

# Shadow Detection and Sun Direction in Photo Collections

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[Snavely et al., IJCV 2007]

Appearance is:



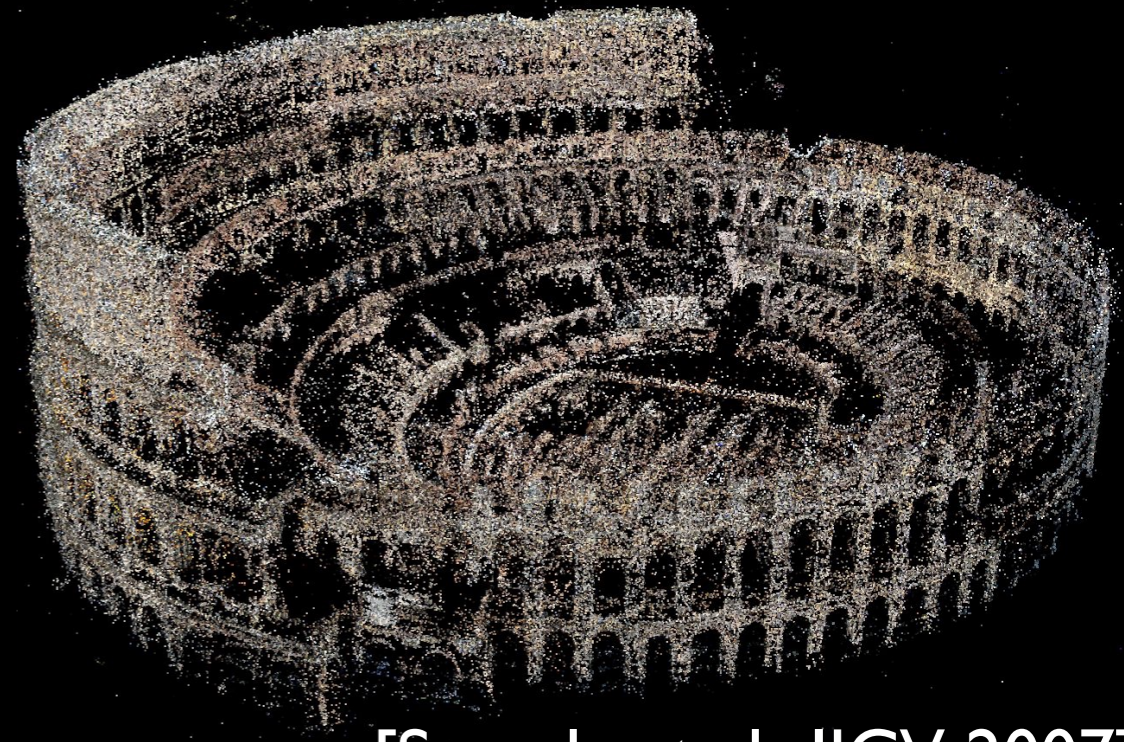
[Snavely et al., IJCV 2007]



[Snavely et al., IJCV 2007]

Appearance is:

- Geometry



[Snavely et al., IJCV 2007]

Appearance is:

- Geometry
- Materials



[Snavely et al., IJCV 2007]

Appearance is:

- Geometry
- Materials
- Illumination



[Snavely et al., IJCV 2007]

Appearance is:

- Geometry (Bundler+PMVS)
- Materials
- Illumination



[Snavely et al., IJCV 2007]

Appearance is:

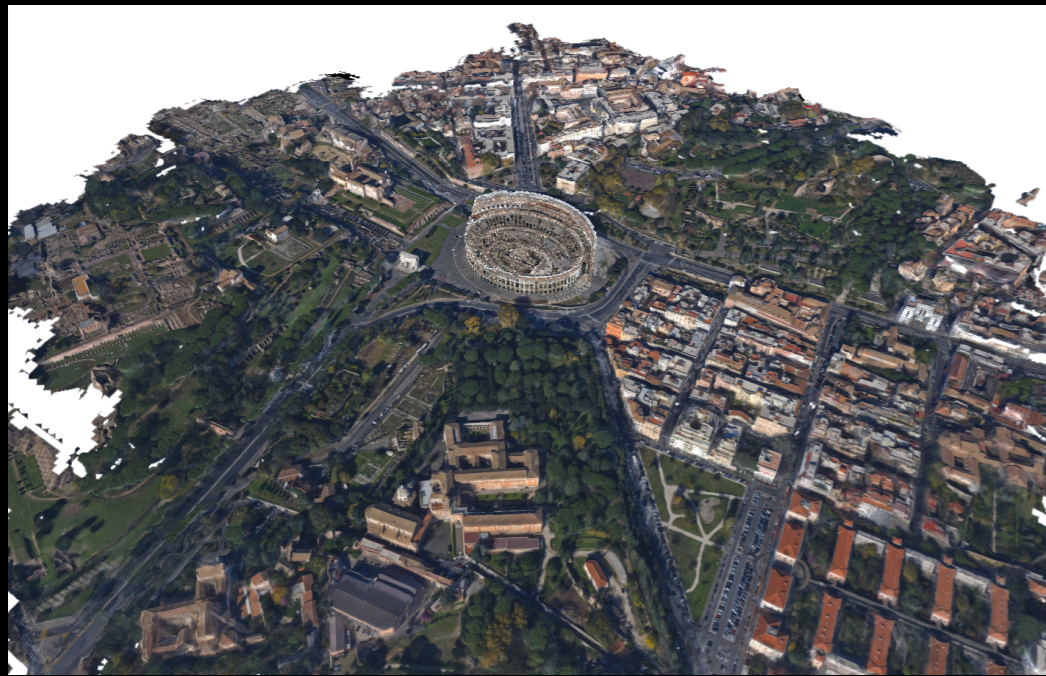
- Geometry (Bundler+PMVS)
- Materials (Assume Lambertian)
- Illumination



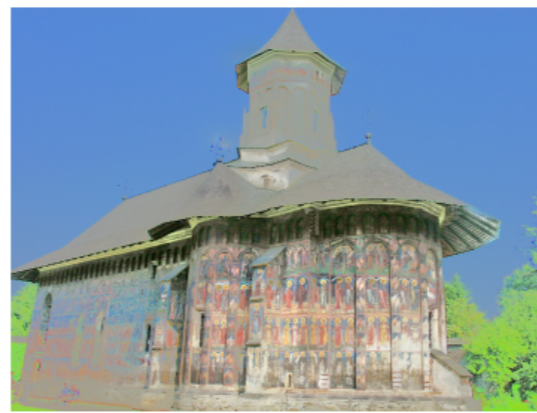
[Snavely et al., IJCV 2007]

Appearance is:

- Geometry (Bundler+PMVS)
- Materials (Assume Lambertian)
- Illumination



[Shan et al., 3DV 2013]



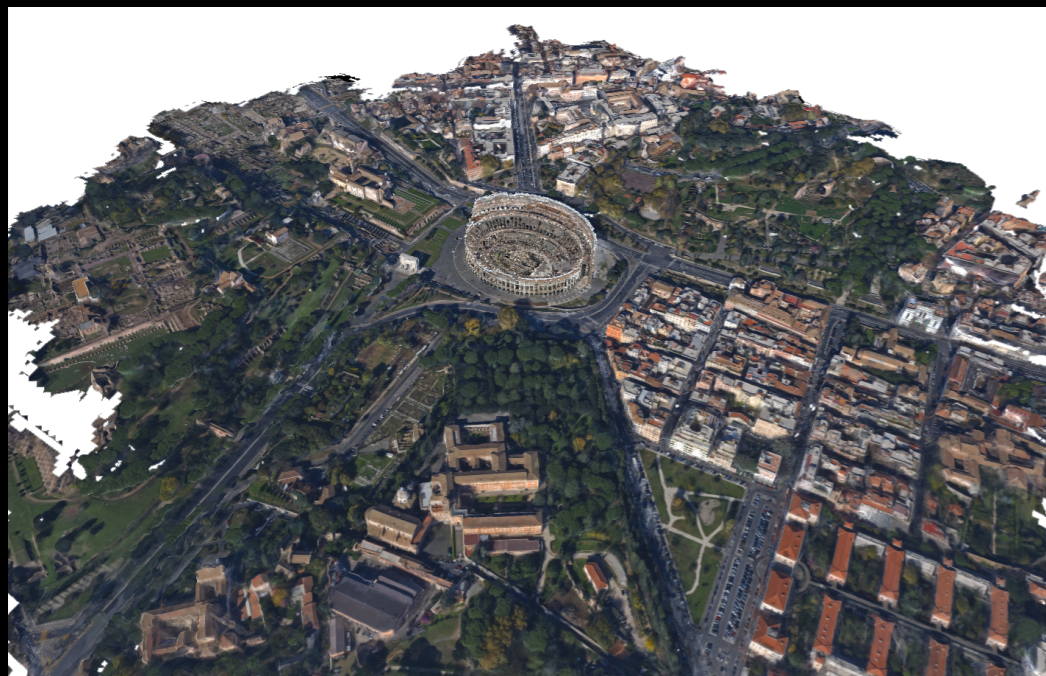
Coherent reflectance



Individual illuminations



[Laffont, Bousseau, Paris, Durand, and Drettakis, SIGGRAPH Asia 2012]



[Shan et al., 3DV 2013]



Coherent reflectance

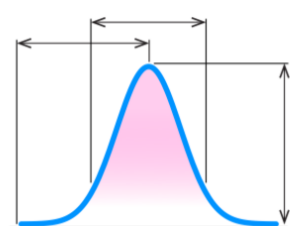


Individual illuminations



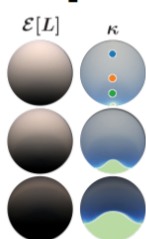
[Laffont, Bousseau, Paris, Durand, and Drettakis, SIGGRAPH Asia 2012]

## Statistics



Point Color Statistics  
( $E[I]$  and  $E[I^2]$ )

+



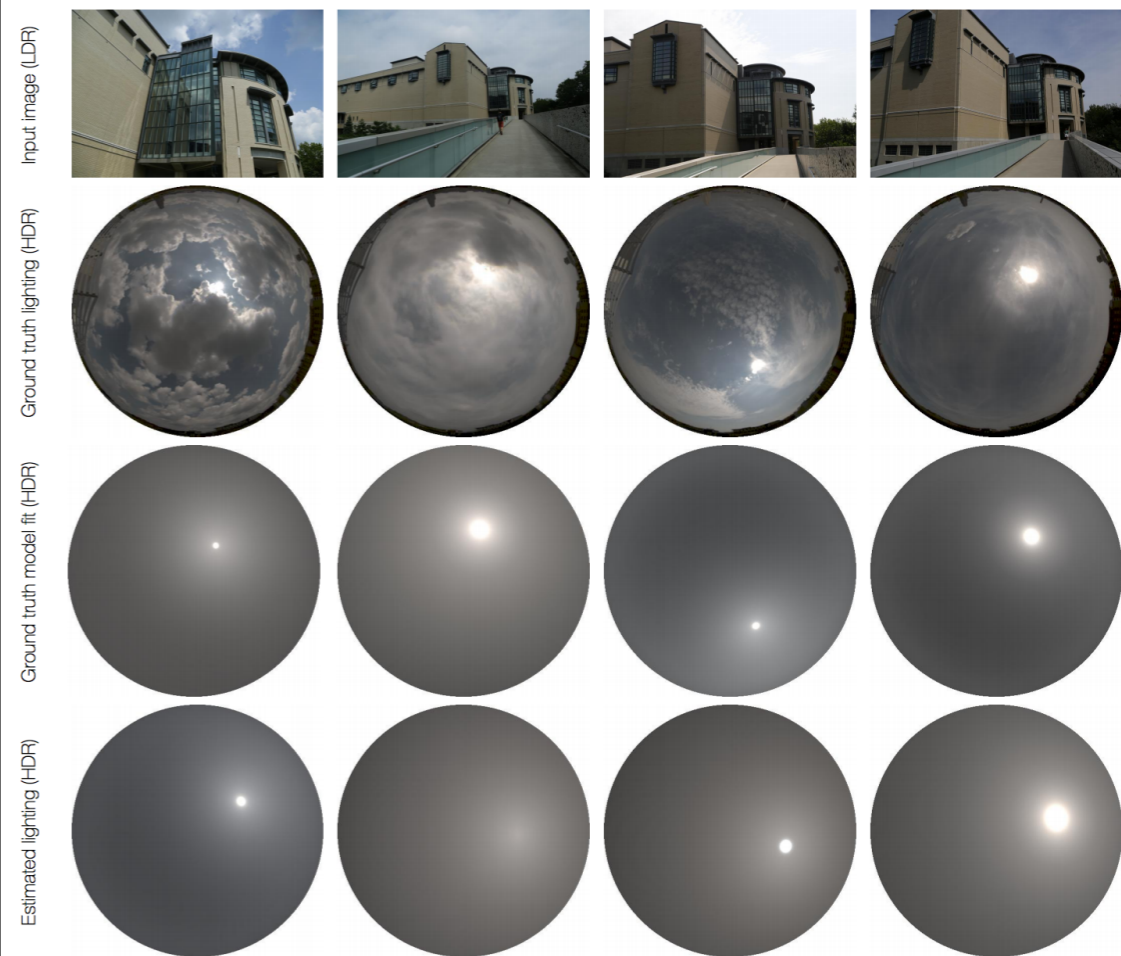
Statistics of Outdoor  
Illumination

## Output



Local Visibility Angle  $\alpha$   
Albedo  
Illumination  
Timestamp

[Hauagge, Wehrwein, Bala, and Snavely, BMVC 2014]



[Lalonde and Matthews, 3DV 2014]





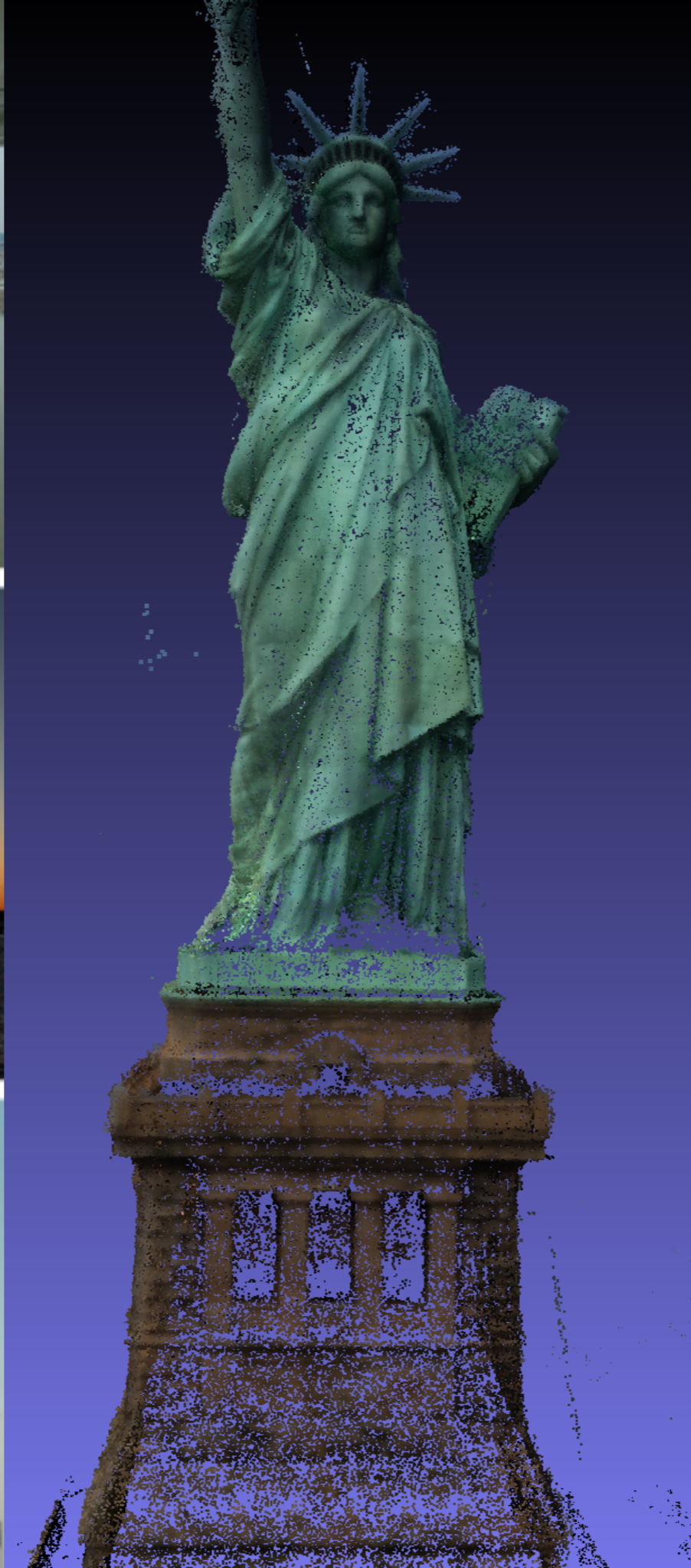


# Outline

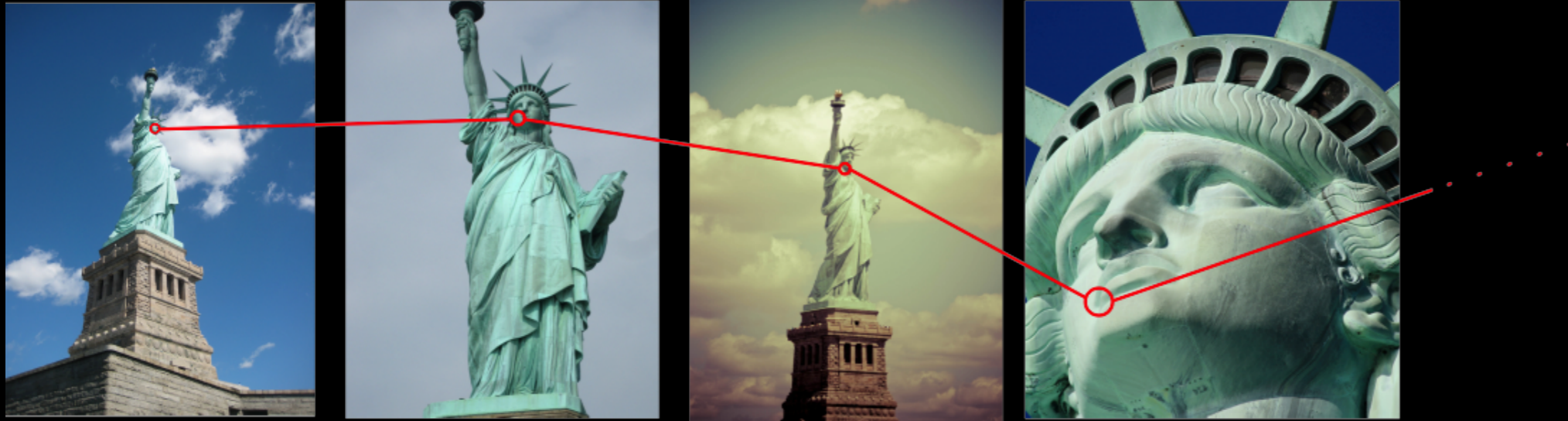
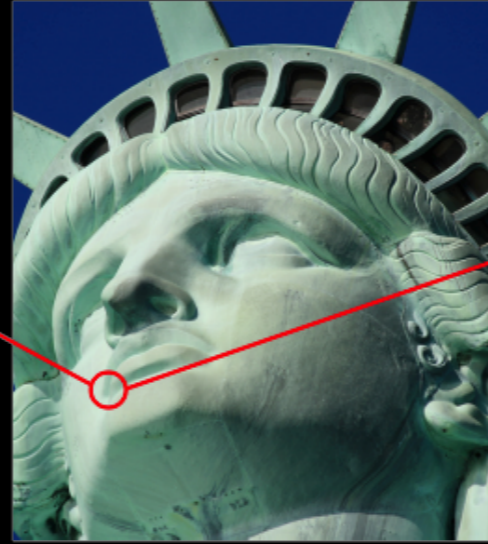
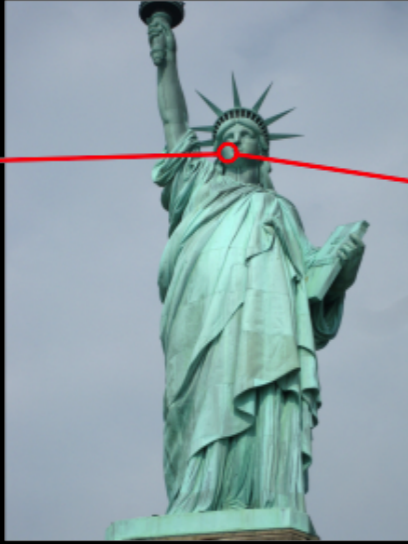
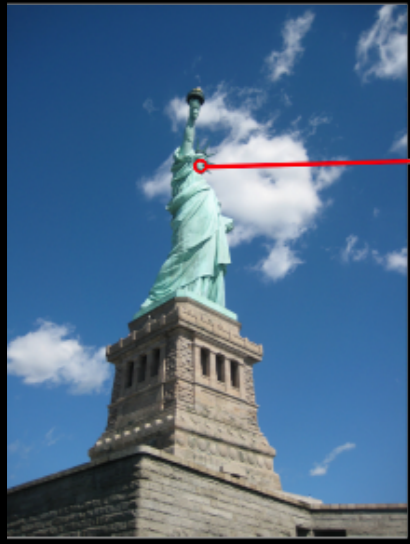
# Outline

1. Input Data
2. The Illumination Ratio
3. Algorithm
4. Sparse to Dense Labels
5. Application: Sun Direction Estimation

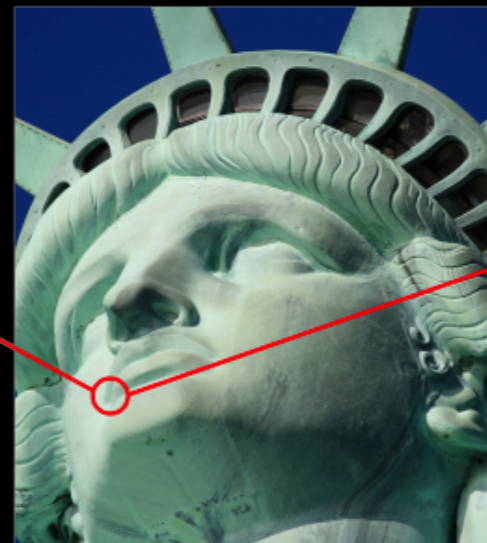
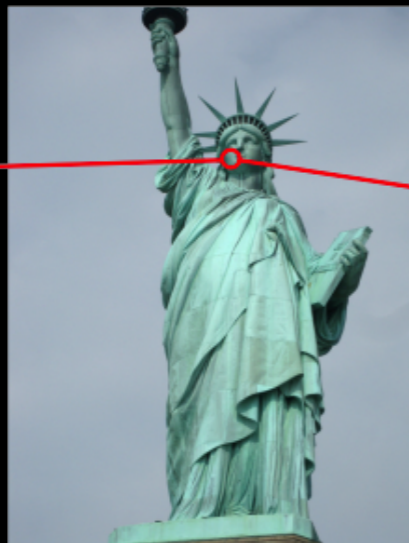




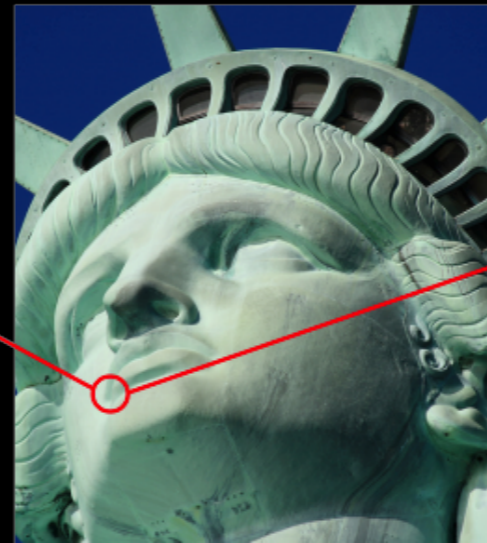
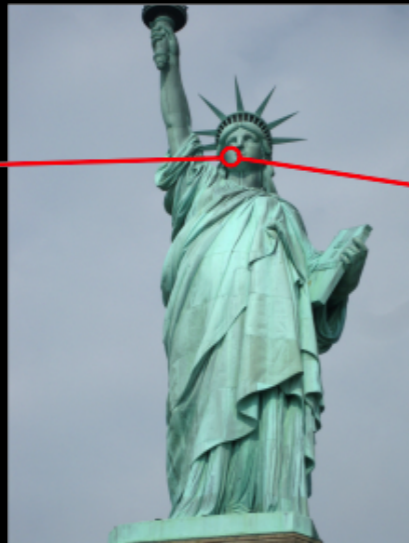
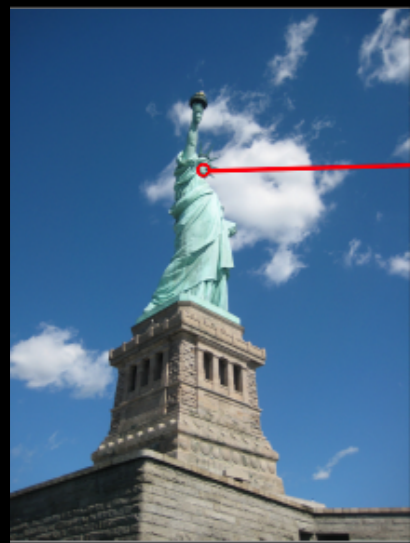
# Input Data



# Input Data

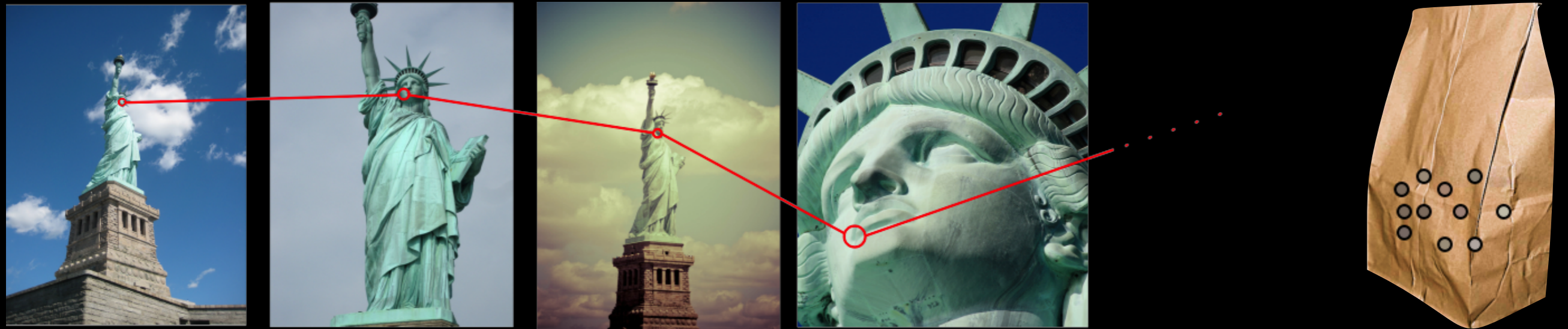


# Input Data



What's in the bag?

