1 Syntax

Constant Qualifiers	q	$\in$	$\mathit{Quals} = \{\mathsf{U}, \mathsf{R}, \mathsf{A}, \mathsf{L}\}$ $\quad \begin{array}{ccc} & \mathsf{L} & \\ \mathsf{A} & \sqsubseteq & \mathsf{R} \\ & \mathsf{U} & \end{array}$
Qualifiers	q	$::=$	$\xi \mid \mathsf{q}$
Locations	l	$\in$	Locs
Expressions	e	$::=$	$l \mid$
			$x \mid {}^{q_a}\lambda x{:}\tau.\, e \mid e_1\, e_2 \mid$
			${}^{q_a}\langle\rangle \mid \mathtt{let}\ \langle\rangle = e_1\ \mathtt{in}\ e_2 \mid {}^{q_a}\langle e_1, e_2\rangle \mid \mathtt{let}\ \langle x_1, x_2\rangle = e_1\ \mathtt{in}\ e_2 \mid$
			${}^{q_a}\llparenthesis\rrparenthesis \mid {}^{q_a}\llparenthesis e_1, e_2\rrparenthesis \mid \mathtt{fst}\, e \mid \mathtt{snd}\, e \mid$
			$\mathtt{abort}\, e \mid {}^{q_a}\mathtt{inl}\, e \mid {}^{q_a}\mathtt{inr}\, e \mid \mathtt{case}\ e_1\ \mathtt{of}\ \mathtt{inl}\, x \Rightarrow e_l \parallel \mathtt{inr}\, y \Rightarrow e_r \mid$
			${}^{q_a}\Lambda\xi.\, e \mid e\,[q] \mid {}^{q_a}\mathtt{pack}(q, e) \mid \mathtt{let}\ \mathtt{pack}(\xi, x) = e_1\ \mathtt{in}\ e_2 \mid$
			${}^{q_a}\Lambda\overline{\alpha}.\, e \mid e\,[\overline{\tau}] \mid {}^{q_a}\mathtt{pack}(\overline{\tau}, e) \mid \mathtt{let}\ \mathtt{pack}(\overline{\alpha}, x) = e_1\ \mathtt{in}\ e_2 \mid$
			${}^{q_a}\Lambda\alpha.\, e \mid e\,[\tau] \mid {}^{q_a}\mathtt{pack}(\tau, e) \mid \mathtt{let}\ \mathtt{pack}(\alpha, x) = e_1\ \mathtt{in}\ e_2 \mid$
			${}^{q_a}\mathtt{fold}\, e \mid \mathtt{unfold}\, e$
Values	v	$::=$	$\lambda x{:}\tau.\, e \mid$
			$\langle\rangle \mid \langle l_1, l_2\rangle \mid$
			$\llparenthesis\rrparenthesis \mid \llparenthesis e_1, e_2\rrparenthesis \mid$
			$\mathtt{inl}\, l \mid \mathtt{inr}\, l \mid$
			$\Lambda\xi.\, e \mid \mathtt{pack}(q, e) \mid \Lambda\overline{\alpha}.\, e \mid \mathtt{pack}(\overline{\tau}, e) \mid \Lambda\alpha.\, e \mid \mathtt{pack}(\tau, e) \mid$
			$\mathtt{fold}\, l$
PreTypes	$\overline{\tau}$	$::=$	$\overline{\alpha} \mid$
			$\tau_1 \multimap \tau_2 \mid$
			$\mathbf{1}_\otimes \mid \tau_1 \otimes \tau_2 \mid$
			$\mathbf{1}_\circledast \mid \tau_1 \circledast \tau_2 \mid$
			$\mathbf{0} \mid \tau_1 \oplus \tau_2 \mid$
			$\forall\xi.\tau \mid \exists\xi.\tau \mid \forall\overline{\alpha}.\tau \mid \exists\overline{\alpha}.\tau \mid \forall\alpha.\tau \mid \exists\alpha.\tau \mid$
			$\mu\overline{\alpha}.\tau$
Types	τ	$::=$	$\alpha \mid {}^{q}\overline{\tau}$
Qualifier/PreType/Type Contexts	Δ	$::=$	$\bullet \mid \Delta, \xi \mid \Delta, \overline{\alpha} \mid \Delta, \alpha$
Value Contexts	Γ	$::=$	$\bullet \mid \Gamma, x{:}\tau$
Flags	f	$\in$	$\{\mathsf{unused}, \mathsf{used}\}$
Stores	σ	$::=$	$\bullet \mid \sigma, l \mapsto (\mathsf{q}, v, f)$
Store Typings	Σ	$::=$	$\bullet \mid \Sigma, l \mapsto \tau$

