

1 Syntax

Constant Qualifiers

$q \in \text{Quals} = \{U, R, A, L\}$



Qualifiers

$q ::= \xi \mid q$

Locations

$l \in \text{Locs}$

Expressions

$e ::= l \mid$
 $x \mid {}^{qa}\lambda x:\tau. e \mid e_1 e_2 \mid$
 ${}^{qa}\langle \rangle \mid \text{let } \langle \rangle = e_1 \text{ in } e_2 \mid {}^{qa}\langle e_1, e_2 \rangle \mid \text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2 \mid$
 ${}^{qa}\langle \rangle \mid {}^{qa}\langle e_1, e_2 \rangle \mid \text{fst } e \mid \text{snd } e \mid$
 $\text{abort } e \mid {}^{qa}\text{inl } e \mid {}^{qa}\text{inr } e \mid \text{case } e_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r \mid$
 ${}^{qa}\Lambda \xi. e \mid e[q] \mid {}^{qa}\text{pack}(q, e) \mid \text{let pack}(\xi, x) = e_1 \text{ in } e_2 \mid$
 ${}^{qa}\Lambda \bar{\alpha}. e \mid e[\bar{\tau}] \mid {}^{qa}\text{pack}(\bar{\tau}, e) \mid \text{let pack}(\bar{\alpha}, x) = e_1 \text{ in } e_2 \mid$
 ${}^{qa}\Lambda \alpha. e \mid e[\tau] \mid {}^{qa}\text{pack}(\tau, e) \mid \text{let pack}(\alpha, x) = e_1 \text{ in } e_2 \mid$
 ${}^{qa}\text{fold } e \mid \text{unfold } e \mid$
 ${}^{qa}\text{new } e \mid \text{free } e \mid \text{read } e \mid \text{swap } e_1 e_2$

Pointers

$p \in \text{Ptrs}$

Values

$v ::=$
 $\lambda x:\tau. e \mid$
 $\langle \rangle \mid \langle l_1, l_2 \rangle \mid$
 $\langle \rangle \mid \langle e_1, e_2 \rangle \mid$
 $\text{inl } l \mid \text{inr } l \mid$
 $\Lambda \xi. e \mid \text{pack}(q, e) \mid \Lambda \bar{\alpha}. e \mid \text{pack}(\bar{\tau}, e) \mid \Lambda \alpha. e \mid \text{pack}(\tau, e) \mid$
 $\text{fold } l$
 $\text{ptr } p$

PreTypes

$\bar{\tau} ::=$
 $\bar{\alpha} \mid$
 $\tau_1 \multimap \tau_2 \mid$
 $\mathbf{1}_{\otimes} \mid \tau_1 \otimes \tau_2 \mid$
 $\mathbf{1}_{\oplus} \mid \tau_1 \oplus \tau_2 \mid$
 $\mathbf{0} \mid \tau_1 \oplus \tau_2 \mid$
 $\forall \xi. \tau \mid \exists \xi. \tau \mid \forall \bar{\alpha}. \tau \mid \exists \bar{\alpha}. \tau \mid \forall \alpha. \tau \mid \exists \alpha. \tau \mid$
 $\mu \bar{\alpha}. \tau \mid$
 $\text{ref } \tau$

Types

$\tau ::= \alpha \mid {}^q \bar{\tau}$

Qualifier/PreType/Type Contexts

$\Delta ::= \bullet \mid \Delta, \xi \mid \Delta, \bar{\alpha} \mid \Delta, \alpha$

Value Contexts

$\Gamma ::= \bullet \mid \Gamma, x:\tau$

Flags

$f \in \{\text{unused}, \text{used}\}$

Stores

$\sigma ::= \bullet \mid \sigma, l \mapsto (q, v, f)$

Store Typings

$\Sigma ::= \bullet \mid \Sigma, l \mapsto \tau$

Heaps

$\psi ::= \bullet \mid \psi, p \mapsto (q, l)$

Heap Typings

$\Psi ::= \bullet \mid \Psi, p \mapsto (q, \tau)$

2 Dynamic Semantics

2.1 $(\sigma; \mathbf{q}; v) \xrightarrow{\text{alloc}} (\sigma'; l')$

$$\frac{l \notin \text{dom}(\sigma)}{(\sigma; \mathbf{q}; v) \xrightarrow{\text{alloc}} (\sigma, l \mapsto (\mathbf{q}, v, \text{unused}); l)}$$