# Input Data

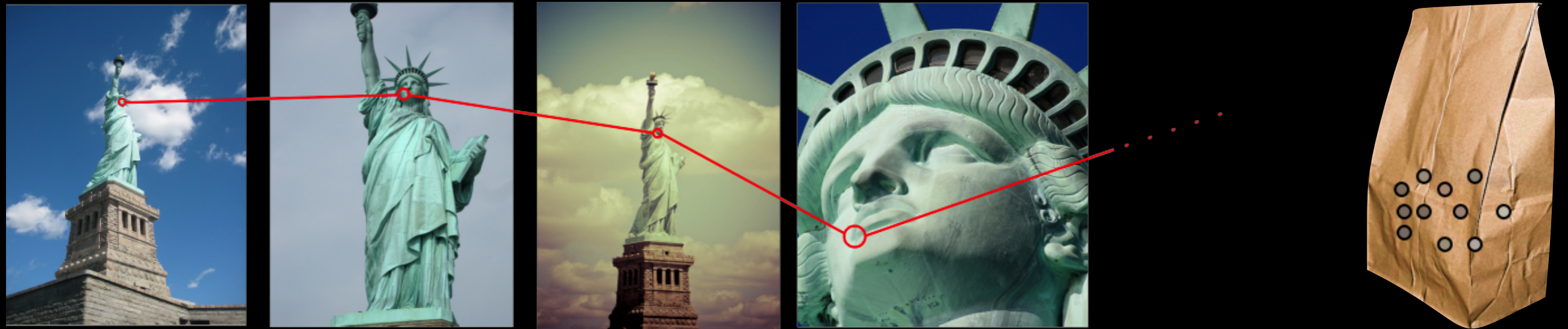


What's in the bag?

**Constants:**

**Variables:**

# Input Data



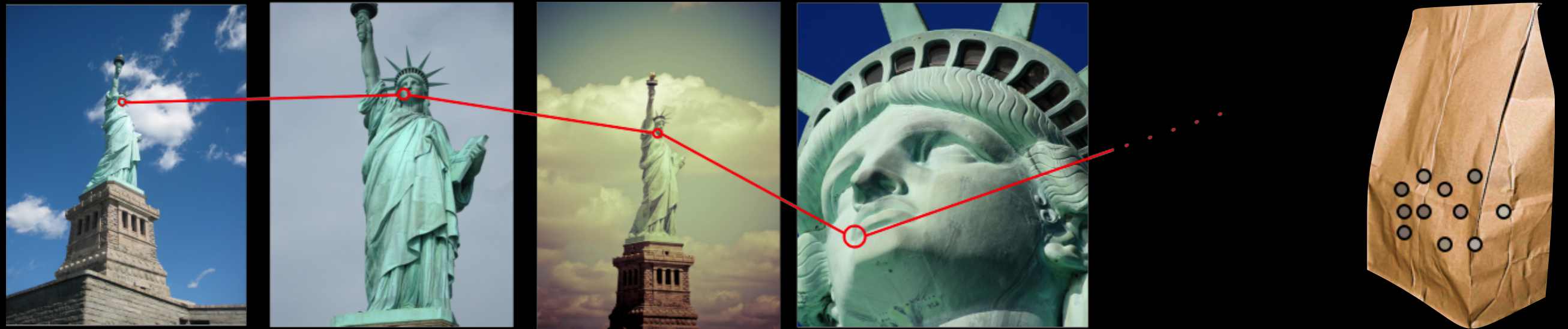
What's in the bag?

**Constants:**

albedo

**Variables:**

# Input Data



What's in the bag?

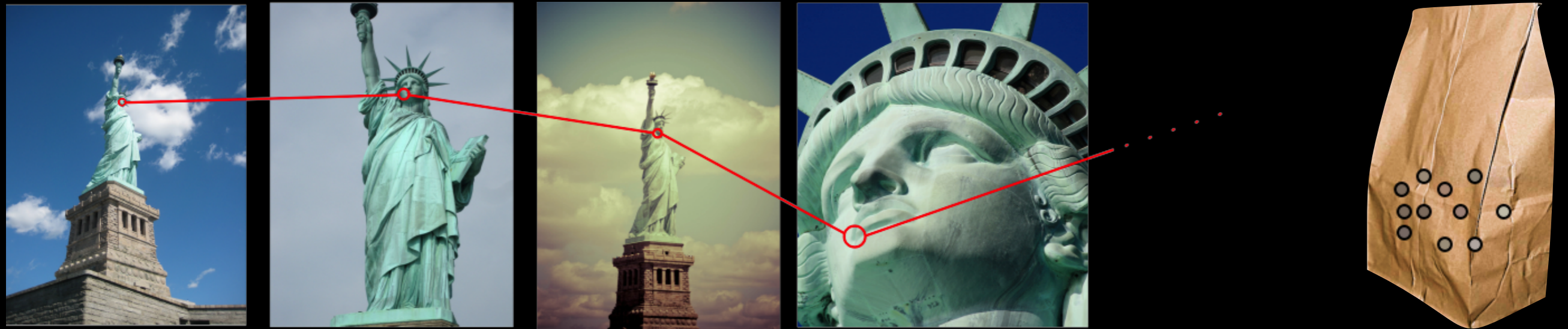
**Constants:**

albedo

**Variables:**

exposure

# Input Data



What's in the bag?

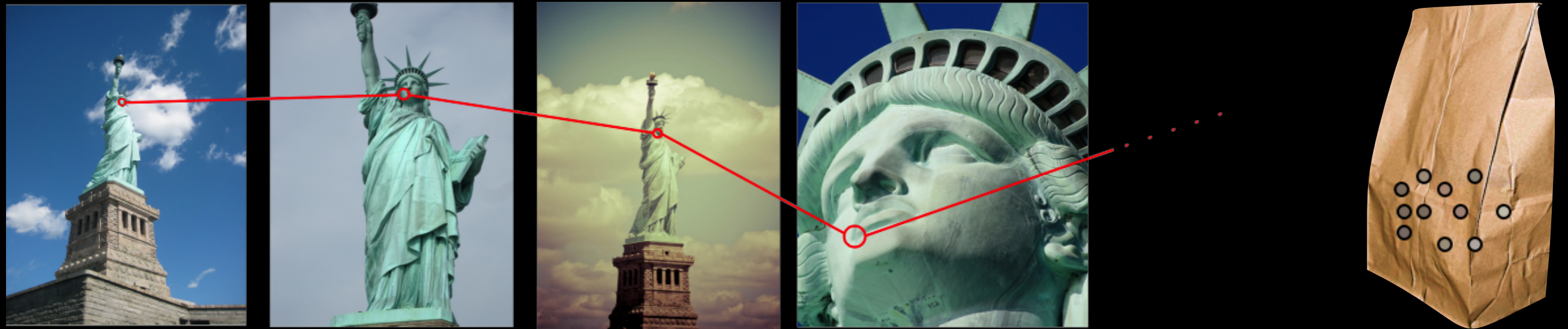
**Constants:**

albedo

**Variables:**

exposure illumination

# Input Data



What's in the bag?

**Constants:**

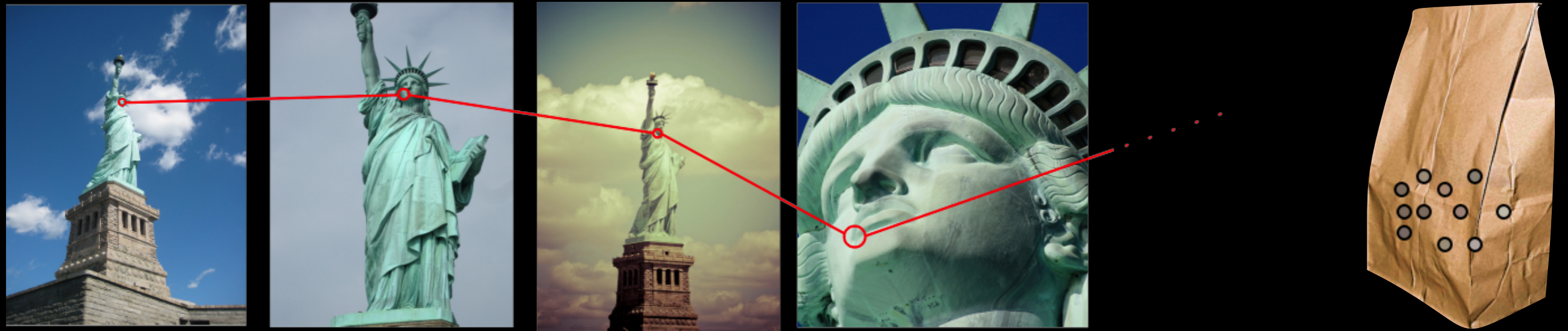
albedo

**Variables:**

exposure illumination

white balance

# Input Data



What's in the bag?

**Constants:**

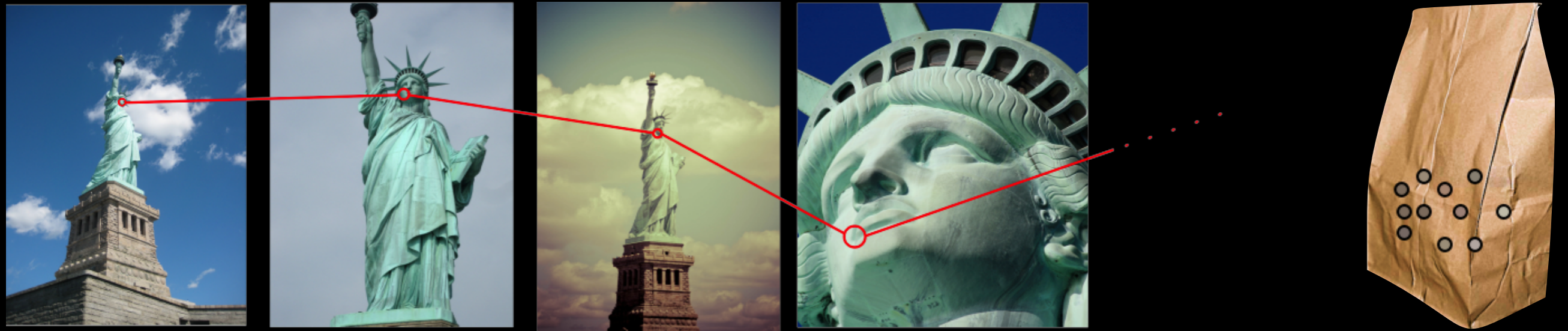
albedo

**Variables:**

exposure illumination

white balance tone curves

# Input Data



What's in the bag?

**Constants:**

albedo

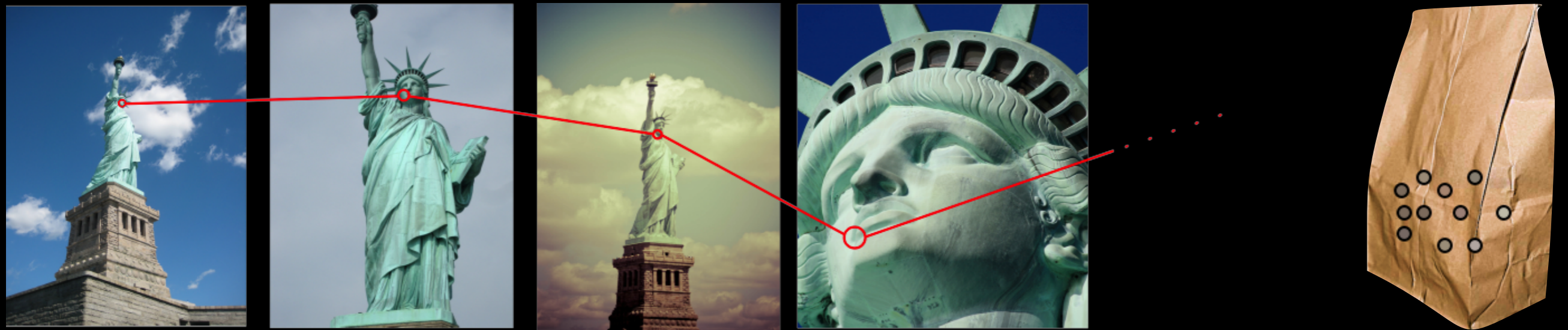
**Variables:**

exposure illumination

white balance tone curves

view direction

# Input Data



What's in the bag?

**Constants:**

albedo

**Variables:**

exposure illumination

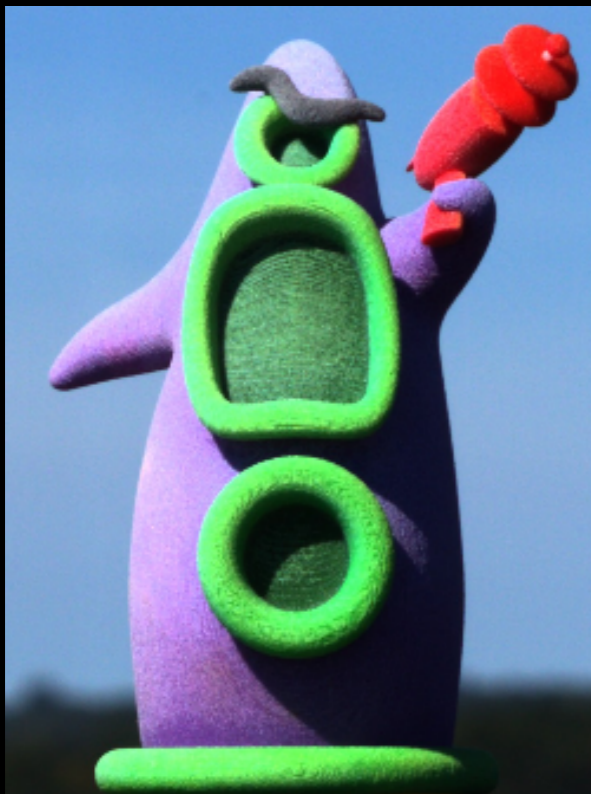
white balance tone curves

view direction

# Outline

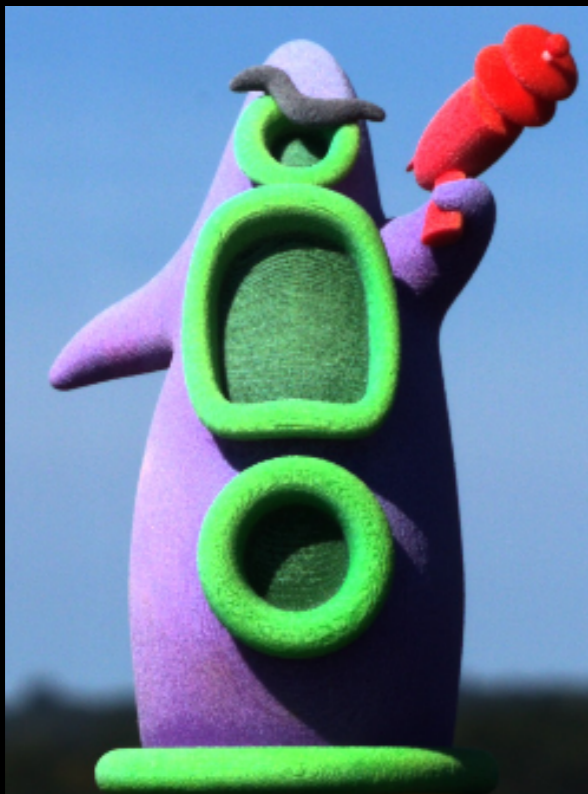
1. Input Data
2. The Illumination Ratio
3. Algorithm
4. Sparse to Dense
5. Application: Sun Direction Estimation

# The Illumination Ratio

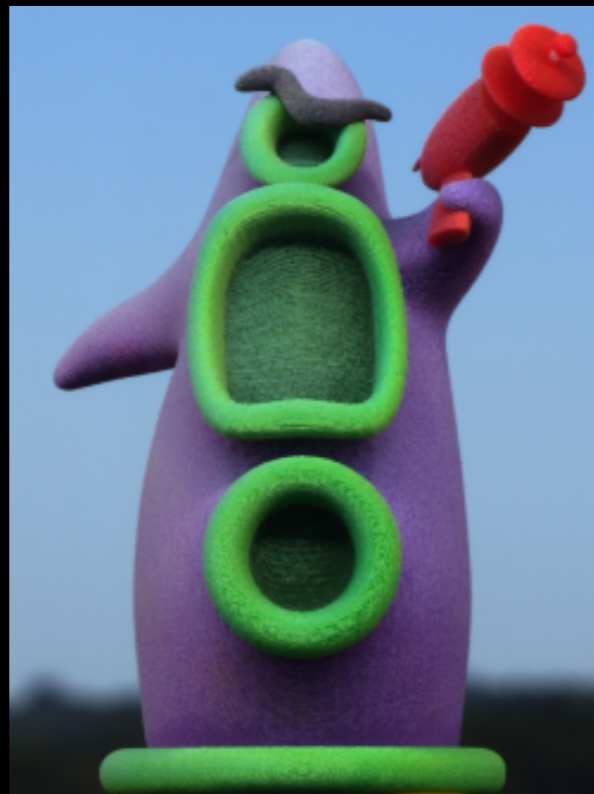


$I_i$

# The Illumination Ratio

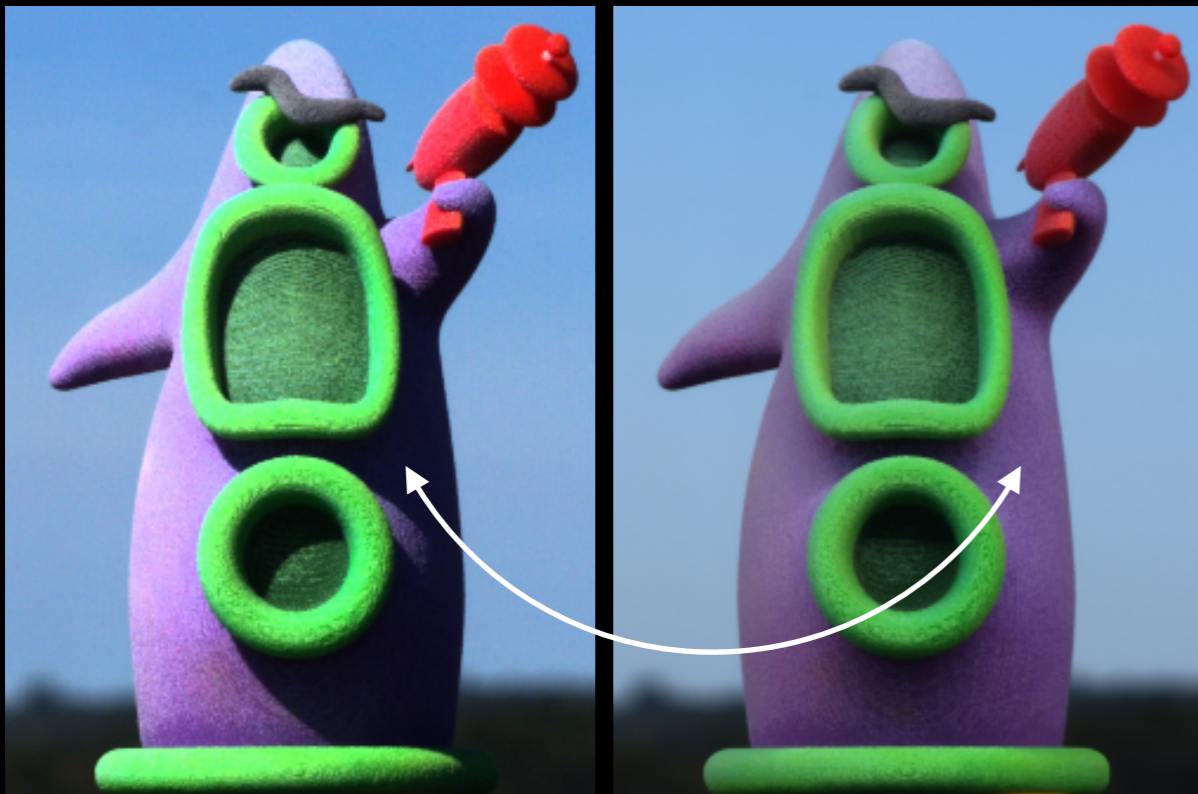


$I_i$



$\mathcal{E}_i[I]$

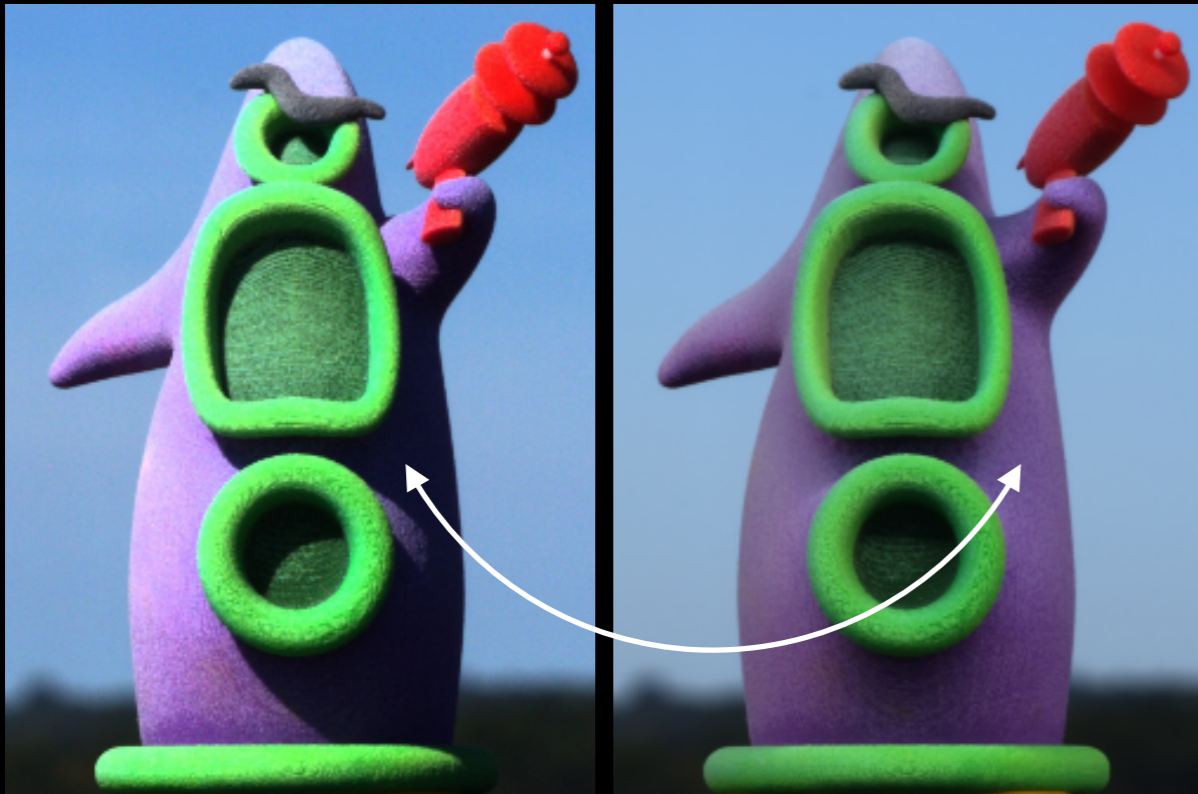
# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