2 Dynamic Semantics

2.1 $(\sigma; \mathsf{q}; v) \xrightarrow{\mathsf{alloc}} (\sigma'; l')$

$$\frac{l \notin dom(\sigma)}{(\sigma; \mathsf{q}; v) \xrightarrow{\mathsf{alloc}} (\sigma, l \mapsto (\mathsf{q}, v, \mathsf{unused}); l)}$$

2.2 $(\sigma; l) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}'; v')$

$$\frac{\mathsf{q} \sqsubseteq \mathsf{R}}{(\sigma, l \mapsto (\mathsf{q}, v, f); l) \xrightarrow{\mathsf{fetch}} (\sigma, l \mapsto (\mathsf{q}, v, \mathsf{used}); \mathsf{q}; v)} \qquad \frac{\mathsf{A} \sqsubseteq \mathsf{q}}{(\sigma, l \mapsto (\mathsf{q}, v, i); l) \xrightarrow{\mathsf{fetch}} (\sigma; \mathsf{q}; v)}$$

$$\frac{l \neq l' \qquad (\sigma_1; l) \xrightarrow{\mathsf{fetch}} (\sigma_2; v)}{(\sigma_1, l' \mapsto (\mathsf{q}', v', f'); l) \xrightarrow{\mathsf{fetch}} (\sigma_2, l' \mapsto (\mathsf{q}', v', f'); v)}$$

2.3 $(\sigma, e) \longmapsto (\sigma', e')$

$$\frac{(\sigma; \mathsf{q}_a; \lambda x{:}\tau.\, e) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathsf{q}_a}\lambda x{:}\tau.\, e) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, e_1\, e_2) \longmapsto (\sigma', e_1'\, e_2)} \qquad \frac{(\sigma, e_2) \longmapsto (\sigma', e_2')}{(\sigma, l_1\, e_2) \longmapsto (\sigma', l_1\, e_2')} \qquad \frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}_a; \lambda x{:}\tau.\, e)}{(\sigma, l_1\, l_2) \longmapsto (\sigma', e[l_2/x])}$$

$$\frac{(\sigma; \mathsf{q}_a; \langle\rangle) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathsf{q}_a}\langle\rangle) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \texttt{let}\ \langle\rangle = e_1\ \texttt{in}\ e_2) \longmapsto (\sigma', \texttt{let}\ \langle\rangle = e_1'\ \texttt{in}\ e_2)} \qquad \frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}_a; \langle\rangle)}{(\sigma, \texttt{let}\ \langle\rangle = l_1\ \texttt{in}\ e_2) \longmapsto (\sigma', e_2)}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, {}^{\mathsf{q}_a}\langle e_1, e_2\rangle) \longmapsto (\sigma', {}^{\mathsf{q}_a}\langle e_1', e_2\rangle)} \qquad \frac{(\sigma, e_2) \longmapsto (\sigma', e_2')}{(\sigma, {}^{\mathsf{q}_a}\langle l_1, e_2\rangle) \longmapsto (\sigma', {}^{\mathsf{q}_a}\langle l_1, e_2'\rangle)} \qquad \frac{(\sigma; \mathsf{q}_a; \langle l_1, l_2\rangle) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathsf{q}_a}\langle l_1, l_2\rangle) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \texttt{let}\ \langle x, y\rangle = e_1\ \texttt{in}\ e_2) \longmapsto (\sigma', \texttt{let}\ \langle x, y\rangle = e_1'\ \texttt{in}\ e_2)} \qquad \frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}_a; \langle l_x, l_y\rangle)}{(\sigma, \texttt{let}\ \langle x, y\rangle = l_1\ \texttt{in}\ e_2) \longmapsto (\sigma', e_2[l_x/x][l_y/y])}$$

