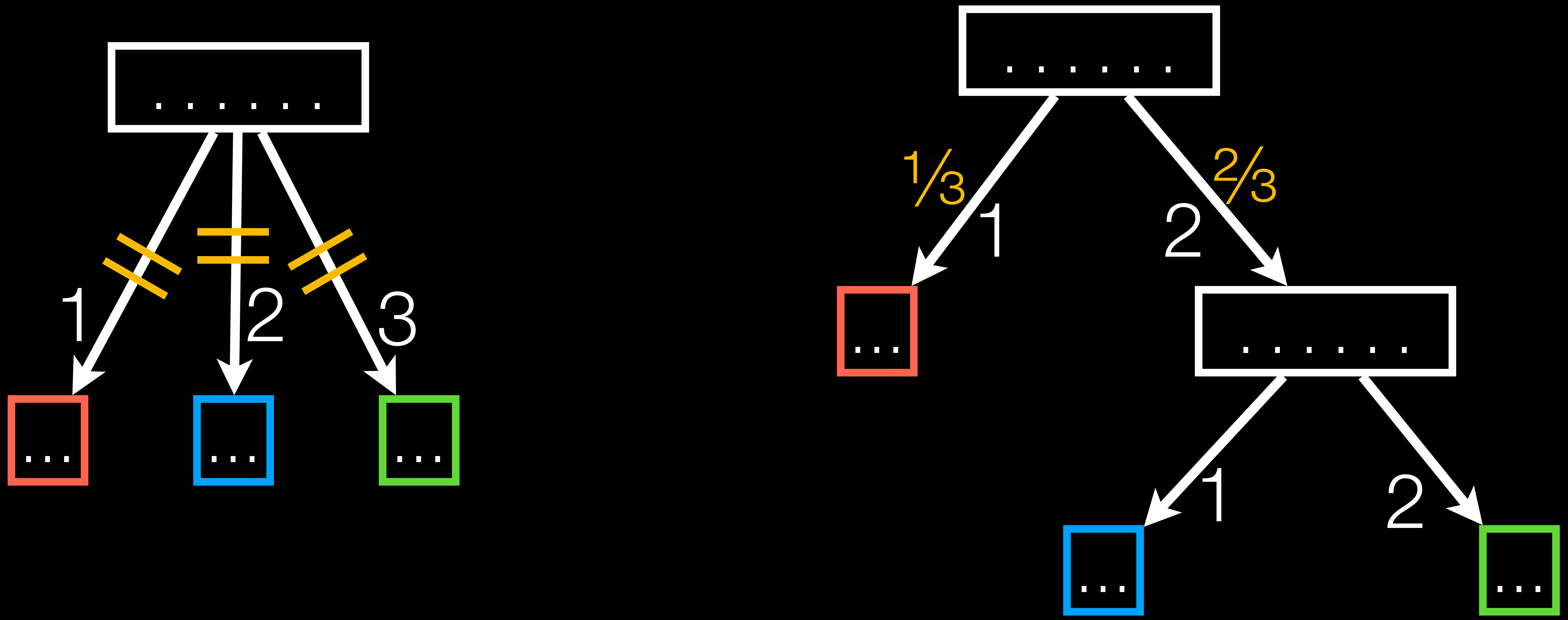

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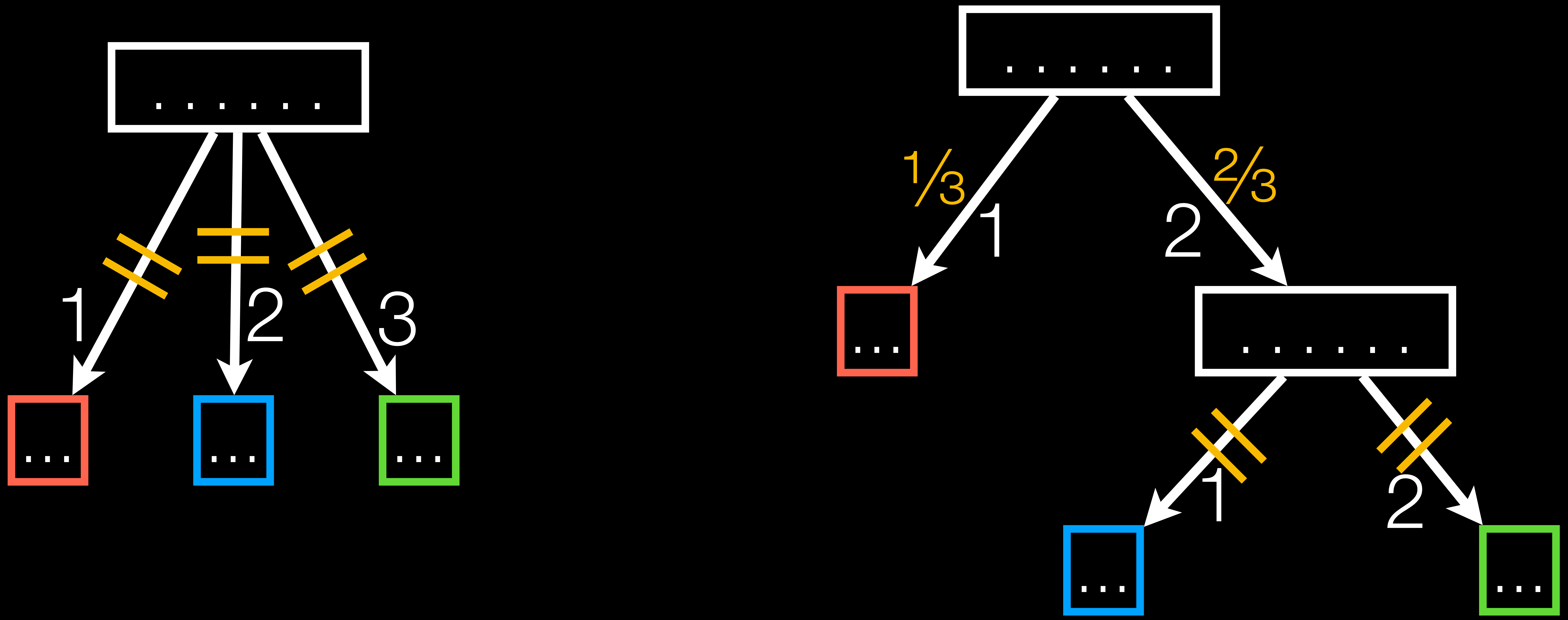

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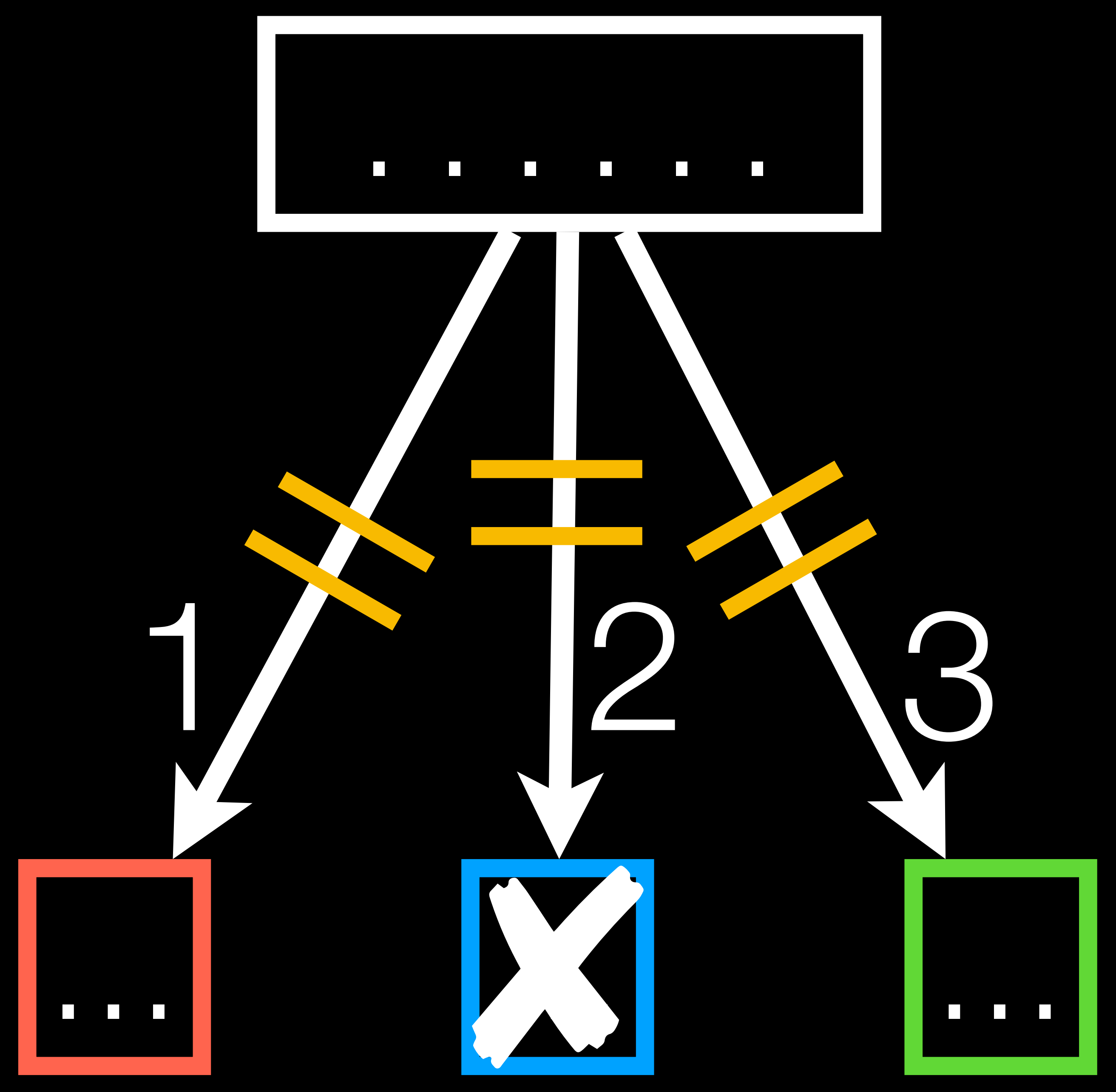

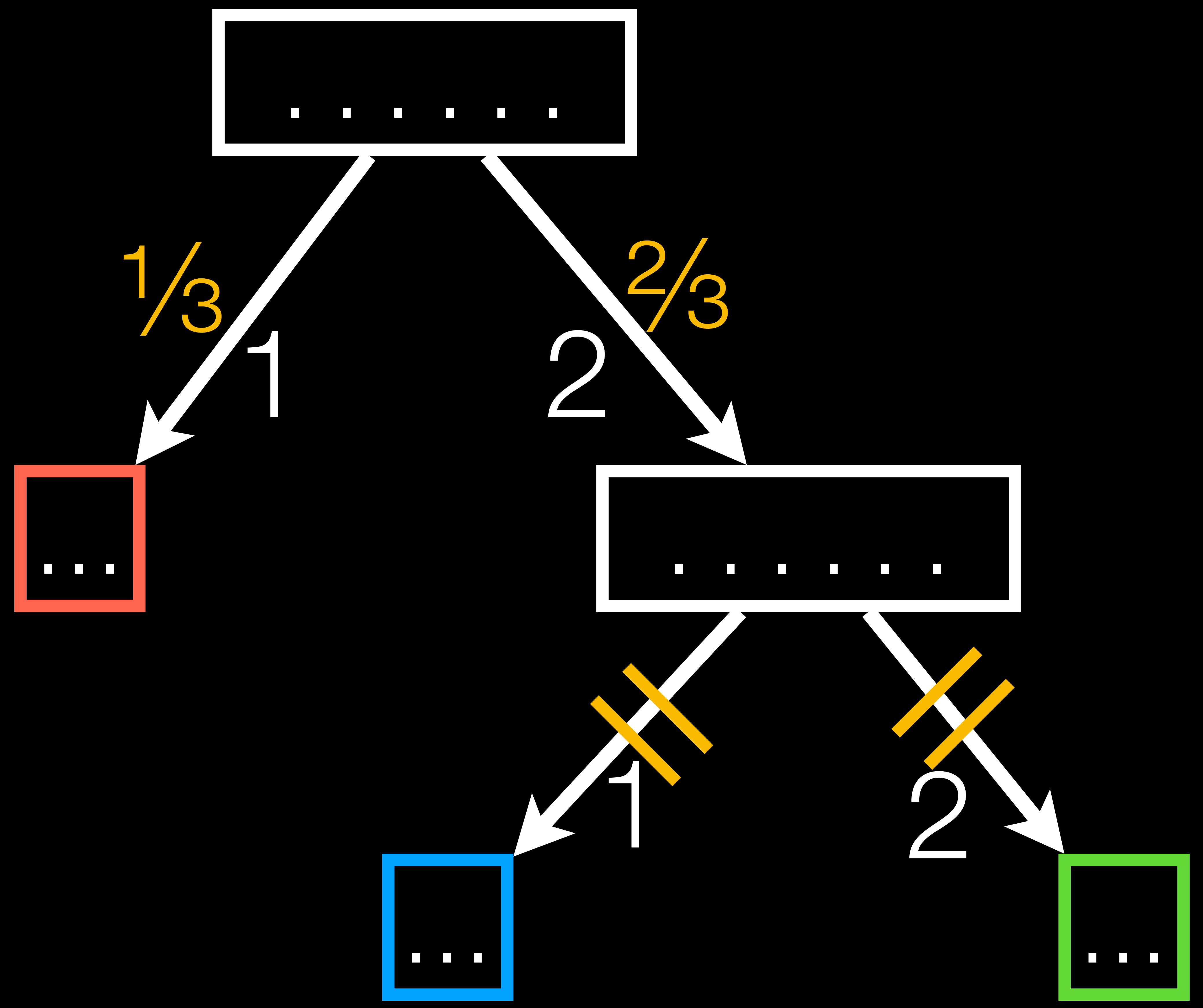

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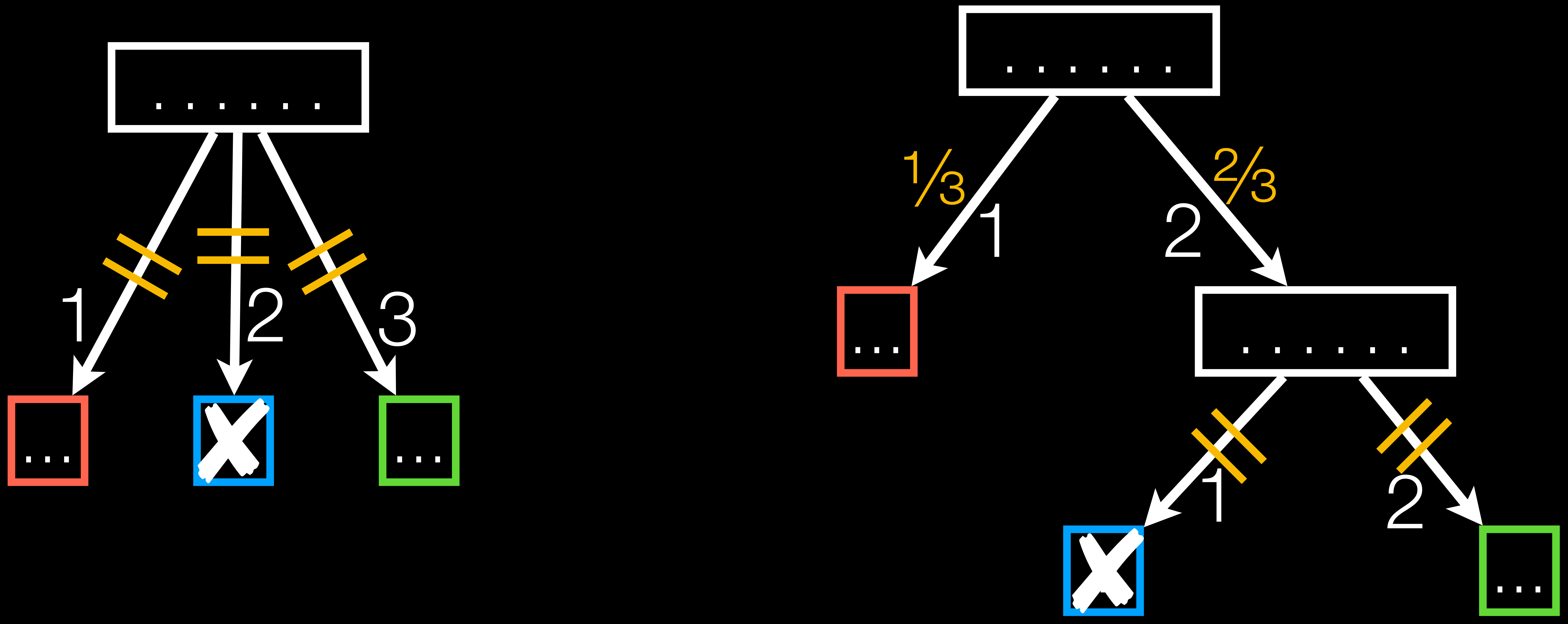

