This is an attempt to explain the idea of the "LP dual" interpretation that David mentioned. It will be hardly intelligible without reading the paper by Elliott, David and Jon first.

We assume that we are talking about a dynamic network and want to route packets from a source s to a sink t. We assume that the adversary has to place all of her tokens on the source s; however, she gets to choose the set of edges active in each time step, as well as capacity of each active edge. This allows the adversary the same power as placing tokens on arbitrary vertices (placing t tokens on vertex v is essentially the same as placing t tokens on s and activating the edge sv with capacity t).

Let f be the max-flow value in the graph G with capacities chosen by the adversary in a particular time step; we still need to keep the condition that the adversary can place at most f tokens on s.

Suppose at some time step the adversary has chosen the set of edges E to activate as well as the capapcities c_e of these edges. The number of tokens she is allowed to place is bounded by the optimal solution to the LP (P)

$$\begin{array}{rcl} \max & & f_{ts} \\ \sum_{e=uv} f_e + \sum_{e=vw} f_e & = & 0 \quad \forall v \in V \\ f_e & \leq & c_e \quad \forall e \in E \\ f_e \geq 0 & & \end{array}$$

(We require flow conservation at all nodes including s and t. We compensate for this by introducing an edge from t to s of infinite capacity, and maximizing the flow in this edge.)

The dual (D) of this max flow LP is

$$\begin{array}{rcl} \min & & \sum_{e \in E} y_e \\ d_v + y_e & \geq & d_u \forall e \in E, e = (uv) \\ d_s \geq 1 + d_t & & \\ y_e \geq 0 & & \end{array}$$

Since nothing changes if we add or subtract a constant from all d_v, we can postulate that $d_t = 0$. Hence we can rewrite the LP (D) as

$$\min \qquad \sum_{e \in E} y_e c_e$$

$$
\begin{aligned}
d_v + y_e &\geq d_u \forall e \in E, e = (uv) \\
d_s \geq 1 & \\
d_t &= 0 \\
y_e \geq 0 &
\end{aligned}
$$

Note that the duals y_e can be interpreted as lengths of edges, and given all y_e, without loss of generality we can put d_v to be the distance of the vertex v from t in the graph with edge lengths given by y. In this light, we want to find a distance function on edges, such that the distance from t to s is at least 1 as to minimize $\sum y_e c_e$.

The invariant David, Elliott and Jon have is something like "for every s-t cut in G, some property holds". Note that for an s-t cut $(S, V - S)$ if we define $d_v = 1$ if $v \in S$ and $d_v = 0$ otherwise, and define $y_e = 1$ for $e = uv$ if $u \in S, v \notin S$ and $y_e = 0$ otherwise, we get a feasible solution to (D). Hence each cut can be viewed as an integral solution to (D).

The proposed new invariant is, instead "for every cut something holds", let us consider "for every feasible dual solution to (D) something holds".

The invariant I proposed (instead of their invariant (3)) is: For every feasible dual solution (y, d) to (D), there is a bijection $\pi : V \rightarrow \{1, 2, \ldots, n\}$ such that

$$\sum_{v \in V} (\gamma \cdot \pi(v) - b(v)) \cdot d(v) \geq 0$$