2.2 $(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}'; v')$

$$\frac{\mathbf{q} \sqsubseteq \mathbf{R}}{(\sigma, l \mapsto (\mathbf{q}, v, f); l) \xrightarrow{\text{fetch}} (\sigma, l \mapsto (\mathbf{q}, v, \text{used}); \mathbf{q}; v)} \quad \frac{\mathbf{A} \sqsubseteq \mathbf{q}}{(\sigma, l \mapsto (\mathbf{q}, v, i); l) \xrightarrow{\text{fetch}} (\sigma; \mathbf{q}; v)}$$

$$\frac{l \neq l' \quad (\sigma_1; l) \xrightarrow{\text{fetch}} (\sigma_2; v)}{(\sigma_1, l' \mapsto (\mathbf{q}', v', f'); l) \xrightarrow{\text{fetch}} (\sigma_2, l' \mapsto (\mathbf{q}', v', f'); v)}$$

2.3 $(\psi; \mathbf{q}; l) \xrightarrow{\text{new}} (\psi'; p')$

$$\frac{p \notin \text{dom}(\psi)}{(\psi; \mathbf{q}; l) \xrightarrow{\text{new}} (\psi, p \mapsto (\mathbf{q}, l); p)}$$

2.4 $(\psi; p) \xrightarrow{\text{free}} (\psi'; l')$

$$\frac{}{(\psi, p \mapsto (\mathbf{q}, l); p) \xrightarrow{\text{free}} (\psi; l)} \quad \frac{p \neq p' \quad (\psi_1; p) \xrightarrow{\text{free}} (\psi_2; l)}{(\psi_1, p' \mapsto (\mathbf{q}', l'); p) \xrightarrow{\text{free}} (\psi_2, p' \mapsto (\mathbf{q}', l'); l)}$$

2.5 $(\psi; p) \xrightarrow{\text{read}} l'$

$$\frac{}{(\psi, p \mapsto (\mathbf{q}, l); p) \xrightarrow{\text{read}} l} \quad \frac{p \neq p' \quad (\psi_1; p) \xrightarrow{\text{read}} l}{(\psi_1, p' \mapsto (\mathbf{q}', l'); p) \xrightarrow{\text{read}} l}$$

2.6 $(\psi; p; l_\star) \xrightarrow{\text{swap}} (\psi'; l')$

$$\frac{}{(\psi, p \mapsto (\mathbf{q}, l); p; l_\star) \xrightarrow{\text{swap}} (\psi, p \mapsto (\mathbf{q}, l_\star); l)} \quad \frac{p \neq p' \quad (\psi_1; p; l_\star) \xrightarrow{\text{swap}} (\psi_2; l)}{(\psi_1, p' \mapsto (\mathbf{q}', l'); p; l_\star) \xrightarrow{\text{swap}} (\psi_2, p' \mapsto (\mathbf{q}', l'); l)}$$

2.7 $(\sigma, \psi, e) \mapsto (\sigma', \psi', e')$

$$\frac{(\sigma; \mathbf{q}_a; \lambda x:\tau. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \lambda x:\tau. e) \mapsto (\sigma', \psi, l')}$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi', e_1 e_2) \mapsto (\sigma', \psi', e'_1 e_2)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, l_1 e_2) \mapsto (\sigma', \psi', l_1 e'_2)} \quad \frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \lambda x:\tau. e)}{(\sigma, \psi, l_1 l_2) \mapsto (\sigma', \psi, e[l_2/x])}$$