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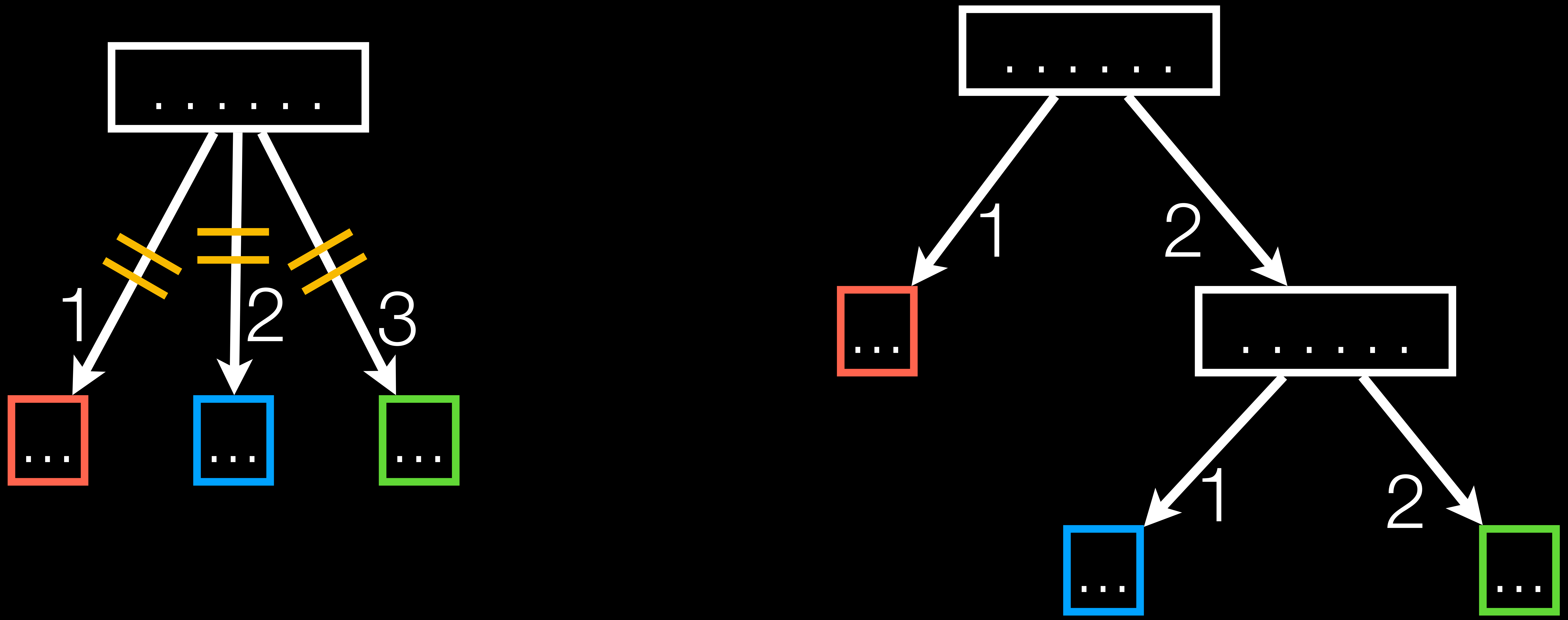

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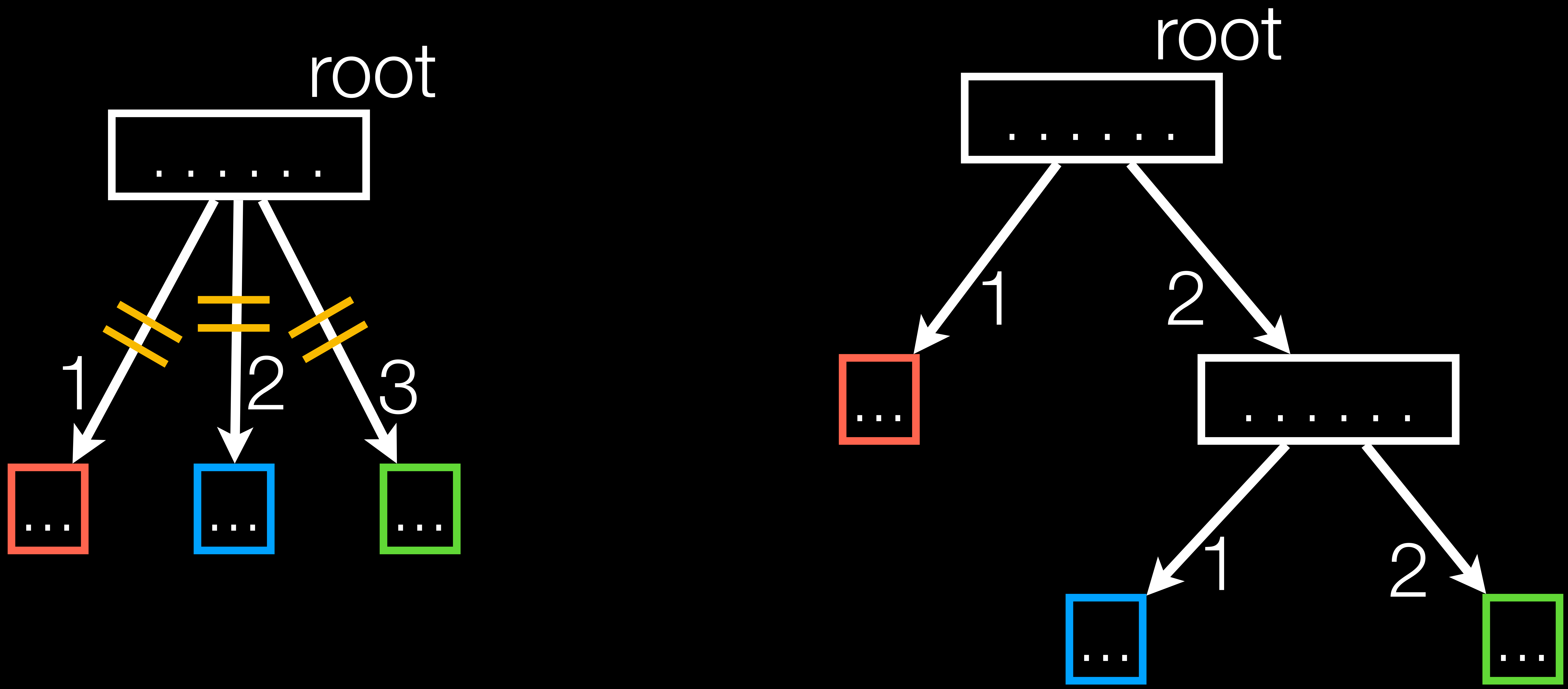

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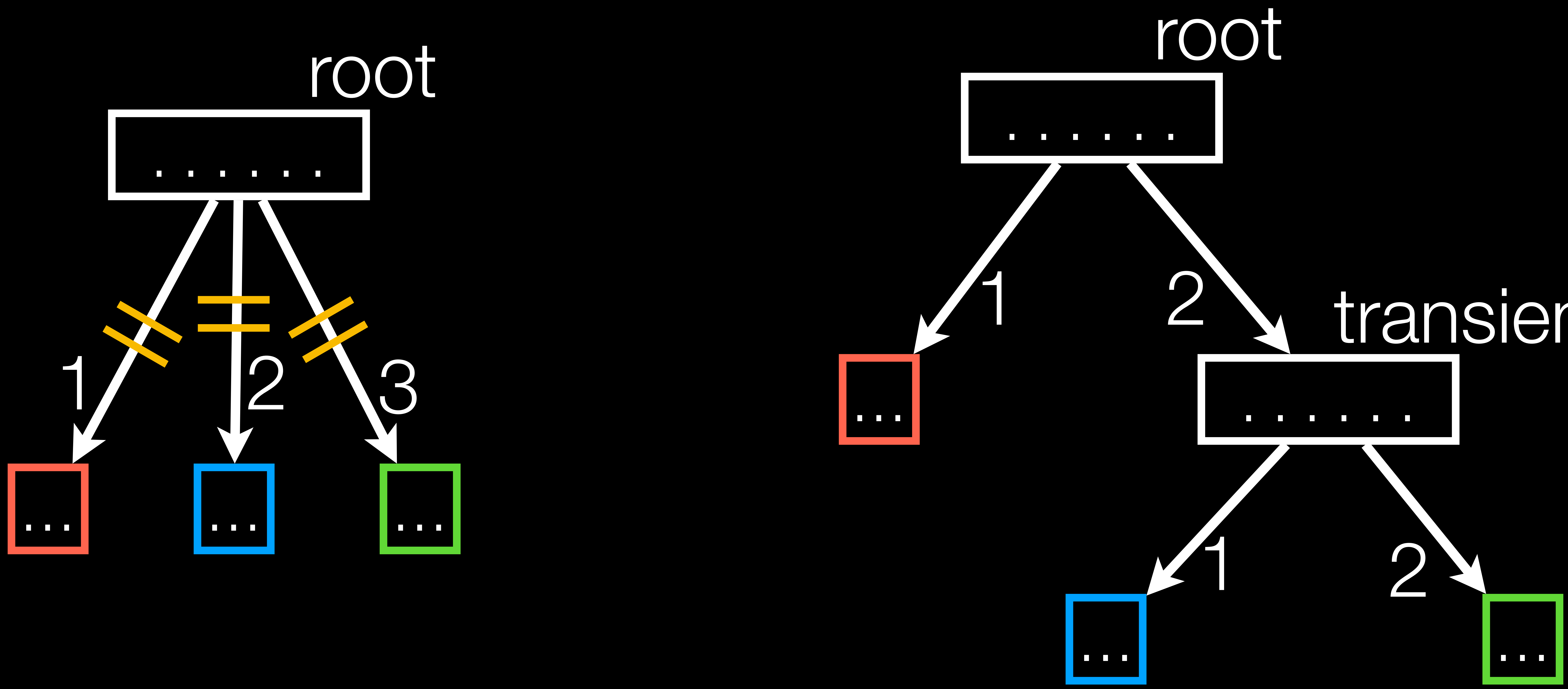

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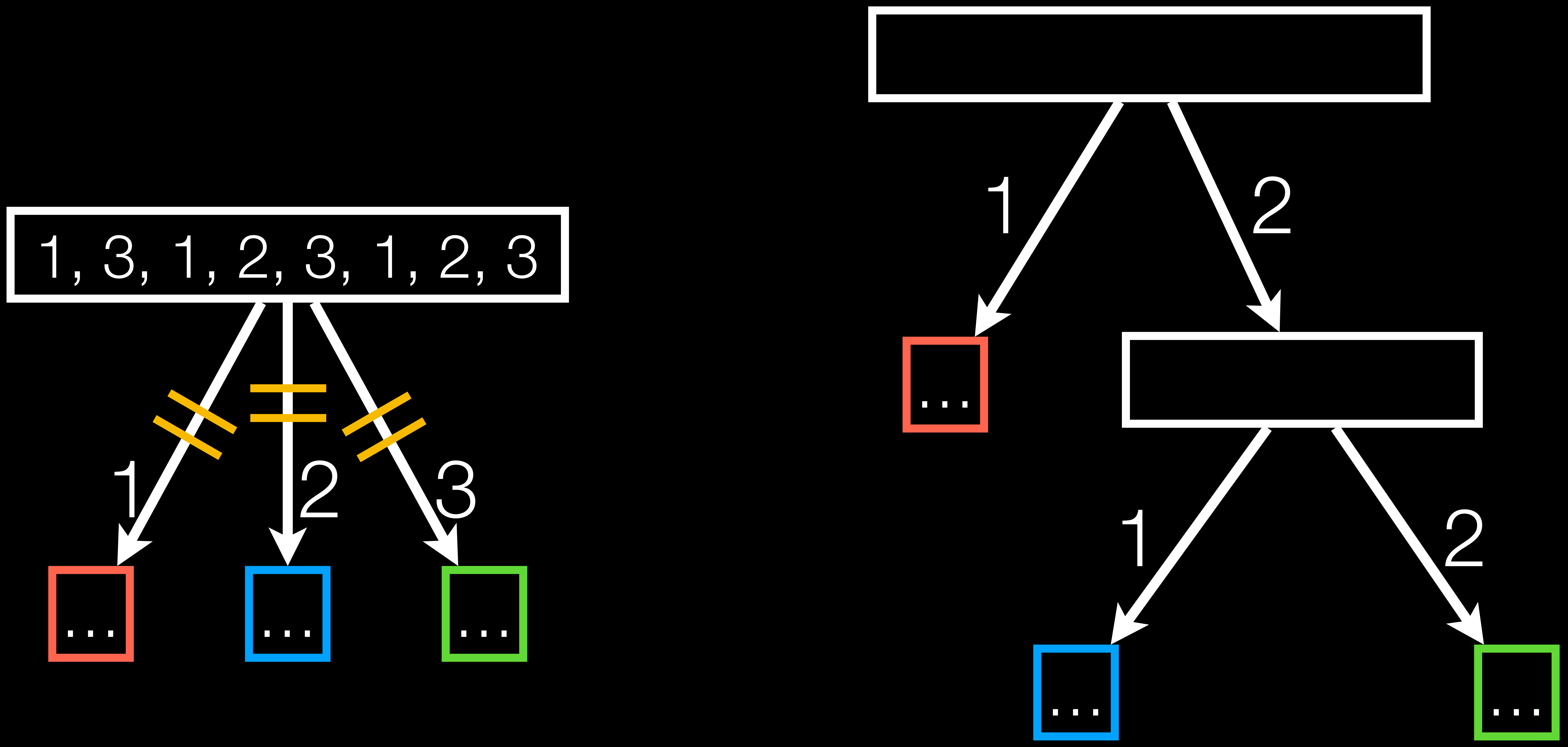

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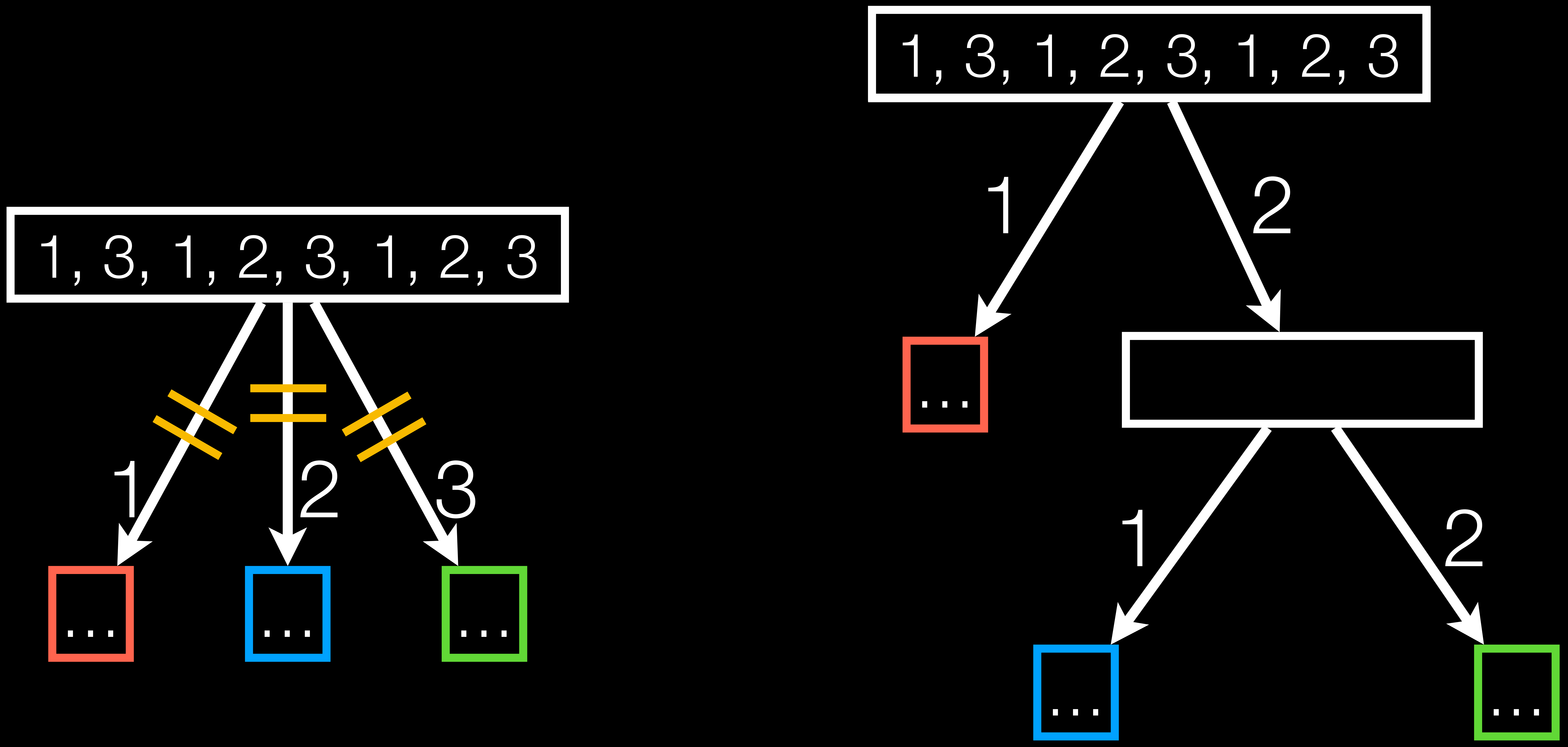

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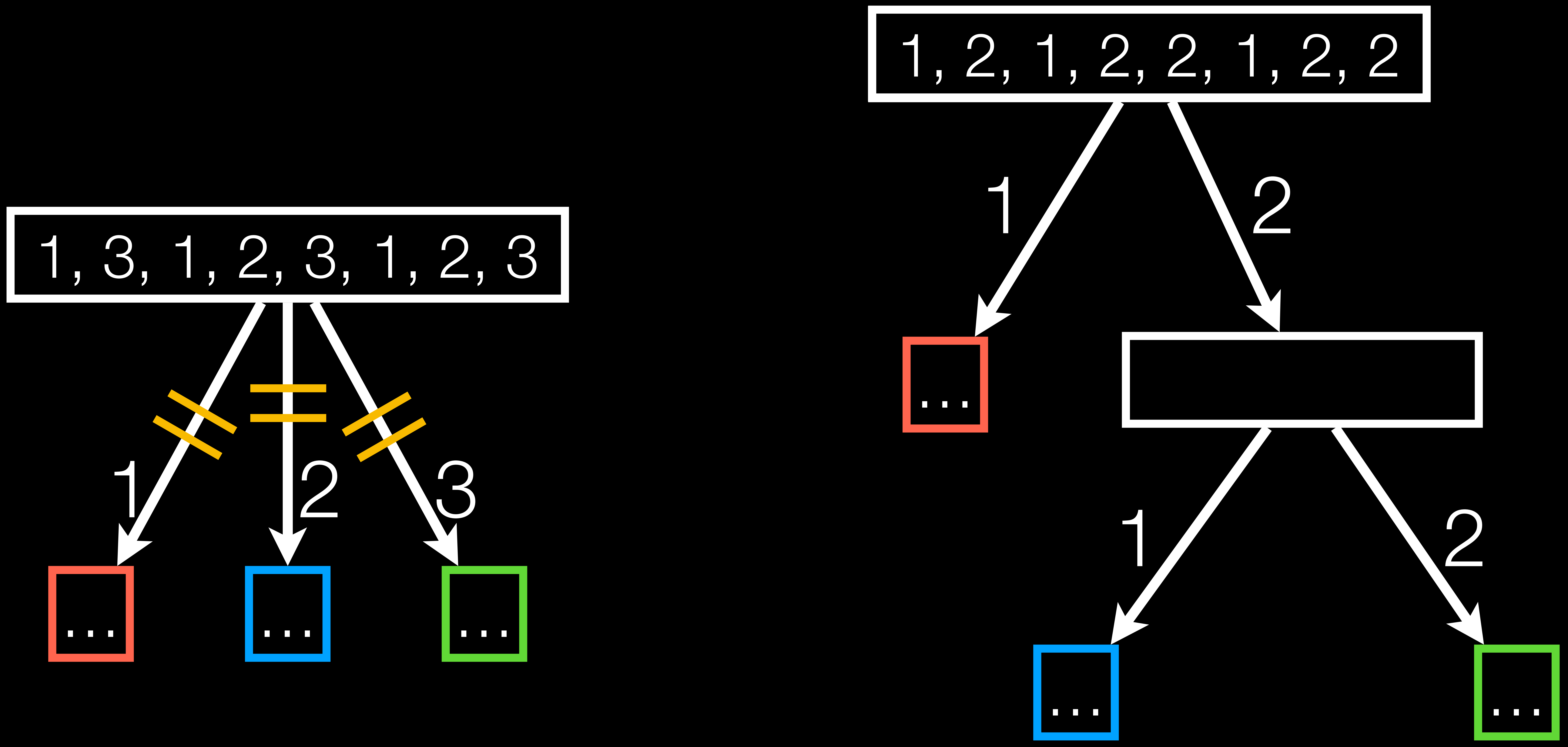