# The Illumination Ratio

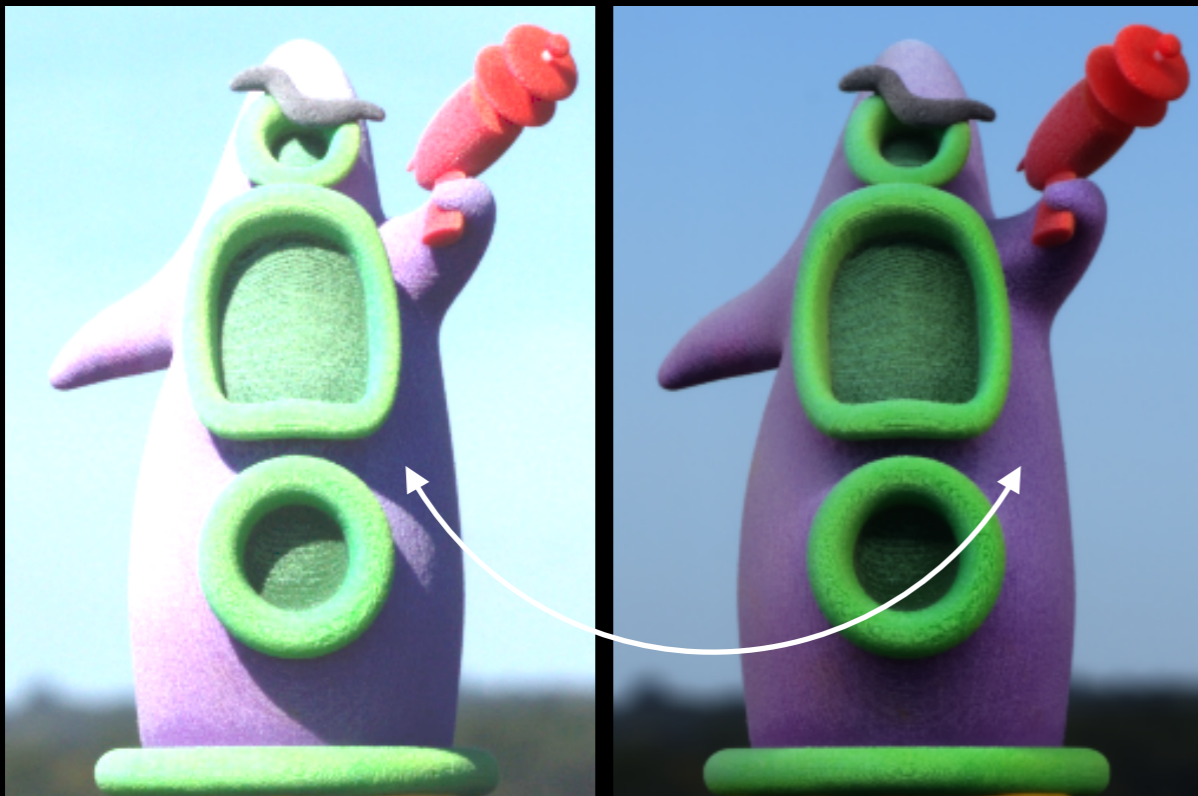


$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

# The Illumination Ratio

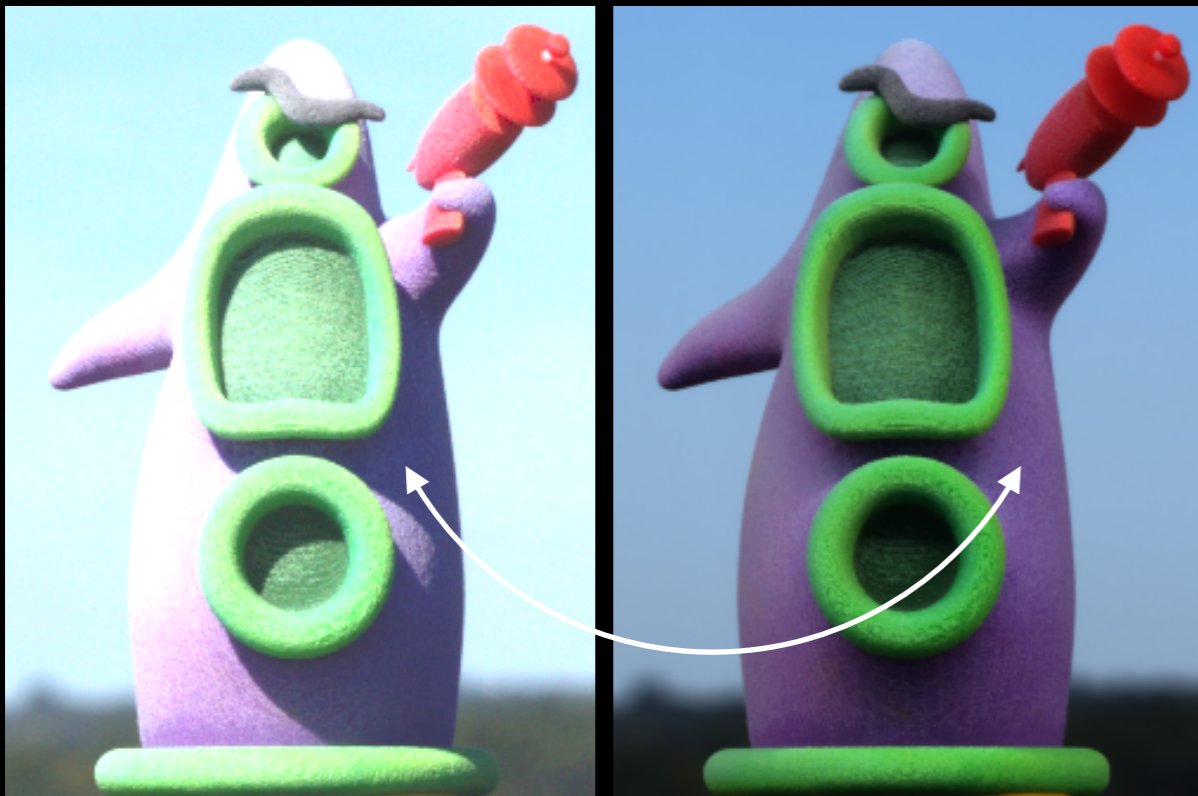


$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

# The Illumination Ratio



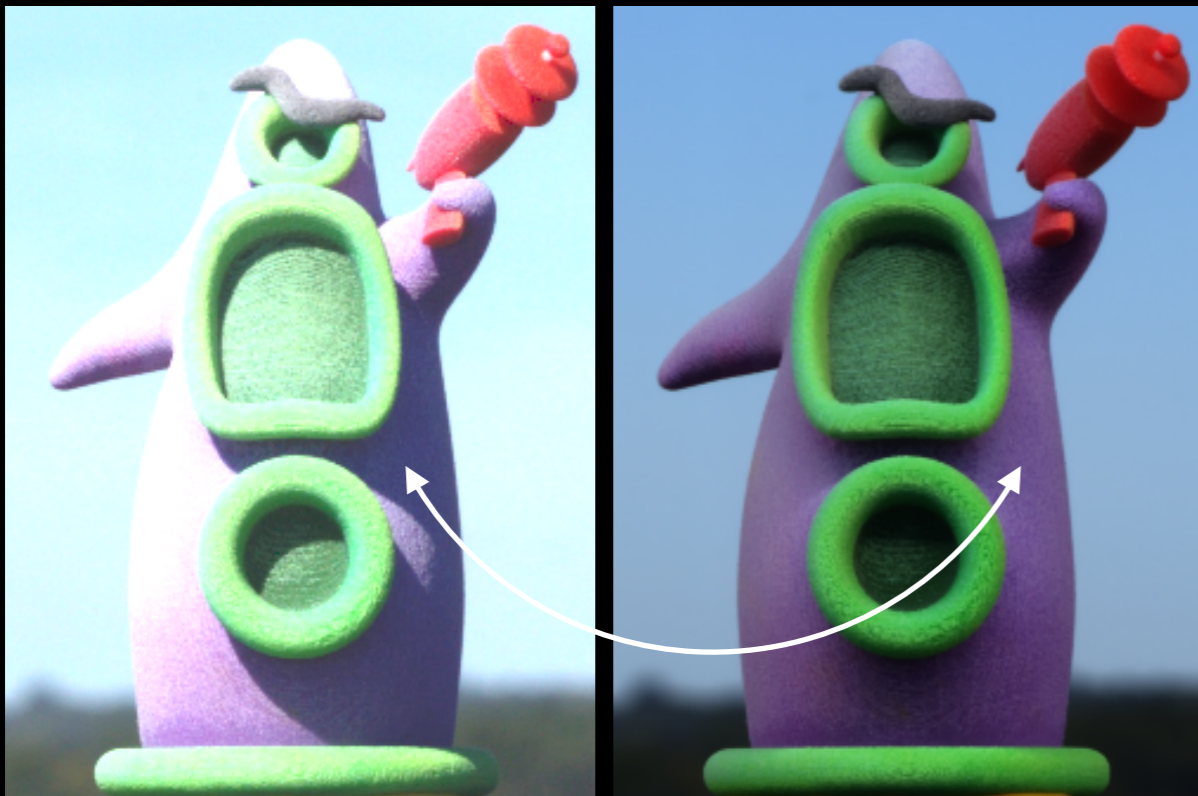
$I_i$

$E_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



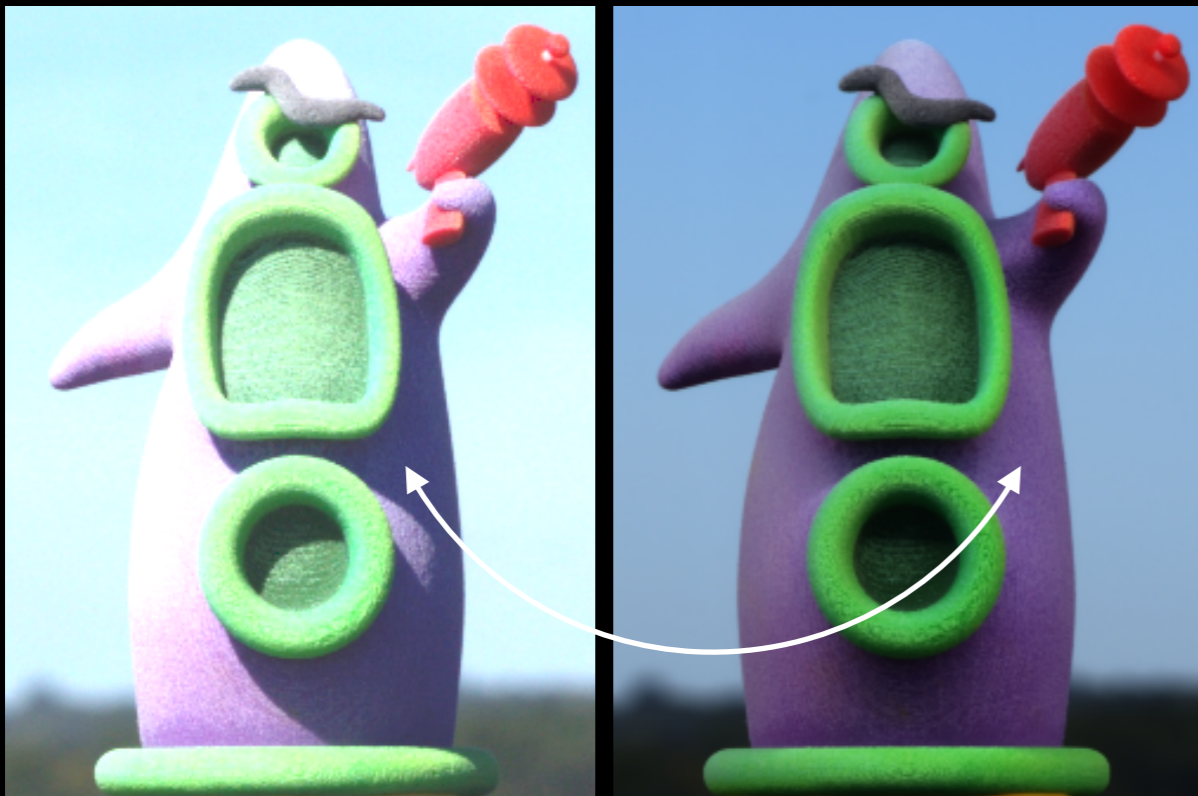
$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]} = \frac{\rho_x E_i L_{i,x}}{\rho_x \mathcal{E}_i[E_i L_{i,x}]}$$

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

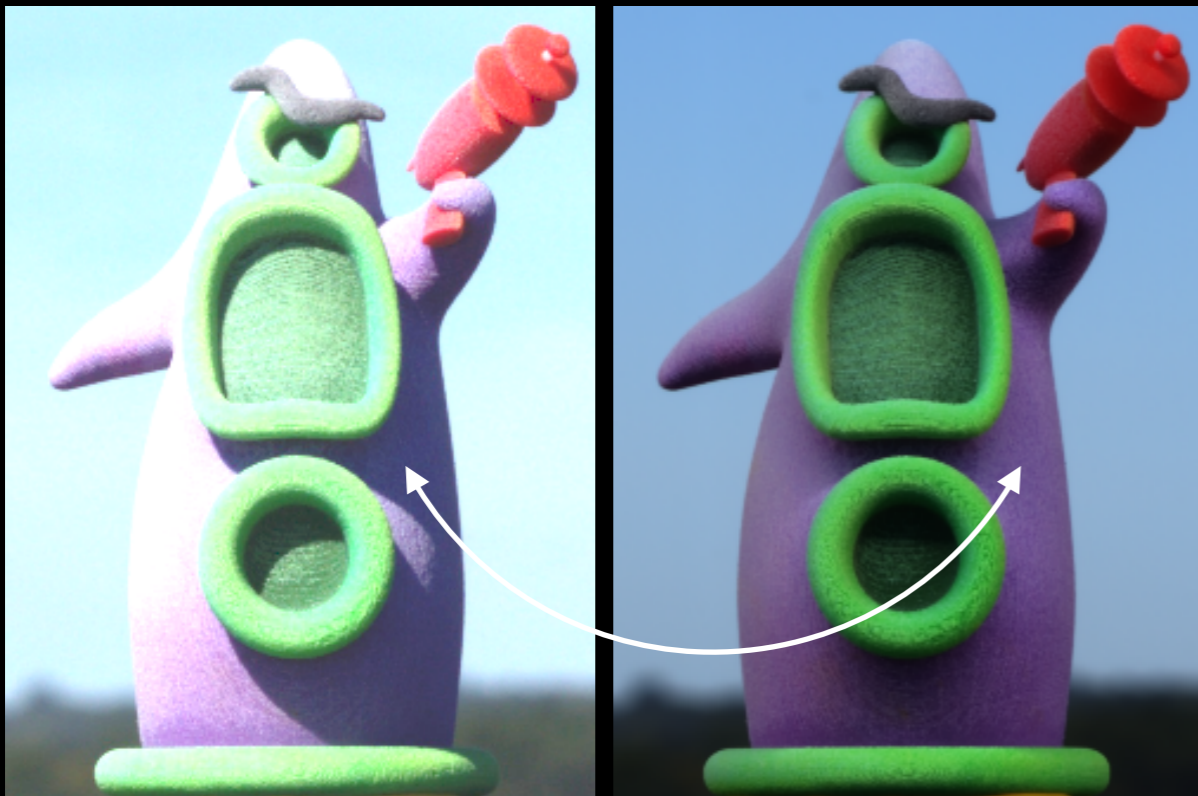
$E_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]} = \frac{\rho_x E_i L_{i,x}}{\rho_x \mathcal{E}_i[E_i L_{i,x}]}$$

Invariant to:

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

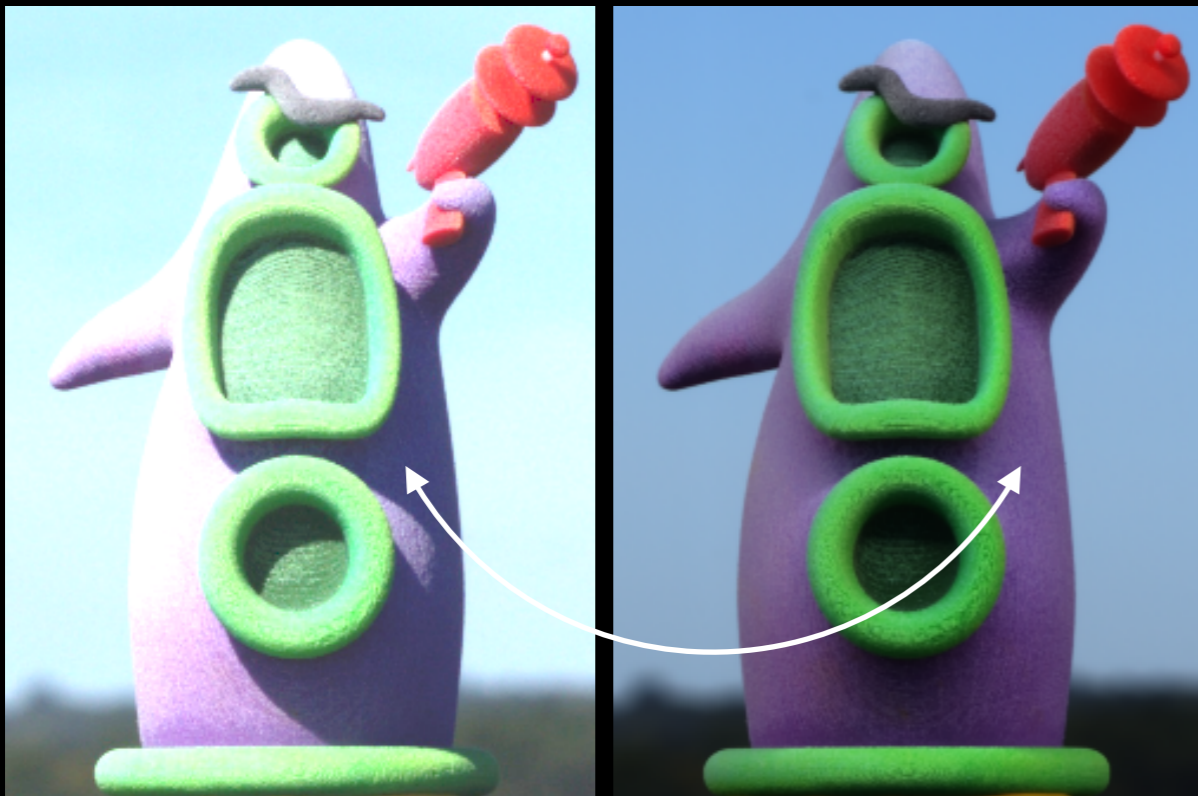
$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]} = \frac{\cancel{\rho_x} E_i L_{i,x}}{\cancel{\rho_x} \mathcal{E}_i[E_i L_{i,x}]}$$

Invariant to:

✓ Albedo

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$E_i[I]$

$$\frac{I_i(x)}{E_i[I(x)]} = \frac{\cancel{\rho_x} \cancel{E_i} L_{i,x}}{\cancel{\rho_x} E_i[E_i L_{i,x}]}$$

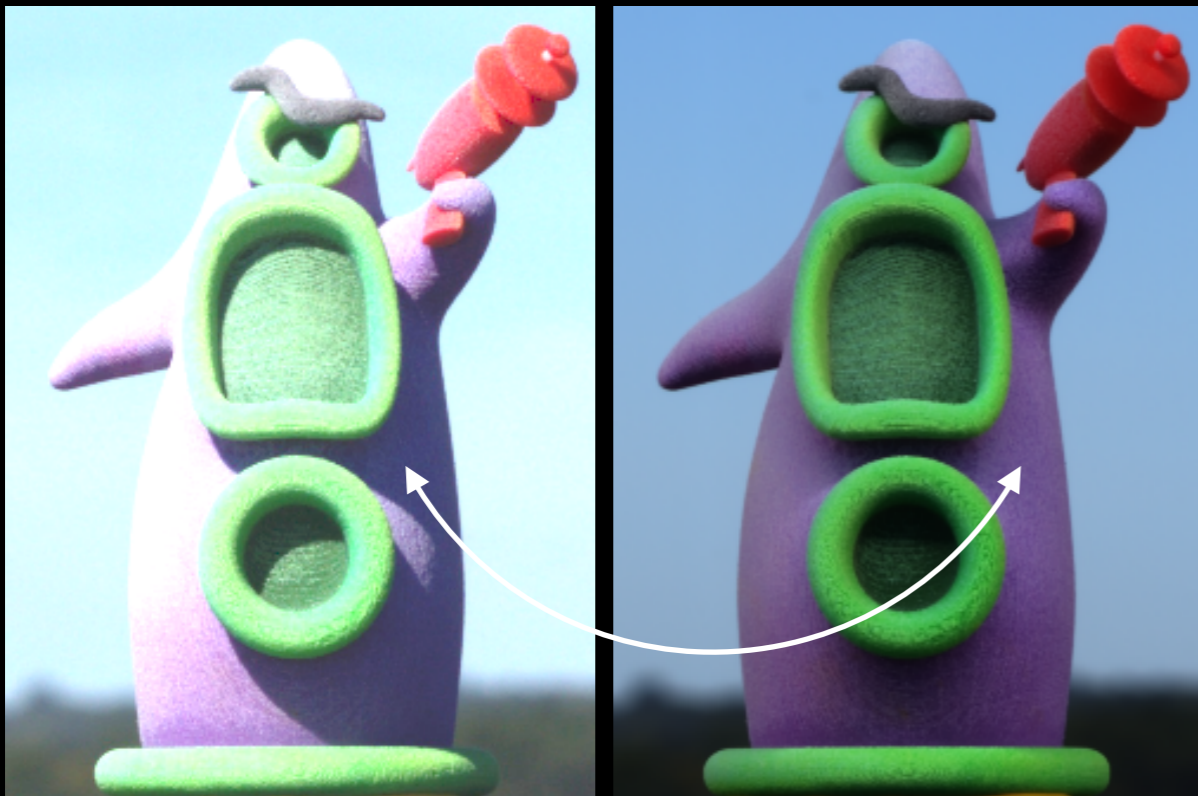
Invariant to:

✓ Albedo

✗ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$E_i[I]$

$$\frac{I_i(x)}{E_i[I(x)]}$$

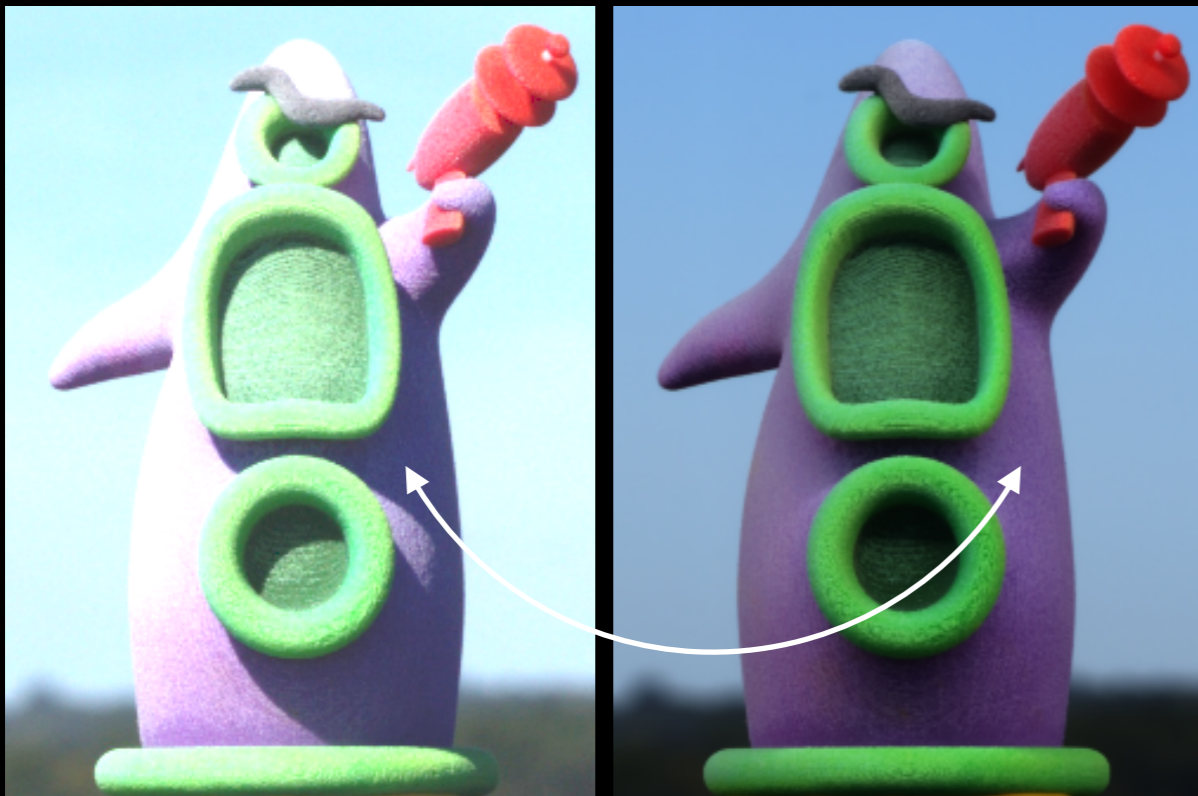
Invariant to:

✓ Albedo

✗ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$E_i[I]$

$$\frac{I_i(x)}{E_i[I(x)]}$$

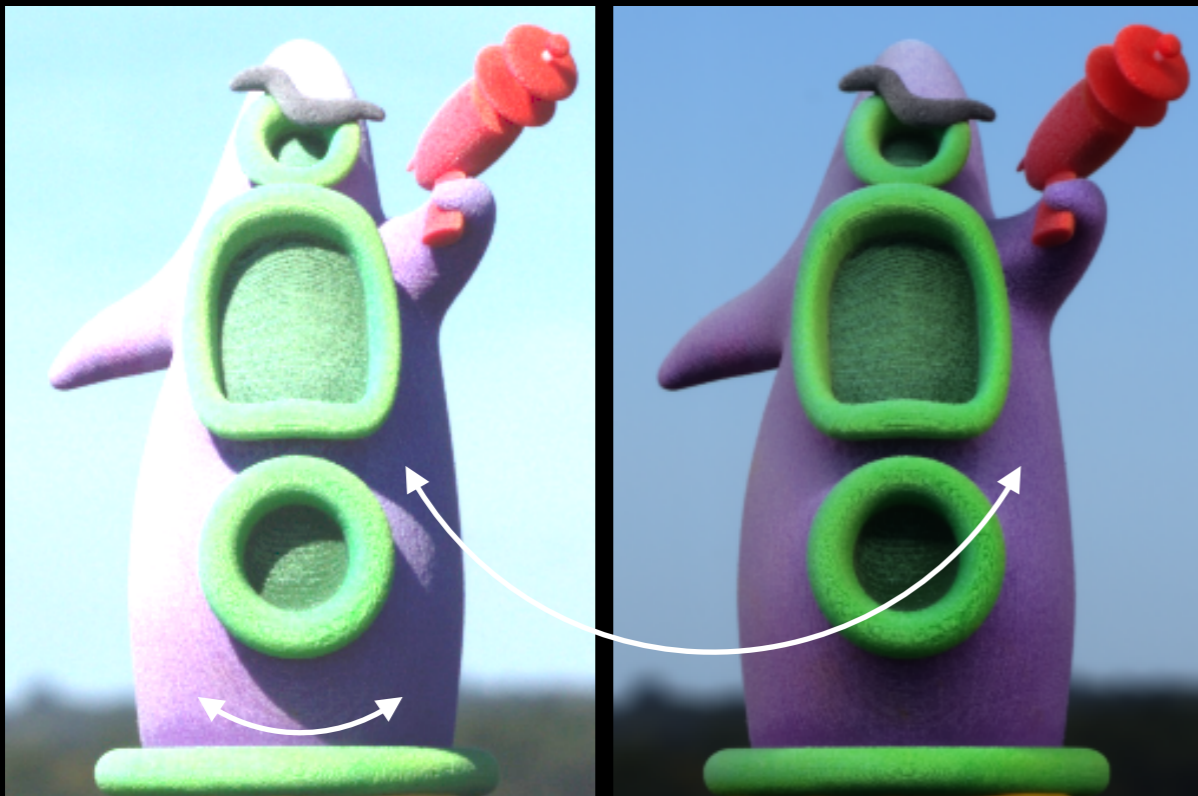
Invariant to:

