3D Pose Estimation from an n-Degree Planar Curved Feature in Two Perspective Views

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Abstract

3D pose estimation from 2D image data is a fundamental problem in computer vision. In this paper, a pose estimation method based on planar-curved features on the surface of an object is presented. This method is linear and generally applicable to any high degree(≥ 2) planar-curved features. So far, the 3D pose estimation methods presented in the literature are based either on point/line features or quadratic-curved features. The methods based on point and line features have to solve the correspondence problem. As there are so many edge points and line segments in an image, to establish the correspondences of these primitives between images is very difficult and time consuming in practice. The methods based on quadratic-curved features have to solve non-linear equations which normally result in many pseudo solutions. To eliminate these extra solutions and choose the right solution is very difficult. Our method is advantageous in these aspects.

1 Introduction

Inferring 3D information from 2D image data is a fundamental problem in computer vision. 3D pose estimation of objects in the scene has been addressed considerably in the literature. Most of the work on 3D pose estimation focuses on polyhedral objects, while some of it tries to deal with curved objects.

For curved objects, pose estimation methods are based on either local features such as the object boundary and edges [1,2] or the object surface itself [3].

For polyhedral objects, the 3D pose estimation methods presented in the literature are based on either point/line features or curved features.

The methods based on point features fall into three categories, which is a 3-point problem [4], a 4-point problem [5] or an N-point problem [6]. The methods based on line features are mathematically similar to the methods based on point features [7]. The method proposed in [8] tends to unify the methods based on point/line features.

Using curved features to determine 3D pose of polyhedral objects is another popular approach [9,10,11,12]. In [9] Haralick et al developed a general method for 3D location estimation based on quadratic-curved features. They consider two cases: either the shape and the size of a quadratic feature are both known, or only its shape is known. Due to the nonlinearity of their optimization process which generally yields a local minimum depending on the initial estimates, it is difficult for them to select the right solution from the many possible pseudo solutions. In [10,11], R. Safaee-Rad et al presented a 3D pose estimation method based on a quadratic-curved feature. Their method is a big improvement as they resolved the ambiguities that existed in [9]. But still they have to solve nonlinear equations which result in extra solutions. In [12], we presented a pose determination method based on conics for a CAD based vision system in which the object models are available and this makes it possible to recognize 3D objects and to determine their poses from a single image. It is shown that if there exist two conics on the surface of an object, the object's pose can be determined by an efficient one-dimensional search. In particular, if two conics are coplanar, a closed-form solution of the object's pose is presented. In this paper, we do not assume a CAD based vision system and a conic feature on the surface. However, we do assume that the world around us is piecewise planar with rich surface patterns. Our point for doing so is that planar-curved features (including quadratic-curved features) are very popular in human made objects [12,13].

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2 3D pose Estimation Based on High Degree Planar-Curved Features

2.1 Problem Formulation

Our 3D pose estimation method can be devided into two steps. First, the 3D orientation of the planar-curved feature's plane is estimated. Second, based on the estimated orientation, the 3D position and the third orientation parameter of the feature is computed. The second step is easy and generally varies from different planar-curved features. Therefore in this paper only the first step is considered.

The problem of 3D-orientation estimation of an object surface which contains the same N-degree planar-curved feature across two views can be expressed as follows: Given camera calibration results, two images of the feature, it is required to estimate the 3D orientation of the feature's plane with respect to the camera coordinate system. This problem can be further reduced to the following one: Given two 3D hypersurfaces defined by two bases (the perspective projections of a high degree planar-curved feature in two image planes) and two vertices (the origins of the two camera coordinate systems) with respect to a camera coordinate system (the first or second one), it is required to determine the 3D orientation of a plane (with respect to the same coordinate system) which intersects the two hypersurfaces and generates a common curve identical to the high degree planar-curved feature.