$$\begin{array}{c}
\frac{(\sigma; \mathbf{q}_a; \langle \rangle) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \rangle) \mapsto (\sigma', \psi', l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let } \langle \rangle = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let } \langle \rangle = e'_1 \text{ in } e_2)} \quad \frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle \rangle)}{(\sigma, \psi, \text{let } \langle \rangle = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2)} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle e_1, e_2 \rangle) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \langle e'_1, e_2 \rangle)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle l_1, e_2 \rangle) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \langle l_1, e'_2 \rangle)} \quad \frac{(\sigma; \mathbf{q}_a; \langle l_1, l_2 \rangle) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle l_1, l_2 \rangle) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let } \langle x, y \rangle = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let } \langle x, y \rangle = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle l_x, l_y \rangle)}{(\sigma, \psi, \text{let } \langle x, y \rangle = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[l_x/x][l_y/y])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \langle \rangle) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \rangle) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma; \mathbf{q}_a; \langle e_1, e_2 \rangle) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle e_1, e_2 \rangle) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \text{fst } e) \mapsto (\sigma', \psi', \text{fst } e')} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle e_1, e_2 \rangle)}{(\sigma, \psi, \text{fst } l) \mapsto (\sigma', \psi, e_1)} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \text{snd } e) \mapsto (\sigma', \psi', \text{snd } e')} \\
\\
\frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle e_1, e_2 \rangle)}{(\sigma, \psi, \text{snd } l) \mapsto (\sigma', \psi, e_2)} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \text{abort } e) \mapsto (\sigma', \psi', \text{abort } e')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{inl } e_1) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{inl } e'_1)} \quad \frac{(\sigma; \mathbf{q}_a; \text{inl } l_1) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{inl } l_1) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{inr } e_2) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{inr } e'_2)} \quad \frac{(\sigma; \mathbf{q}_a; \text{inr } l_2) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{inr } l_2) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{case } e_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r) \mapsto (\sigma', \psi', \text{case } e'_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{inl } l_x)}{(\sigma, \psi, \text{case } l_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r) \mapsto (\sigma', \psi, e_l[l_x/x])} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{inr } l_y)}{(\sigma, \psi, \text{case } l_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r) \mapsto (\sigma', \psi, e_r[l_y/y])}
\end{array}$$

$$\begin{array}{c}
\frac{(\sigma; \mathbf{q}_a; \Lambda \xi. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \xi. e) \mapsto (\sigma', \psi, l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e[q]) \mapsto (\sigma', \psi', e'[q])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \xi. e)}{(\sigma, \psi, l[q]) \mapsto (\sigma', \psi, e[q/\xi])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(q, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(q, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(q, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(q, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\xi, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\xi, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(q_1, l_1))}{(\sigma, \psi, \text{let pack}(\xi, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[q_1/\xi][l_1/x])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \Lambda \bar{\alpha}. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \bar{\alpha}. e) \mapsto (\sigma', \psi', l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e[\bar{\tau}]) \mapsto (\sigma', \psi', e'[\bar{\tau}])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \bar{\alpha}. e)}{(\sigma, \psi, l[\bar{\tau}]) \mapsto (\sigma', \psi, e[\bar{\tau}/\bar{\alpha}])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(\bar{\tau}, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\bar{\alpha}, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\bar{\alpha}, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(\bar{\tau}_1, l_1))}{(\sigma, \psi, \text{let pack}(\bar{\alpha}, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[\bar{\tau}_1/\bar{\alpha}][l_1/x])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \Lambda \alpha. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \alpha. e) \mapsto (\sigma', \psi, l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e[\tau]) \mapsto (\sigma', \psi', e'[\tau])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \alpha. e)}{(\sigma, \psi, l[\tau]) \mapsto (\sigma', \psi, e[\tau/\alpha])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\tau, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(\tau, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(\tau, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\tau, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\alpha, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\alpha, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(\tau_1, l_1))}{(\sigma, \psi, \text{let pack}(\alpha, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[\tau_1/\alpha][l_1/x])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{fold } e) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{fold } e')} \quad \frac{(\sigma; \mathbf{q}_a; \text{fold } l) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{fold } l) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \text{unfold } e) \mapsto (\sigma', \psi', \text{unfold } e')} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{fold } l')}{(\sigma, \psi, \text{unfold } l) \mapsto (\sigma', \psi', l')}
\end{array}$$