$$\frac{(\sigma; \mathsf{q}_a; \llparenthesis\rrparenthesis) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathsf{q}_a}\llparenthesis\rrparenthesis) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma; \mathsf{q}_a; \llparenthesis e_1, e_2\rrparenthesis) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathsf{q}_a}\llparenthesis e_1, e_2\rrparenthesis) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, \texttt{fst}\, e) \longmapsto (\sigma', \texttt{fst}\, e')} \qquad \frac{(\sigma; l) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}_a; \llparenthesis e_1, e_2\rrparenthesis)}{(\sigma, \texttt{fst}\, l) \longmapsto (\sigma', e_1)} \qquad \frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, \texttt{snd}\, e) \longmapsto (\sigma', \texttt{snd}\, e')} \qquad \frac{(\sigma; l) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathsf{q}_a; \llparenthesis e_1, e_2\rrparenthesis)}{(\sigma, \texttt{snd}\, l) \longmapsto (\sigma', e_2)}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, \texttt{abort}\, e) \longmapsto (\sigma', \texttt{abort}\, e')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, {}^{q_a}\texttt{inl}\, e_1) \longmapsto (\sigma', {}^{q_a}\texttt{inl}\, e_1')} \qquad \frac{(\sigma; \mathtt{q}_a; \texttt{inl}\, l_1) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\texttt{inl}\, l_1) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_2) \longmapsto (\sigma', e_2')}{(\sigma, {}^{q_a}\texttt{inr}\, e_2) \longmapsto (\sigma', {}^{q_a}\texttt{inr}\, e_2')} \qquad \frac{(\sigma; \mathtt{q}_a; \texttt{inr}\, l_2) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\texttt{inr}\, l_2) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \texttt{case}\ e_1\ \texttt{of}\ \texttt{inl}\, x \Rightarrow e_l \parallel \texttt{inr}\, y \Rightarrow e_r) \longmapsto (\sigma', \texttt{case}\ e_1'\ \texttt{of}\ \texttt{inl}\, x \Rightarrow e_l \parallel \texttt{inr}\, y \Rightarrow e_r)}$$

$$\frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \texttt{inl}\, l_x)}{(\sigma, \texttt{case}\ l_1\ \texttt{of}\ \texttt{inl}\, x \Rightarrow e_l \parallel \texttt{inr}\, y \Rightarrow e_r) \longmapsto (\sigma', e_l[l_x/x])}$$

$$\frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \texttt{inr}\, l_y)}{(\sigma, \texttt{case}\ l_1\ \texttt{of}\ \texttt{inl}\, x \Rightarrow e_l \parallel \texttt{inr}\, y \Rightarrow e_r) \longmapsto (\sigma', e_r[l_y/y])}$$

$$\frac{(\sigma; \mathtt{q}_a; \Lambda\xi.\, e) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\Lambda\xi.\, e) \longmapsto (\sigma', l')} \qquad \frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, e\,[q]) \longmapsto (\sigma', e'\,[q])} \qquad \frac{(\sigma; l) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \Lambda\xi.\, e)}{(\sigma, l\,[q]) \longmapsto (\sigma', e[q/\xi])}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, {}^{q_a}\texttt{pack}(q, e)) \longmapsto (\sigma', {}^{q_a}\texttt{pack}(q, e'))} \qquad \frac{(\sigma; \mathtt{q}_a; \texttt{pack}(q, l)) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\texttt{pack}(q, l)) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \texttt{let pack}(\xi, x) = e_1\ \texttt{in}\ e_2) \longmapsto (\sigma', \texttt{let pack}(\xi, x) = e_1'\ \texttt{in}\ e_2)}$$

$$\frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \texttt{pack}(q_1, l_1))}{(\sigma, \texttt{let pack}(\xi, x) = l_1\ \texttt{in}\ e_2) \longmapsto (\sigma', e_2[q_1/\xi][l_1/x])}$$