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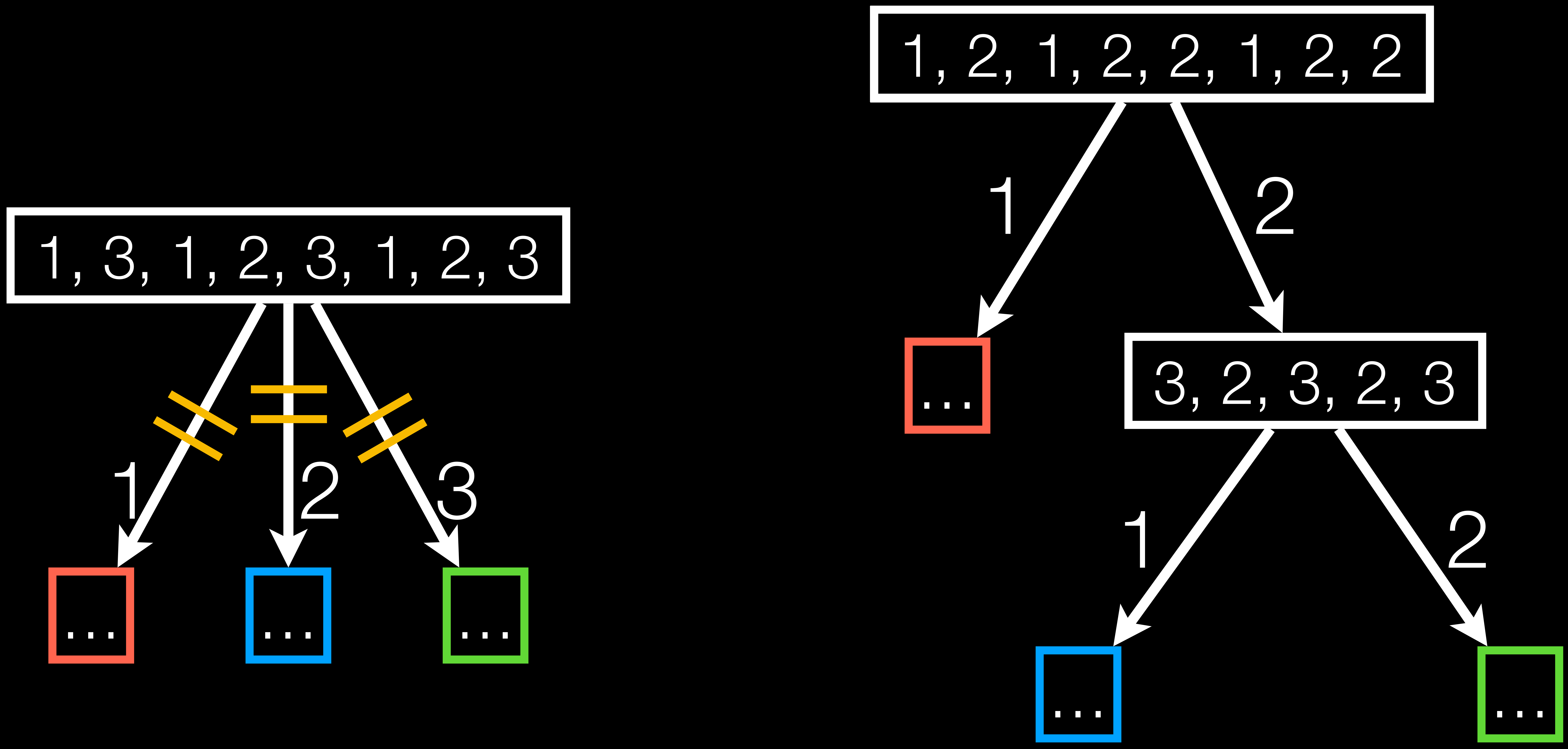

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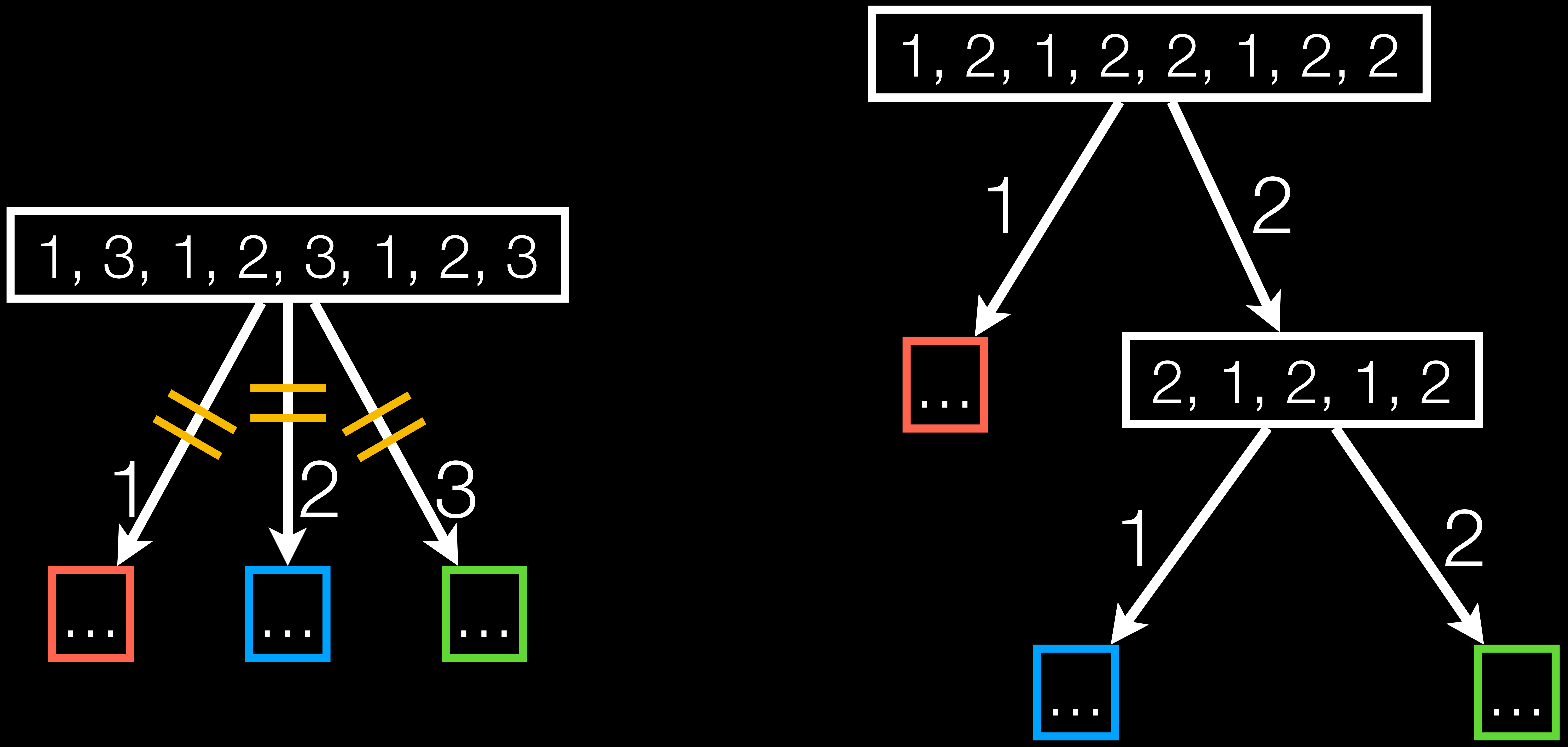

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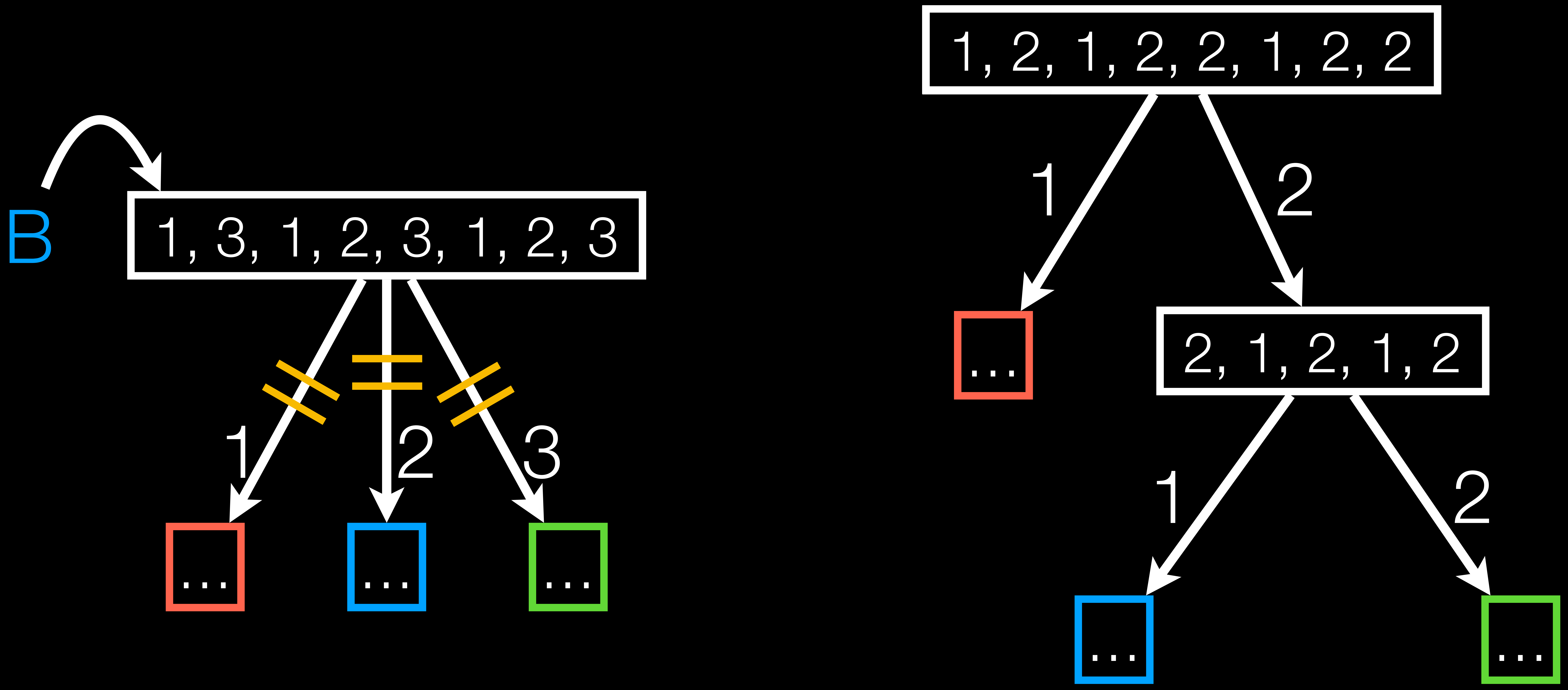

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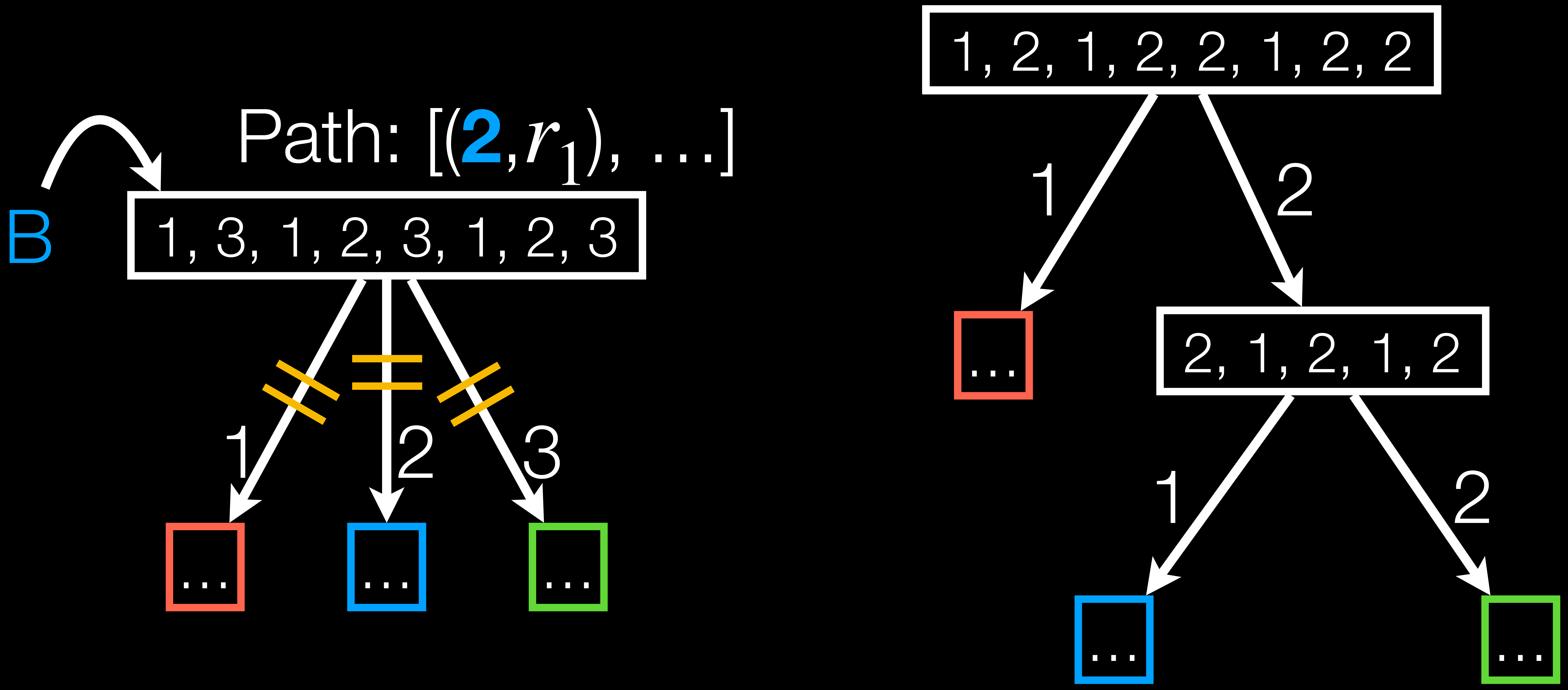

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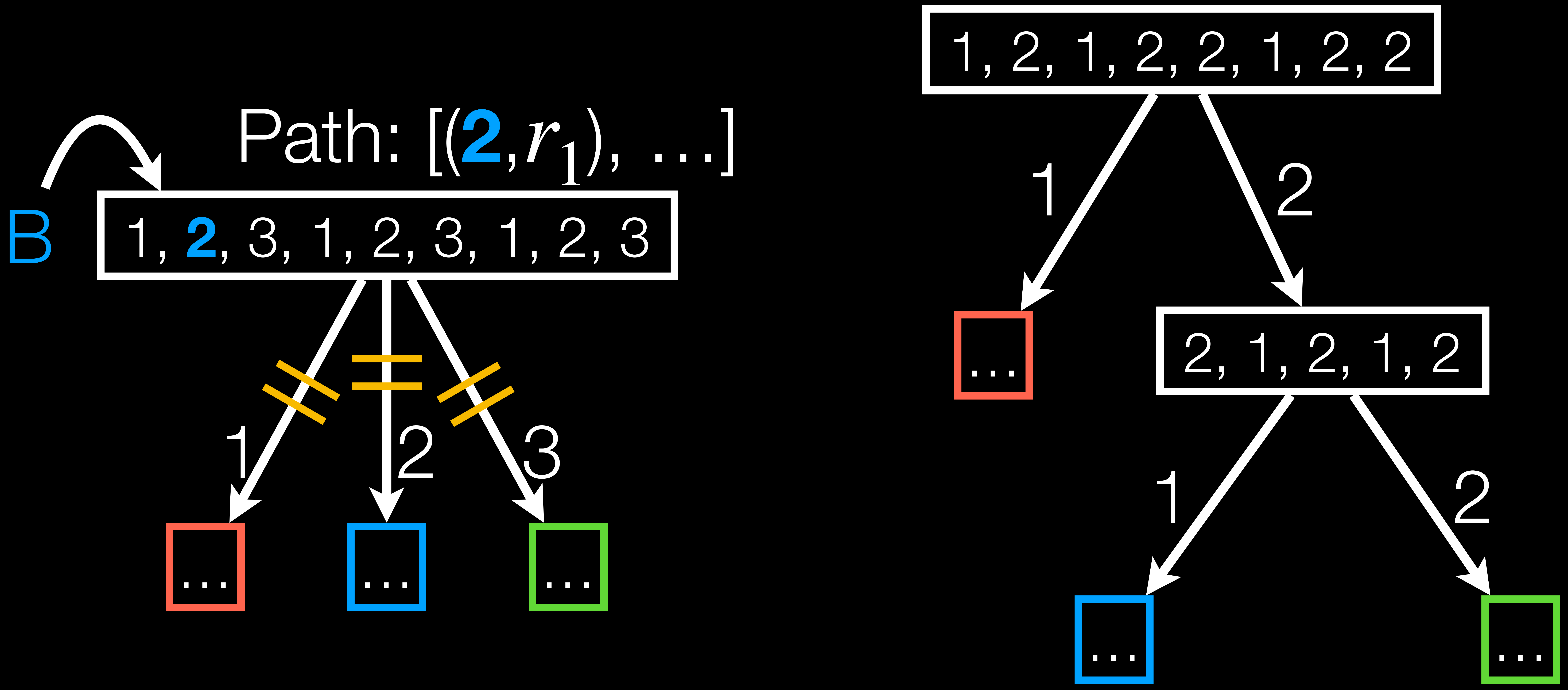