$$\begin{array}{c}
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{qa}\mathbf{new} e) \mapsto (\sigma', \psi', {}^{qa}\mathbf{new} e')} \qquad \frac{(\psi; \mathbf{q}_a; l) \xrightarrow{\mathbf{new}} (\psi'; p') \quad (\sigma; \mathbf{q}_a; \mathbf{ptr} p') \xrightarrow{\mathbf{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{qa}\mathbf{new} l) \mapsto (\sigma', \psi', l')} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \mathbf{free} e) \mapsto (\sigma', \psi', \mathbf{free} e')} \qquad \frac{(\sigma; l) \xrightarrow{\mathbf{fetch}} (\sigma'; \mathbf{q}_a; \mathbf{ptr} p') \quad (\psi; p') \xrightarrow{\mathbf{free}} (\psi'; l_{p'})}{(\sigma, \psi, \mathbf{free} l) \mapsto (\sigma', \psi', l_{p'})} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \mathbf{read} e) \mapsto (\sigma', \psi', \mathbf{read} e')} \qquad \frac{(\sigma; l) \xrightarrow{\mathbf{fetch}} (\sigma'; \mathbf{q}_a; \mathbf{ptr} p') \quad (\psi; p') \xrightarrow{\mathbf{read}} l_{p'} \quad (\sigma'; \mathbf{q}_a; \mathbf{ptr} p') \xrightarrow{\mathbf{alloc}} (\sigma''; l'') \quad (\sigma''; \mathbf{L}; \langle l'', l_{p'} \rangle) \xrightarrow{\mathbf{alloc}} (\sigma'''; l''')}{(\sigma, \psi, \mathbf{read} l) \mapsto (\sigma''', \psi, l''')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{swap} e_1 e_2) \mapsto (\sigma', \psi', \mathbf{swap} e'_1 e_2)} \qquad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{swap} l_1 e_2) \mapsto (\sigma', \psi', \mathbf{swap} l_1 e'_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\mathbf{fetch}} (\sigma'; \mathbf{q}_a; \mathbf{ptr} p') \quad (\psi; p'; l_2) \xrightarrow{\mathbf{swap}} (\psi'; l_{p'}) \quad (\sigma'; \mathbf{q}_a; \mathbf{ptr} p') \xrightarrow{\mathbf{alloc}} (\sigma''; l'') \quad (\sigma''; \mathbf{L}; \langle l'', l_{p'} \rangle) \xrightarrow{\mathbf{alloc}} (\sigma'''; l''')}{(\sigma, \psi, \mathbf{swap} l_1 l_2) \mapsto (\sigma''', \psi', l''')}
\end{array}$$

2.8 $(\sigma, \psi, e) \mapsto^* (\sigma', \psi', e')$

$$\begin{array}{c}
\frac{}{(\sigma, \psi, e) \mapsto^* (\sigma, \psi, e)} \qquad \frac{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_2, \psi_2, e_2) \quad (\sigma_2, \psi_2, e_2) \mapsto^* (\sigma_3, \psi_3, e_3)}{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_3, \psi_3, e_3)} \\
\\
\frac{(\sigma_1, \psi_1, e_1) \mapsto (\sigma_2, \psi_2, e_2)}{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_2, \psi_2, e_2)}
\end{array}$$

3 Static Semantics

3.1 $\Delta \vdash q \preceq q'$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash \mathbf{U} \preceq q} \quad \frac{q_1 \sqsubseteq q_2}{\Delta \vdash q_1 \preceq q_2} \quad \frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq \mathbf{L}}$$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq q} \quad \frac{\Delta \vdash q_1 \preceq q_2 \quad \Delta \vdash q_2 \preceq q_3}{\Delta \vdash q_1 \preceq q_3}$$

3.2 $\Delta \vdash \tau \preceq q'$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta \vdash \tau \preceq \mathbf{L}} \quad \frac{\Delta \vdash q \preceq q' \quad FV(\bar{\tau}) \subseteq \Delta}{\Delta \vdash {}^q\bar{\tau} \preceq q'}$$