$$\frac{(\sigma; \mathtt{q}_a; \Lambda\overline{\alpha}.\, e) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\Lambda\overline{\alpha}.\, e) \longmapsto (\sigma', l')} \qquad \frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, e\,[\overline{\tau}]) \longmapsto (\sigma', e'\,[\overline{\tau}])} \qquad \frac{(\sigma; l) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \Lambda\overline{\alpha}.\, e)}{(\sigma, l\,[\overline{\tau}]) \longmapsto (\sigma', e[\overline{\tau}/\overline{\alpha}])}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, {}^{q_a}\texttt{pack}(\overline{\tau}, e)) \longmapsto (\sigma', {}^{q_a}\texttt{pack}(\overline{\tau}, e'))} \qquad \frac{(\sigma; \mathtt{q}_a; \texttt{pack}(\overline{\tau}, l)) \xrightarrow{\mathsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\texttt{pack}(\overline{\tau}, l)) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \texttt{let pack}(\overline{\alpha}, x) = e_1\ \texttt{in}\ e_2) \longmapsto (\sigma', \texttt{let pack}(\overline{\alpha}, x) = e_1'\ \texttt{in}\ e_2)}$$

$$\frac{(\sigma; l_1) \xrightarrow{\mathsf{fetch}} (\sigma'; \mathtt{q}_a; \texttt{pack}(\overline{\tau_1}, l_1))}{(\sigma, \texttt{let pack}(\overline{\alpha}, x) = l_1\ \texttt{in}\ e_2) \longmapsto (\sigma', e_2[\overline{\tau_1}/\overline{\alpha}][l_1/x])}$$

$$\frac{(\sigma; \mathtt{q}_a; \Lambda\alpha.\, e) \xrightarrow{\textsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\Lambda\alpha.\, e) \longmapsto (\sigma', l')} \qquad \frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, e\,[\tau]) \longmapsto (\sigma', e'\,[\tau])} \qquad \frac{(\sigma; l) \xrightarrow{\textsf{fetch}} (\sigma'; \mathtt{q}_a; \Lambda\alpha.\, e)}{(\sigma, l\,[\tau]) \longmapsto (\sigma', e[\tau/\alpha])}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, {}^{q_a}\mathtt{pack}(\tau, e)) \longmapsto (\sigma', {}^{q_a}\mathtt{pack}(\tau, e'))} \qquad \frac{(\sigma; \mathtt{q}_a; \mathtt{pack}(\tau, l)) \xrightarrow{\textsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\mathtt{pack}(\tau, l)) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e_1) \longmapsto (\sigma', e_1')}{(\sigma, \mathtt{let\ pack}(\alpha, x) = e_1\ \mathtt{in}\ e_2) \longmapsto (\sigma', \mathtt{let\ pack}(\alpha, x) = e_1'\ \mathtt{in}\ e_2)}$$

$$\frac{(\sigma; l_1) \xrightarrow{\textsf{fetch}} (\sigma'; \mathtt{q}_a; \mathtt{pack}(\tau_1, l_1))}{(\sigma, \mathtt{let\ pack}(\alpha, x) = l_1\ \mathtt{in}\ e_2) \longmapsto (\sigma', e_2[\tau_1/\alpha][l_1/x])}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, {}^{q_a}\mathtt{fold}\, e) \longmapsto (\sigma', {}^{q_a}\mathtt{fold}\, e')} \qquad \frac{(\sigma; \mathtt{q}_a; \mathtt{fold}\, l) \xrightarrow{\textsf{alloc}} (\sigma'; l')}{(\sigma, {}^{\mathtt{q}_a}\mathtt{fold}\, l) \longmapsto (\sigma', l')}$$

$$\frac{(\sigma, e) \longmapsto (\sigma', e')}{(\sigma, \mathtt{unfold}\, e) \longmapsto (\sigma', \mathtt{unfold}\, e')} \qquad \frac{(\sigma; l) \xrightarrow{\textsf{fetch}} (\sigma'; \mathtt{q}_a; \mathtt{fold}\, l')}{(\sigma, \mathtt{unfold}\, l) \longmapsto (\sigma', l')}$$

