Verified C Implementation of Dijksta's Algorithm

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NUS
National University of Singapore

APLAS Student Research Competition
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Verified
Software
Toolchain

Certifying Graph-Manipulating C Programs via Localizations within Data Structures

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VST + CompCert + 25000 LOC library

Powerful enough to verify executable code
against realistic specifications
expressed with mathematical graphs

[Wang *et. al.*, PACMPL OOPSLA 2019]

This Work

Certifying Graph-Manipulating C Programs via Localizations within Data Structures

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Never used edge labels...
Not unlike vertex labels, but let's try anyway

Verify Dijkstra's one-to-all shortest path algorithm

Challenges

Using CompCert C

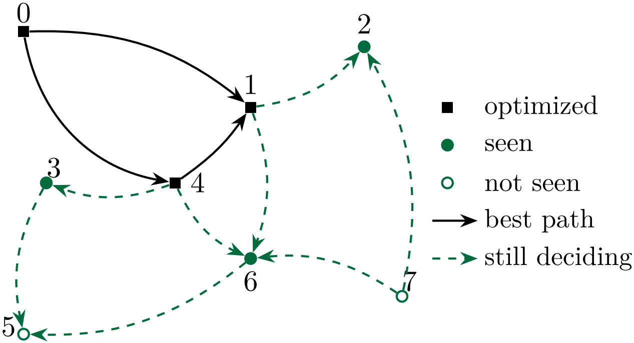
executable and realistic

real-world complications

Aiming for full functional correctness

and not just safety

Refresher



Instantiating DijkGraph

A PreGraph is a hextuple (`VType`, `EType`, `vvalid`, `evalid`, `src`, `dst`)

$$\textbf{Dijk\_PG}(\gamma) \stackrel{\text{def}}{=} \begin{array}{l} \texttt{VType := Z} \\ \texttt{EType := VType * VType} \\ \texttt{src := fst} \\ \texttt{dst := snd} \\ \forall v.\ \texttt{vvalid}(\gamma, v) \Leftrightarrow 0 \leqslant v < \texttt{SIZE} \\ \forall s, d.\ \texttt{evalid}(\gamma, (s, d)) \Leftrightarrow \texttt{vvalid}(\gamma, s) \wedge \texttt{vvalid}(\gamma, d) \end{array}$$

Instantiating DijkGraph

A LabeledGraph is a quadruple (PreGraph, `VL`, `EL`, `GL`)

$$\textbf{Dijk\_LG}(\gamma) \stackrel{\text{def}}{=} \text{Dijk\_PG as shown}$$

```
VL := list EL
EL := Z
GL := unit
```

Instantiating DijkGraph

A GeneralGraph adds arbitrary soundness conditions

$$
\begin{aligned}
\textbf{DijkGraph}(\gamma) \overset{\text{def}}{=}\ & \text{Dijk\_LG as shown, and} \\
& \mathit{FiniteGraph}(\gamma) \wedge \\
& \forall i, j.\ \texttt{vvalid}(\gamma, i) \wedge \texttt{vvalid}(\gamma, j) \Rightarrow \\
& \qquad i = j \Rightarrow \texttt{elabel}(\gamma, (i, j)) = 0 \wedge \\
& \qquad i \neq j \Rightarrow 0 \leqslant \texttt{elabel}(\gamma, (i, j)) \leqslant \lfloor \texttt{MAX/SIZE} \rfloor
\end{aligned}
$$

Representing DijkGraph in Memory

$$\mathsf{list\_rep}(\gamma, i) \stackrel{\text{def}}{=} \mathtt{data\_at\ array\ graph2mat}(\gamma)[i]\ \mathtt{list\_addr}(\gamma, i)$$

$$\mathsf{graph\_rep}(\gamma) \stackrel{\text{def}}{=} \mathop{\circledast}_{\mathtt{vvalid}(\gamma,v)} v \mapsto \mathsf{list\_rep}(\gamma, v)$$