3.3 $\Delta \vdash \Gamma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'} \quad \frac{\Delta \vdash \Gamma \preceq q' \quad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Gamma, x:\tau \preceq q'}$$

3.4 $\Delta \vdash \Sigma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'} \quad \frac{\Delta \vdash \Sigma \preceq q' \quad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Sigma, l \mapsto \tau \preceq q'}$$

3.5 $\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma$

$$\overline{\Delta \vdash \bullet \boxdot \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1, x:\tau \boxdot \Gamma_2 \rightsquigarrow \Gamma, x:\tau} \quad \frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1 \boxdot \Gamma_2, x:\tau \rightsquigarrow \Gamma, x:\tau}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \tau \preceq \mathbf{R}}{\Delta \vdash \Gamma_1, x:\tau \boxdot \Gamma_2, x:\tau \rightsquigarrow \Gamma, x:\tau}$$

3.6 $\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma$

$$\overline{\Delta \vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2 \rightsquigarrow \Sigma, l \mapsto \tau} \quad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1 \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta \vdash \tau \preceq \mathbf{R}}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

3.7 $\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi$

$$\overline{\vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi}{\vdash \Psi_1, p \mapsto (q, \tau) \odot \Psi_2 \rightsquigarrow \Psi, p \mapsto (q, \tau)} \quad \frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi}{\vdash \Psi_1 \odot \Psi_2, p \mapsto (q, \tau) \rightsquigarrow \Psi, p \mapsto (q, \tau)}$$

$$\frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi \quad q \sqsubseteq R}{\vdash \Psi_1, p \mapsto (q, \tau) \odot \Psi_2, p \mapsto (q, \tau) \rightsquigarrow \Psi, p \mapsto (q, \tau)}$$

3.8 $\Delta; \Sigma \vdash l : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, l \mapsto \tau \vdash l : \tau}$$

3.9 $\Delta; \Gamma \vdash x : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, x : \tau \vdash x : \tau}$$

3.10 $\Delta; \Gamma; \Sigma \vdash e : \tau$

$$\frac{\Delta; \Sigma \vdash l : \tau}{\Delta; \bullet; \Sigma \vdash l : \tau} \quad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta \vdash \Sigma_1 \preceq A \quad \Delta; \Gamma; \Sigma_2 \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}$$

$$\frac{\Delta; \Gamma \vdash x : \tau}{\Delta; \Gamma; \bullet \vdash x : \tau} \quad \frac{\Delta; \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta; \Gamma_1 \preceq A \quad \Delta; \Gamma_2; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta; \Gamma; \Sigma \vdash e : \tau} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\tau_x \multimap \tau) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_x}{\Delta; \Gamma; \Sigma \vdash e_1 e_2 : \tau}$$

$$\frac{FV(q_a) \subseteq \Delta}{\Delta; \bullet; \bullet \vdash {}^{q_a}\langle \rangle : {}^{q_a}\mathbf{1}_{\otimes}} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}\mathbf{1}_{\otimes} \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \langle \rangle = e_1 \mathbf{in} e_2 : \tau}$$

$$\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : \tau_1 \quad \Delta \vdash \tau_1 \preceq q_a \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\langle e_1, e_2 \rangle : {}^{q_a}(\tau_1 \otimes \tau_2)}$$

$$\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\tau_x \otimes \tau_y) \quad \Delta; \Gamma_2, x : \tau_x, y : \tau_y; \Sigma_2 \vdash e_2 : \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \langle x, y \rangle = e_1 \mathbf{in} e_2 : \tau}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\langle \rangle : {}^{q_a}\mathbf{1}_{\otimes}}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \quad \Delta; \Gamma; \Sigma \vdash e_2 : \tau_2}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\langle e_1, e_2 \rangle : {}^{q_a}(\tau_1 \otimes \tau_2)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\tau_1 \otimes \tau_2)}{\Delta; \Gamma; \Sigma \vdash \mathbf{fst} e : \tau_1} \quad \frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\tau_1 \otimes \tau_2)}{\Delta; \Gamma; \Sigma \vdash \mathbf{snd} e : \tau_2}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{qa}\mathbf{0}}{\Delta; \Gamma; \Sigma \vdash \mathbf{abort} \, e : \tau}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \quad \Delta \vdash \tau_1 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{inl} \, e_1 : {}^{qa}(\tau_1 \oplus \tau_2)} \quad \frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{inr} \, e_2 : {}^{qa}(\tau_1 \oplus \tau_2)}$$