✓ Albedo

✗ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$E_i[I]$

$$\frac{I_i(x)}{E_i[I(x)]}$$

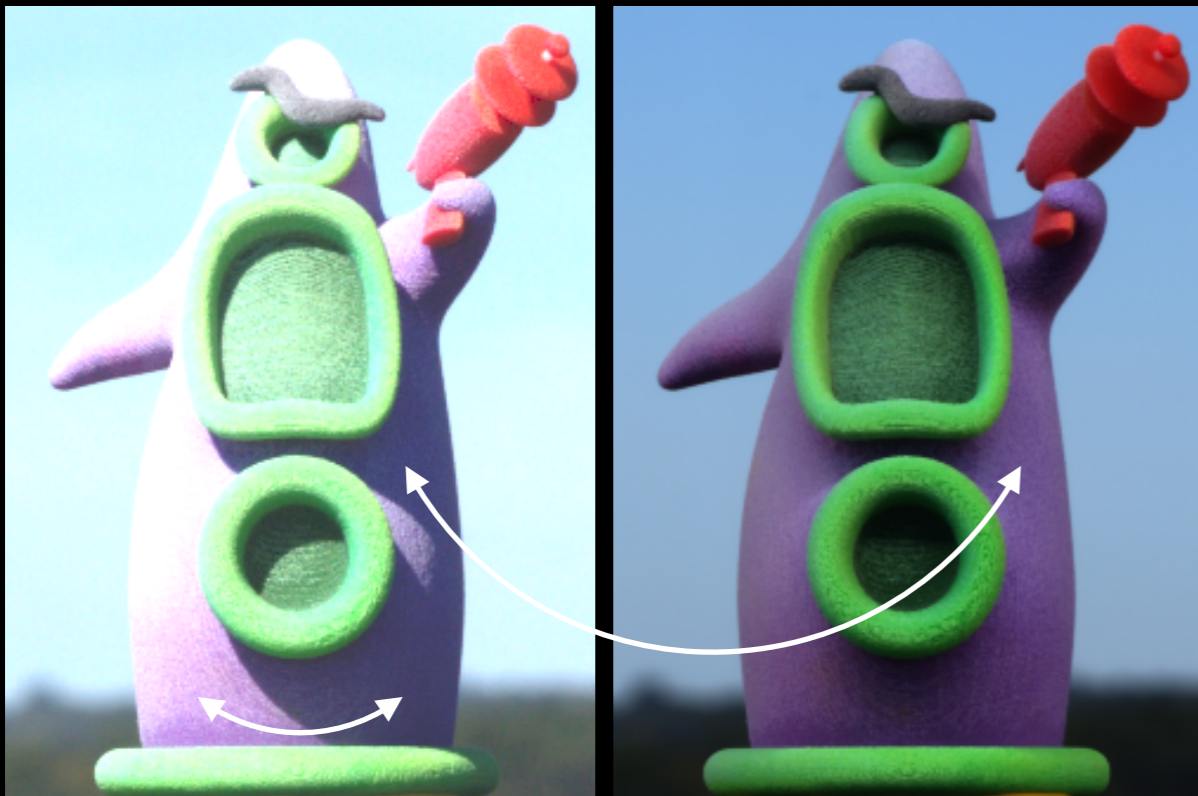
Invariant to:

✓ Albedo

✗ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

$$\frac{I_i(x)}{I_i(y)}$$

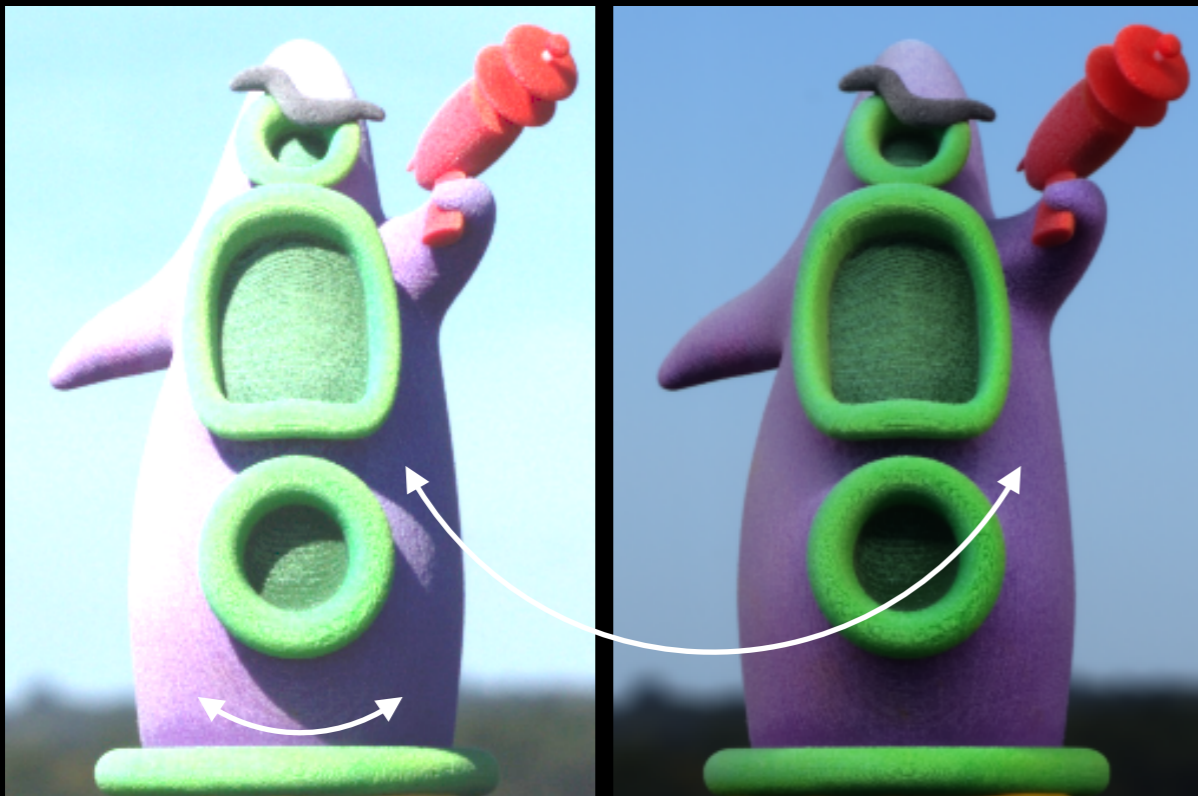
Invariant to:

✓ Albedo

✗ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

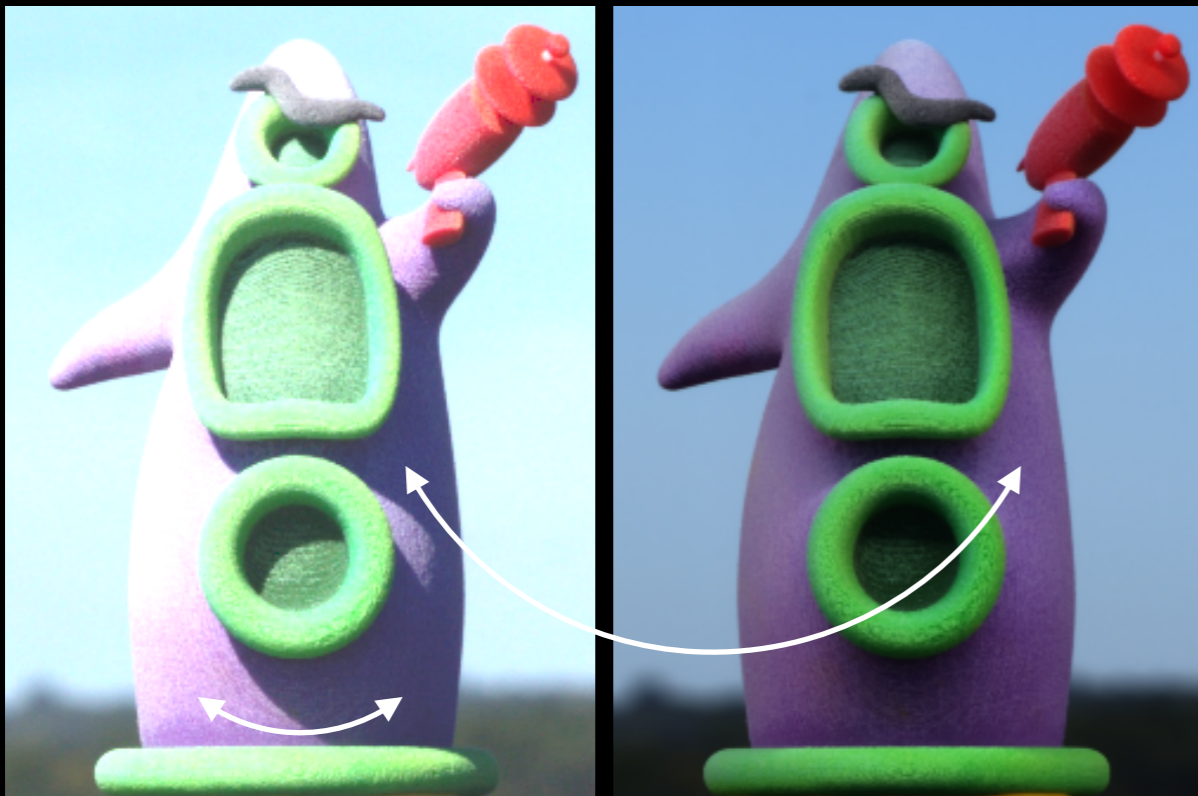
✓ Albedo

✗ Exposure

$$\frac{I_i(x)}{I_i(y)} = \frac{\rho_x E_i L_{i,x}}{\rho_y E_i L_{i,y}}$$

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

✓ Albedo

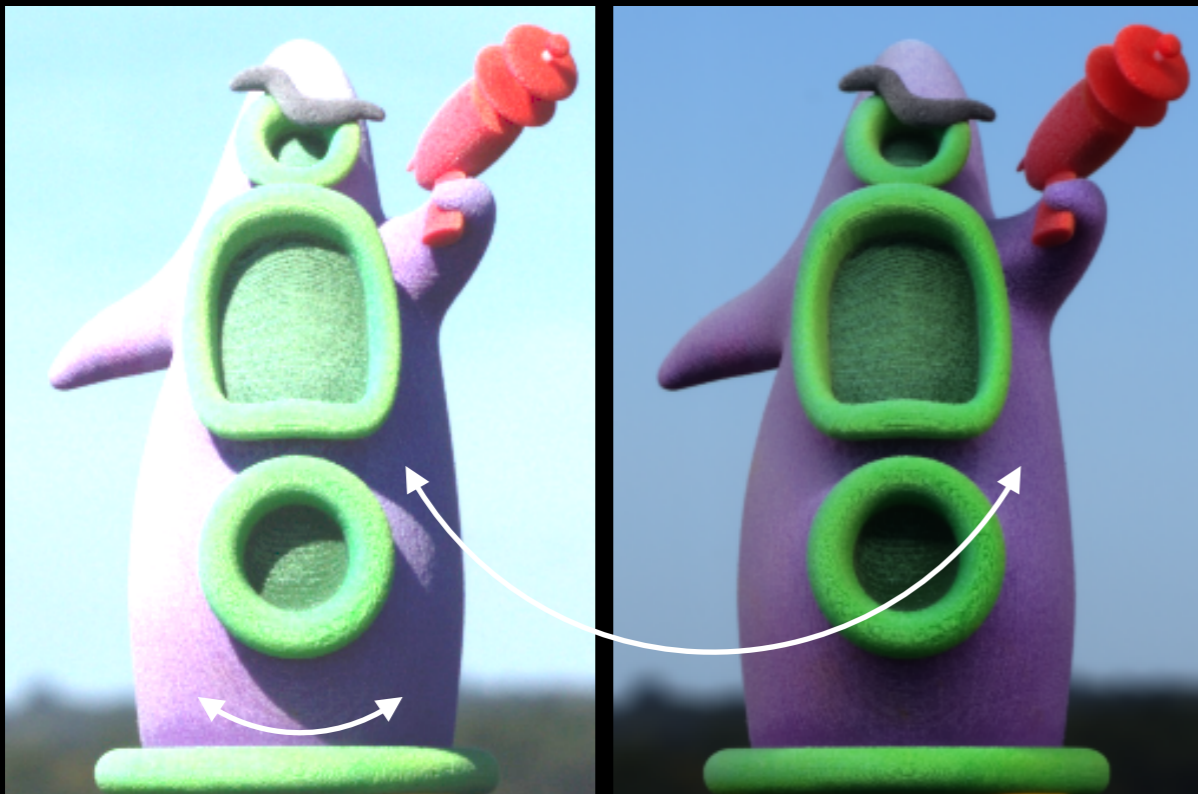
✗ Exposure

$$\frac{I_i(x)}{I_i(y)} = \frac{\rho_x E_i L_{i,x}}{\rho_y E_i L_{i,y}}$$

Invariant to:

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$E_i[I]$

$$\frac{I_i(x)}{E_i[I(x)]}$$

Invariant to:

✓ Albedo

✗ Exposure

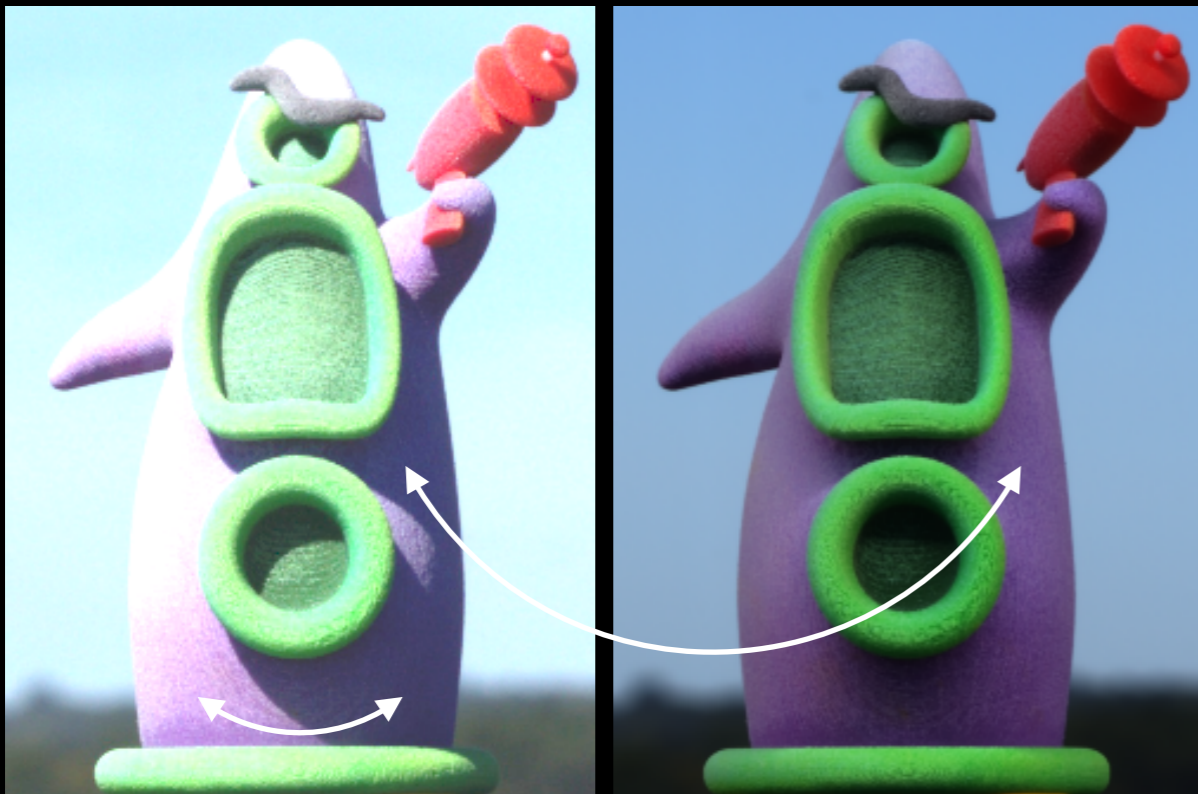
$$\frac{I_i(x)}{I_i(y)} = \frac{\rho_x \cancel{E_i} L_{i,x}}{\rho_y \cancel{E_i} L_{i,y}}$$

Invariant to:

✓ Exposure

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

✓ Albedo

✗ Exposure

$$\frac{I_i(x)}{I_i(y)} = \frac{\rho_x \cancel{E_i} L_{i,x}}{\rho_y \cancel{E_i} L_{i,y}}$$

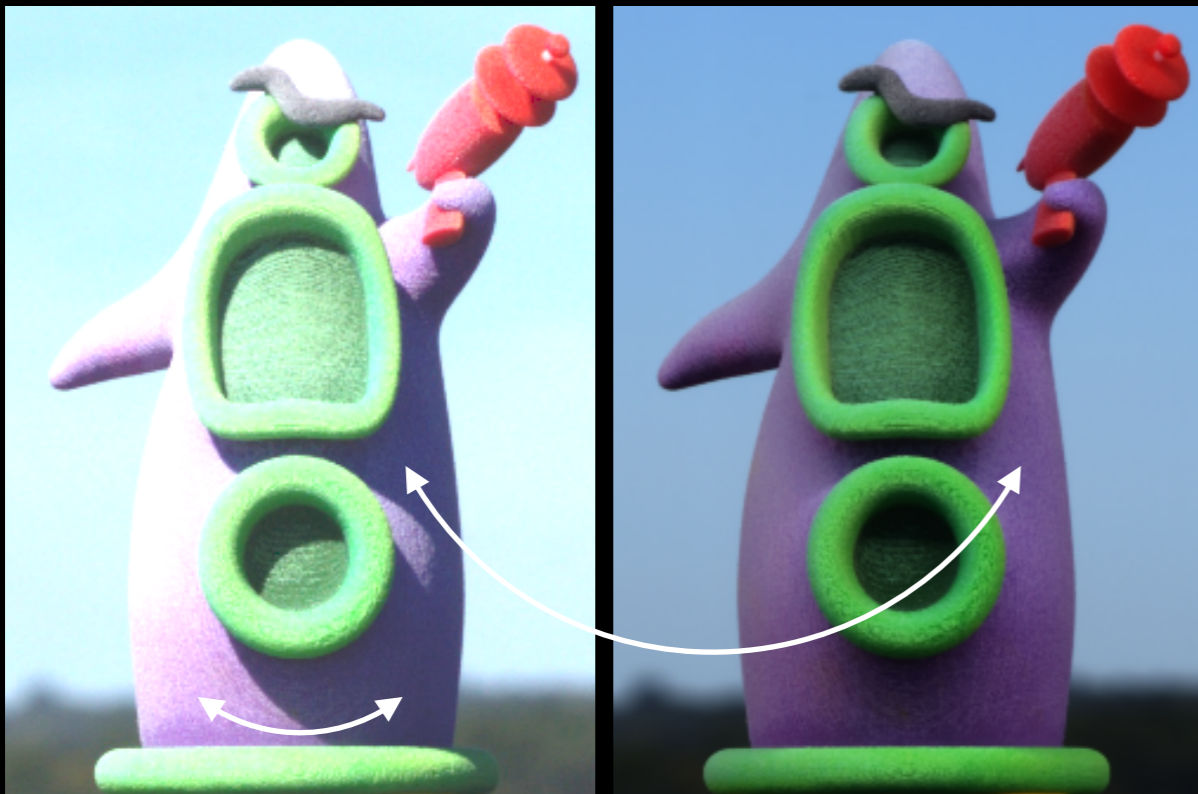
Invariant to:

✓ Exposure

✗ Albedo

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

✓ Albedo

✗ Exposure

$$\frac{I_i(x)}{I_i(y)}$$

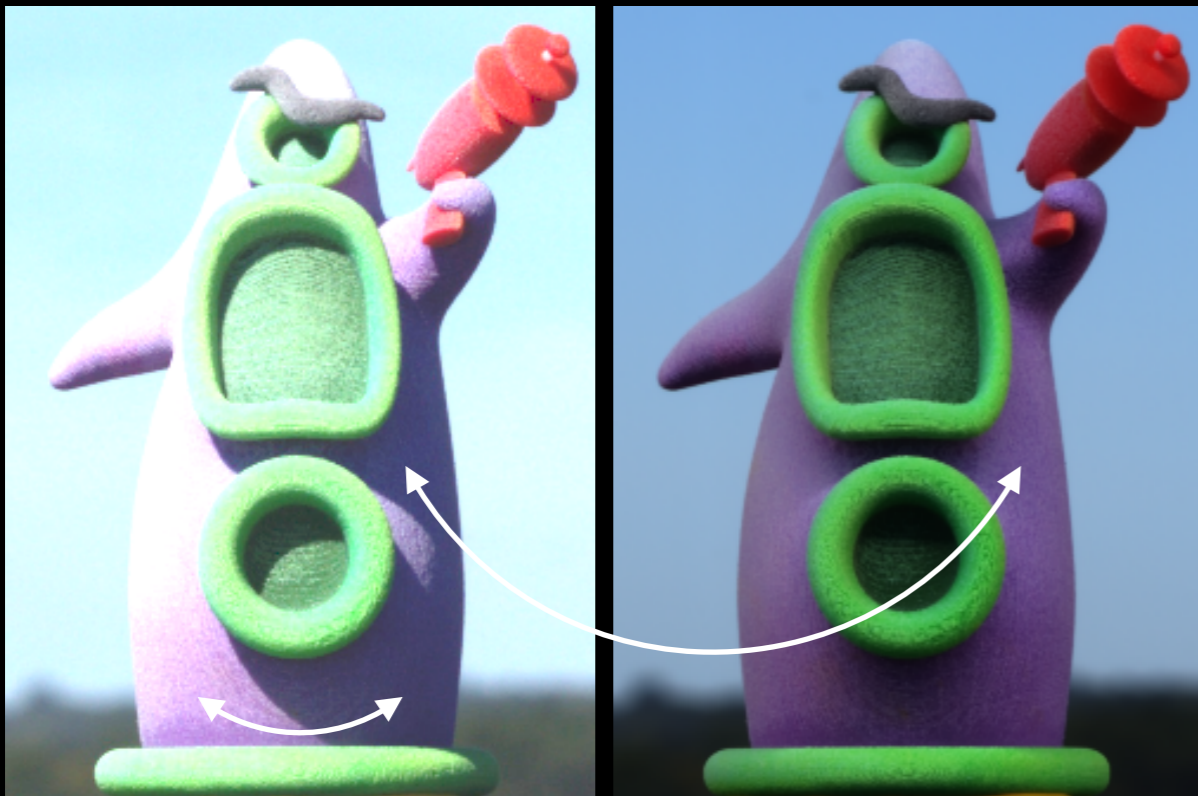
Invariant to:

✓ Exposure

✗ Albedo

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

✓ Albedo

✗ Exposure

$$\frac{I_i(x)}{I_i(y)}$$

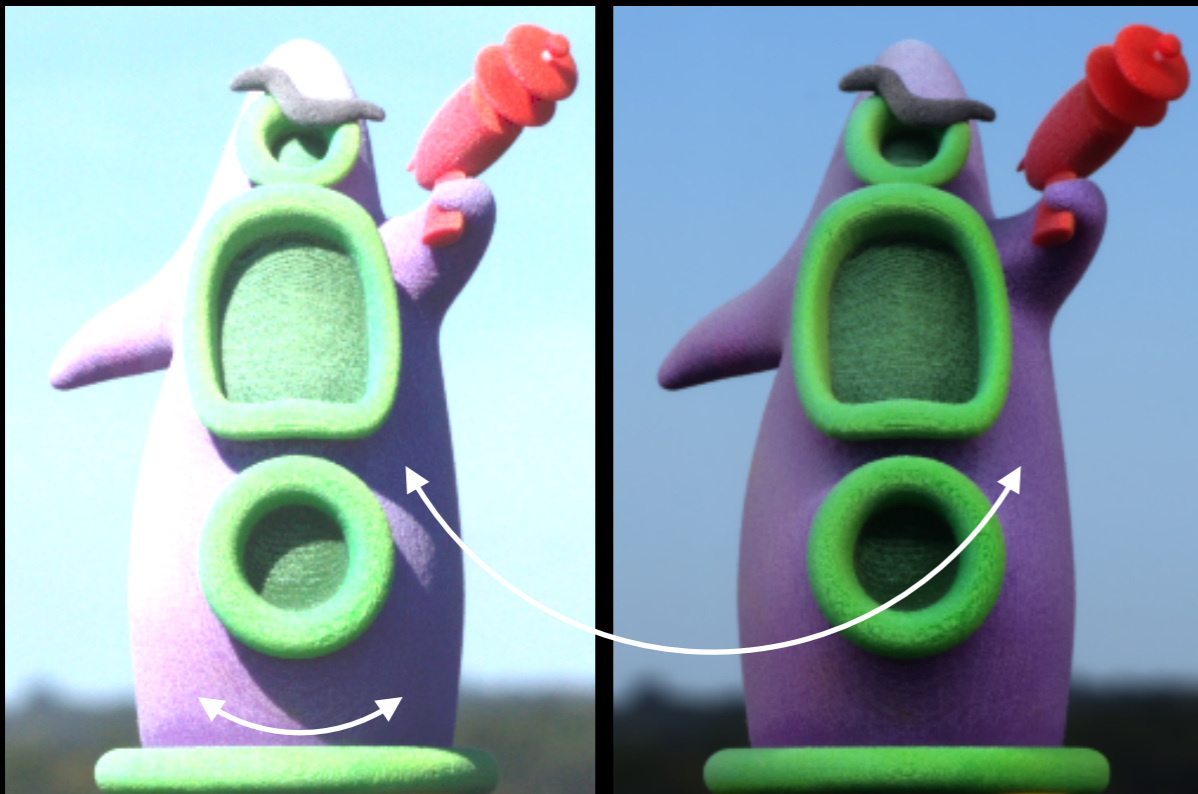
Invariant to:

✓ Exposure

✗ Albedo

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# The Illumination Ratio



$I_i$

$\mathcal{E}_i[I]$

$$\frac{I_i(x)}{\mathcal{E}_i[I(x)]}$$

Invariant to:

✓ Albedo

✗ Exposure

$$\frac{I_i(x)}{I_i(y)}$$

Invariant to:

✓ Exposure

✗ Albedo

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)]}}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)]}}$$

Image formation model:  $I_i(x) = \rho_x E_i L_{i,x}$

# Analyzing R

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)]}}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)]}}$$

# Analyzing R

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)] \approx \rho_x}}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)] \approx \rho_y}}$$

# Analyzing R

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)] \approx \rho_x}}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)] \approx \rho_y}}$$



# Analyzing R

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)] \approx \rho_x}}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)] \approx \rho_y}} \approx \frac{L_{i,x}}{L_{i,y}}$$



# Analyzing R

$$R(x, y) = \frac{\frac{I_i(x)}{\mathcal{E}_i[I_i(x)]} \approx \rho_x}{\frac{I_i(y)}{\mathcal{E}_i[I_i(y)]} \approx \rho_y} \approx \frac{L_{i,x}}{L_{i,y}}$$

Illumination model:

$$L_x = C_x L_{sun} \cos(\phi_x) + L_{sky}$$



# Analyzing R

$$R(x, y) \approx \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$

Illumination model:

$$L_x = C_x L_{sun} \cos(\phi_x) + L_{sky}$$



# Analyzing R

$$R(x, y) \approx \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$

$$C_x = \begin{cases} 1 & \text{if } x \text{ is sunlit} \\ 0 & \text{if } x \text{ is in shadow} \end{cases}$$

Illumination model:

$$L_x = C_x L_{sun} \cos(\phi_x) + L_{sky}$$



# Analyzing R

$$R(x, y) \approx \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$

$$C_x = \begin{cases} 1 & \text{if } x \text{ is sunlit} \\ 0 & \text{if } x \text{ is in shadow} \end{cases} \quad f = \frac{L_{sky}}{L_{sun}}$$