Code and Specification

```
#define IFTY INT_MAX - INT_MAX/SIZE
void dijkstra (int graph[SIZE][SIZE], int src,
                         int *dist, int *prev) {
```

$\{$ DijkGraph$(\gamma)\}$

```
 int pq[SIZE];
 int i, j, u, cost;
 for (i = 0; i < SIZE; i++)
 {  dist[i] = INF; prev[i] = INF; pq[i] = INF;  }
 dist[src] = 0; pq[src] = 0; prev[src] = src;
```

$\{$ DijkGraph$(\gamma) \wedge$ $dijk\_correct(\gamma, src, prev, dist, priq)\}$

```
 // big while loop
```

Code and Specification

$\{\ \mathsf{DijkGraph}(\gamma) \land \mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq})\}$

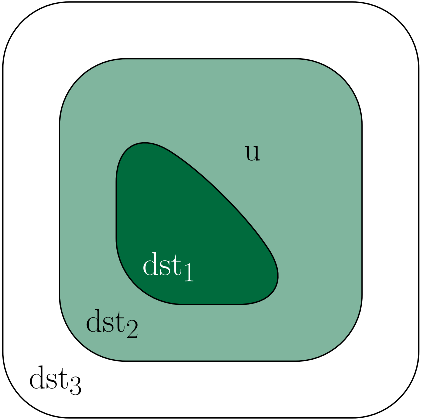
```
 while (!pq_emp(pq)) {
  u = popMin(pq);
  for (i = 0; i < SIZE; i++) {
   cost = graph[u][i];
    if (cost < INF) {
     if (dist[i] > dist[u] + cost) {
      dist[i] = dist[u] + cost; prev[i] = u; pq[i] = dist[i];
 }}}}
```

$\left\{\begin{array}{l} \mathsf{DijkGraph}(\gamma) \land \forall \mathit{dst} \in \mathit{priq}.\ \mathit{priq}[\mathit{dst}] = \mathtt{INF} \land \\ \mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq}) \end{array}\right\}$

```
return;
}
```

$$
\begin{aligned}
&\textit{dijk\_correct}(\gamma, \textit{src}, \textit{prev}, \textit{dist}, \textit{priq}) \stackrel{\text{def}}{=} \\
&\forall \textit{dst}.\ \textit{dst} \in \textit{popped}(\textit{priq}) \Rightarrow \\
&\qquad\qquad \exists \textit{path}.\ \textit{path\_correct}(\gamma, \textit{prev}, \textit{dist}, \textit{path}) \wedge \\
&\qquad\qquad \textit{path\_glob\_optimal}(\gamma, \textit{dist}, \textit{path}) \wedge \\
&\qquad\qquad \textit{path\_entirely\_in\_popped}(\gamma, \textit{prev}, \textit{path}) \wedge \\
&\qquad \textit{priq}[\textit{dst}] < \texttt{INF} \Rightarrow \\
&\qquad\qquad \texttt{let}\ m\ \texttt{:=}\ \textit{prev}[\textit{dst}]\ \texttt{in}\ m \in \textit{popped}(\textit{priq}) \wedge \\
&\qquad\qquad \forall m' \in \textit{popped}(\textit{priq}).\ \textit{cost}(\textit{path2m}+ :: (m, \textit{dst})) \leqslant \\
&\qquad\qquad\qquad\qquad \textit{cost}(\textit{path2m}'+ :: (m', \textit{dst})) \wedge \\
&\qquad \textit{priq}[\textit{dst}] = \texttt{INF} \Rightarrow \\
&\qquad\qquad \forall m \in \textit{popped}(\textit{priq}).\ \textit{cost}(\textit{path2m}+ :: (m, \textit{dst})) = \texttt{INF}
\end{aligned}
$$