$$\frac{\Delta \vdash \Gamma_1 \sqcup \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\tau_1 \oplus \tau_2) \quad \Delta; \Gamma_2, x:\tau_1 \vdash e_l \vdash \tau \quad \Delta; \Gamma_2, y:\tau_2 \vdash e_r \vdash \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{case} \, e_1 \, \mathbf{of} \, \mathbf{inl} \, x \Rightarrow e_l \parallel \mathbf{inr} \, y \Rightarrow e_r : \tau}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta, \xi; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\Lambda \xi. e : {}^{qa}(\forall \xi. \tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{qa}(\forall \xi. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [q_2] : \tau[q_2/\xi]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[q_1/\xi] \quad \Delta \vdash \tau[q_1/\xi] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{pack}(q_1, e_2) : {}^{qa}(\exists \xi. \tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \sqcup \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\exists \xi. \tau) \quad \Delta, \xi; \Gamma_2, x:\tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \, \mathbf{pack}(\xi, x) = e_1 \, \mathbf{in} \, e_2 : \tau'}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta, \bar{\alpha}; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\Lambda \bar{\alpha}. e : {}^{qa}(\forall \bar{\alpha}. \tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{qa}(\forall \bar{\alpha}. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [\bar{\tau}_2] : \tau[\bar{\tau}_2/\bar{\alpha}]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\bar{\tau}_1/\bar{\alpha}] \quad \Delta \vdash \tau[\bar{\tau}_1/\bar{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{pack}(\bar{\tau}_1, e_2) : {}^{qa}(\exists \bar{\alpha}. \tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \sqcup \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\exists \bar{\alpha}. \tau) \quad \Delta, \bar{\alpha}; \Gamma_2, x:\tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \, \mathbf{pack}(\bar{\alpha}, x) = e_1 \, \mathbf{in} \, e_2 : \tau'}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta, \alpha; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\Lambda \alpha. e : {}^{qa}(\forall \alpha. \tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{qa}(\forall \alpha. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [\tau_2] : \tau[\tau_2/\alpha]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\tau_1/\alpha] \quad \Delta \vdash \tau[\tau_1/\alpha] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{pack}(\tau_1, e_2) : {}^{qa}(\exists \alpha. \tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \sqcup \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\exists \alpha. \tau) \quad \Delta, \alpha; \Gamma_2, x:\tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \, \mathbf{pack}(\alpha, x) = e_1 \, \mathbf{in} \, e_2 : \tau'}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \quad \Delta \vdash \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\mathbf{fold} \, e : {}^{qa}(\mu \bar{\alpha}. \tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{qa}(\mu \bar{\alpha}. \tau)}{\Delta; \Gamma; \Sigma \vdash \mathbf{unfold} \, e : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}]}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma; \Sigma \vdash e : \tau \quad \Delta \vdash \tau \preceq A}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\text{new } e : {}^{qa}(\text{ref } \tau)} \quad \frac{\Delta \vdash R \preceq q_a \quad \Delta; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa}\text{new } e : {}^{qa}(\text{ref } \tau)} \\
\\
\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{qa}(\text{ref } \tau) \quad \Delta \vdash A \preceq q_a}{\Delta; \Gamma; \Sigma \vdash \text{free } e : \tau} \\
\\
\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{qa}(\text{ref } \tau) \quad \Delta \vdash \tau \preceq R}{\Delta; \Gamma; \Sigma \vdash \text{read } e : {}^L({}^{qa}(\text{ref } \tau) \otimes \tau)} \\
\\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\text{ref } \tau_1) \quad \Delta \vdash A \preceq q_a}{\Delta; \Gamma; \Sigma \vdash \text{swap } e_1 e_2 : {}^L({}^{qa}(\text{ref } \tau_2) \otimes \tau_1)} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\text{ref } \tau) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau}{\Delta; \Gamma; \Sigma \vdash \text{swap } e_1 e_2 : {}^L({}^{qa}(\text{ref } \tau) \otimes \tau)}
\end{array}$$