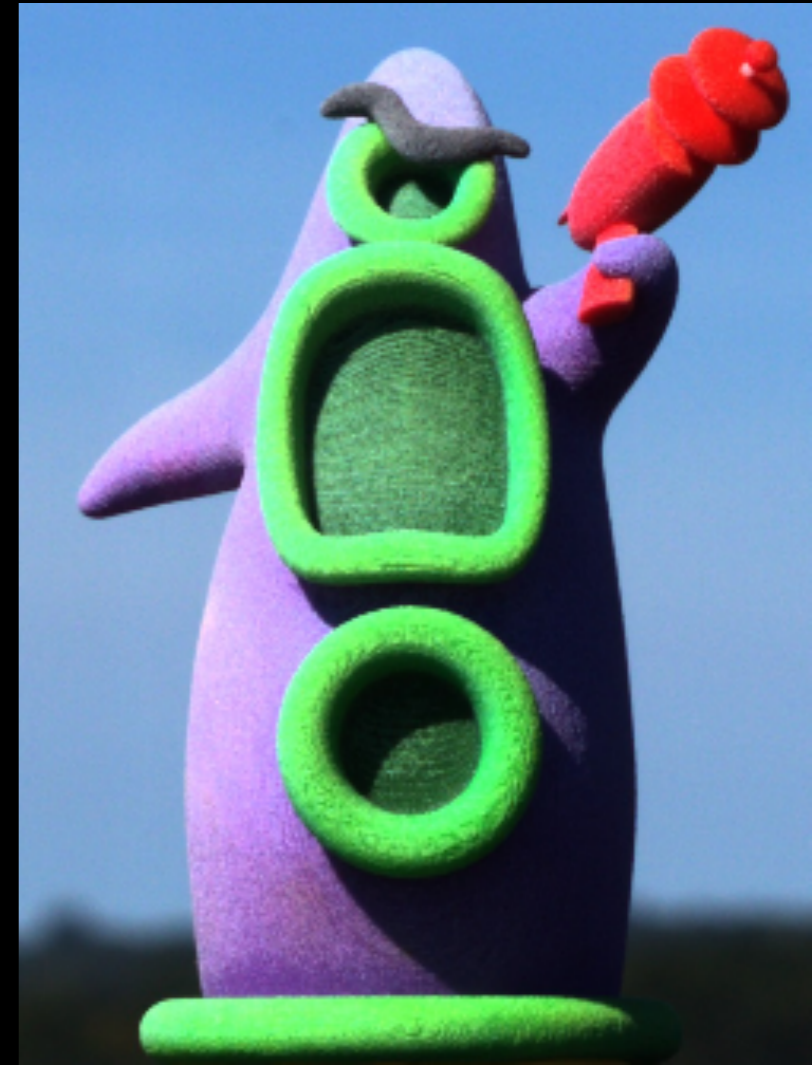
Illumination model:

$$L_x = C_x L_{sun} \cos(\phi_x) + L_{sky}$$



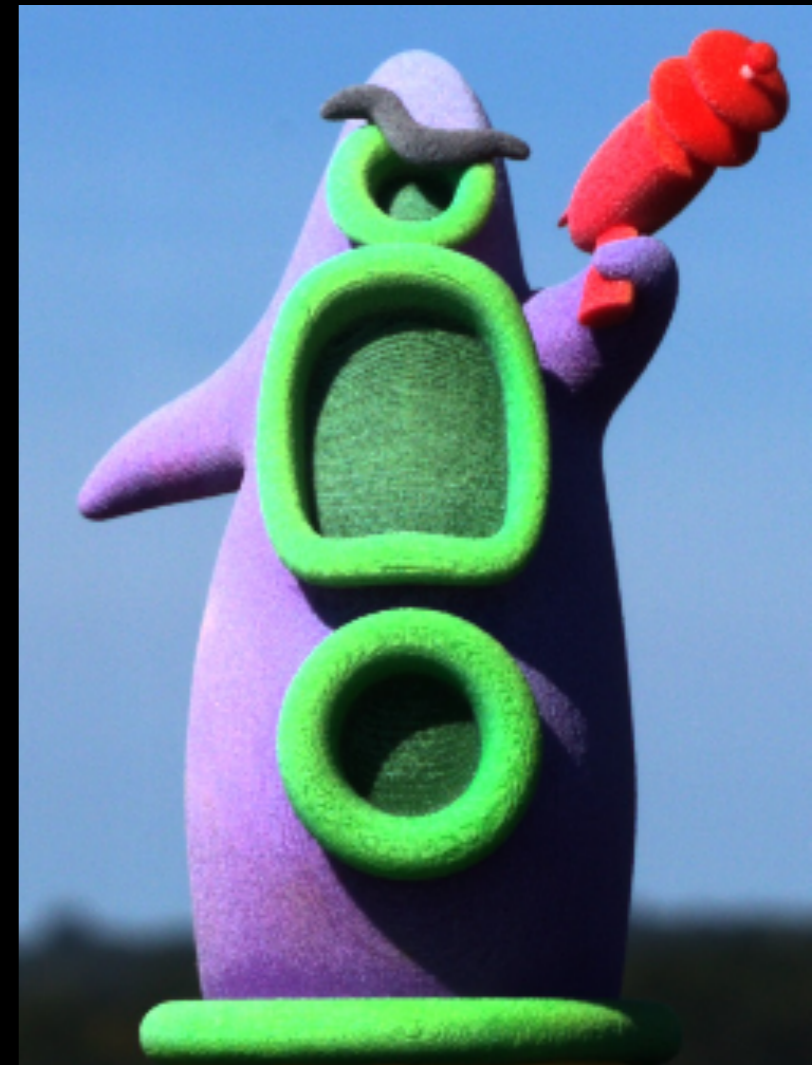
# Analyzing R

$$R(x, y) = \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$



# Analyzing R

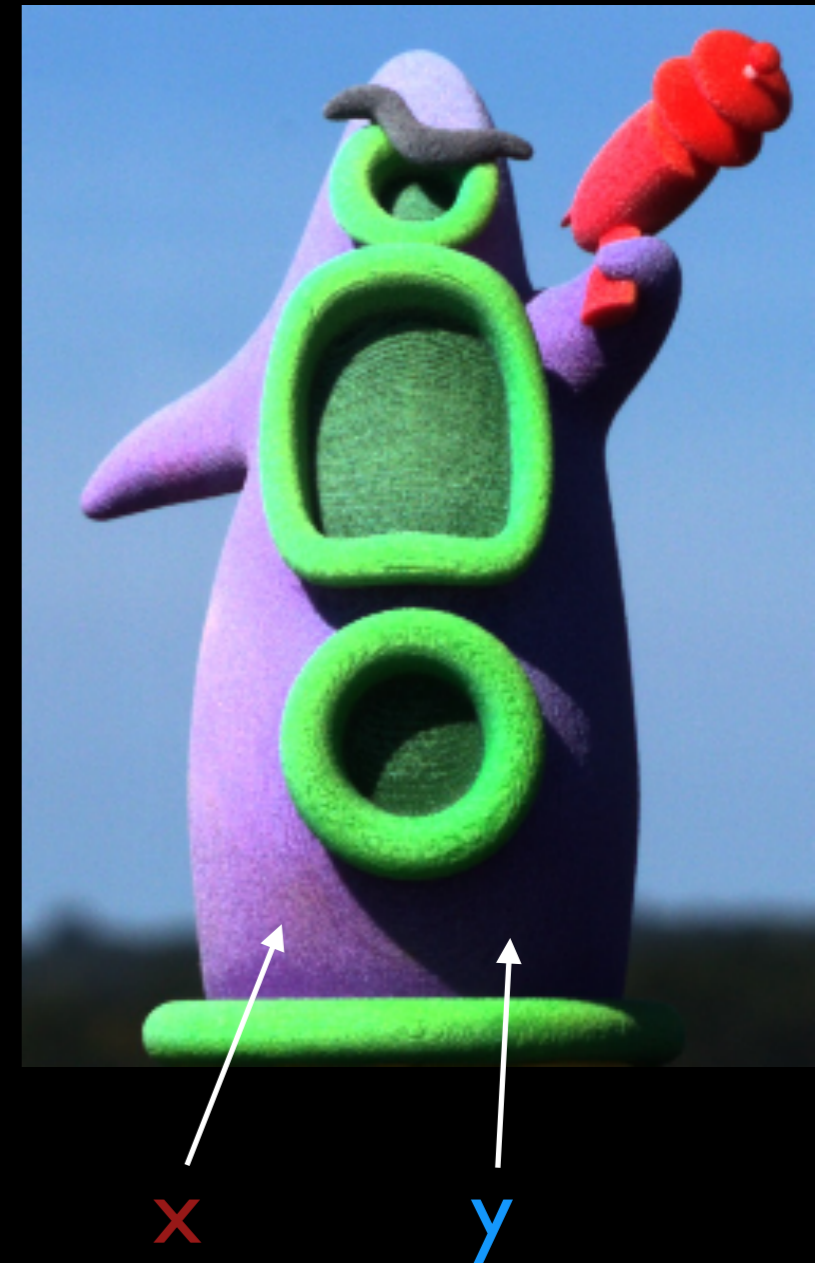
$$R(x, y) = \frac{C_x + f}{C_y + f}$$



# Analyzing R

$$R(x, y) = \frac{C_x + f}{C_y + f}$$

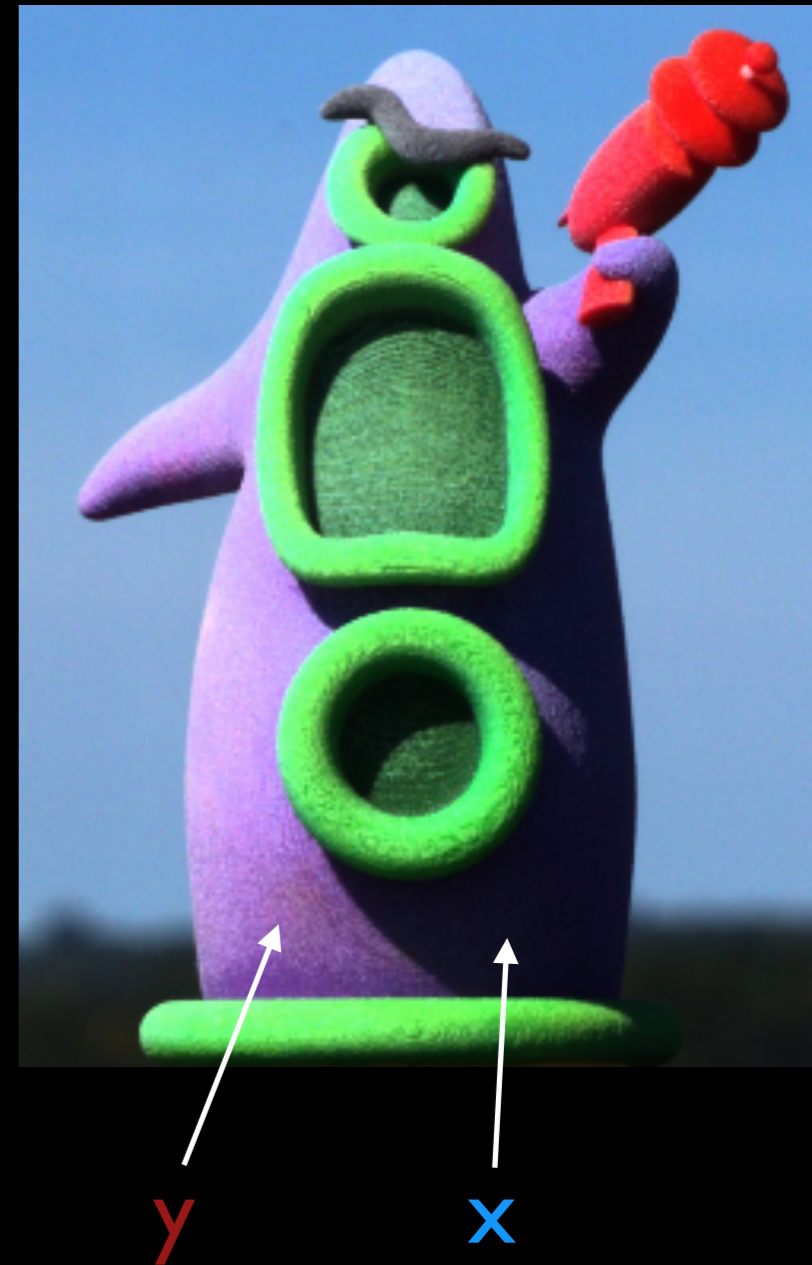
- **x**, **y**: “large”



# Analyzing R

$$R(x, y) = \frac{C_x + f}{C_y + f}$$

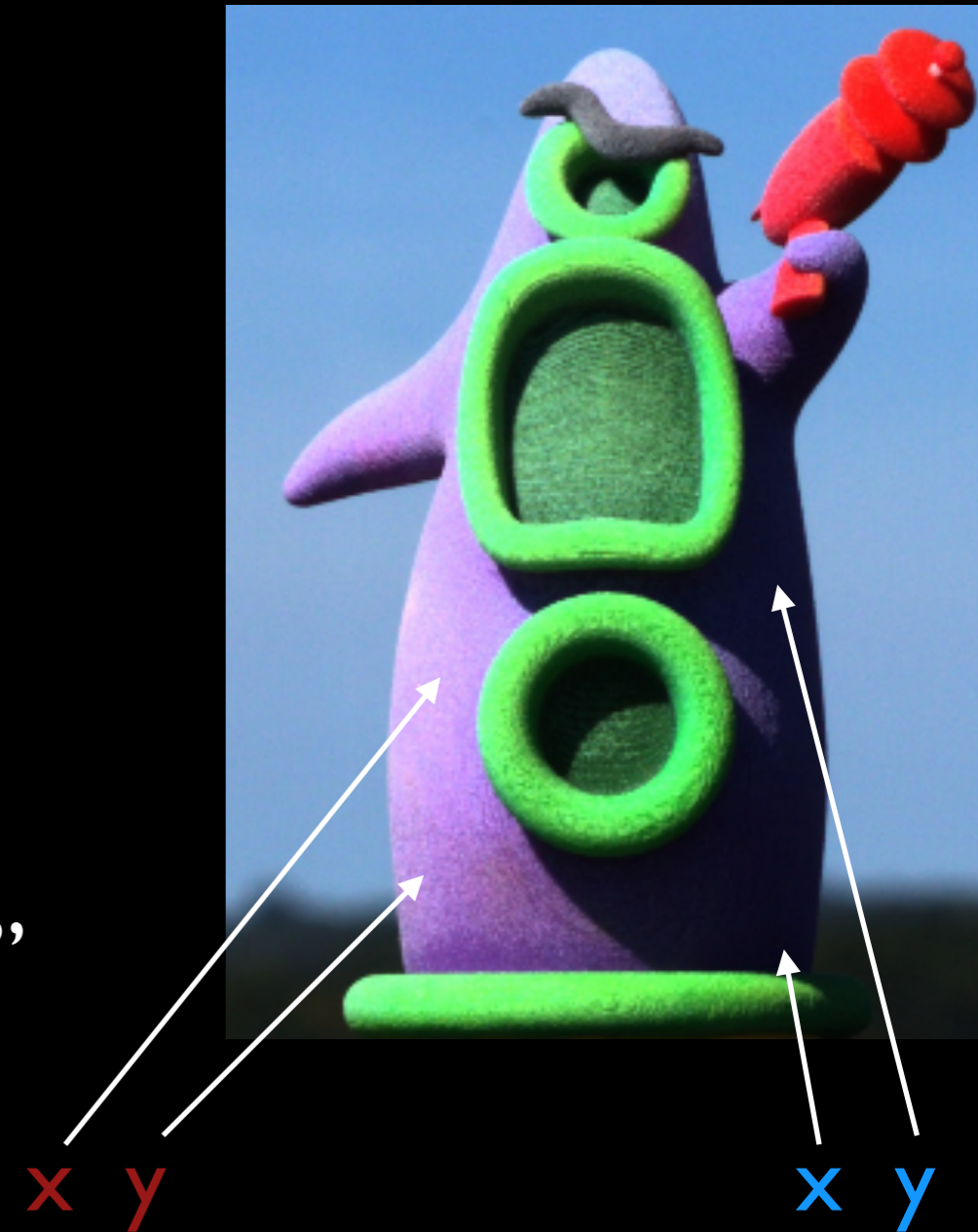
- **x**, **y**: “large”
- **x**, **y**: “small”



# Analyzing R

$$R(x, y) = \frac{C_x + f}{C_y + f}$$

- **x**, **y**: “large”
- **x**, **y**: “small”
- **x**, **y** or **x**, **y**: “one”



# Outline

1. Input Data
2. The Illumination Ratio
3. Algorithm
4. Sparse to Dense
5. Application: Sun Direction Estimation

# Shadow Detection Algorithm

# Shadow Detection Algorithm

- I. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$

# Shadow Detection Algorithm

1. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$
2. For each  $y_j$ , compute  $R(x, y_j)$ .

# Shadow Detection Algorithm

1. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$
2. For each  $y_j$ , compute  $R(x, y_j)$ .
  - If  $R(x, y_j) > T$ , vote  $x$  is sunlit

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  - If  $R(x, y_j) < \frac{1}{T}$ , vote  $x$  is shaded

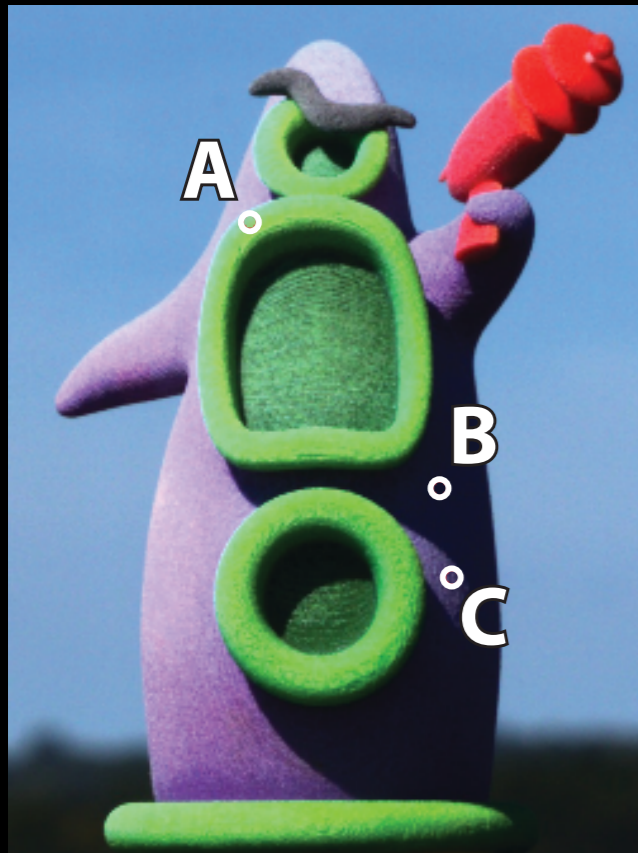
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  - Otherwise, cast no vote and continue.

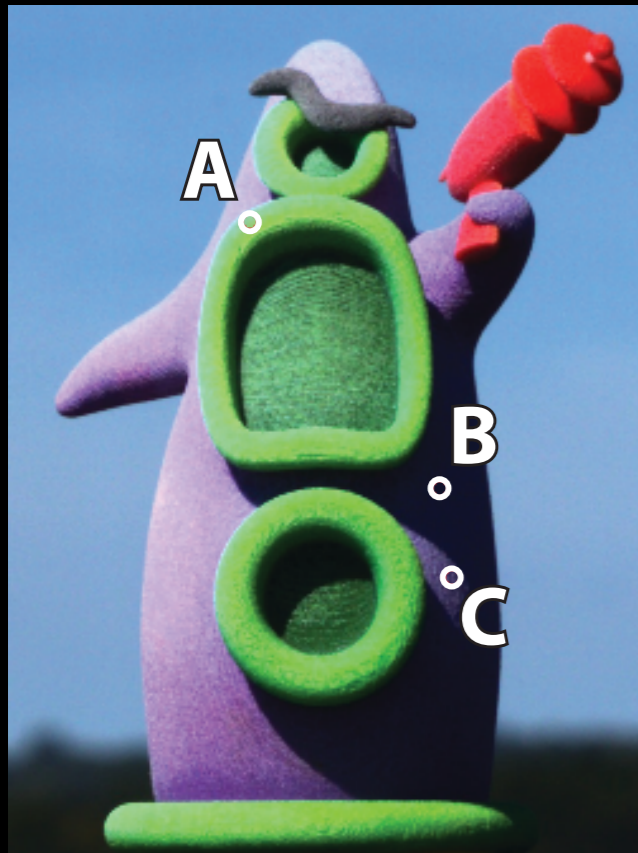
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3. Assign a label to  $x$  according to majority vote

# Shadow Detection



# Shadow Detection



$R(A, *)$



$R(B, *)$



$R(C, *)$

$(R > T)$   Vote sunlit

$(R < 1/T)$   Vote shadowed

$(1/T \leq R \leq T)$   Uninformative; no vote

# Shadow Detection Algorithm

1. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$
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# Shadow Detection Algorithm

1. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$  What about T?
2. For each  $y_j$ , compute  $R(x, y_j)$ .
  - If  $R(x, y_j) > T$ , vote  $x$  is sunlit
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# Shadow Detection Algorithm

1. For each point  $x$ , pick  $K$  random other points  $y_1, y_2, \dots, y_K$

What about  $T$ ?

we use  $T = 3$ .

2. For each  $y_j$ , compute  $R(x, y_j)$ .
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Why?

- If  $R(x, y_j) > T$ , vote  $x$  is sunlit
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we use  $T = 3$ .

2. For each  $y_j$ , compute  $R(x, y_j)$ .

Why?

$$R(x, y) = \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$

- If  $R(x, y_j) > T$ , vote  $x$  is sunlit
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What about  $T$ ?

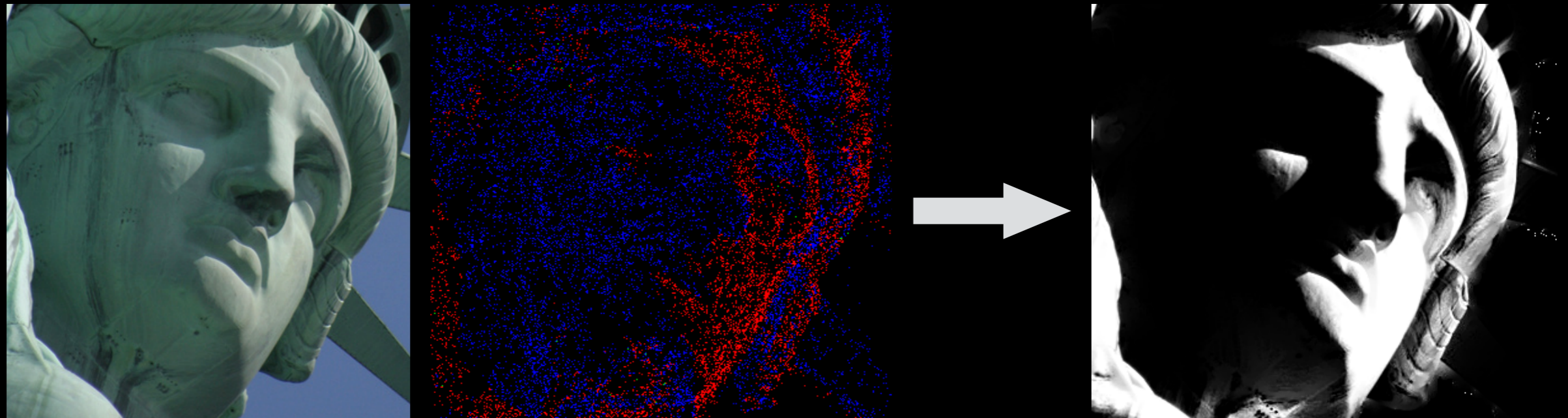
we use  $T = 3$ .

Why?

$$R(x, y) = \frac{C_x \cos(\phi_x) + f}{C_y \cos(\phi_y) + f}$$

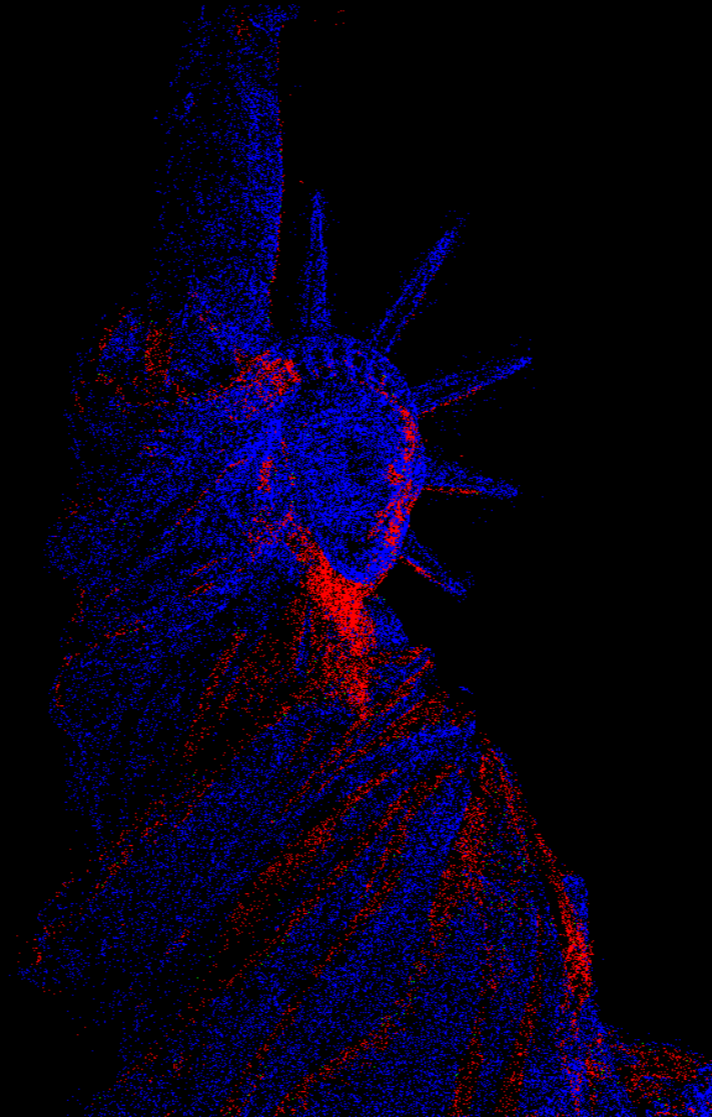
See the paper for detailed analysis.