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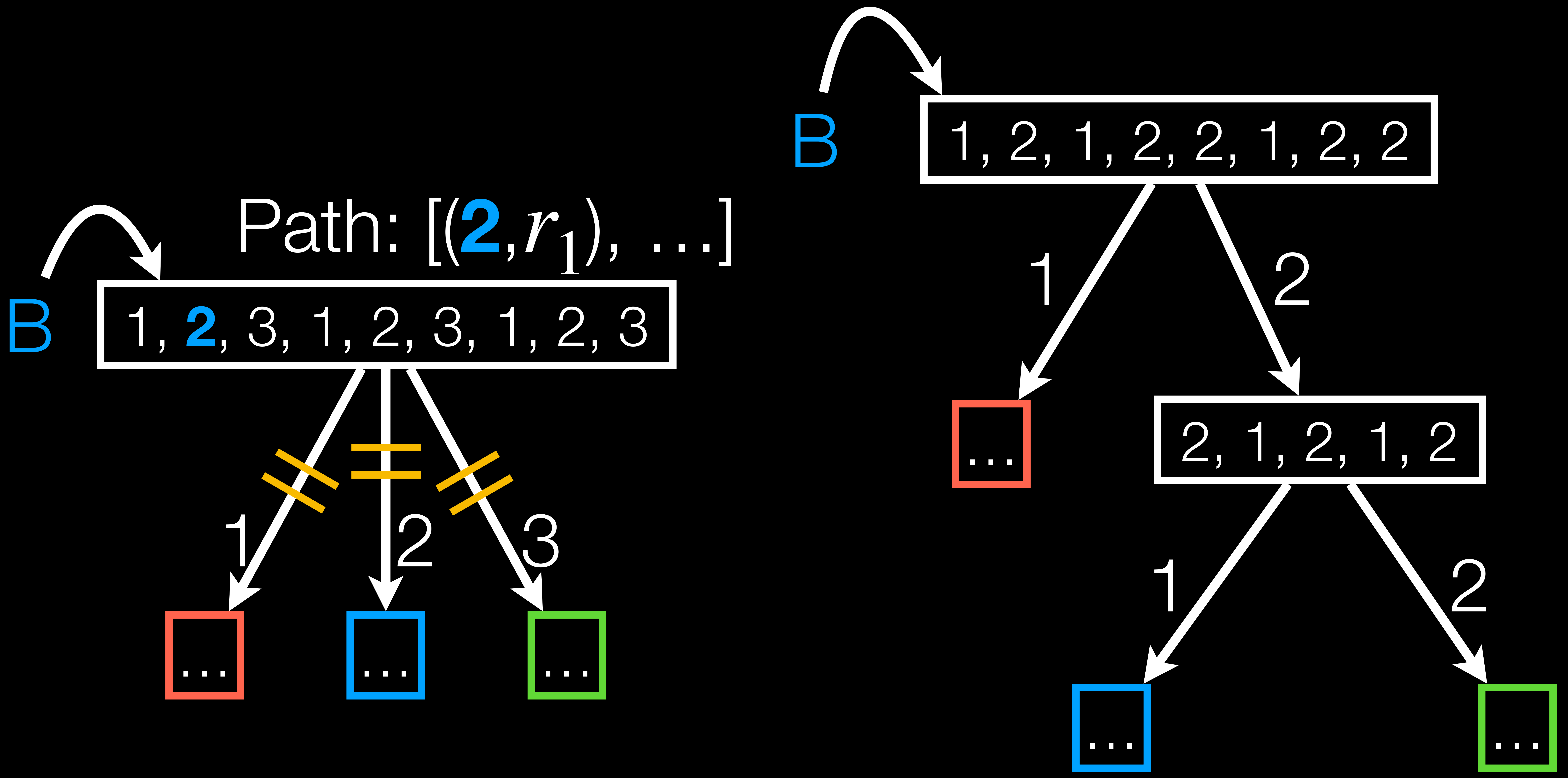

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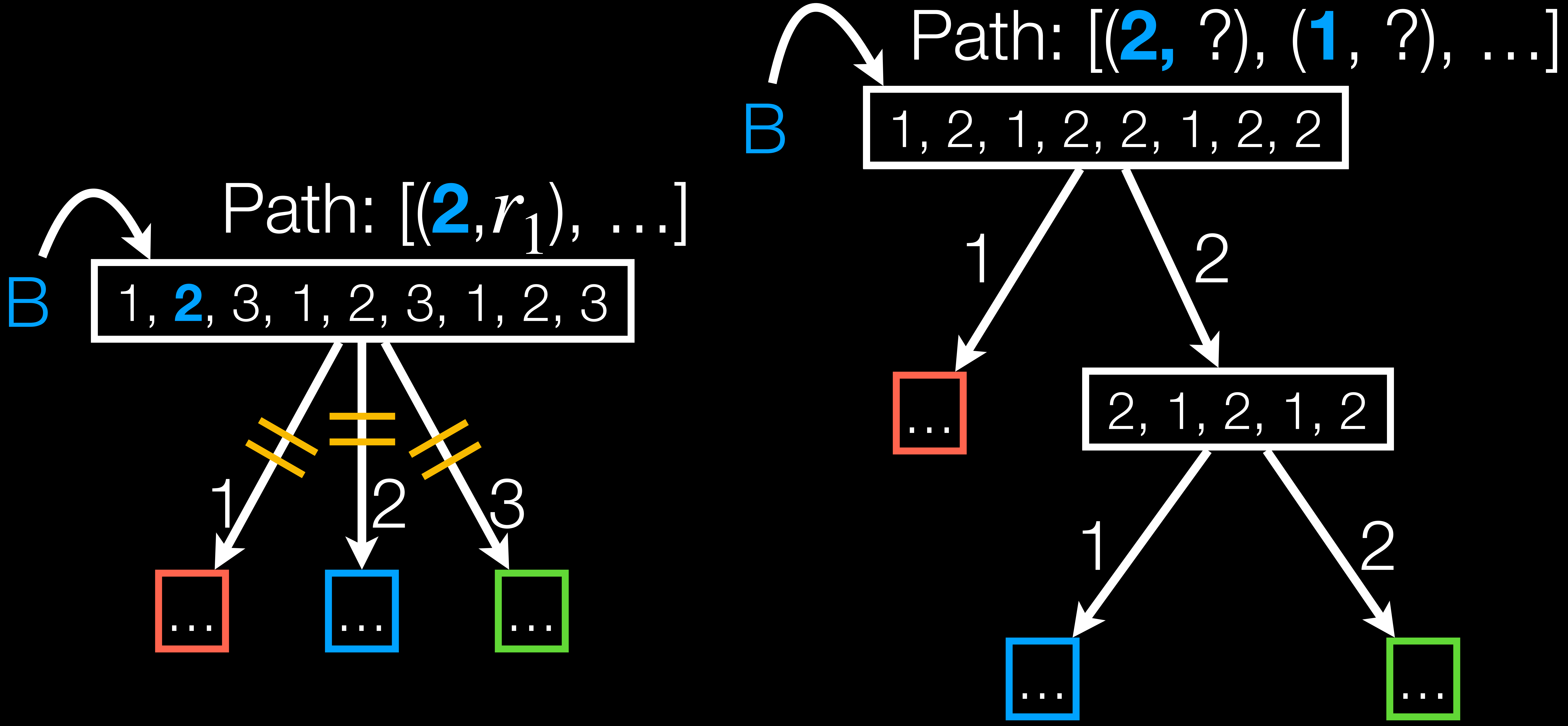

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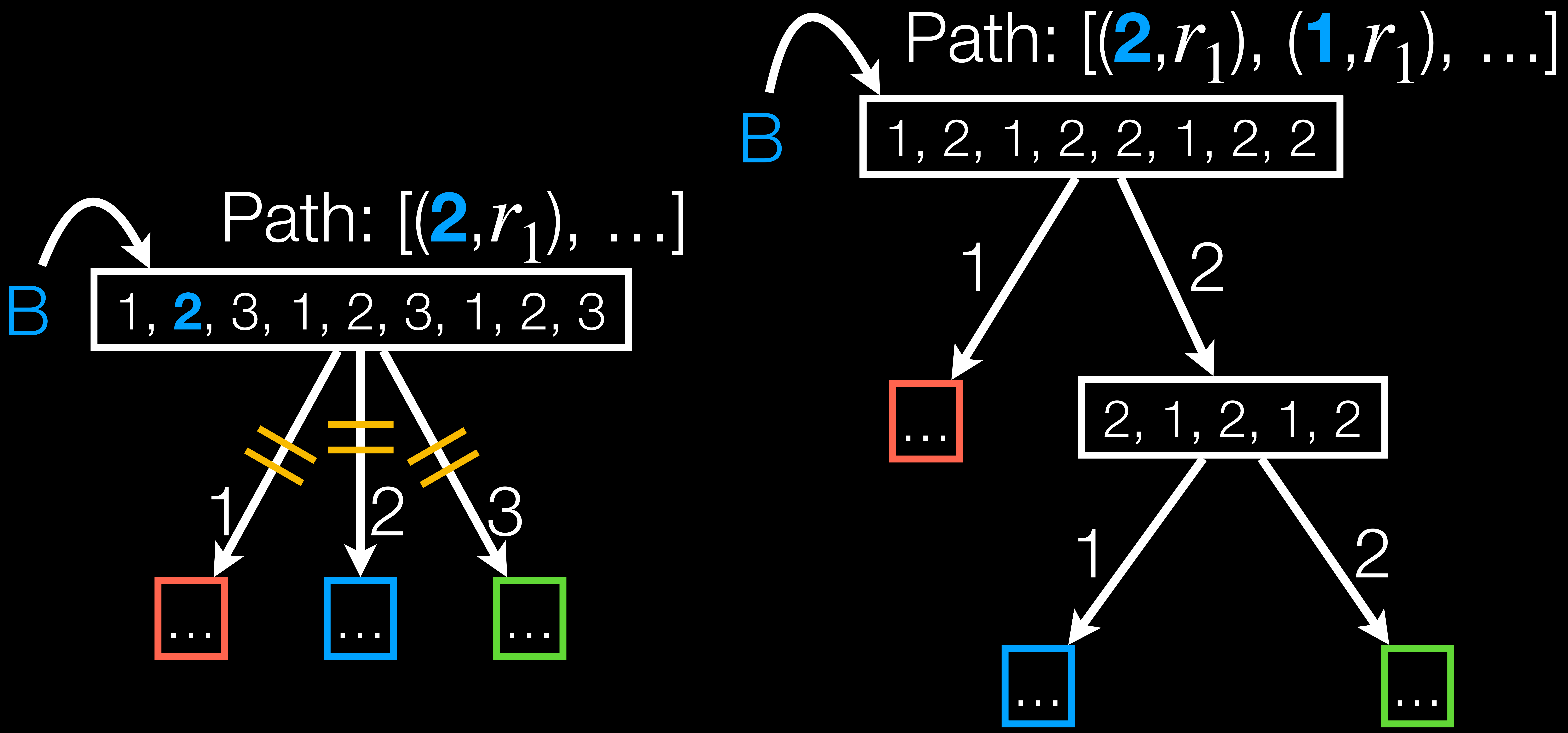

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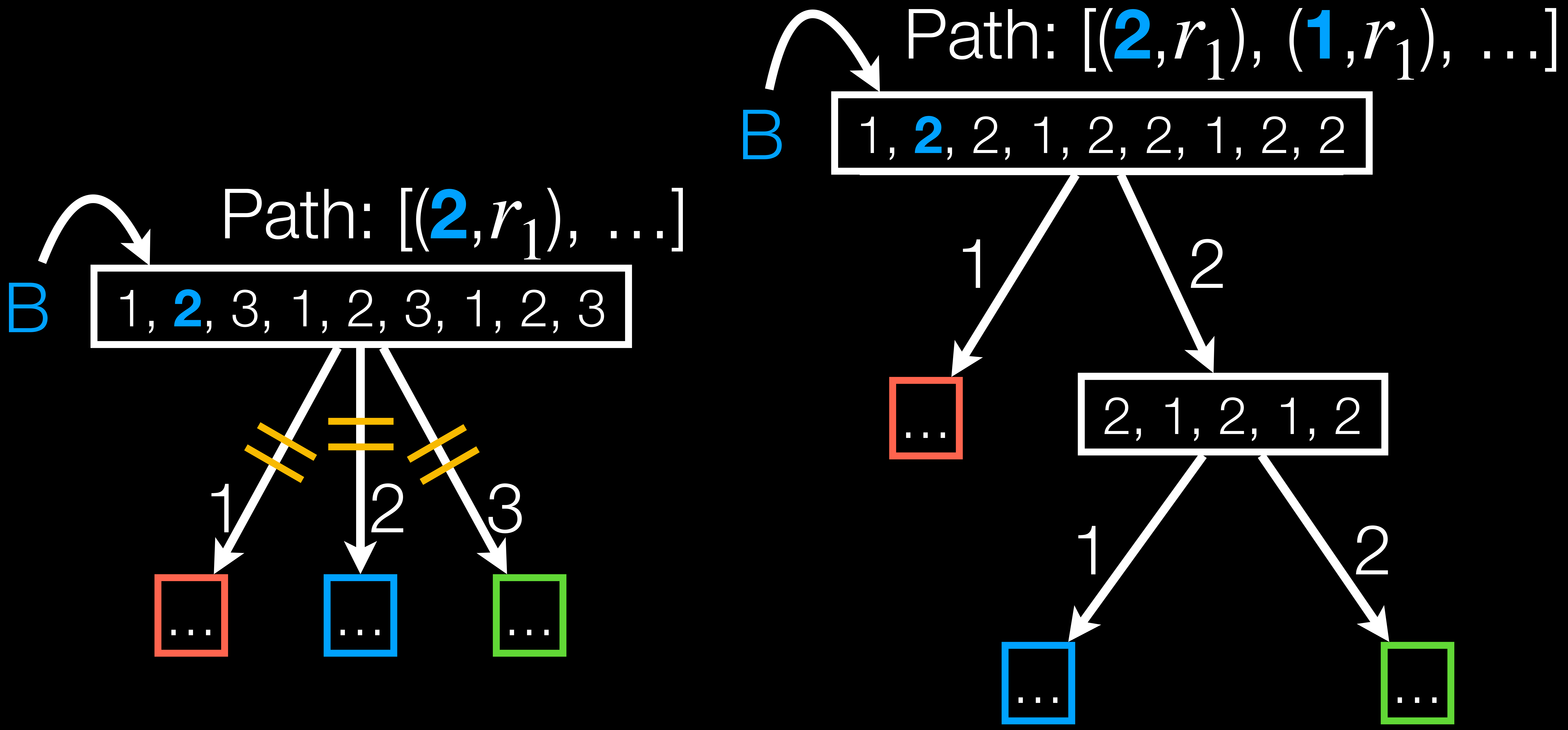