3.11 $\Psi \vdash p : (q, \tau)$

$$\overline{\bullet, p \mapsto (q, \tau) \vdash p : (q, \tau)}$$

3.12 $\Sigma; \Psi \vdash (q, v) : \tau$

$$\begin{array}{c}
\frac{\bullet \vdash \Sigma \preceq q \quad \bullet; \bullet, x : \tau_x; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (q, \lambda x : \tau_x. e) : {}^q(\tau_x \multimap \tau)} \\
\\
\frac{\bullet; \bullet \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \bullet; \Sigma_1 \vdash l_1 : \tau_1 \quad \bullet; \Sigma_2 \vdash l_2 : \tau_2}{\Sigma; \bullet \vdash (q, \langle l_1, l_2 \rangle) : {}^q(\tau_1 \otimes \tau_2)} \\
\\
\overline{\bullet; \bullet \vdash (q, \langle \rangle) : {}^q \mathbf{1}_\otimes} \quad \frac{\bullet \vdash \Sigma \preceq q}{\Sigma; \bullet \vdash (q, \langle \rangle) : {}^q \mathbf{1}_\otimes} \quad \frac{\bullet \vdash \Sigma \preceq q \quad \bullet; \bullet; \Sigma \vdash e_1 : \tau_1 \quad \bullet; \bullet; \Sigma \vdash e_2 : \tau_2}{\Sigma; \bullet \vdash (q, \langle e_1, e_2 \rangle) : {}^q(\tau_1 \otimes \tau_2)} \\
\\
\frac{\bullet; \Sigma \vdash l : \tau_1 \quad \bullet \vdash \tau_1 \preceq q}{\Sigma; \bullet \vdash (q, \text{inl } l) : {}^q(\tau_1 \oplus \tau_2)} \quad \frac{\bullet; \Sigma \vdash l : \tau_2 \quad \bullet \vdash \tau_2 \preceq q}{\Sigma; \bullet \vdash (q, \text{inr } l) : {}^q(\tau_1 \oplus \tau_2)} \\
\\
\frac{\bullet \vdash \Sigma \preceq q \quad \bullet, \xi; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (q, \Lambda \xi. e) : {}^q(\forall \xi. \tau)} \quad \frac{\bullet; \Sigma \vdash l_2 : \tau[q_1/\xi] \quad \bullet \vdash \tau[q_1/\xi] \preceq q}{\Sigma; \bullet \vdash (q, \text{pack}(q_1, l_2)) : {}^q(\exists \xi. \tau)} \\
\\
\frac{\bullet \vdash \Sigma \preceq q \quad \bullet, \bar{\alpha}; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (q, \Lambda \bar{\alpha}. e) : {}^q(\forall \bar{\alpha}. \tau)} \quad \frac{\bullet; \Sigma \vdash l_2 : \tau[\bar{\tau}_1/\bar{\alpha}] \quad \bullet \vdash \tau[\bar{\tau}_1/\bar{\alpha}] \preceq q}{\Sigma; \bullet \vdash (q, \text{pack}(\bar{\tau}_1, l_2)) : {}^q(\exists \bar{\alpha}. \tau)} \\
\\
\frac{\bullet \vdash \Sigma \preceq q \quad \bullet, \alpha; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (q, \Lambda \alpha. e) : {}^q(\forall \alpha. \tau)} \quad \frac{\bullet; \Sigma \vdash l_2 : \tau[\tau_1/\alpha] \quad \bullet \vdash \tau[\tau_1/\alpha] \preceq q}{\Sigma; \bullet \vdash (q, \text{pack}(\tau_1, l_2)) : {}^q(\exists \alpha. \tau)} \\
\\
\frac{\bullet; \Sigma \vdash l : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \quad \bullet \vdash \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \preceq q}{\Sigma; \bullet \vdash (q, \text{fold } l) : {}^q(\mu \bar{\alpha}. \tau)} \\
\\
\frac{\Psi \vdash p : (q, \tau)}{\bullet; \Psi \vdash (q, \text{ptr } p) : {}^q(\text{ref } \tau)}
\end{array}$$