2.4 $(\sigma, e) \longmapsto^* (\sigma', e')$

$$\frac{}{(\sigma, e) \longmapsto^* (\sigma, e)} \qquad \frac{(\sigma_1, e_1) \longmapsto^* (\sigma_2, e_2) \qquad (\sigma_2, e_2) \longmapsto^* (\sigma_3, e_3)}{(\sigma_1, e_1) \longmapsto^* (\sigma_3, e_3)} \qquad \frac{(\sigma_1, e_1) \longmapsto (\sigma_2, e_2)}{(\sigma_1, e_1) \longmapsto^* (\sigma_2, e_2)}$$

3 Static Semantics

3.1 $\Delta \vdash q \preceq q'$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash \mathsf{U} \preceq q} \qquad \frac{\mathfrak{q}_1 \sqsubseteq \mathfrak{q}_2}{\Delta \vdash \mathfrak{q}_1 \preceq \mathfrak{q}_2} \qquad \frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq \mathsf{L}}$$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq q} \qquad \frac{\Delta \vdash q_1 \preceq q_2 \qquad \Delta \vdash q_2 \preceq q_3}{\Delta \vdash q_1 \preceq q_3}$$

3.2 $\Delta \vdash \tau \preceq q'$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta \vdash \tau \preceq \mathsf{L}} \qquad \frac{\Delta \vdash q \preceq q' \qquad FV(\overline{\tau}) \subseteq \Delta}{\Delta \vdash {}^{q}\overline{\tau} \preceq q'}$$

3.3 $\Delta \vdash \Gamma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'} \qquad \frac{\Delta \vdash \Gamma \preceq q' \qquad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Gamma, x{:}\tau \preceq q'}$$

3.4 $\Delta \vdash \Sigma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'} \qquad \frac{\Delta \vdash \Sigma \preceq q' \qquad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Sigma, l \mapsto \tau \preceq q'}$$

3.5 $\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma$

$$\frac{}{\Delta \vdash \bullet \boxdot \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1, x{:}\tau \boxdot \Gamma_2 \rightsquigarrow \Gamma, x{:}\tau} \qquad \frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1 \boxdot \Gamma_2, x{:}\tau \rightsquigarrow \Gamma, x{:}\tau}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \tau \preceq \mathsf{R}}{\Delta \vdash \Gamma_1, x{:}\tau \boxdot \Gamma_2, x{:}\tau \rightsquigarrow \Gamma, x{:}\tau}$$

3.6 $\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma$

$$\frac{}{\Delta \vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2 \rightsquigarrow \Sigma, l \mapsto \tau} \qquad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1 \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \Delta \vdash \tau \preceq \mathsf{R}}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

3.7 $\Delta; \Sigma \vdash l : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, l \mapsto \tau \vdash l : \tau}$$

3.8 $\Delta; \Gamma \vdash x : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, x{:}\tau \vdash x : \tau}$$

3.9 $\Delta; \Gamma; \Sigma \vdash e : \tau$

$$\frac{\Delta; \Sigma \vdash l : \tau}{\Delta; \bullet; \Sigma \vdash l : \tau} \qquad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \Delta \vdash \Sigma_1 \preceq \mathsf{A} \qquad \Delta; \Gamma; \Sigma_2 \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}$$

$$\frac{\Delta; \Gamma \vdash x : \tau}{\Delta; \Gamma; \bullet \vdash x : \tau} \qquad \frac{\Delta; \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta; \Gamma_1 \preceq \mathsf{A} \qquad \Delta; \Gamma_2; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}$$

$$\frac{\begin{array}{c}\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a \\ \Delta; \Gamma, x{:}\tau_x; \Sigma \vdash e : \tau\end{array}}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\lambda x{:}\tau_x.\, e : {}^{q_a}(\tau_x \multimap \tau)} \qquad \frac{\begin{array}{c}\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \\ \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\tau_x \multimap \tau) \qquad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_x\end{array}}{\Delta; \Gamma; \Sigma \vdash e_1\, e_2 : \tau}$$