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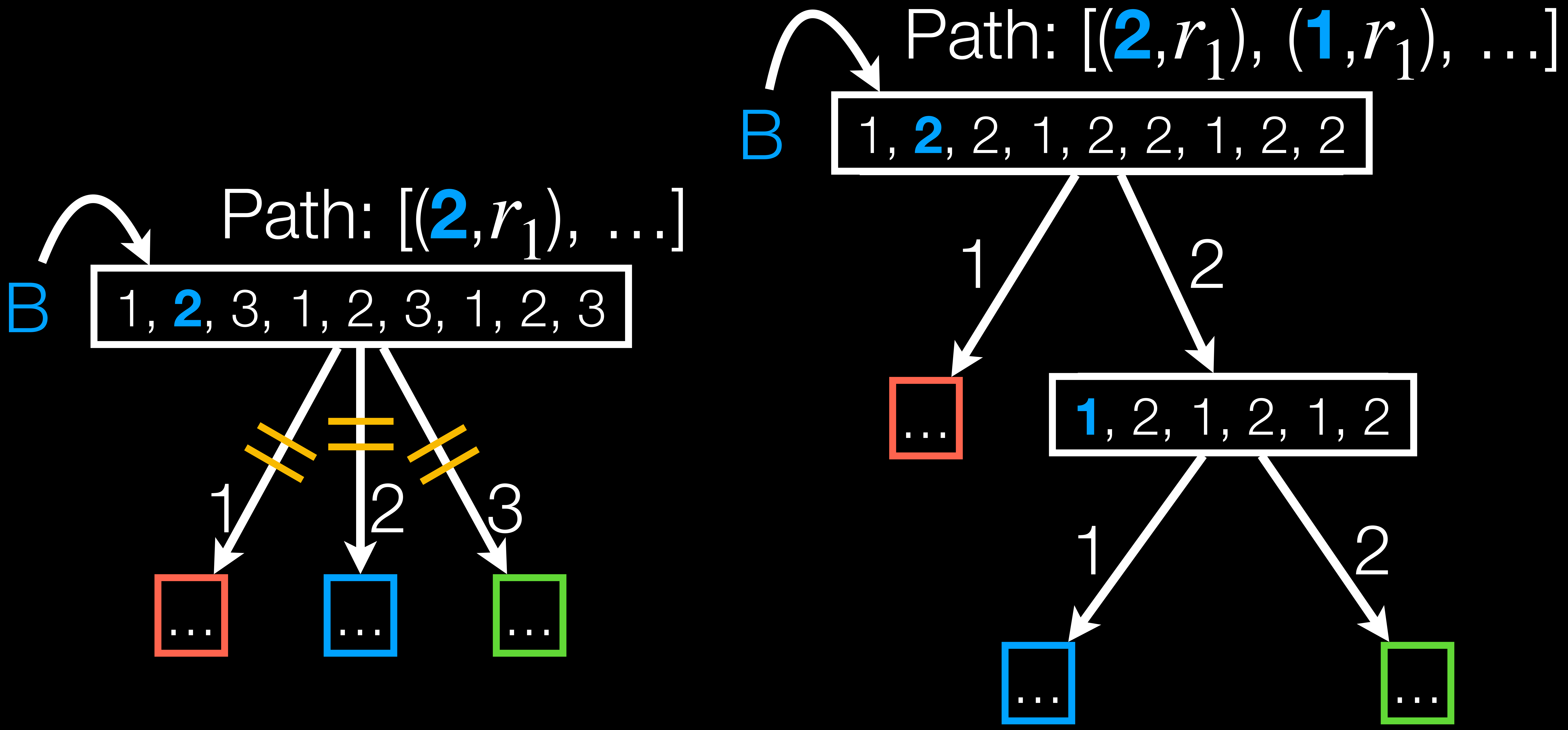
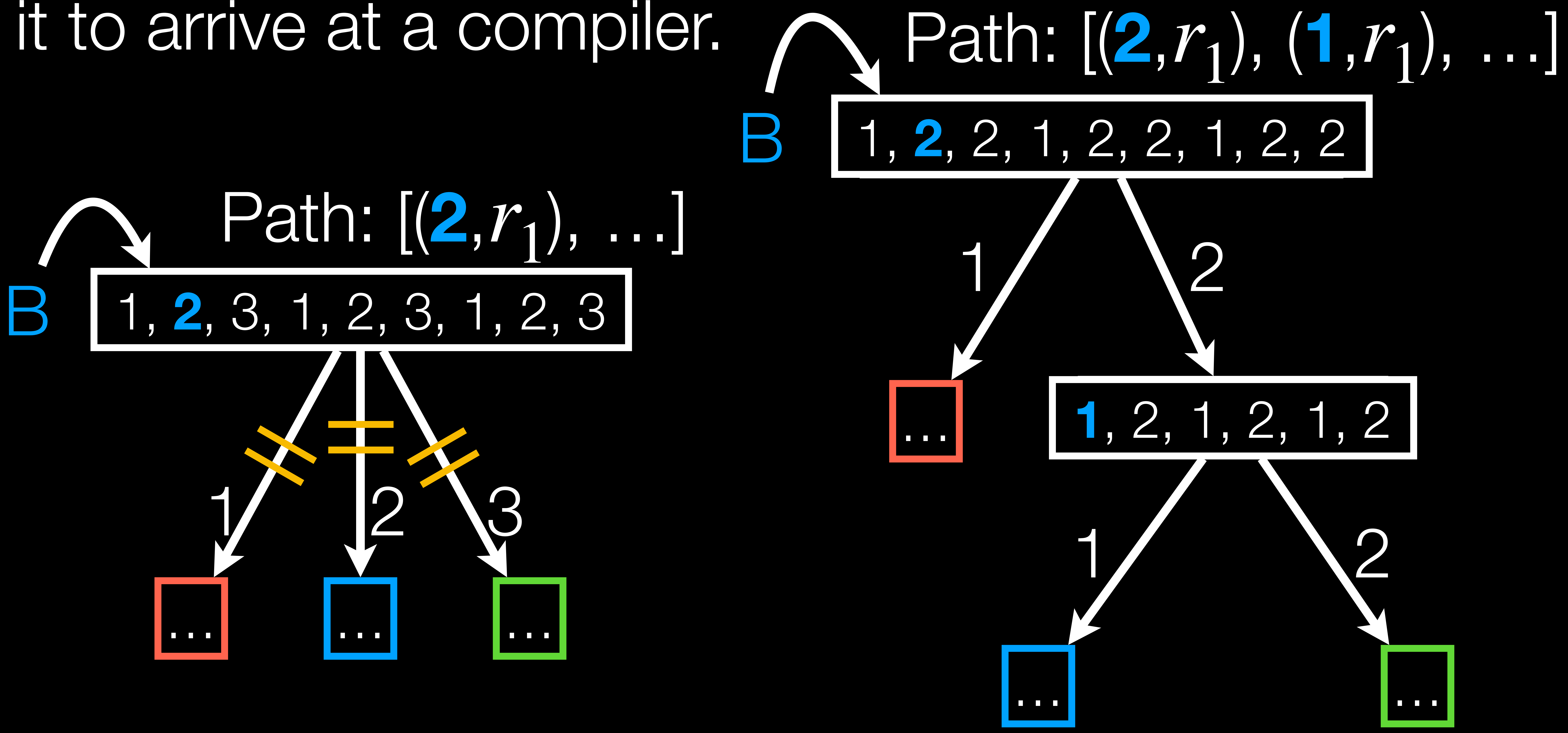


Given an embedding, we *lift* it to arrive at a compiler.



Generating embeddings automatically!

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Homomorphic embedding.

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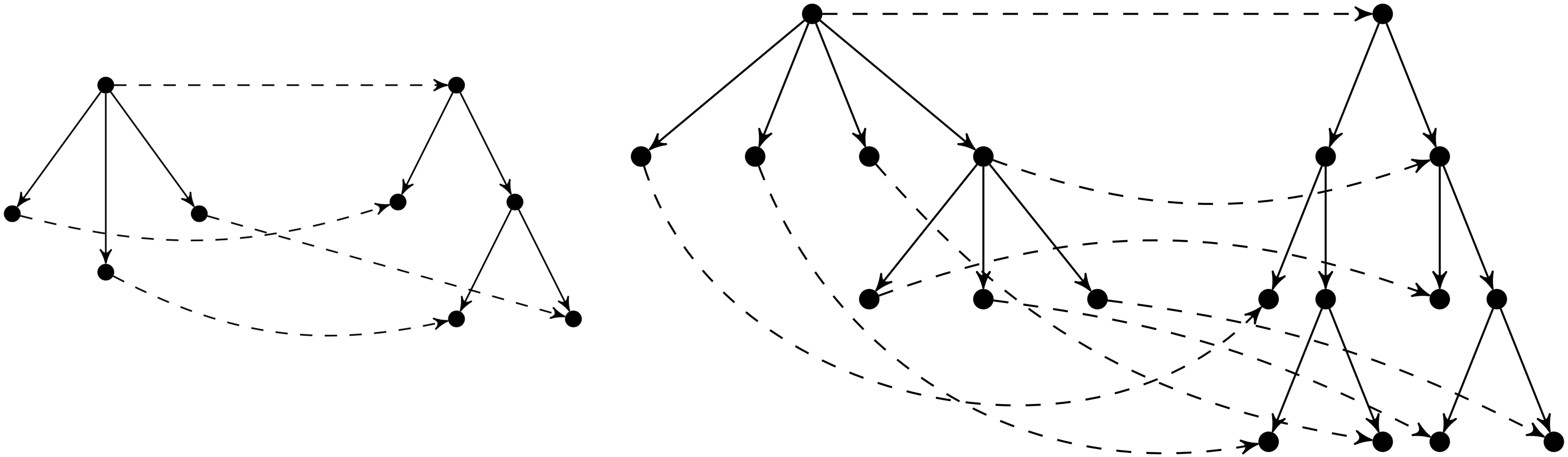

Generating embeddings automatically!

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Two new algorithms,
both starting with heterogeneous source trees.

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1. If target tree is regular *d*-ary for some *d*.

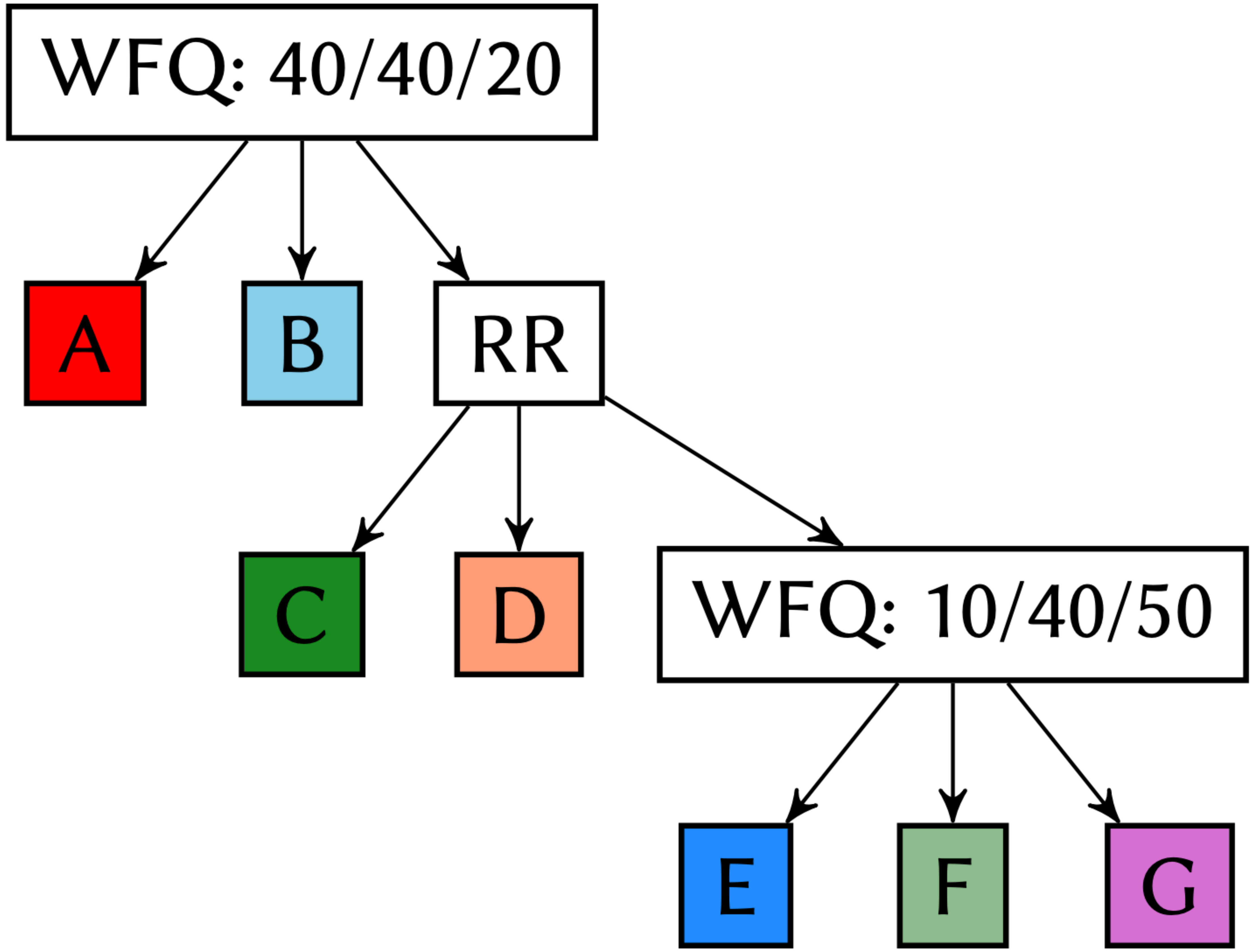
Generating embeddings automatically!

Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.

Two new algorithms,
both starting with heterogeneous source trees.

1. If target tree is regular *d*-ary for some *d*.
2. If target tree is itself heterogeneous.

Workflow



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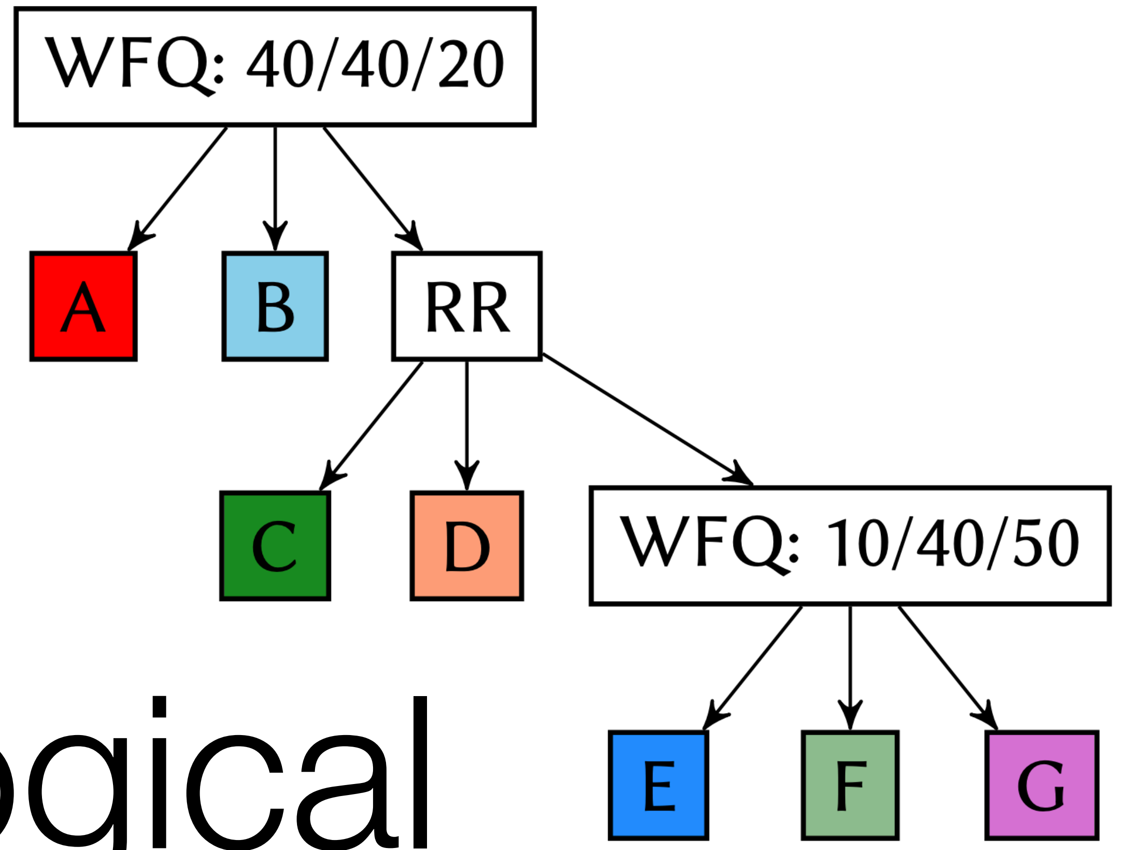
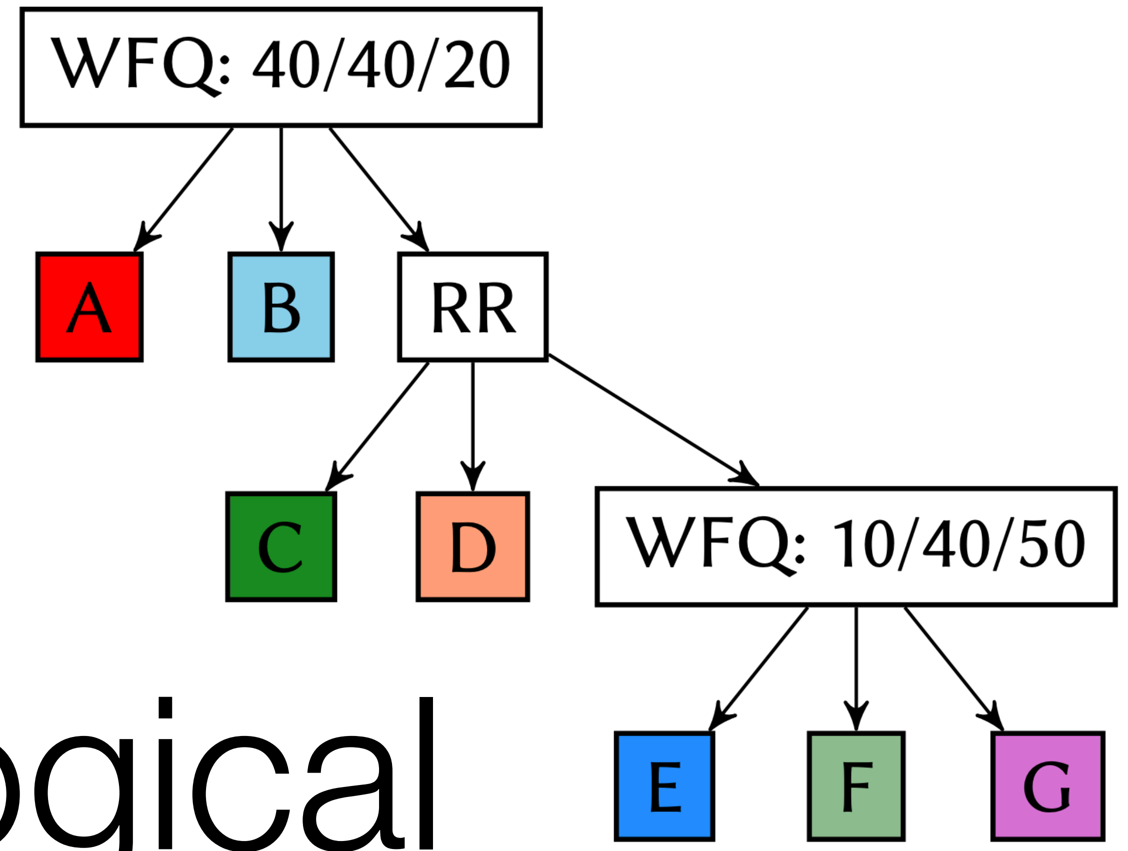