3.13 $\Psi \vdash \sigma : \Sigma$

$$\frac{}{\bullet \vdash \bullet : \bullet} \quad \frac{\Sigma_v; \Psi_v \vdash (\mathbf{q}, v) : \tau \quad \bullet \vdash \Sigma_v \odot \Sigma \leadsto \Sigma_* \quad \Psi_* \vdash \sigma : \Sigma_* \quad \vdash \Psi_* \odot \Psi_v \leadsto \Psi}{\Psi \vdash \sigma, l \mapsto (\mathbf{q}, v, f) : \Sigma, l \mapsto \tau}$$

$$\frac{\Psi \vdash \sigma : \Sigma \quad \mathbf{q} \sqsubseteq R}{\Psi \vdash \sigma, l \mapsto (\mathbf{q}, v, \text{used}) : \Sigma} \quad \frac{\Psi \vdash \sigma : \Sigma \quad \mathbf{q} \sqsubseteq A}{\Psi \vdash \sigma, l \mapsto (\mathbf{q}, v, f) : \Sigma}$$

3.14 $\vdash \tau \downarrow \mathbf{q}$

$$\frac{\bullet \vdash \tau \preceq A}{\vdash \tau \downarrow \mathbf{U}} \quad \frac{}{\vdash \tau \downarrow \mathbf{R}} \quad \frac{\bullet \vdash \tau \preceq A}{\vdash \tau \downarrow \mathbf{A}} \quad \frac{}{\vdash \tau \downarrow \mathbf{L}}$$

3.15 $\Sigma \vdash \psi : \Psi$

$$\frac{}{\bullet \vdash \bullet : \bullet} \quad \frac{\Sigma_* \vdash \psi : \Psi \quad \vdash \tau \downarrow \mathbf{q} \quad \bullet; \Sigma_l \vdash l : \tau \quad \bullet \vdash \Sigma_l \odot \Sigma_* \leadsto \Sigma}{\Sigma \vdash \psi, p \mapsto (\mathbf{q}, l) : \Psi, p \mapsto (\mathbf{q}, \tau)}$$

$$\frac{\Sigma \vdash \psi : \Psi \quad \mathbf{q} \sqsubseteq A}{\Sigma \vdash \psi, p \mapsto (\mathbf{q}, l) : \Psi}$$

3.16 $\vdash (\sigma, \psi) : \Sigma$

$$\frac{\Sigma_1 \vdash \psi : \Psi \quad \bullet \vdash \Sigma_1 \odot \Sigma_2 \leadsto \Sigma \quad \Psi \vdash \sigma : \Sigma}{\vdash (\sigma, \psi) : \Sigma_2}$$

4 Safety

Theorem 1 (Preservation)

If $(\sigma_1, \psi_1, e_1) \mapsto^ (\sigma_2, \psi_2, e_2)$ and $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$,
then there exists Σ_2 such that $\vdash (\sigma_2, \psi_2) : \Sigma_2$ and $\bullet; \bullet; \Sigma_2 \vdash e_2 : \tau$.*

Theorem 2 (Progress)

*If $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$,
then either there exists l such that $e_1 \equiv l$
or there exists σ_2 and ψ_2 and e_2 such that $(\sigma_1, \psi_1, e_1) \mapsto (\sigma_2, \psi_2, e_2)$.*

Theorem 3 (Safety)

If $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$ and $(\sigma_1, \psi_1, e_1) \mapsto^ (\sigma_2, \psi_2, e_2)$,
then either there exists l such that $e_2 \equiv l$
or there exists σ_3 and ψ_3 and e_3 such that $(\sigma_2, \psi_2, e_2) \mapsto (\sigma_3, \psi_3, e_3)$.*