$$\frac{FV(q_a) \subseteq \Delta}{\Delta; \bullet; \bullet \vdash {}^{q_a}\langle\rangle : {}^{q_a}\mathbf{1}_{\otimes}} \qquad \frac{\begin{array}{c}\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \\ \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}\mathbf{1}_{\otimes} \qquad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau\end{array}}{\Delta; \Gamma; \Sigma \vdash \texttt{let}\ \langle\rangle = e_1\ \texttt{in}\ e_2 : \tau}$$

$$\frac{\begin{array}{c}\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \\ \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : \tau_1 \qquad \Delta \vdash \tau_1 \preceq q_a \qquad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_2 \qquad \Delta \vdash \tau_2 \preceq q_a\end{array}}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\langle e_1, e_2\rangle : {}^{q_a}(\tau_1 \otimes \tau_2)}$$

$$\frac{\begin{array}{c}\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \\ \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\tau_x \otimes \tau_y) \qquad \Delta; \Gamma_2, x{:}\tau_x, y{:}\tau_y; \Sigma_2 \vdash e_2 : \tau\end{array}}{\Delta; \Gamma; \Sigma \vdash \texttt{let}\ \langle x, y\rangle = e_1\ \texttt{in}\ e_2 : \tau}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}(\!|\,|\!) : {}^{q_a}\mathbf{1}_{\circledast}}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a \qquad \Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \qquad \Delta; \Gamma; \Sigma \vdash e_2 : \tau_2}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}(\!| e_1, e_2 |\!) : {}^{q_a}(\tau_1 \circledast \tau_2)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\tau_1 \circledast \tau_2)}{\Delta; \Gamma; \Sigma \vdash \texttt{fst}\, e : \tau_1} \qquad \frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\tau_1 \circledast \tau_2)}{\Delta; \Gamma; \Sigma \vdash \texttt{snd}\, e : \tau_2}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}\mathbf{0}}{\Delta; \Gamma; \Sigma \vdash \texttt{abort}\, e : \tau}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \qquad \Delta \vdash \tau_1 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\texttt{inl}\, e_1 : {}^{q_a}(\tau_1 \oplus \tau_2)} \qquad \frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau_2 \qquad \Delta \vdash \tau_2 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\texttt{inl}\, e_2 : {}^{q_a}(\tau_1 \oplus \tau_2)}$$

$$\frac{\begin{array}{c}\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \\ \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\tau_1 \oplus \tau_2) \\ \Delta; \Gamma_2, x{:}\tau_1 \vdash e_l \vdash \tau \qquad \Delta; \Gamma_2, y{:}\tau_2 \vdash e_r \vdash \tau\end{array}}{\Delta; \Gamma; \Sigma \vdash \texttt{case}\ e_1\ \texttt{of}\ \texttt{inl}\, x \Rightarrow e_l \parallel \texttt{inr}\, y \Rightarrow e_r : \tau}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a \qquad \Delta, \xi; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\Lambda\xi.\, e : {}^{q_a}(\forall \xi.\tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \xi.\tau)}{\Delta; \Gamma; \Sigma \vdash e_1\,[q_2] : \tau[q_2/\xi]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[q_1/\xi] \qquad \Delta \vdash \tau[q_1/\xi] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\mathtt{pack}(q_1, e_2) : {}^{q_a}(\exists \xi.\tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \xi.\tau) \qquad \Delta, \xi; \Gamma_2, x{:}\tau; \Sigma_2 \vdash e_2 : \tau' \qquad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathtt{let\ pack}(\xi, x) = e_1\ \mathtt{in}\ e_2 : \tau'}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a \qquad \Delta, \overline{\alpha}; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\Lambda\overline{\alpha}.\, e : {}^{q_a}(\forall \overline{\alpha}.\tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \overline{\alpha}.\tau)}{\Delta; \Gamma; \Sigma \vdash e_1\,[\overline{\tau_2}] : \tau[\overline{\tau_2}/\overline{\alpha}]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\overline{\tau_1}/\overline{\alpha}] \qquad \Delta \vdash \tau[\overline{\tau_1}/\overline{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\mathtt{pack}(\overline{\tau_1}, e_2) : {}^{q_a}(\exists \overline{\alpha}.\tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \overline{\alpha}.\tau) \qquad \Delta, \overline{\alpha}; \Gamma_2, x{:}\tau; \Sigma_2 \vdash e_2 : \tau' \qquad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathtt{let\ pack}(\overline{\alpha}, x) = e_1\ \mathtt{in}\ e_2 : \tau'}$$