logical

Workflow



logical

But the hardware supports
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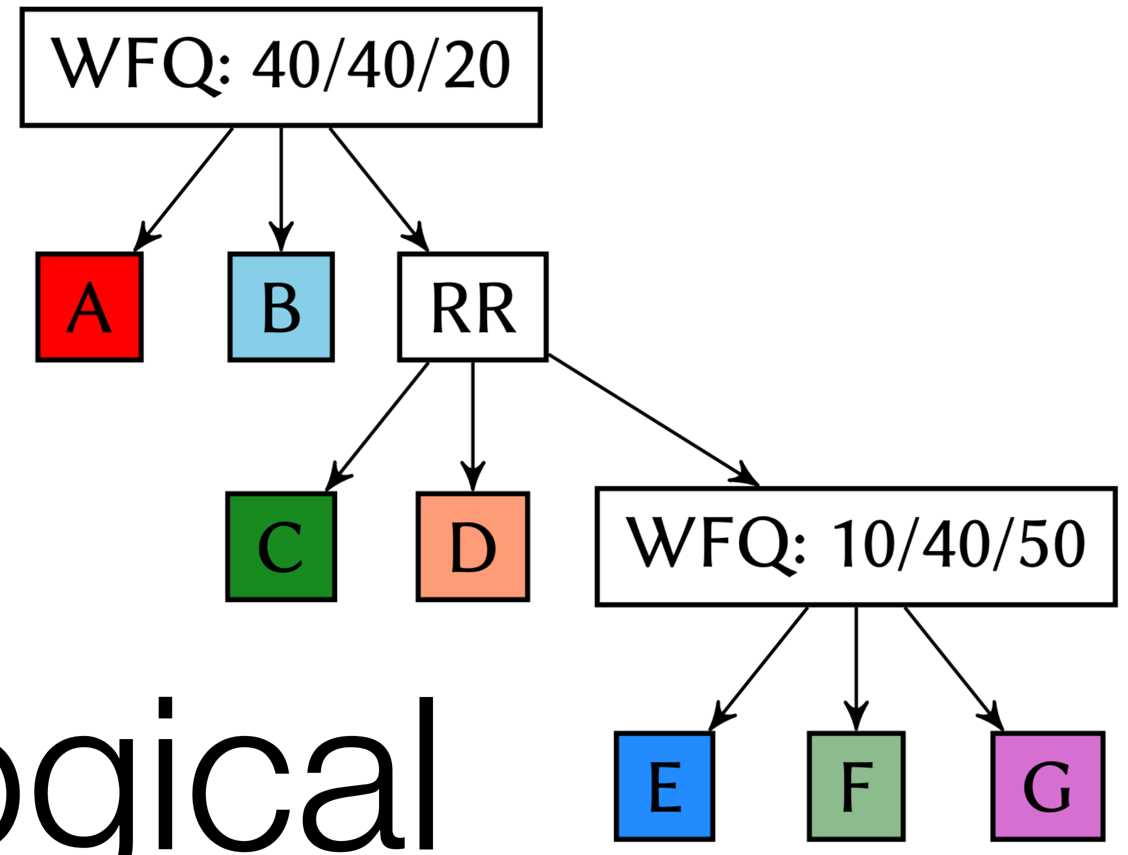
Workflow



logical

But the hardware supports
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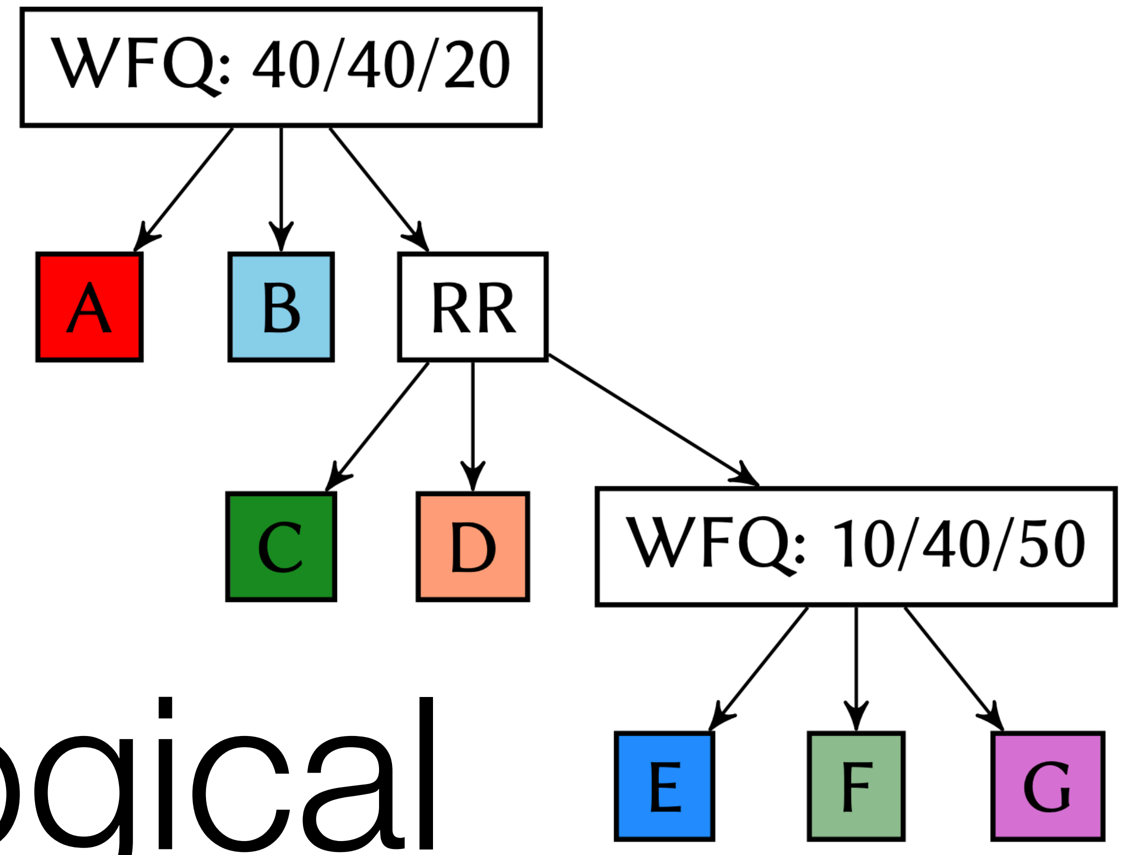
No problem.
Here's how I'll use that tree.

Workflow



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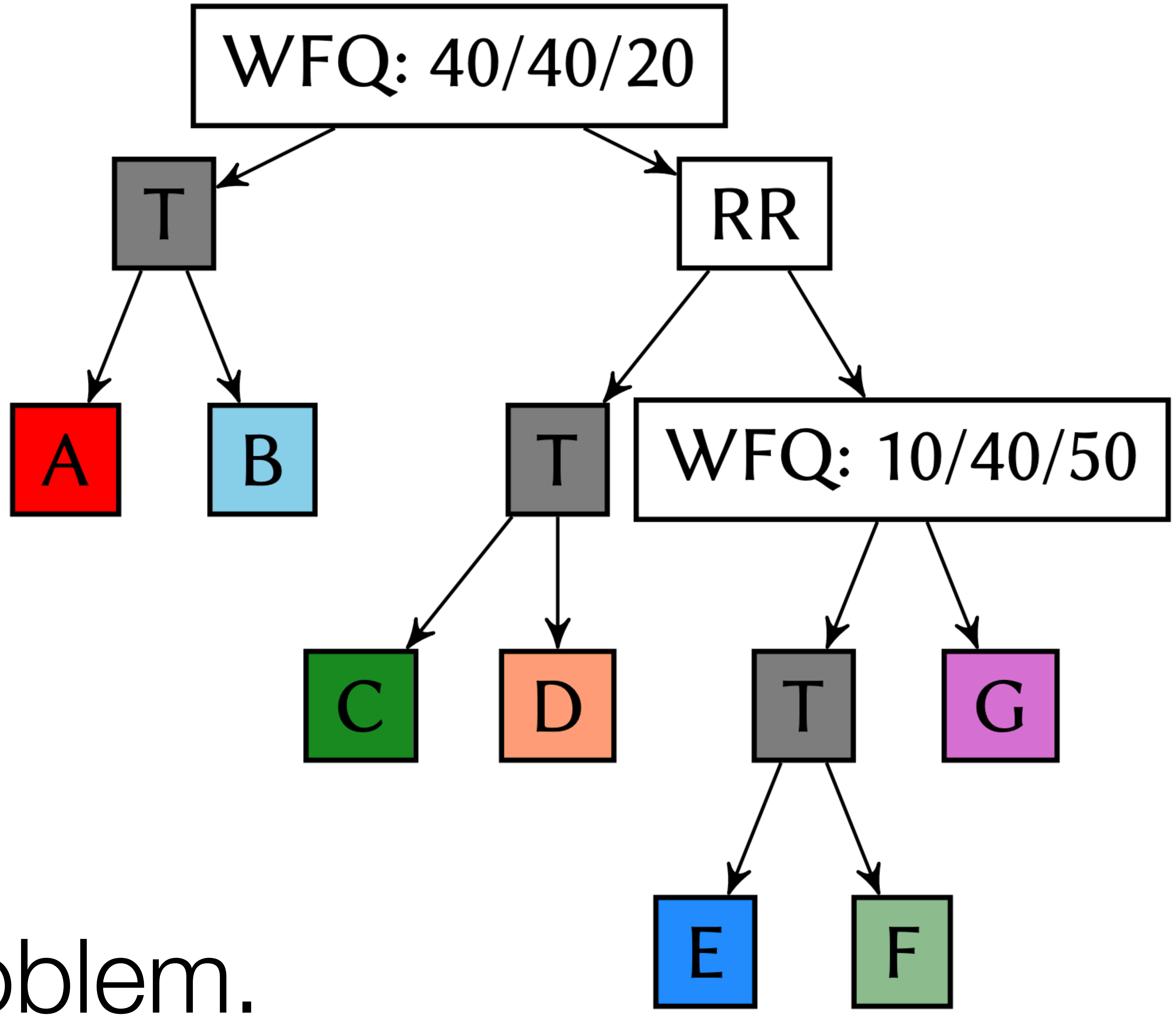

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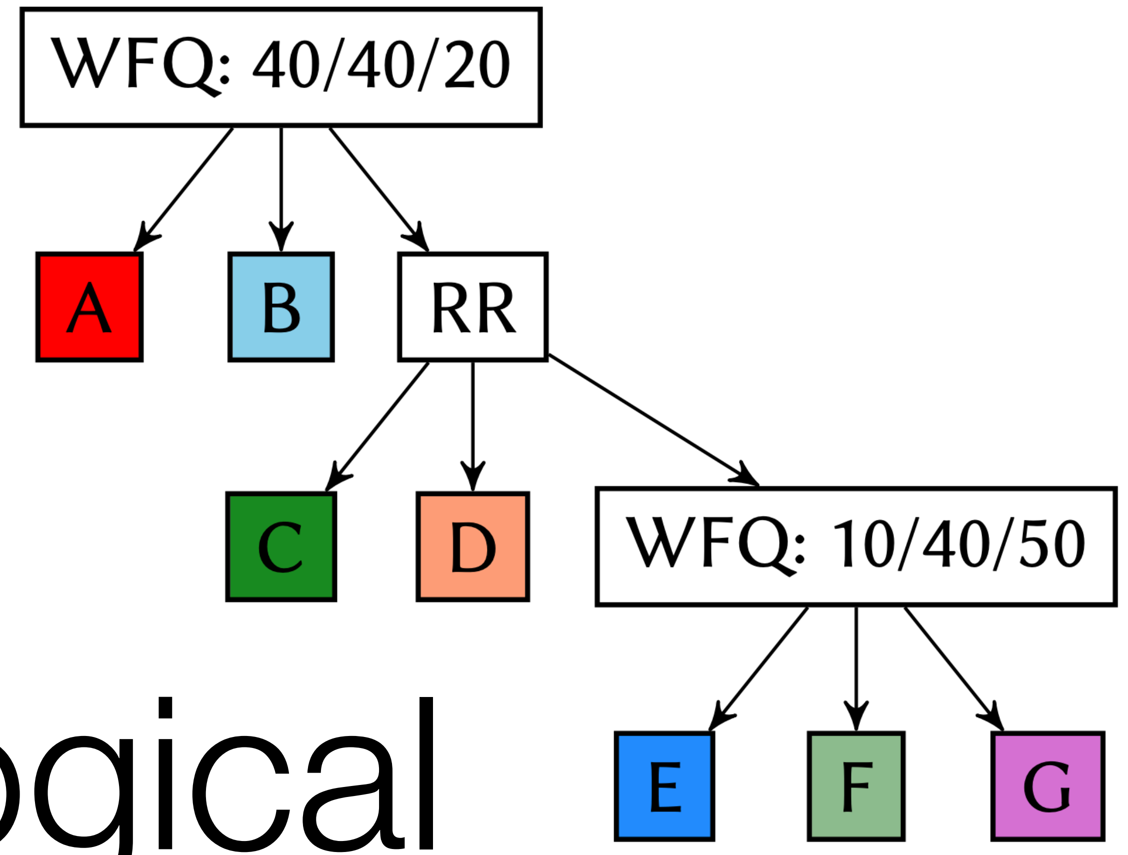

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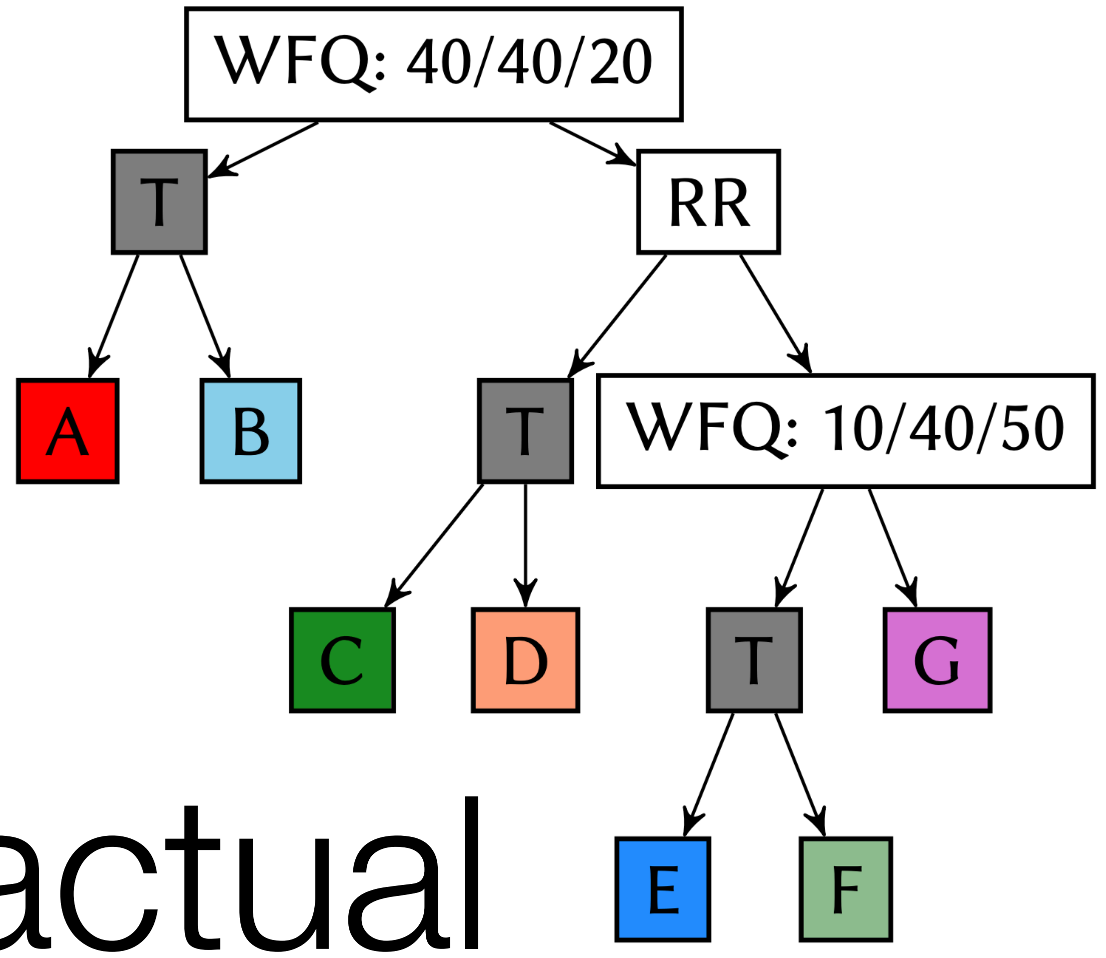
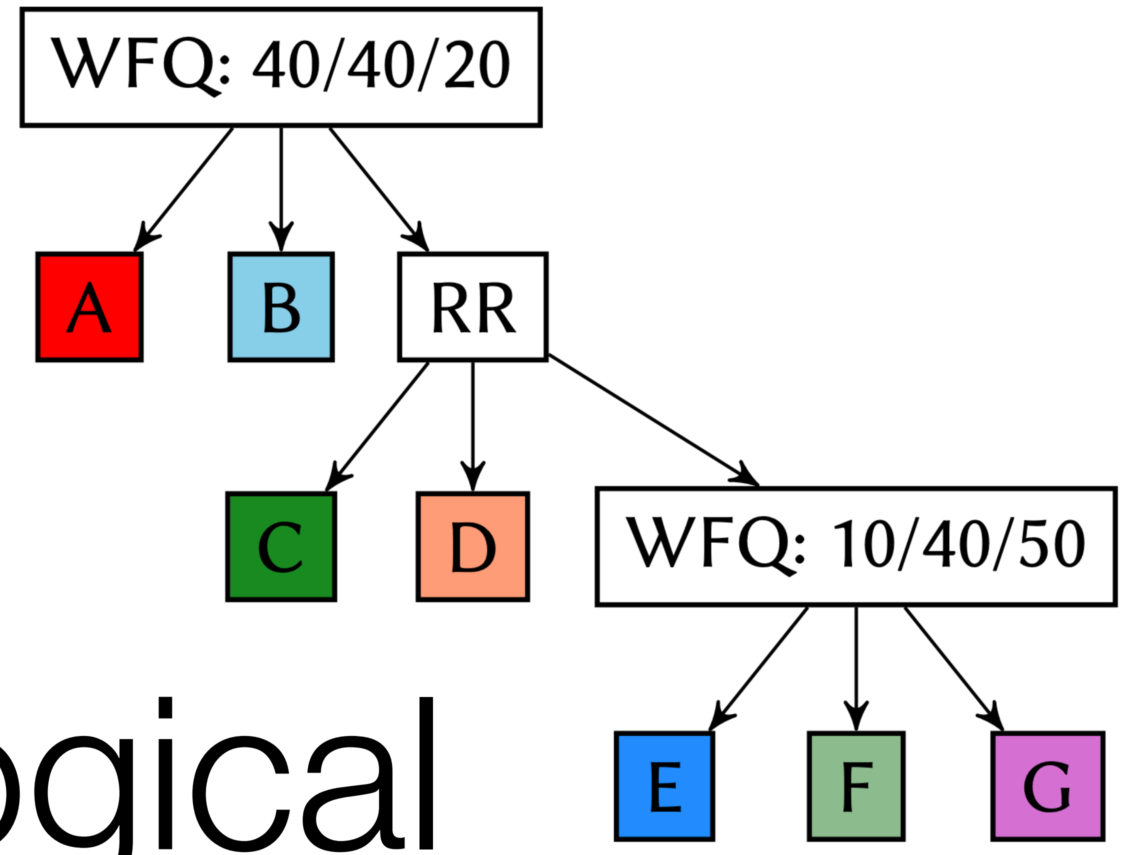