# 3D to 2D Shadow Labels

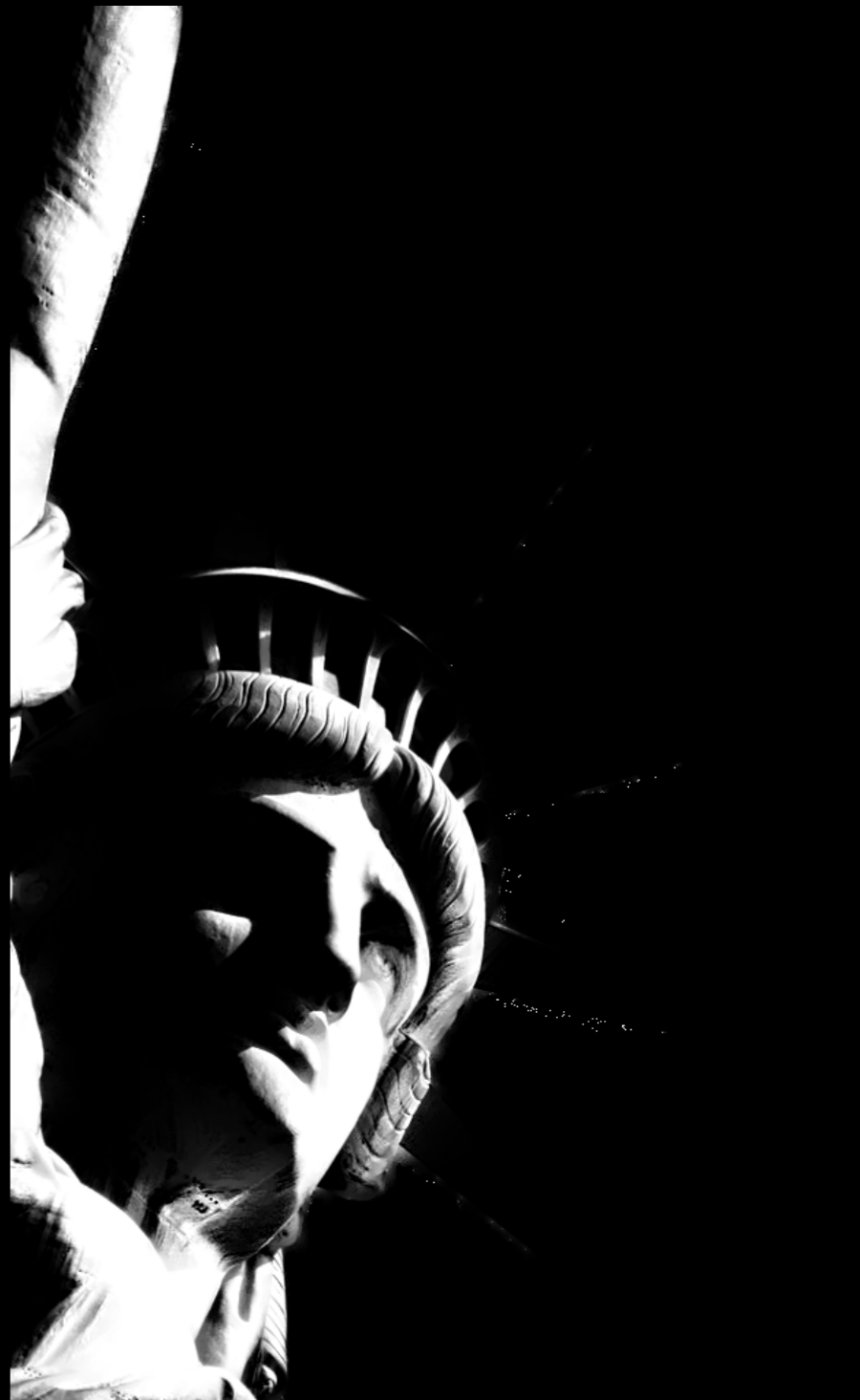
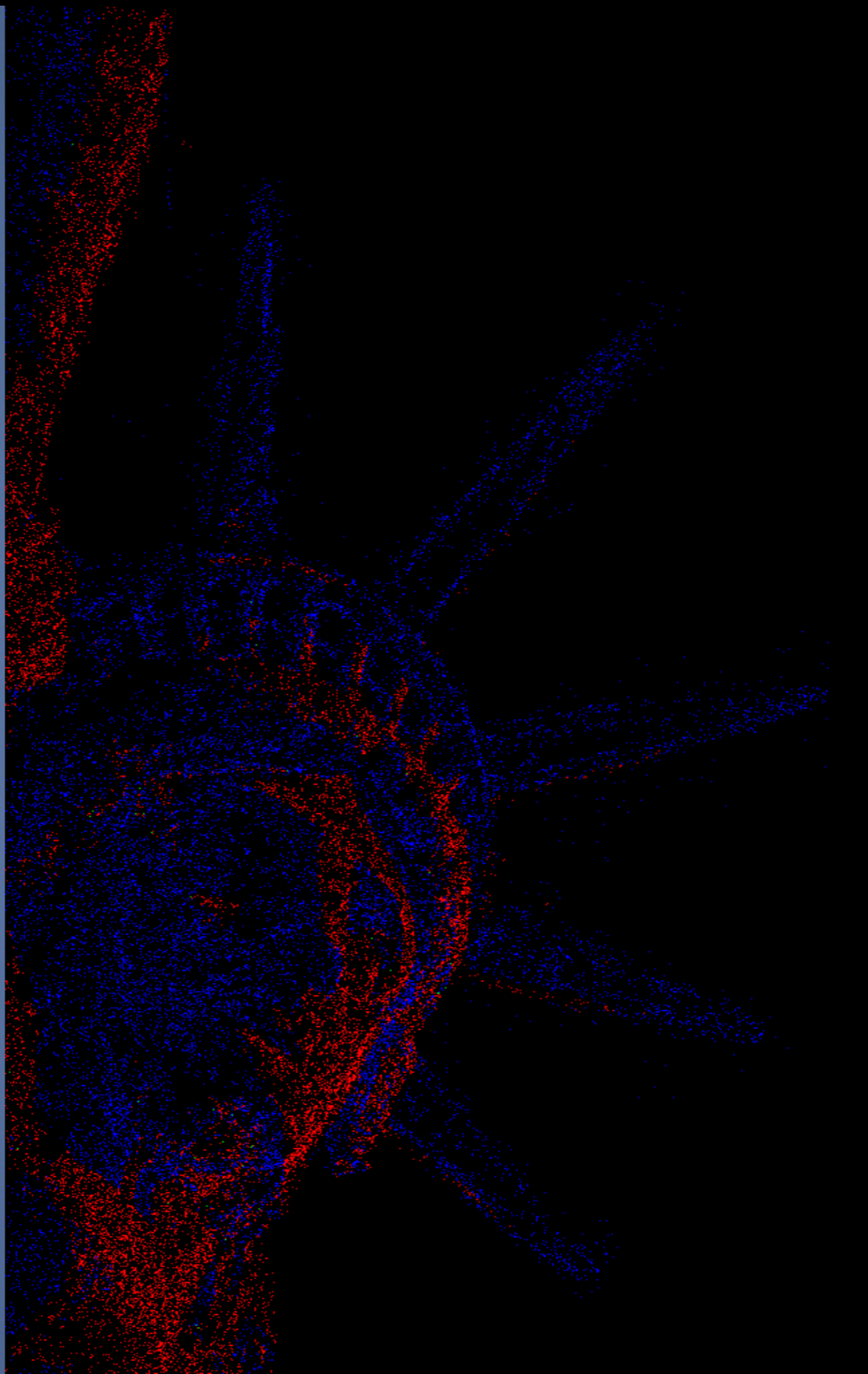


- Sparse 3D to dense 2D using the Matting Laplacian [Levin et al. 2008]
- Use high-confidence points (where vote is decisive) as constraints
- Resulting alpha map gives dense and continuous labels

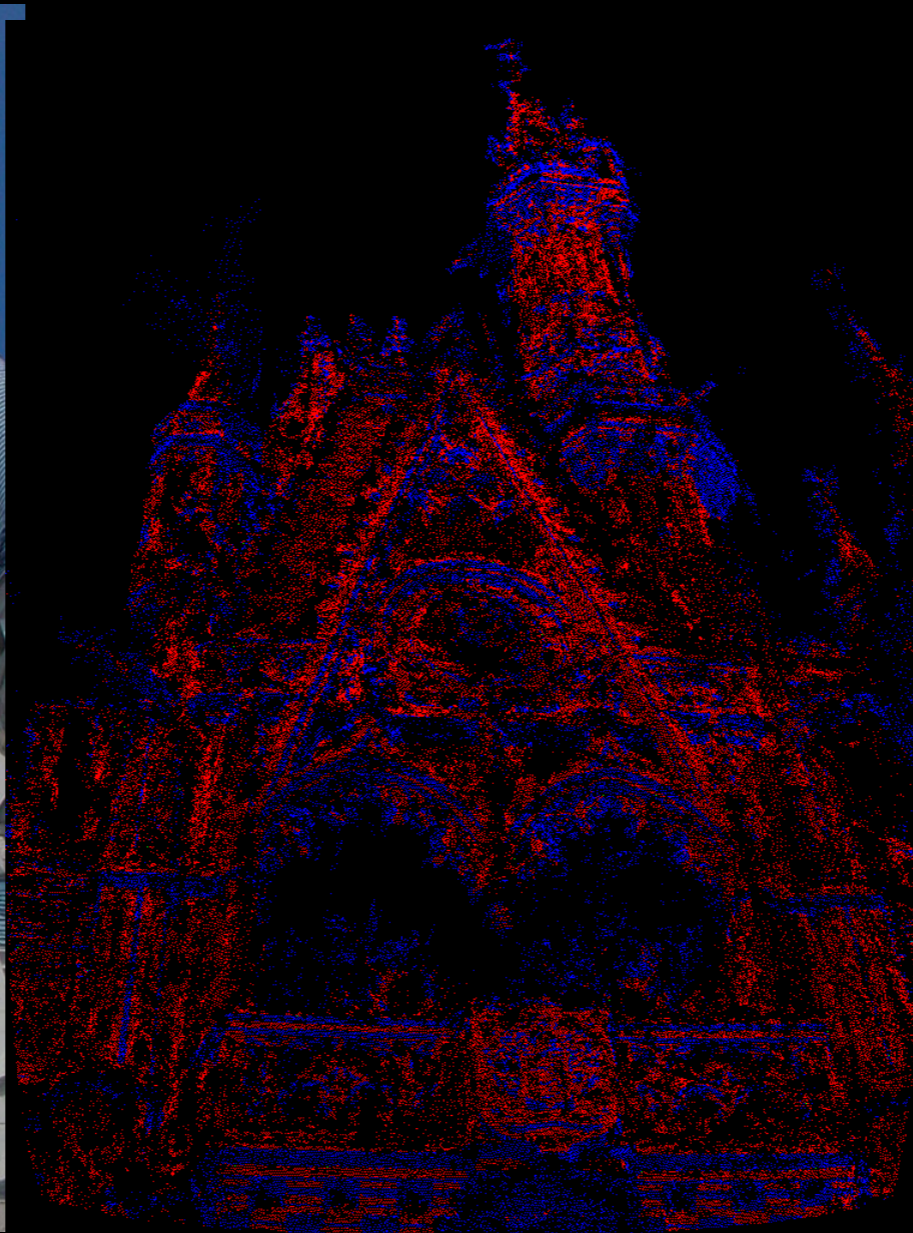
# Shadow Detection Results



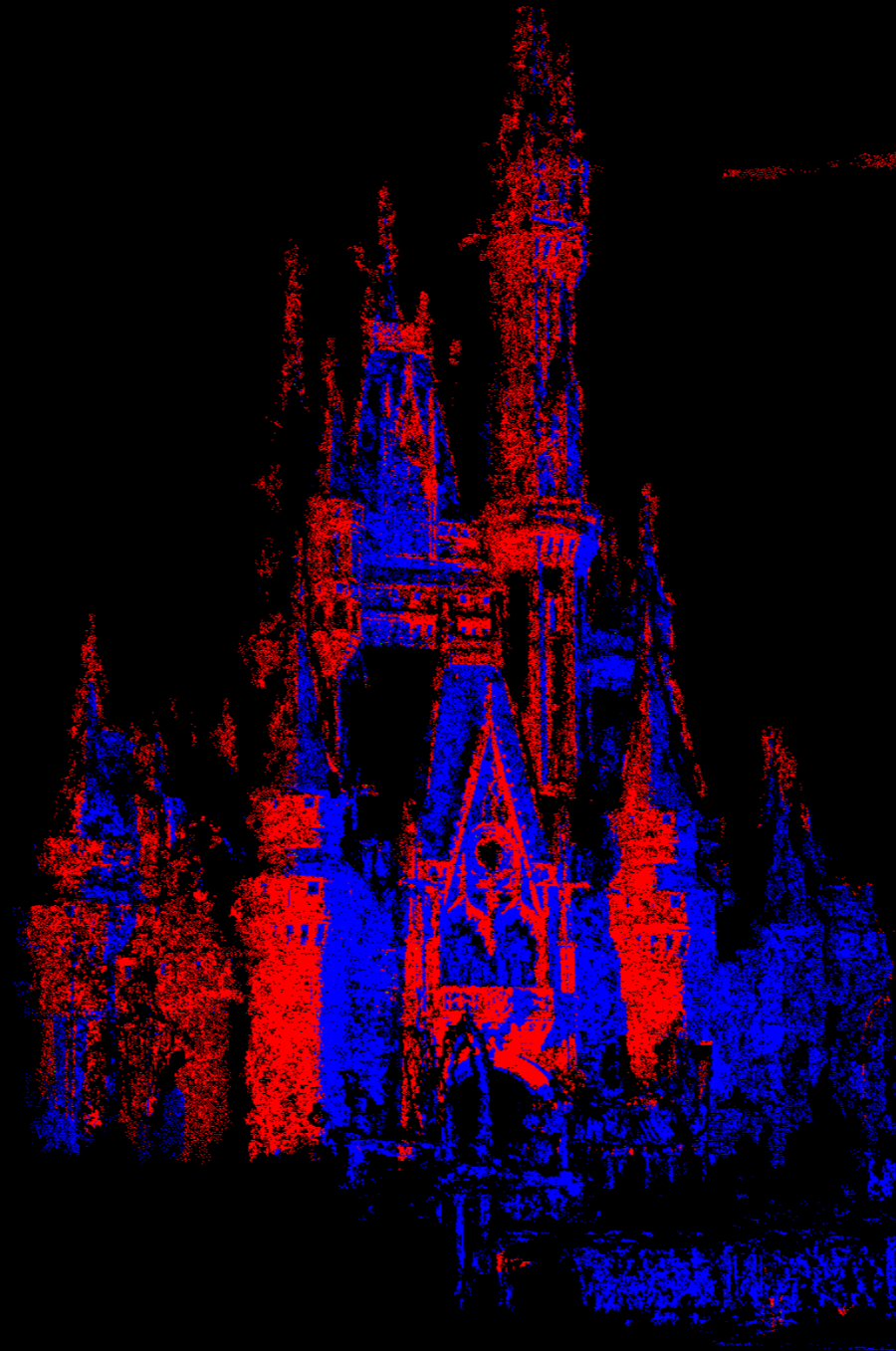
# Shadow Detection Results



# Shadow Detection Results



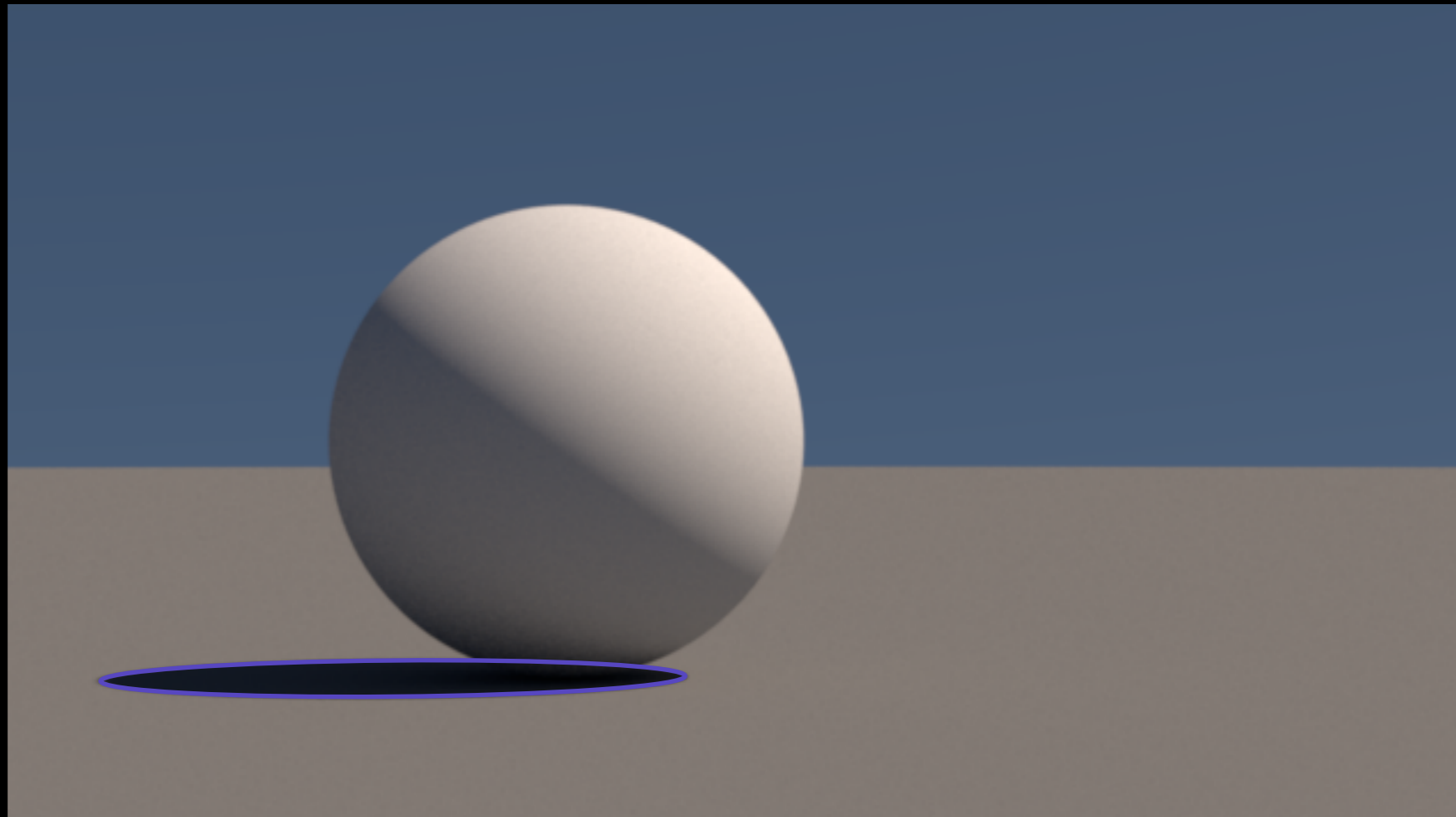
# Shadow Detection Results



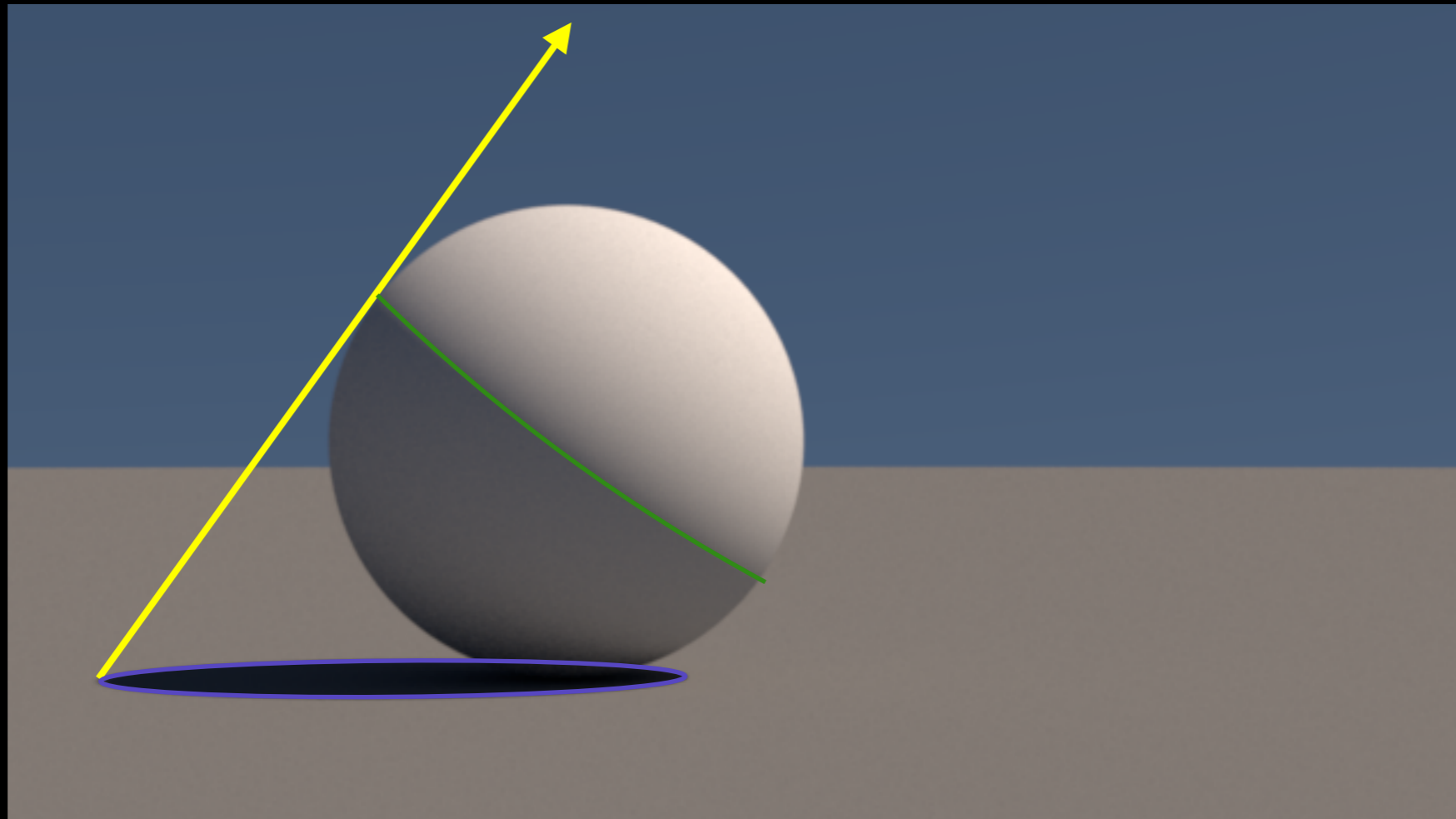
# Outline

1. Input Data
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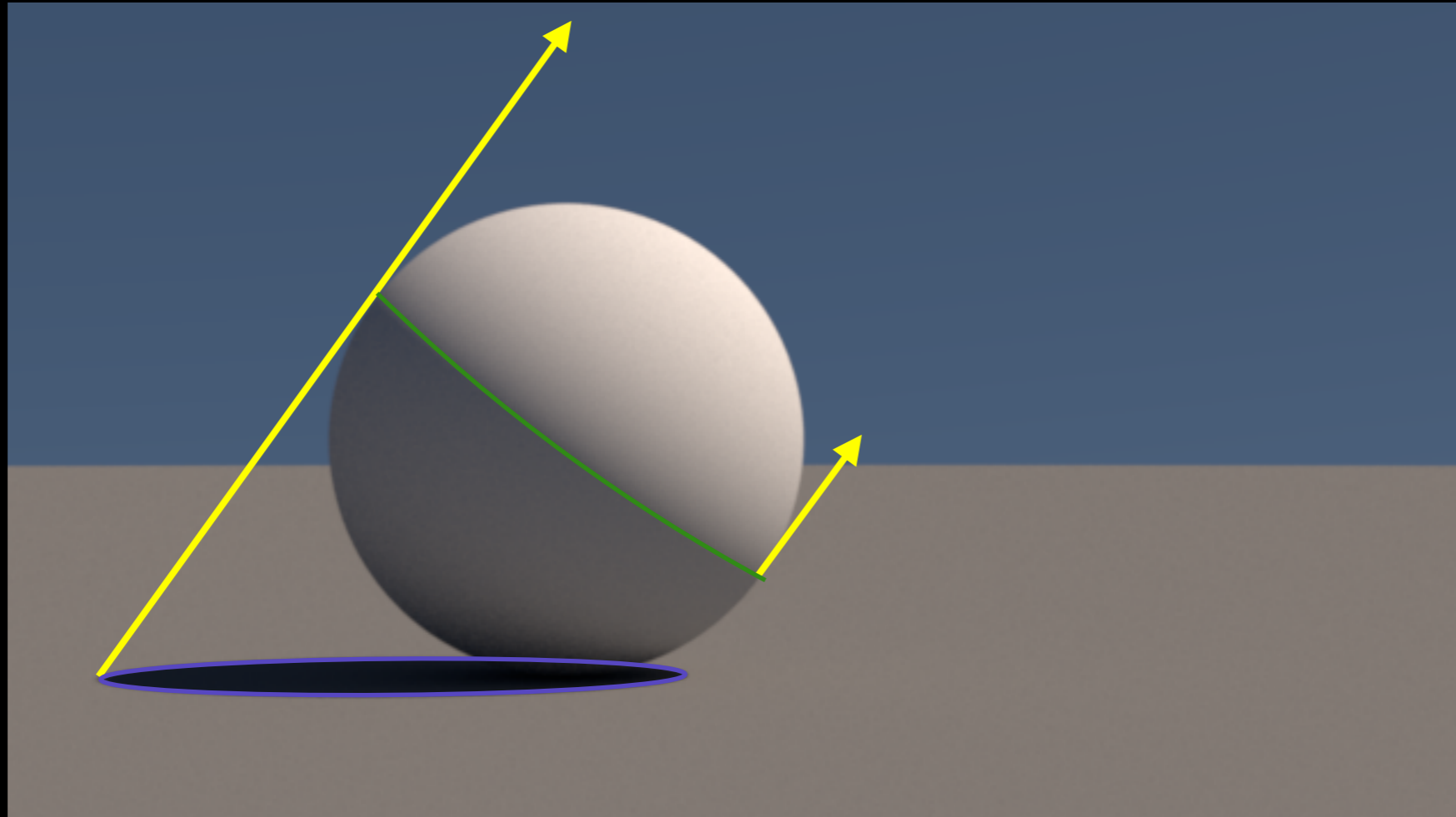
# Sun Direction from Shadows