$$\frac{\Delta \vdash \Gamma \preceq q_a \qquad \Delta \vdash \Sigma \preceq q_a \qquad \Delta, \alpha; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\Lambda\alpha.\, e : {}^{q_a}(\forall \alpha.\tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \alpha.\tau)}{\Delta; \Gamma; \Sigma \vdash e_1\,[\tau_2] : \tau[\tau_2/\alpha]}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\tau_1/\alpha] \qquad \Delta \vdash \tau[\tau_1/\alpha] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\mathtt{pack}(\tau_1, e_2) : {}^{q_a}(\exists \alpha.\tau)}$$

$$\frac{\Delta \vdash \Gamma_1 \boxdot \Gamma_2 \rightsquigarrow \Gamma \qquad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \alpha.\tau) \qquad \Delta, \alpha; \Gamma_2, x{:}\tau; \Sigma_2 \vdash e_2 : \tau' \qquad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \mathtt{let\ pack}(\alpha, x) = e_1\ \mathtt{in}\ e_2 : \tau'}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : \tau[\mu\overline{\alpha}.\tau/\overline{\alpha}] \qquad \Delta \vdash \tau[\mu\overline{\alpha}.\tau/\overline{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\mathtt{fold}\, e : {}^{q_a}(\mu\overline{\alpha}.\tau)}$$

$$\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\mu\overline{\alpha}.\tau)}{\Delta; \Gamma; \Sigma \vdash \mathtt{unfold}\, e : \tau[\mu\overline{\alpha}.\tau/\overline{\alpha}]}$$

3.10 $\Sigma \vdash (\mathsf{q}, v) : \tau$

$$\frac{\bullet \vdash \Sigma \preceq \mathsf{q} \qquad \bullet; \bullet, x{:}\tau_x; \Sigma \vdash e : \tau}{\Sigma \vdash (\mathsf{q}, \lambda x{:}\tau_x.\, e) : {}^{\mathsf{q}}(\tau_x \multimap \tau)}$$

$$\frac{}{\bullet \vdash (\mathsf{q}, \langle\rangle) : {}^{\mathsf{q}}\mathbf{1}_{\otimes}} \qquad \frac{\bullet \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \qquad \bullet; \Sigma_1 \vdash l_1 : \tau_1 \qquad \bullet; \Sigma_2 \vdash l_2 : \tau_2}{\Sigma \vdash (\mathsf{q}, \langle l_1, l_2 \rangle) : {}^{\mathsf{q}}(\tau_1 \otimes \tau_2)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, (\!|\!)) : {}^{\mathsf{q}}\mathbf{1}_{\circledast}} \qquad \frac{\bullet \vdash \Sigma \preceq \mathsf{q} \qquad \bullet; \bullet; \Sigma \vdash e_1 : \tau_1 \qquad \bullet; \bullet; \Sigma \vdash e_2 : \tau_2}{\Sigma \vdash (\mathsf{q}, (\!| e_1, e_2 |\!)) : {}^{\mathsf{q}}(\tau_1 \circledast \tau_2)}$$

$$\frac{\bullet; \Sigma \vdash l : \tau_1 \qquad \bullet \vdash \tau_1 \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{inl}\, l) : {}^{\mathsf{q}}(\tau_1 \oplus \tau_2)} \qquad \frac{\bullet; \Sigma \vdash l : \tau_2 \qquad \bullet \vdash \tau_2 \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{inr}\, l) : {}^{\mathsf{q}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\begin{array}{c}\bullet \vdash \Sigma \preceq \mathsf{q} \\ \bullet, \xi; \bullet; \Sigma \vdash e : \tau\end{array}}{\Sigma \vdash (\mathsf{q}, \Lambda\xi.\, e) : {}^{\mathsf{q}}(\forall \xi.\tau)} \qquad \frac{\bullet; \Sigma \vdash l_2 : \tau[q_1/\xi] \qquad \bullet \vdash \tau[q_1/\xi] \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{pack}(q_1, l_2)) : {}^{\mathsf{q}}(\exists \xi.\tau)}$$