actual

Simulation



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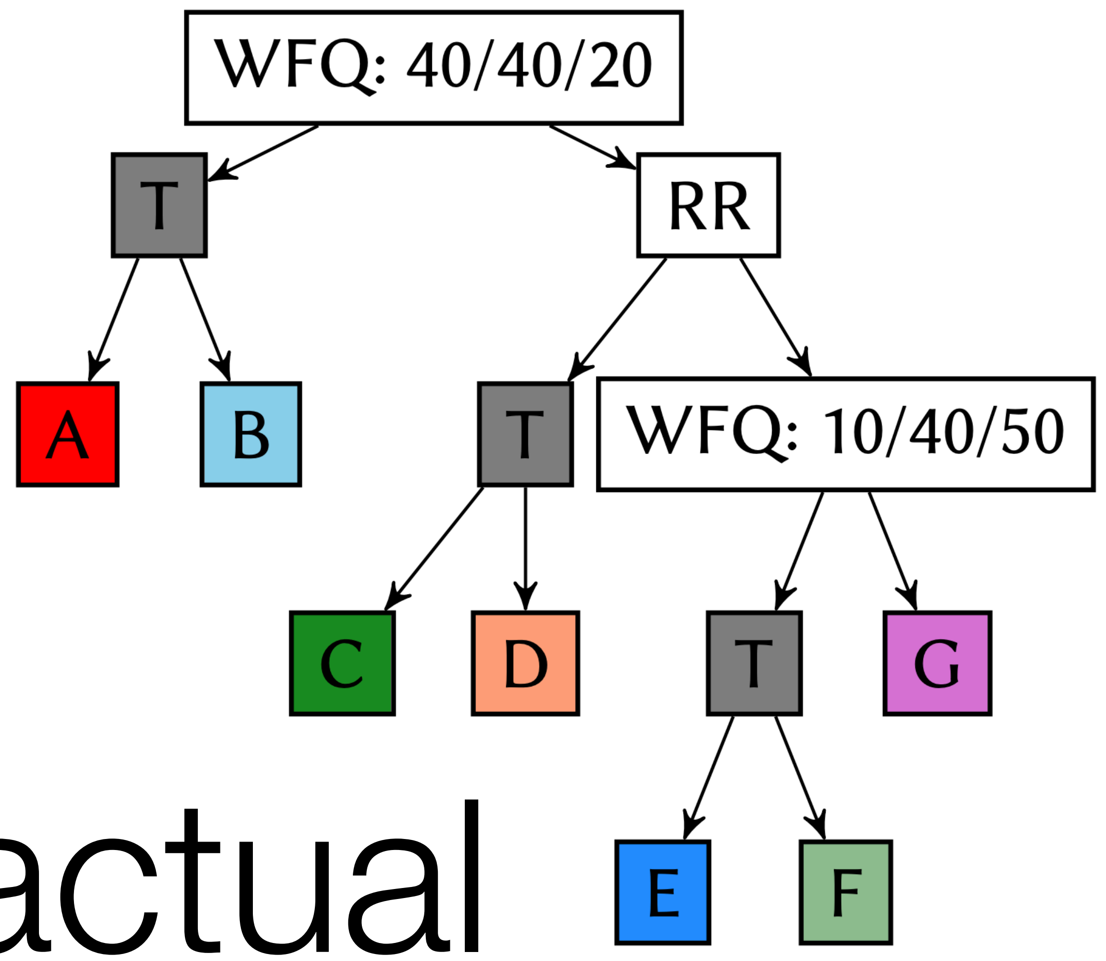

actual

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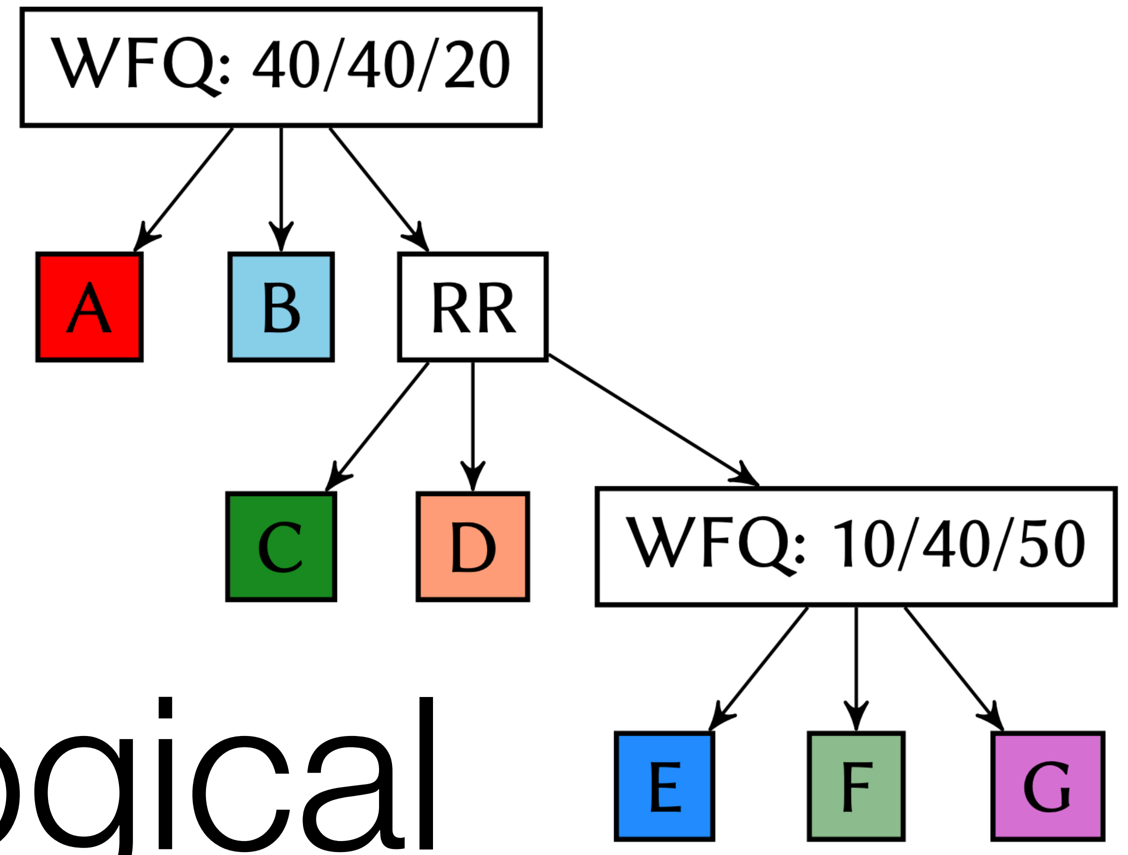

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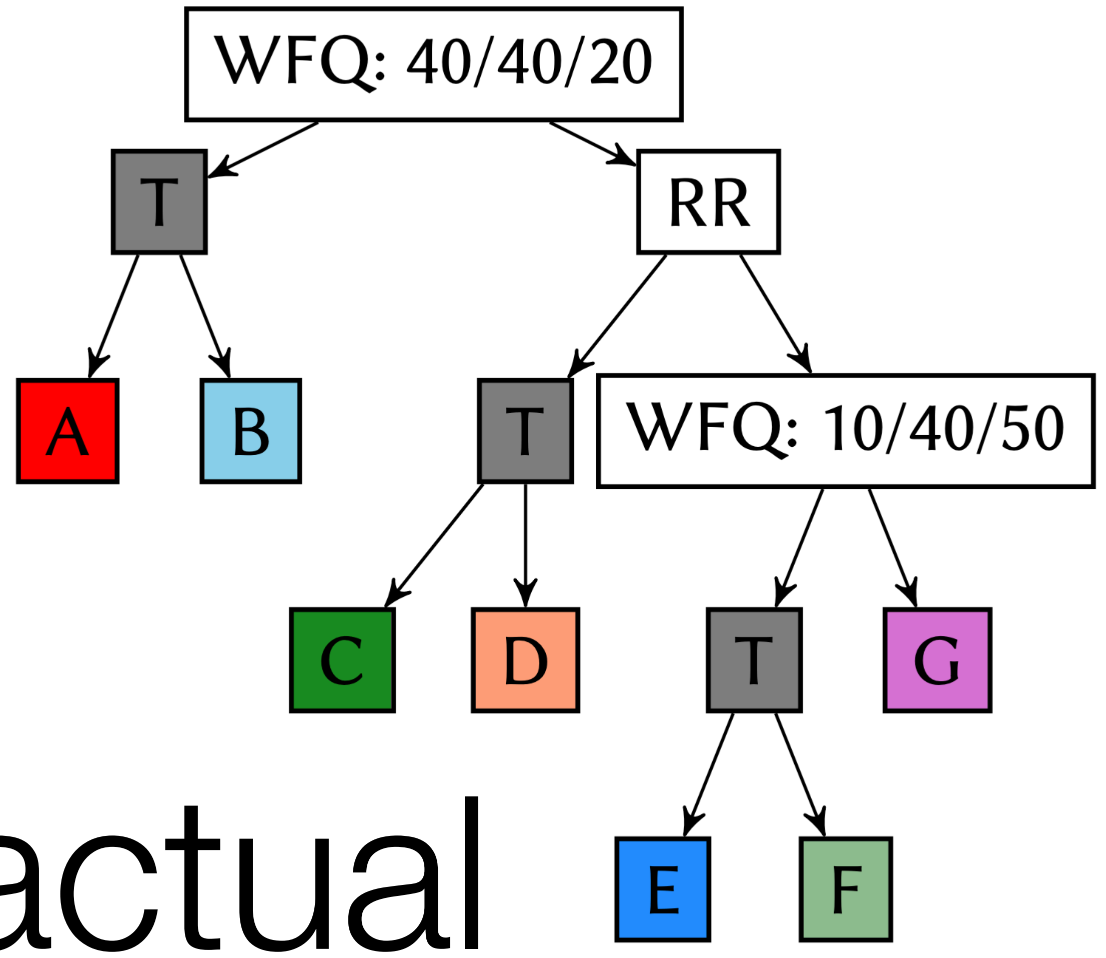

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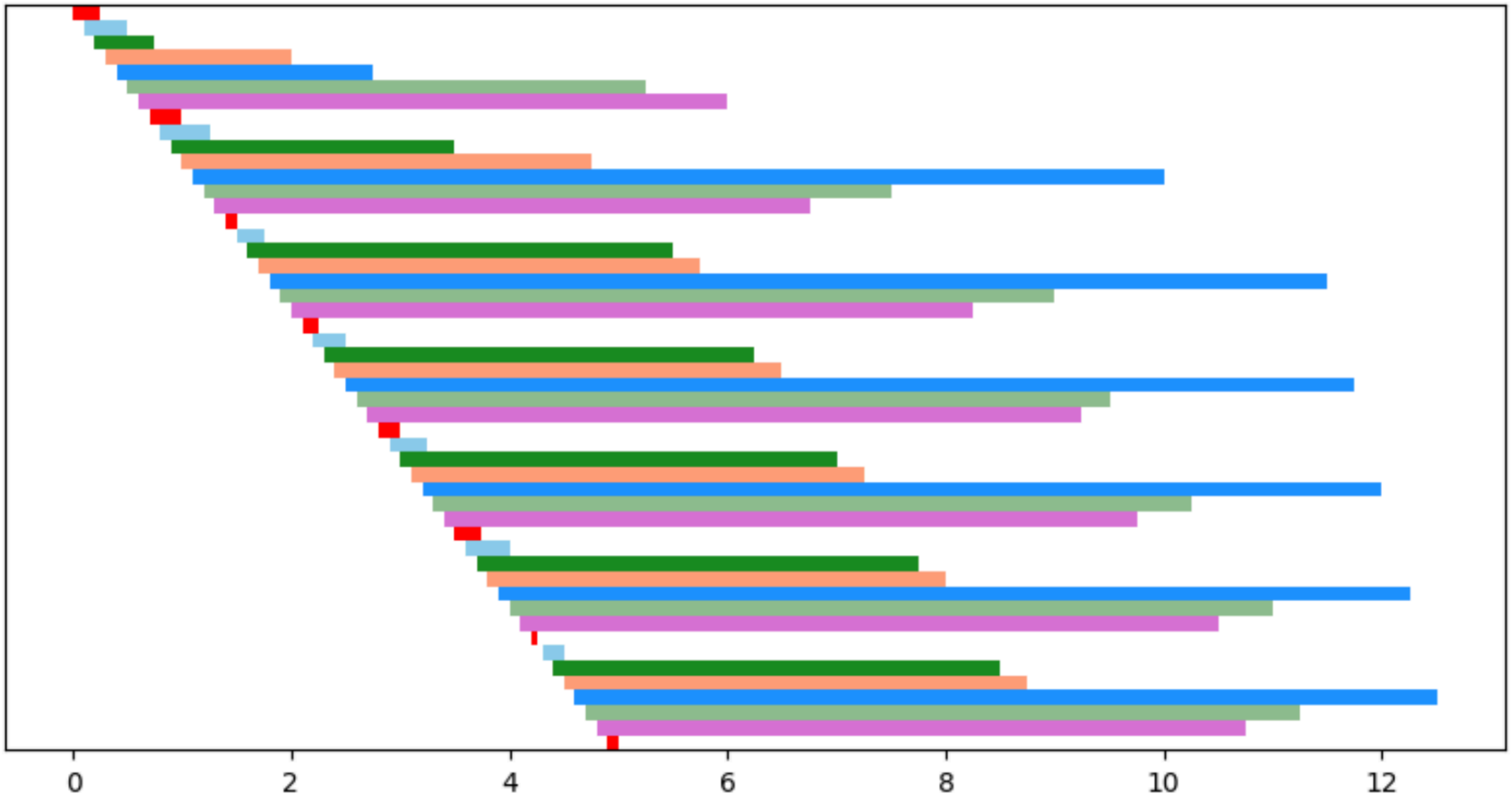
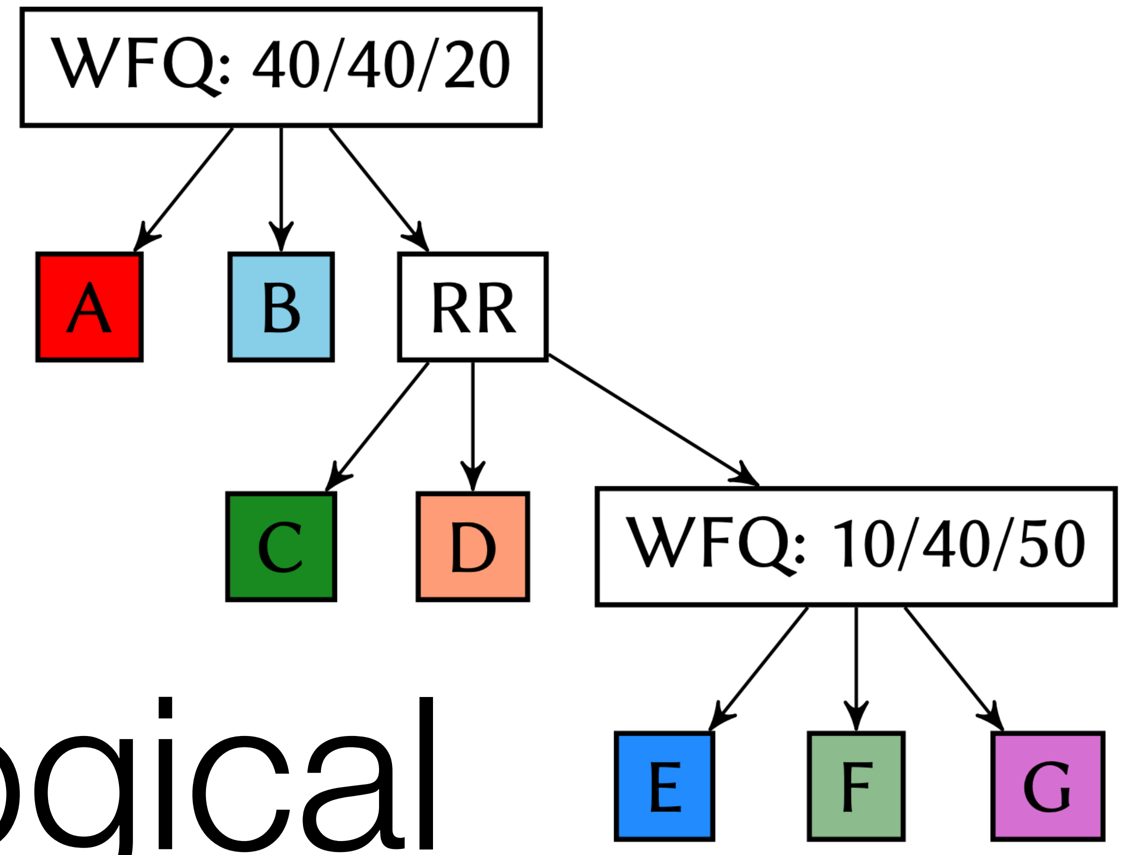