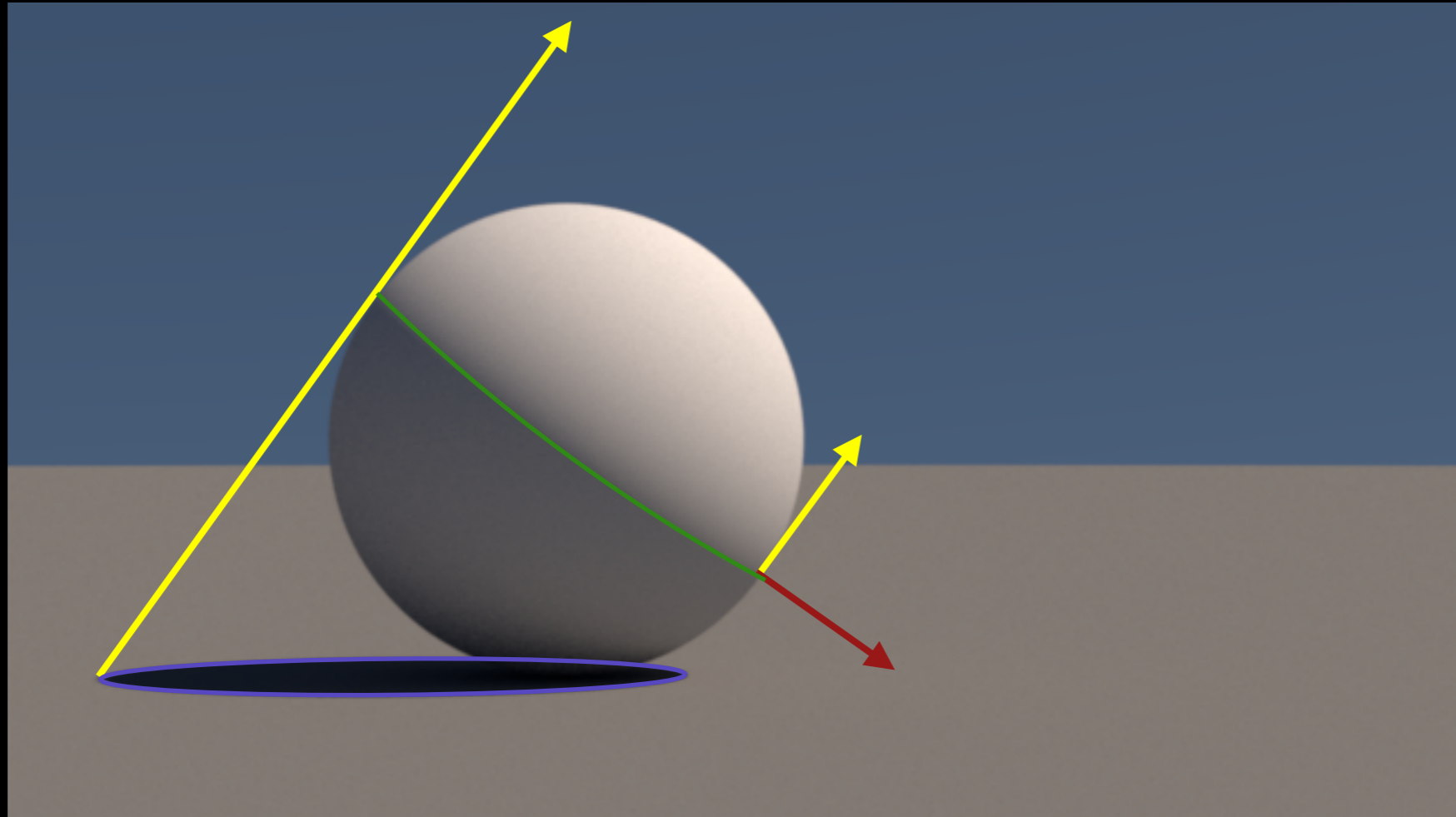
# Sun Direction from Shadows



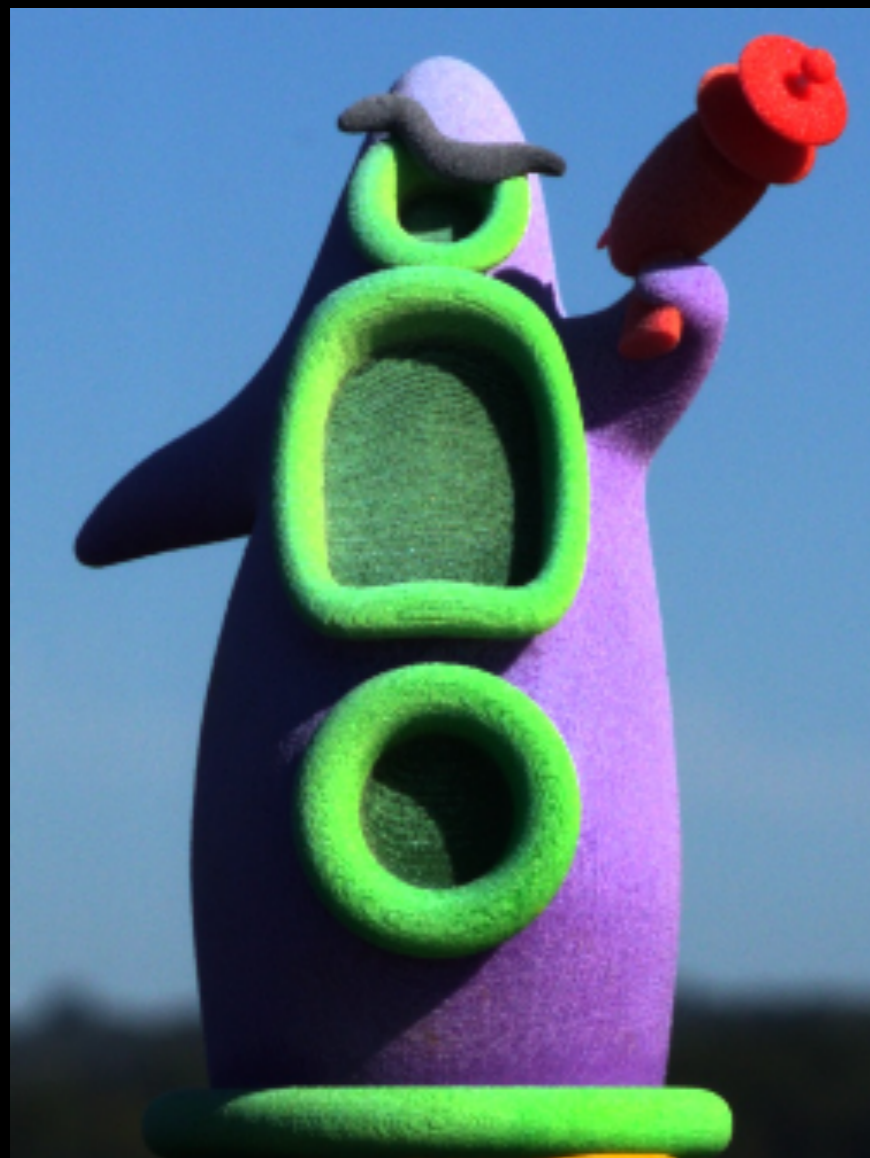
# Sun Direction from Shadows



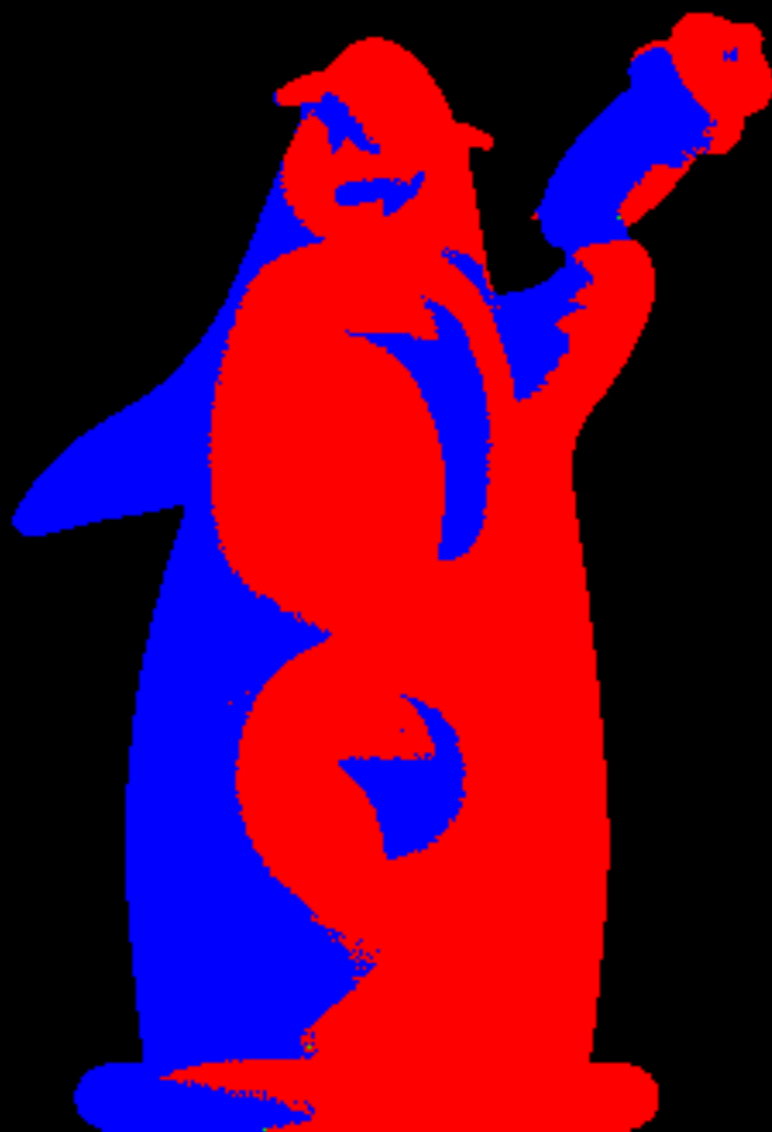
# Sun Direction from Shadows



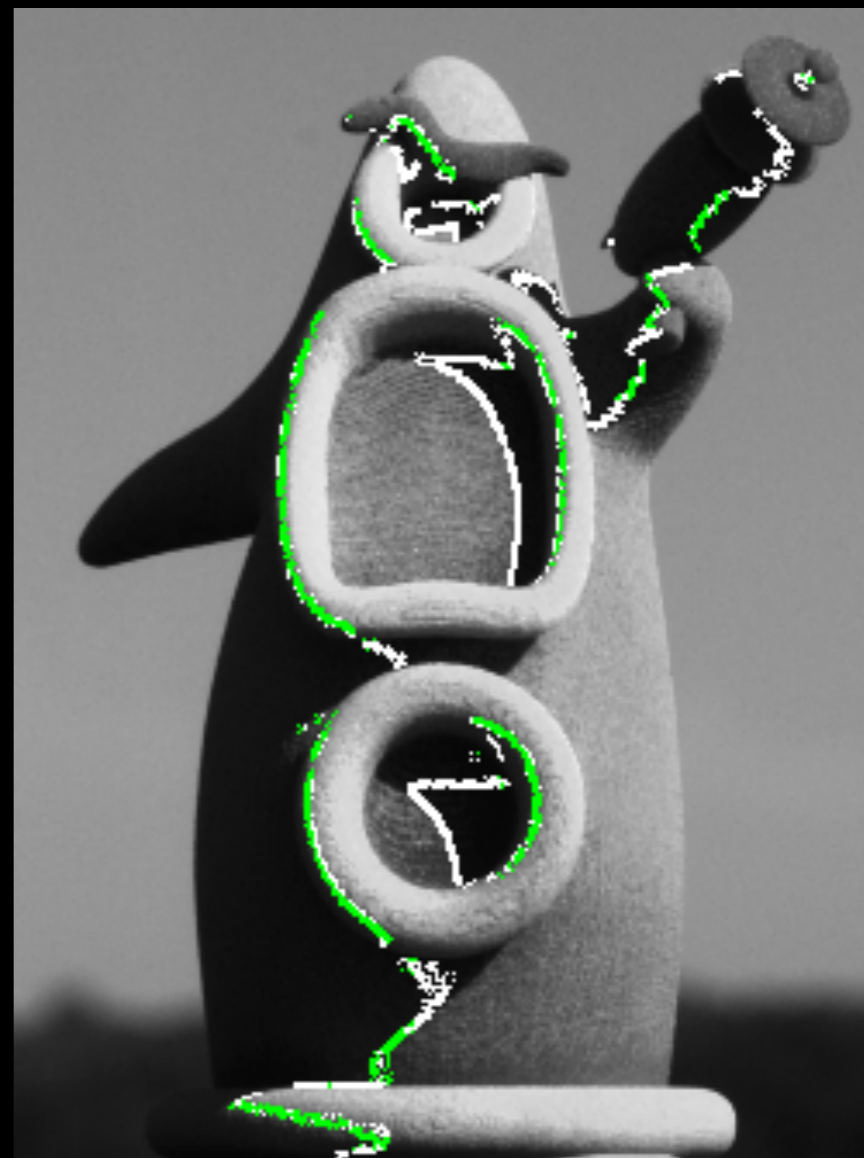
# Sun Direction: Steps



Input

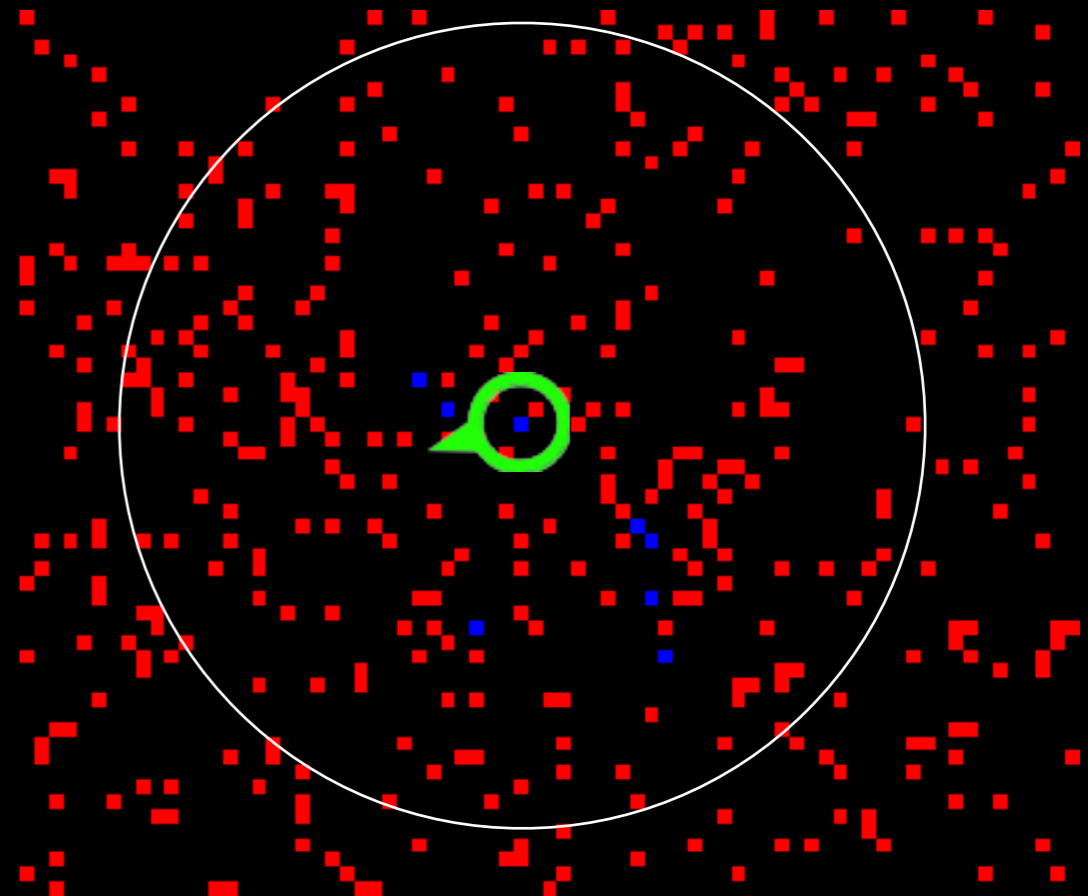
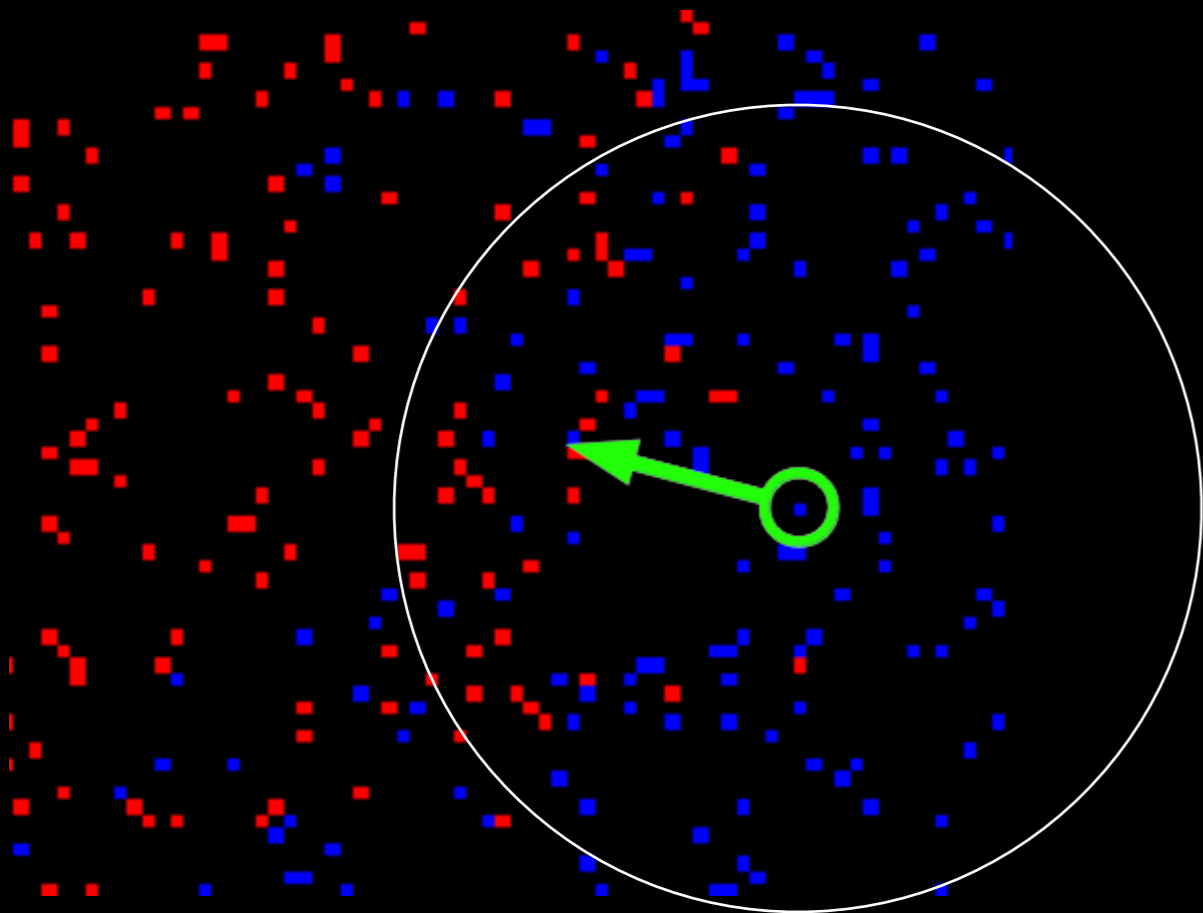


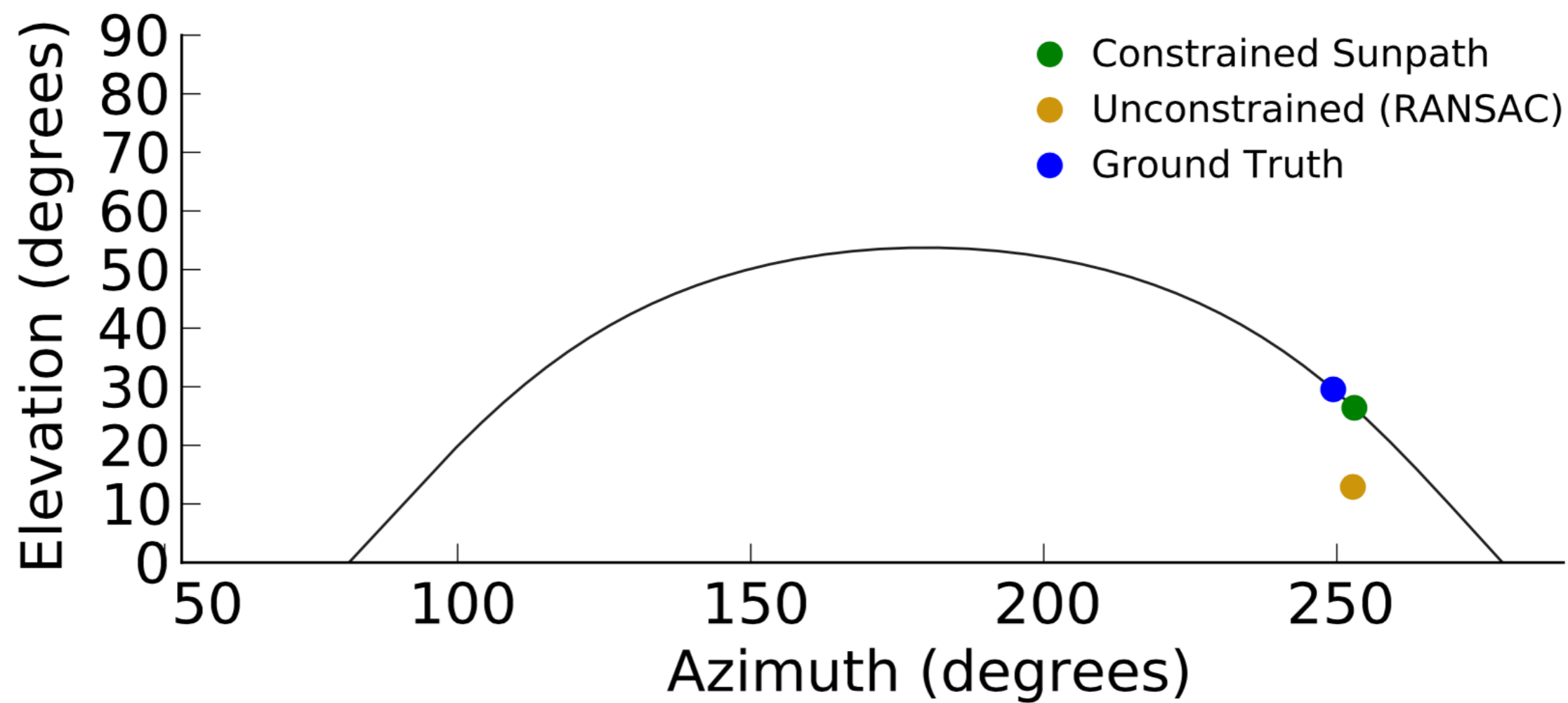
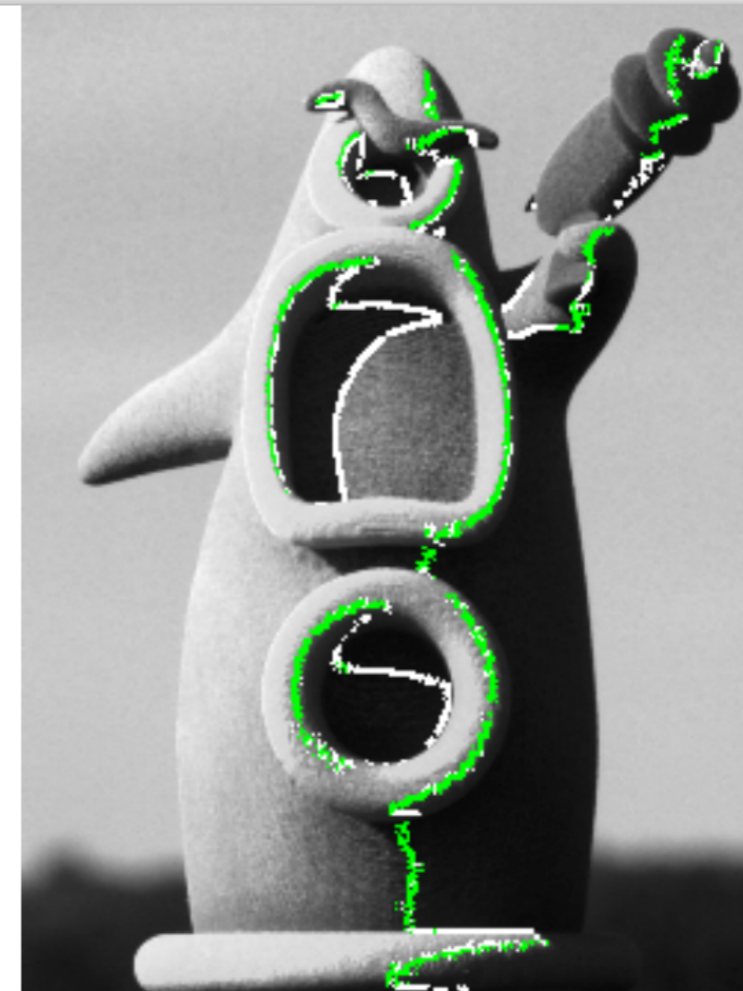
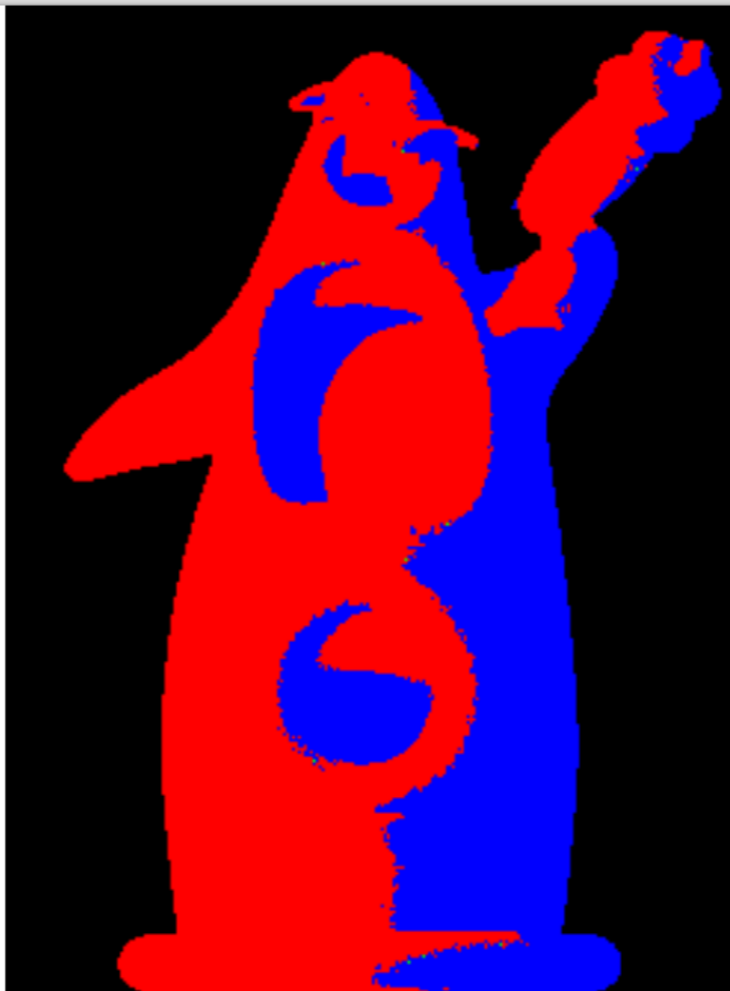
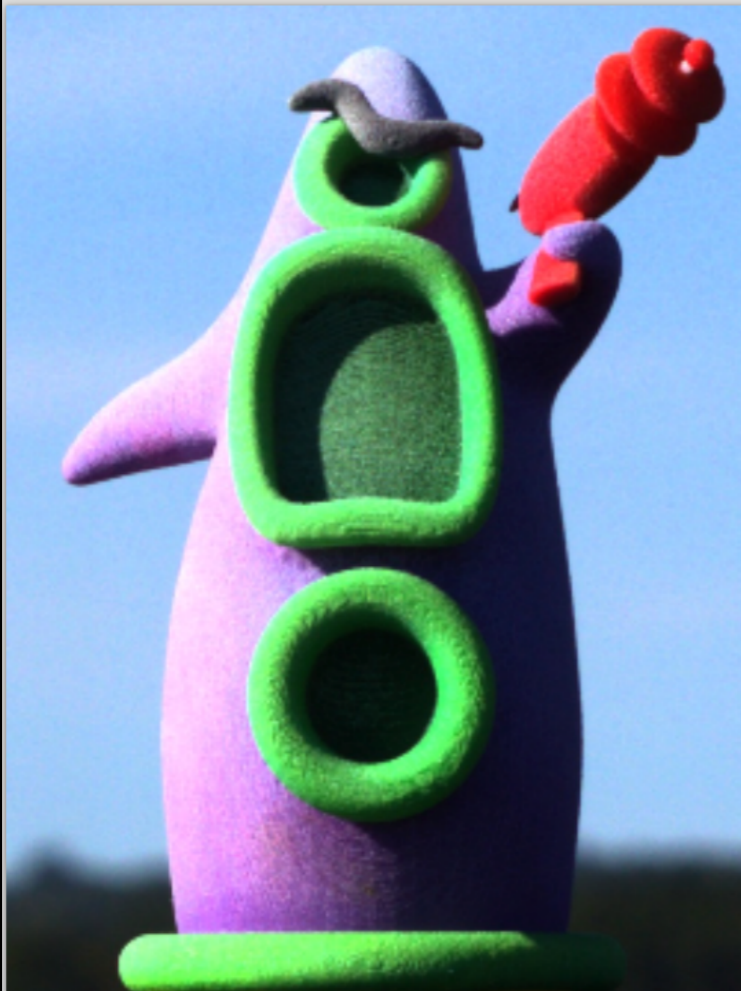
Shadow Labels