$$\frac{\begin{array}{c}\bullet \vdash \Sigma \preceq \mathsf{q} \\ \bullet, \overline{\alpha}; \bullet; \Sigma \vdash e : \tau\end{array}}{\Sigma \vdash (\mathsf{q}, \Lambda\overline{\alpha}.\, e) : {}^{\mathsf{q}}(\forall \overline{\alpha}.\tau)} \qquad \frac{\bullet; \Sigma \vdash l_2 : \tau[\overline{\tau_1}/\overline{\alpha}] \qquad \bullet \vdash \tau[\overline{\tau_1}/\overline{\alpha}] \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{pack}(\overline{\tau_1}, l_2)) : {}^{\mathsf{q}}(\exists \overline{\alpha}.\tau)}$$

$$\frac{\begin{array}{c}\bullet \vdash \Sigma \preceq \mathsf{q} \\ \bullet, \alpha; \bullet; \Sigma \vdash e : \tau\end{array}}{\Sigma \vdash (\mathsf{q}, \Lambda\alpha.\, e) : {}^{\mathsf{q}}(\forall \alpha.\tau)} \qquad \frac{\bullet; \Sigma \vdash l_2 : \tau[\tau_1/\alpha] \qquad \bullet \vdash \tau[\tau_1/\alpha] \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{pack}(\tau_1, l_2)) : {}^{\mathsf{q}}(\exists \alpha.\tau)}$$

$$\frac{\bullet; \Sigma \vdash l : \tau[\mu\overline{\alpha}.\tau/\overline{\alpha}] \qquad \bullet \vdash \tau[\mu\overline{\alpha}.\tau/\overline{\alpha}] \preceq \mathsf{q}}{\Sigma \vdash (\mathsf{q}, \mathtt{fold}\, l) : {}^{\mathsf{q}}(\mu\overline{\alpha}.\tau)}$$

3.11 $\vdash \sigma : \Sigma$

$$\frac{}{\vdash \bullet : \bullet} \qquad \frac{\vdash \sigma : \Sigma_\star \qquad \bullet \vdash \Sigma_v \odot \Sigma \rightsquigarrow \Sigma_\star \qquad \Sigma_v \vdash (\mathsf{q}, v) : \tau}{\vdash \sigma, l \mapsto (\mathsf{q}, v, f) : \Sigma, l \mapsto \tau}$$

$$\frac{\vdash \sigma : \Sigma \qquad \mathsf{q} \sqsubseteq \mathsf{R}}{\vdash \sigma, l \mapsto (\mathsf{q}, v, \mathsf{used}) : \Sigma} \qquad \frac{\vdash \sigma : \Sigma \qquad \mathsf{q} \sqsubseteq \mathsf{A}}{\vdash \sigma, l \mapsto (\mathsf{q}, v, f) : \Sigma}$$

4 Safety

Theorem 1 (Preservation)

If $(\sigma_1, e_1) \longmapsto^* (\sigma_2, e_2)$ *and* $\vdash \sigma_1 : \Sigma_1$ *and* $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$,
then there exists Σ_2 *such that* $\vdash \sigma_2 : \Sigma_2$ *and* $\bullet; \bullet; \Sigma_2 \vdash e_2 : \tau$.

Theorem 2 (Progress)

If $\vdash \sigma_1 : \Sigma_1$ *and* $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$,
then either there exists l *such that* $e_1 \equiv l$
or there exists σ_2 *and* e_2 *such that* $(\sigma_1, e_1) \longmapsto (\sigma_2, e_2)$.

Theorem 3 (Safety)

If $\vdash \sigma_1 : \Sigma_1$ *and* $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$ *and* $(\sigma_1, e_1) \longmapsto^* (\sigma_2, e_2)$,
then either there exists l *such that* $e_2 \equiv l$
or there exists σ_3 *and* e_3 *such that* $(\sigma_2, e_2) \longmapsto (\sigma_3, e_3)$.