Simulation



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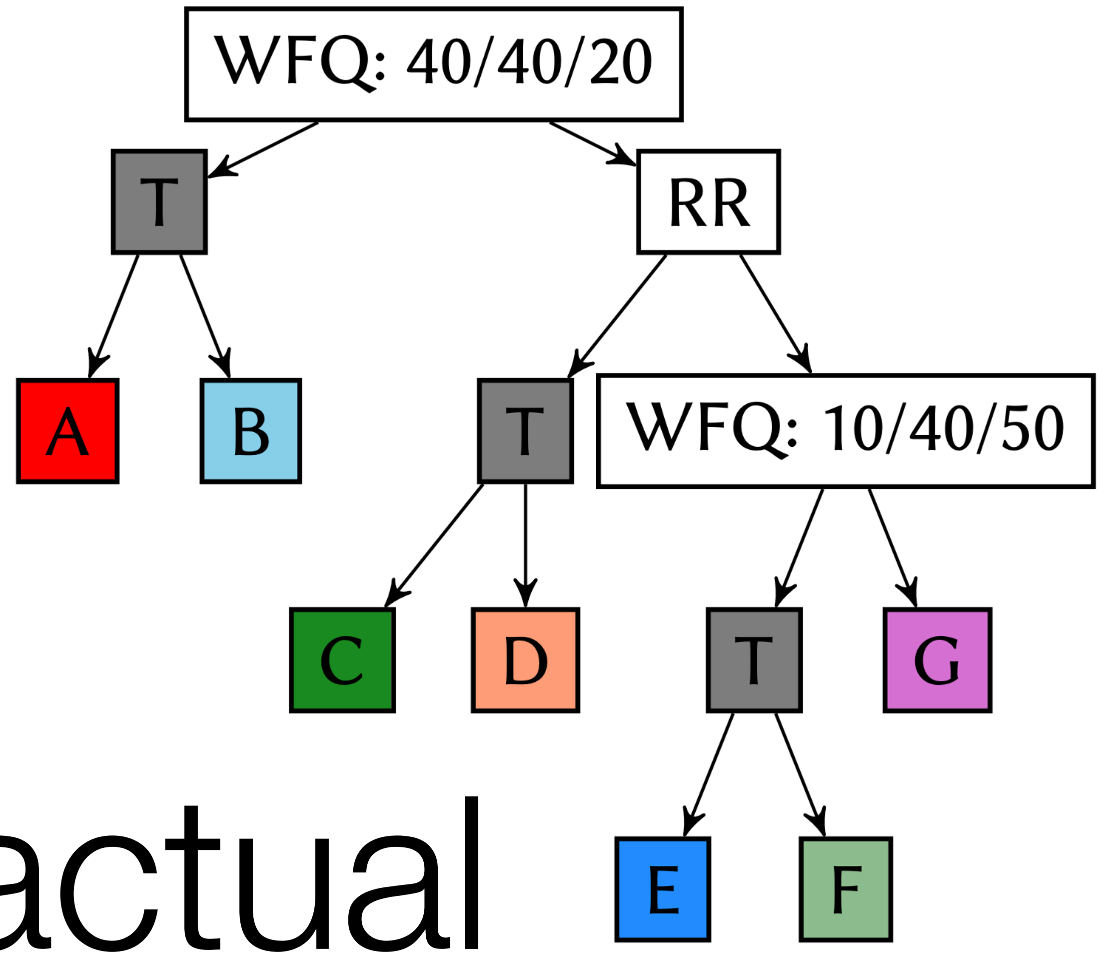

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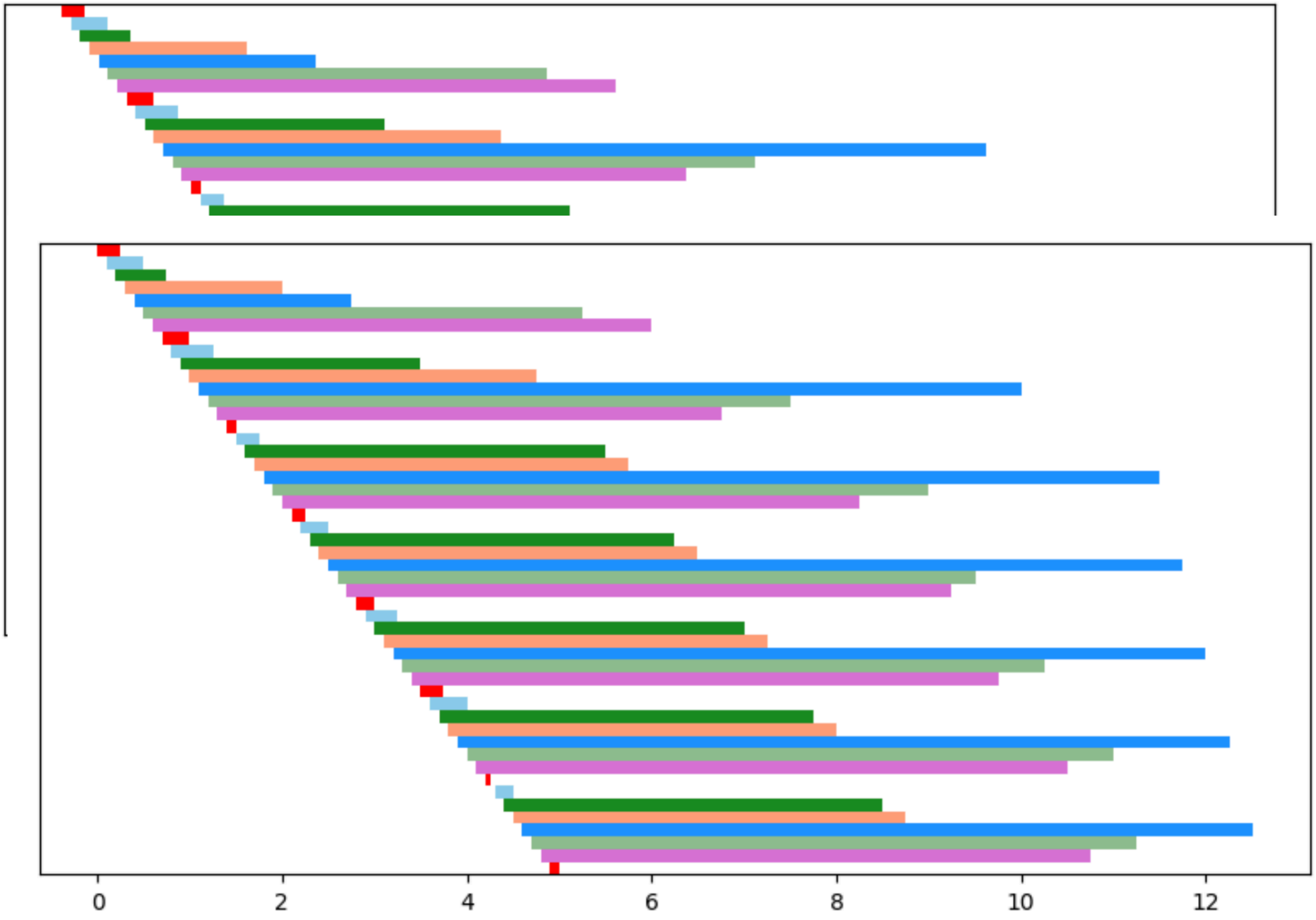

Underlying formalism

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$$\frac{}{* \in \mathsf{Topo}} \qquad \frac{n \in \mathbb{N} \qquad ts \in \mathsf{Topo}^n}{\mathsf{Node}(ts) \in \mathsf{Topo}}$$

$$\frac{p \in \mathsf{PIFO}(\mathsf{Pkt})}{\mathsf{Leaf}(p) \in \mathsf{PIFOTree}(\underline{*})} \qquad \frac{n \in \mathbb{N} \qquad ts \in \mathsf{Topo}^n \qquad p \in \mathsf{PIFO}(\{1,\ldots,n\}) \quad \forall 1 \leq i \leq n.\ qs[i] \in \mathsf{PIFOTree}(ts[i])}{\mathsf{Internal}(qs, p) \in \mathsf{PIFOTree}(\underline{\mathsf{Node}(ts)})}$$

Topo Topo

$$\frac{r \in \mathsf{Rk}}{r \in \mathsf{Path}(\underline{*})} \qquad \frac{ts \in \mathsf{Topo}^n \qquad 1 \leq i \leq n \qquad r \in \mathsf{Rk} \qquad pt \in \mathsf{Path}(ts[i])}{(i, r) :: pt \in \mathsf{Path}(\underline{\mathsf{Node}(ts)})}$$

Topo Topo

$$\frac{\textsc{push}(p, pkt, r) = p'}{\mathsf{push}(\mathsf{Leaf}(p), pkt, r) = \underline{\mathsf{Leaf}(p')}} \qquad \frac{\mathsf{push}(qs[i], pkt, pt) = q' \qquad \textsc{push}(p, i, r) = p'}{\mathsf{push}(\mathsf{Internal}(qs, p), pkt, \underline{(i, r) :: pt}) = \underline{\mathsf{Internal}(qs[i/q'], p')}}$$

PIFOTree Path PIFOTree

A general way to deploy PIFO trees.

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Let the hardware support some tree.

A general way to deploy PIFO trees.

Let the human program against some tree.

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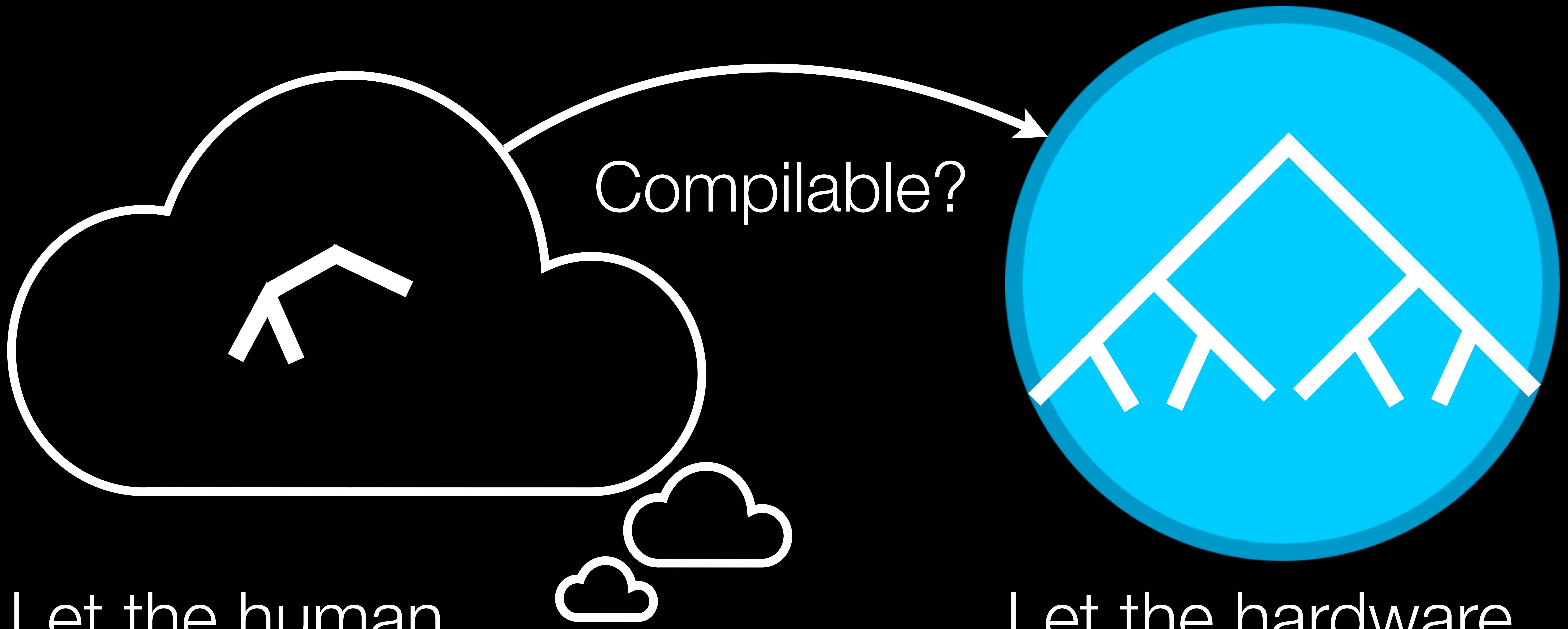

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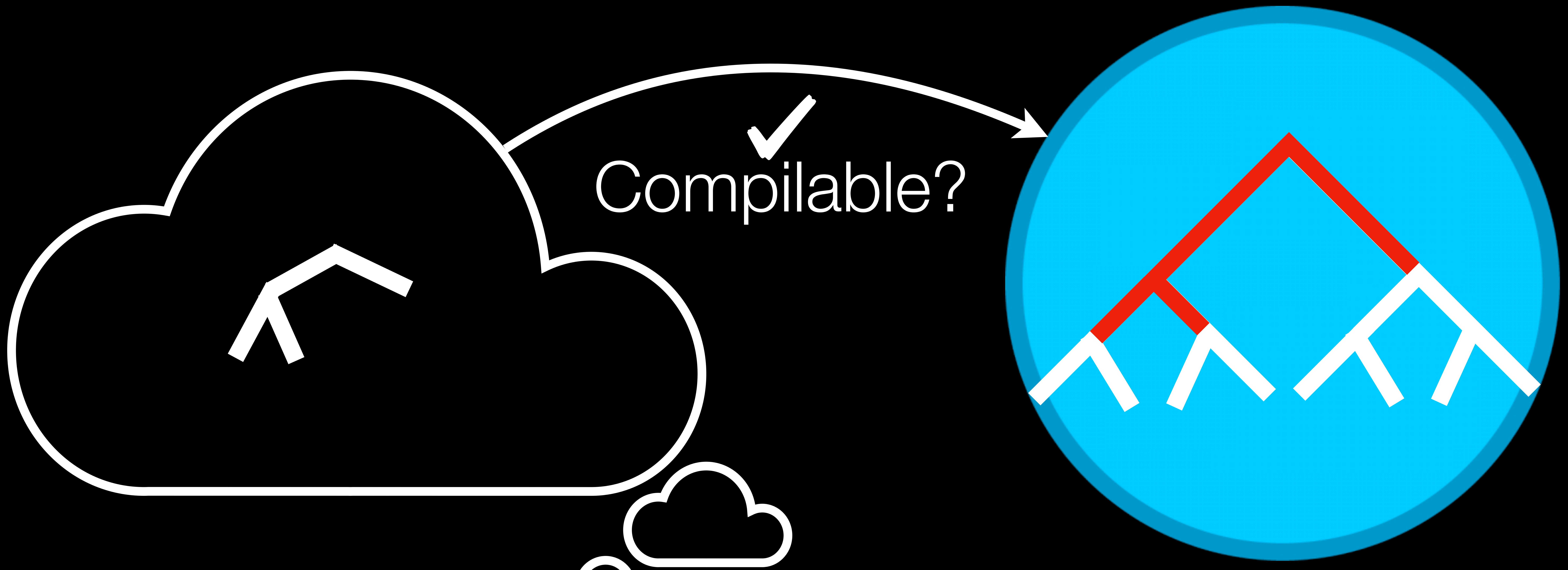

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Formal Abstractions for Packet Scheduling

Mohan, Liu, Foster, Kappé, Kozen

cs.cornell.edu/~amohan

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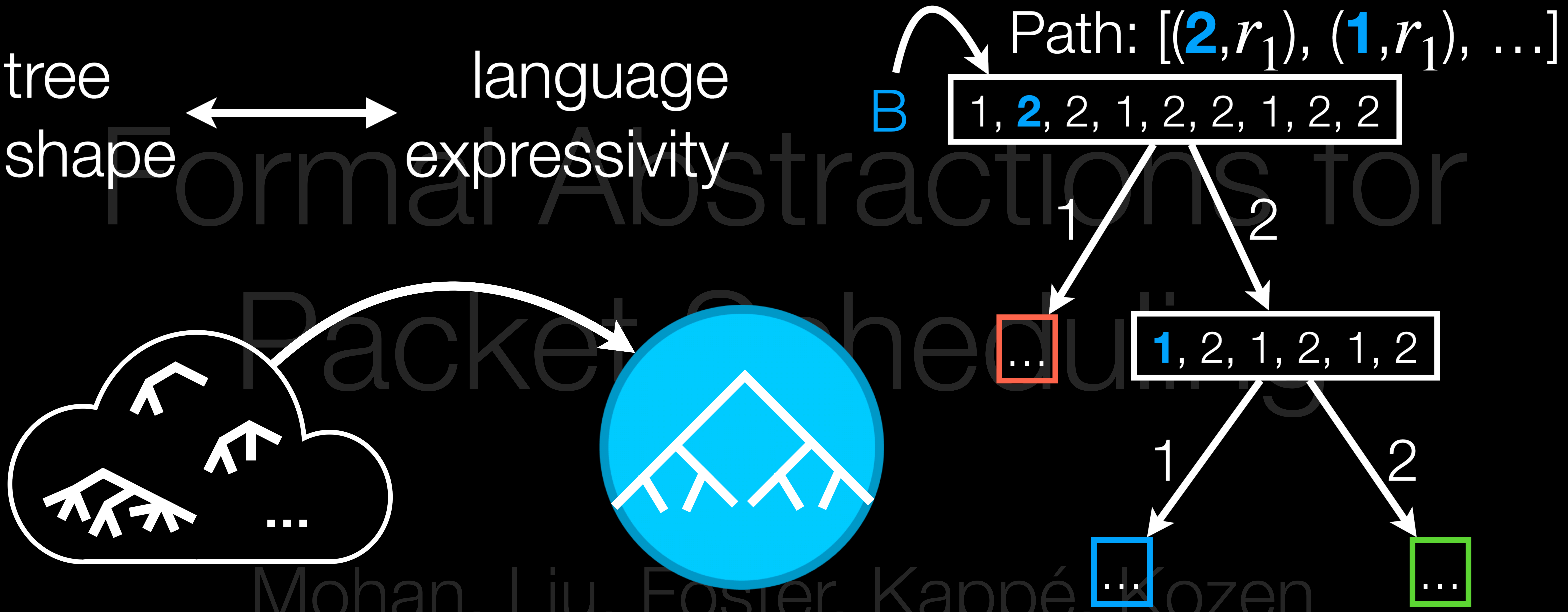

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