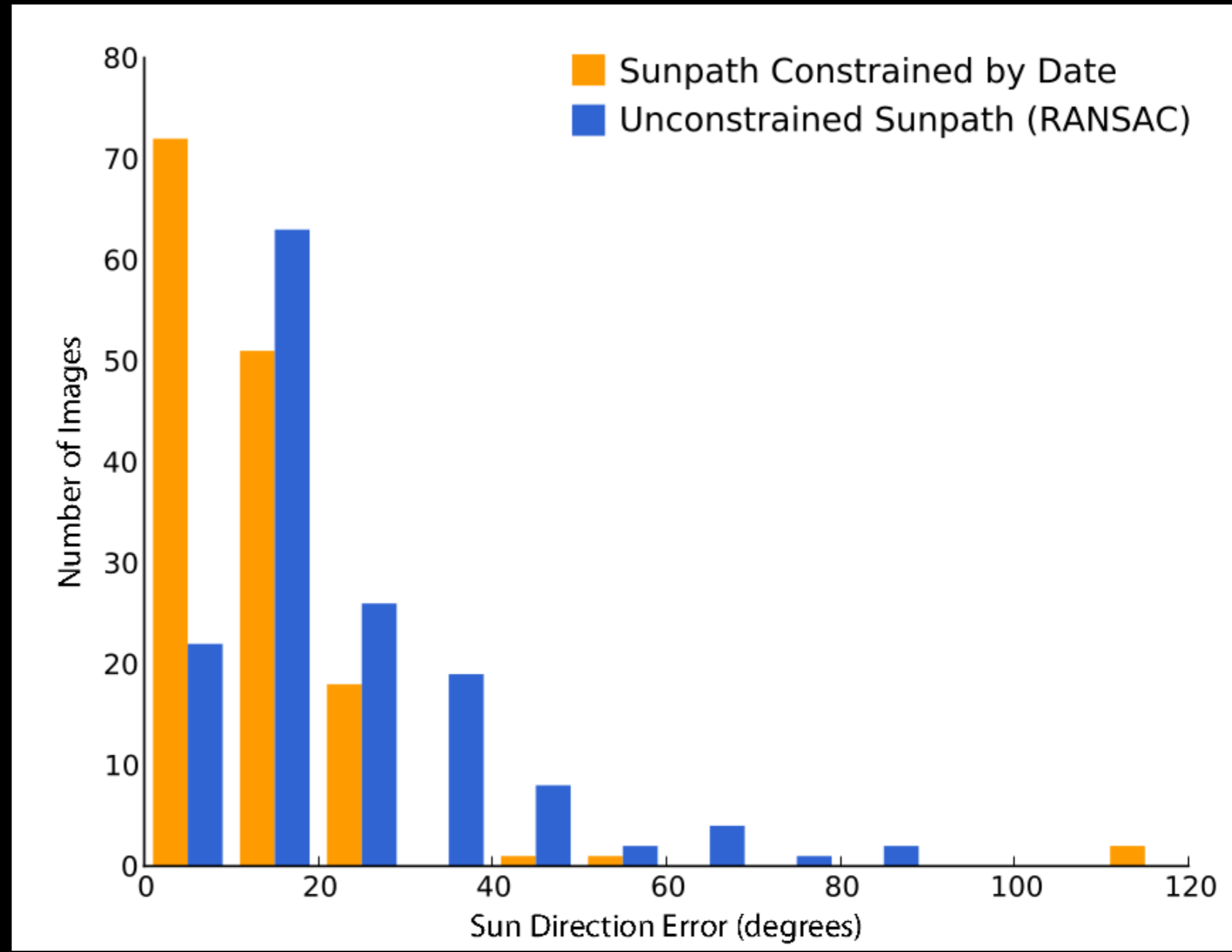
Boundary Points  
(inliers in green)

# Boundary Score





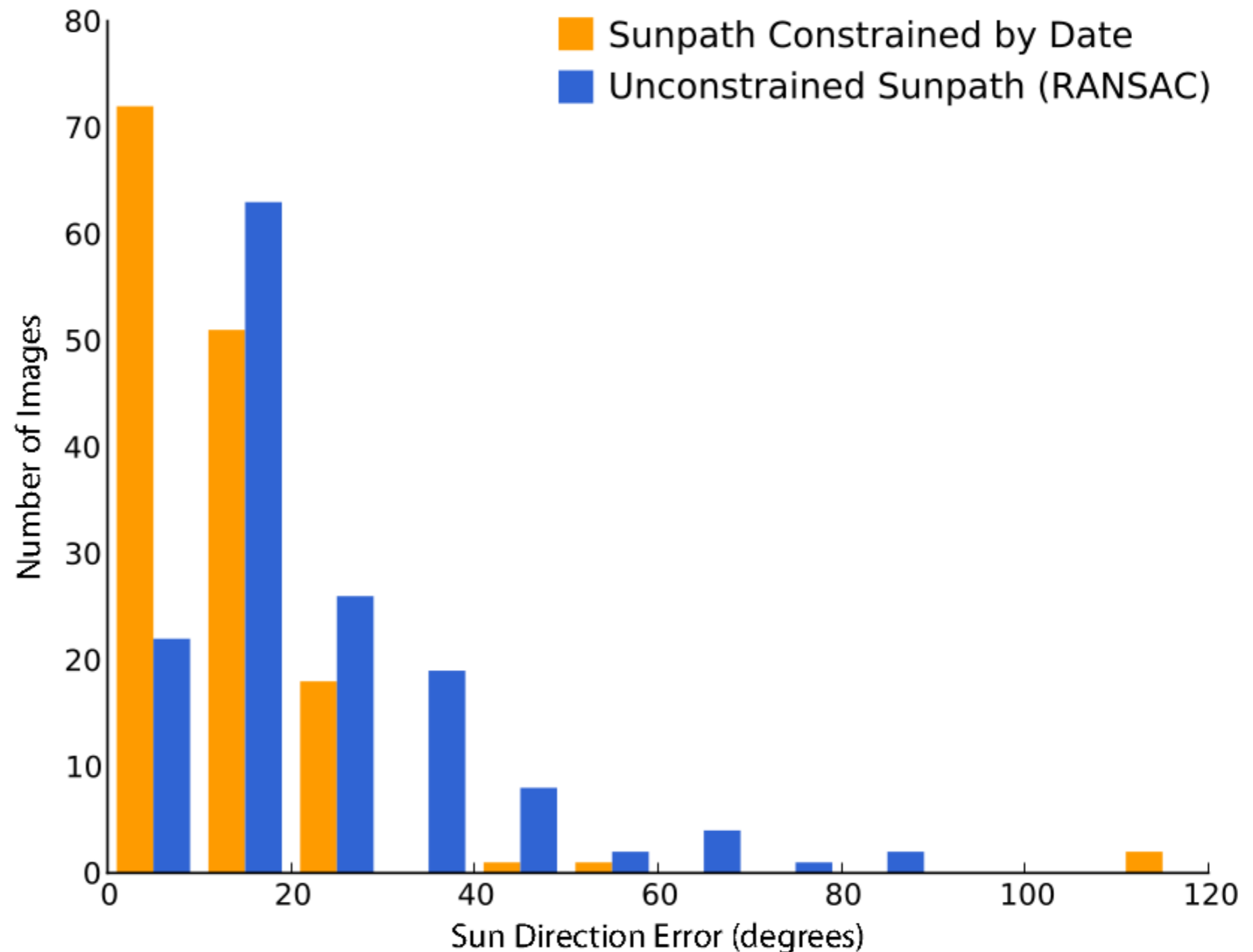
# Sun Direction Results: Tentacle



# Sun Direction Results: Tentacle

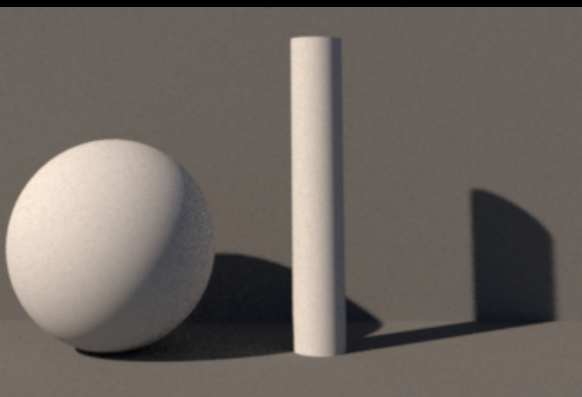
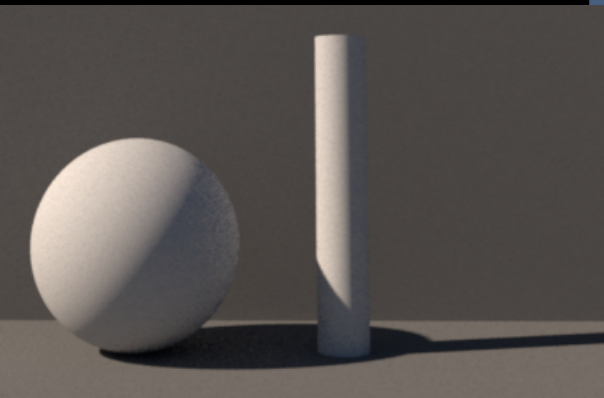
Median: 17.5°

Median: 10.9°

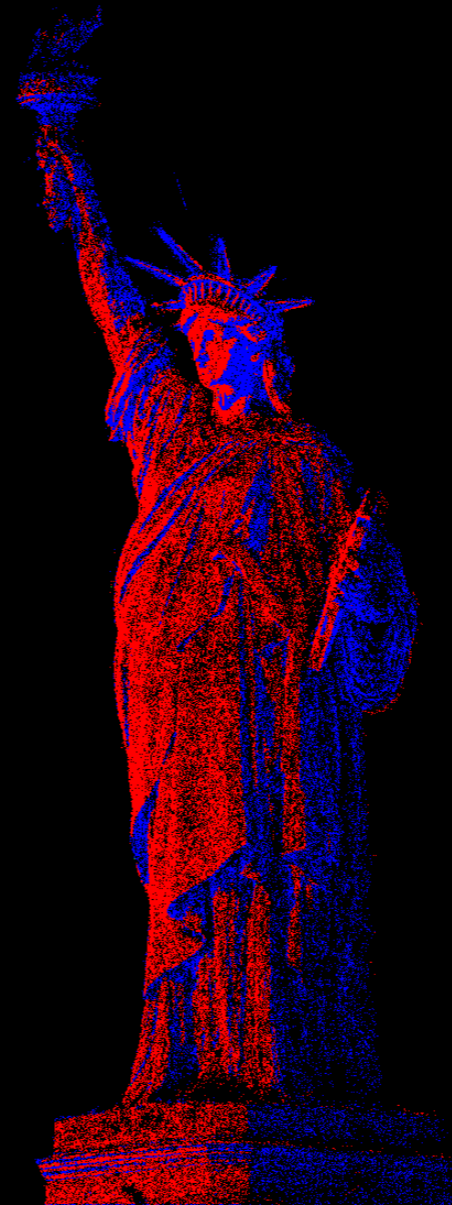


# Sun Direction Results: Statue

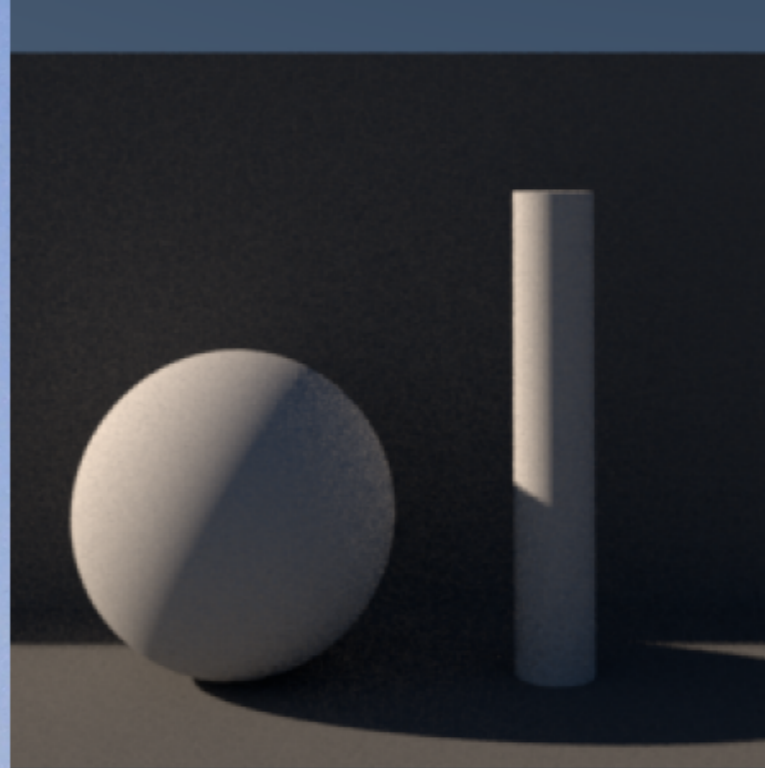
Constrained



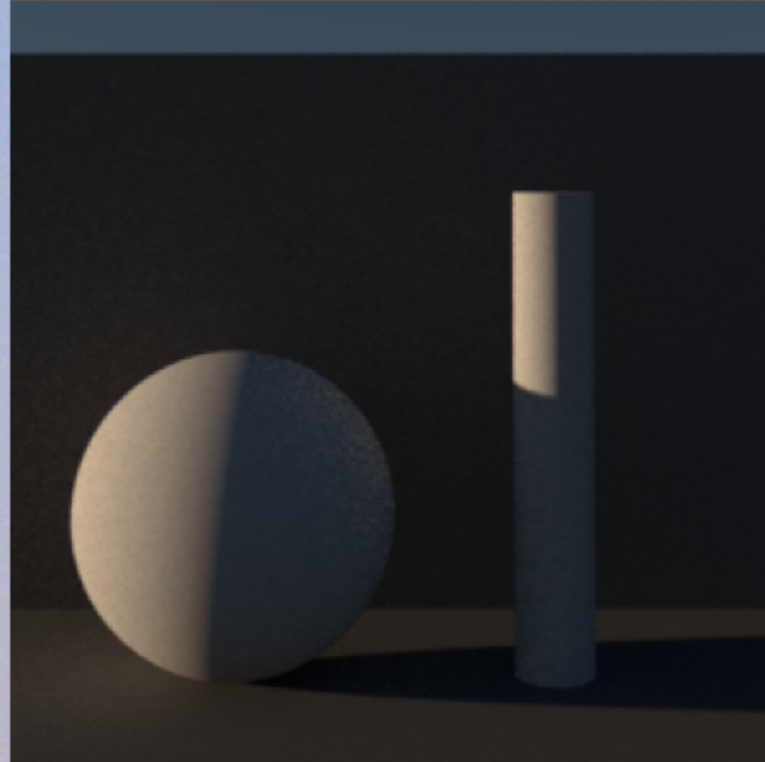
RANSAC



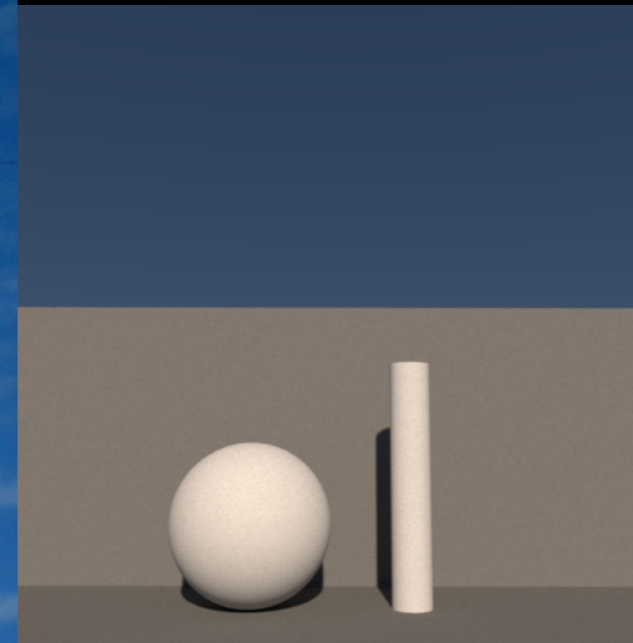




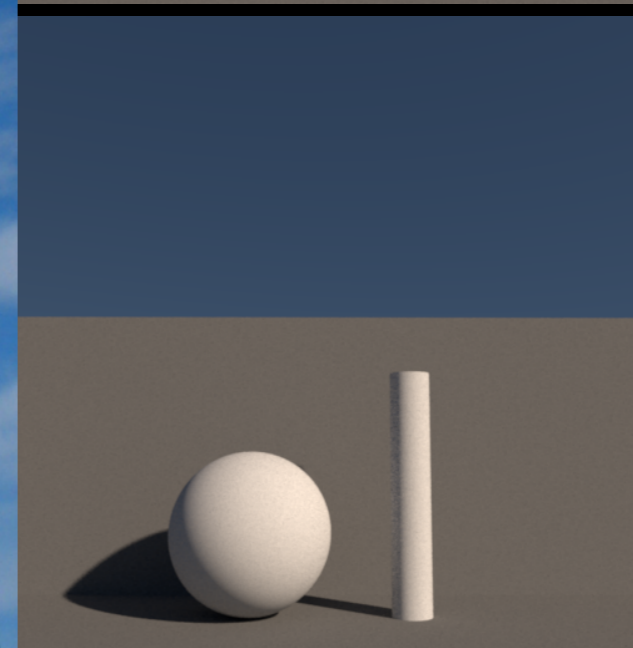
Constrained



RANSAC



Constrained



RANSAC

# Limitations

- Requires a good visibility graph
- Specular materials cause incorrect votes
- Highly occluded points are always shaded
- Threshold choice affects sharpness and location of attached shadow boundaries

# Recap

# Recap

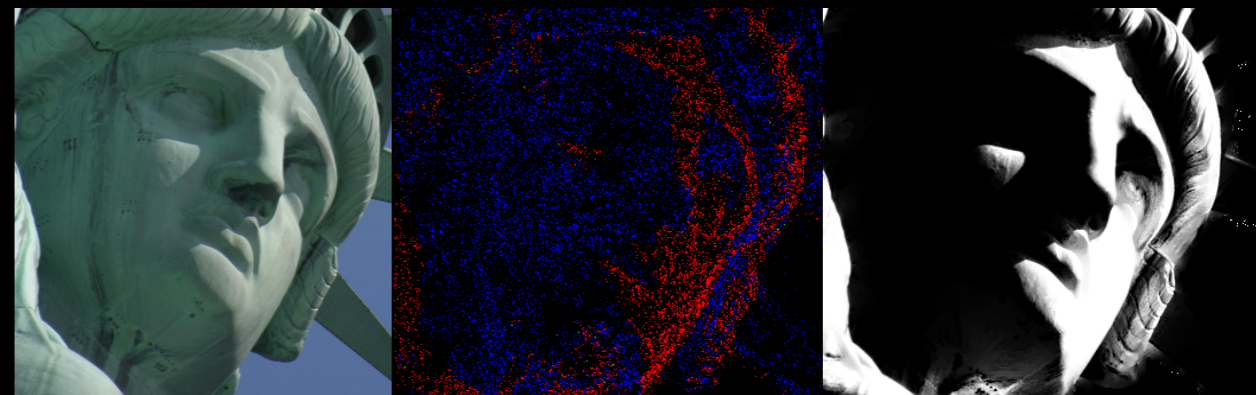
- Analysis of the Illumination Ratio

$$R(x, y) = \frac{C_x + f}{C_y + f}$$

# Recap

- Analysis of the Illumination Ratio
- Fast, efficient method for shadow detection in photo collections

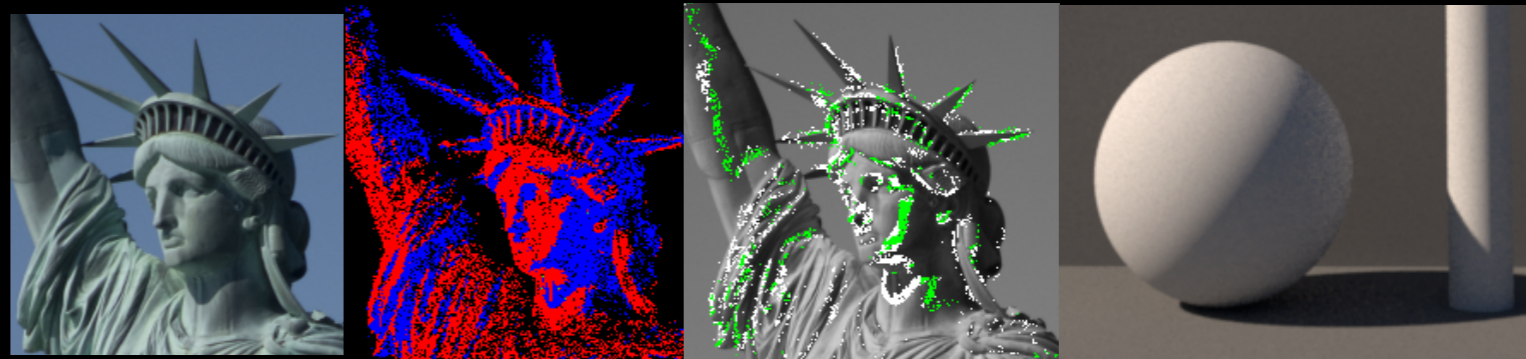
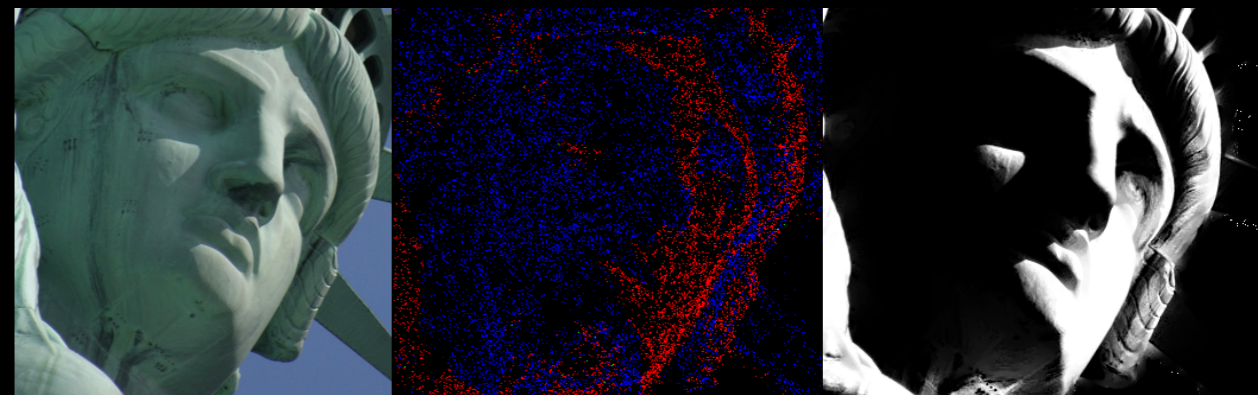
$$R(x, y) = \frac{C_x + f}{C_y + f}$$



# Recap

- Analysis of the Illumination Ratio
- Fast, efficient method for shadow detection in photo collections
- Method for estimating sun direction from shadows

$$R(x, y) = \frac{C_x + f}{C_y + f}$$



# Thank you.

<http://www.cs.cornell.edu/projects/shadows>