Key Transformation: Growing the Subgraph



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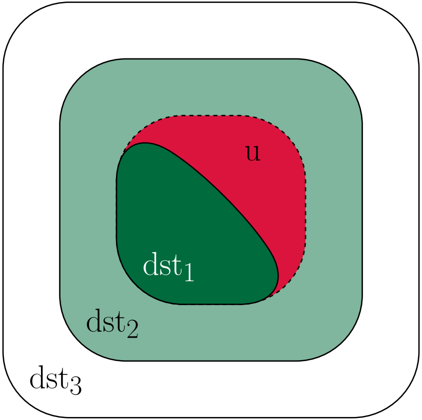

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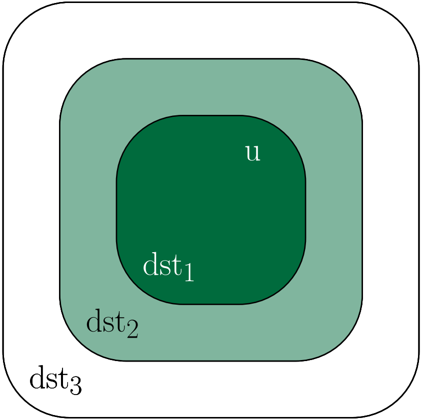
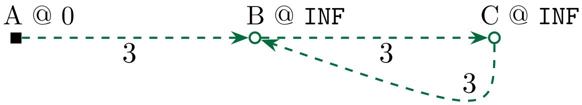


Overflow Strikes Again

The longest optimal path has `SIZE-1` links
so say we set `elabel`'s upper bound to $\lfloor$`MAX/(SIZE-1)`$\rfloor$

`MAX` $= 7$, `SIZE` $= 3$, so $0 \leqslant$ `elabel`$(\gamma, e) \leqslant 3$.



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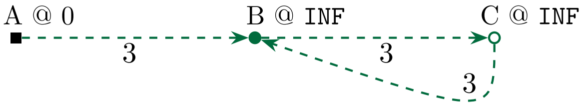

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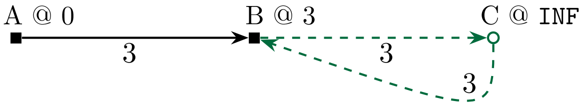

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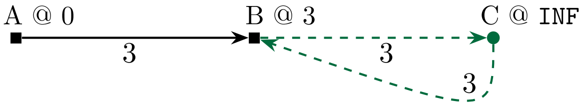

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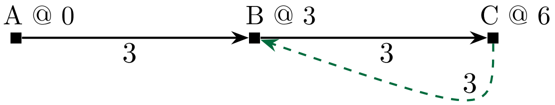

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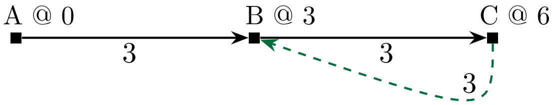

if $3 > 9$ then relax C $\rightsquigarrow$ B

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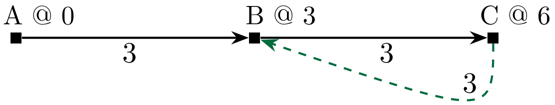

if $3 > 1$ then relax C $\rightsquigarrow$ B

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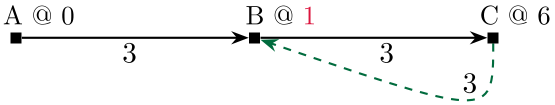

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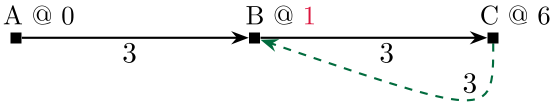

if $3 > 1$ then relax C $\rightsquigarrow$ B

One solution: Conservatively set upper bound to $\lfloor$`MAX/SIZE`$\rfloor$

Max path cost is then $\lfloor$`MAX/SIZE`$\rfloor$ `* (SIZE-1) = MAX -` $\lfloor$`MAX/SIZE`$\rfloor$

Overflow Strikes Again

There are other ways to fix this!

- Refactor troublesome addition as subtraction
- Never look back into optimized part
- Your suggestion here
- Your suggestion here

That is not the point

- Intuition supports `INF = MAX`
- No reason to do any of the above... until today