2.2 Solution in the Canonical Coordinate Systems

The canonical coordinate systems are defined as any pair of coordinate systems as long as the following conditions are met: (I) each of their two origins coincides with each of the origins of the original camera coordinate systems; (II) one of their axes (in this paper it's the X axis) are aligned with the translation vector of the original two camera coordinate systems; (III) the Y (Z) axis of one canonical coordinate system parallels to the Y (Z) axis of the other canonical coordinate system.

The two hypersurfaces expressed in the canonical coordinate systems are as follows:

$$f_a: \quad \sum_{i+j+k=n} a_{ijk} x_l^i y^j z^k = 0 \tag{1}$$

$$f_b: \quad \sum_{i+j+k=n} b_{ijk} x_r^i y^j z^k = 0 \tag{2}$$

where $x_l = x_r + d$, d is the distance between the left and right canonical coordinate systems. Given equations (1) and (2), it is required to find the parameters r, s, t of the following plane equation:

$$r x_r + sy + tz + d = 0 \tag{3}$$

Theorem 1

Equations (1) and (3) represent the planar-curved feature. From the viewpoint of the right canonical coordinate system, hypersurface f_b has the following expression:

$$\begin{aligned}
& a_{n00}(1-r)^n x_r^n + [C_n^1 a_{n00}(-s) + a_{n-1,10}](1-r)^{n-1} x_r^{n-1} y + \\
& (C_n^2 a_{n00} s^2 - C_{n-1}^1 a_{n-1,10} s + a_{n-2,20})(1-r)^{n-2} x_r^{n-2} y^2 + \\
& (C_n^1 a_{n00}(-t) + a_{n-1,01})(1-r)^{n-1} x_r^{n-1} z + \\
& (C_n^2 a_{n00} t^2 - C_{n-1}^1 a_{n-1,01} t + a_{n-2,02})(1-r)^{n-2} x_r^{n-2} z^2 + \\
& (2 C_n^2 a_{n00} st - C_{n-1}^1 a_{n-1,10} t - C_{n-1}^1 a_{n-1,01} s + a_{n-2,11}) \\
& (1-r)^{n-2} x_r^{n-2} yz + \cdots = 0
\end{aligned} \tag{4}$$

Altogether it has $(n+1)(n+2)/2$ terms. This theorem can be easily (although very tedious) proved by applying Wu's method proposed in [14].

Equations (4) and (2) represent the same hypersurface. By comparing the coefficients of the two equations, we obtain $(n+1)(n+2)/2$ equations. We can always choose a canonical pair of coordinate systems such that: $a_{n00} b_{n00} \neq 0$. Let $\alpha_{ijk} = a_{ijk}/a_{n00}$, $\beta_{ijk} = b_{ijk}/b_{n00}$, we have :

$$\begin{cases}
C_n^1(-s) + \alpha_{n-1,10} = \beta_{n-1,10}(1-r) \\
C_n^2 s^2 - C_{n-1}^1 \alpha_{n-1,10} s + \alpha_{n-2,20} = \beta_{n-2,20}(1-r)^2 \\
C_n^1(-t) + \alpha_{n-1,01} = \beta_{n-1,01}(1-r) \\
C_n^2 t^2 - C_{n-1}^1 \alpha_{n-1,01} t + \alpha_{n-2,02} = \beta_{n-2,02}(1-r)^2 \\
2 C_n^2 st - C_{n-1}^1 \alpha_{n-1,10} t - C_{n-1}^1 \alpha_{n-1,01} s + \\
\alpha_{n-2,11} = \beta_{n-2,11}(1-r)^2
\end{cases} \tag{5}$$

A simple manipulation of the above equations shows that:

$$\frac{(C_n^1)^2 \alpha_{n-2,20} - C_n^2 \alpha_{n-1,10}^2}{(C_n^1)^2 \beta_{n-2,20} - C_n^2 \beta_{n-1,10}^2} = \frac{(C_n^1)^2 \alpha_{n-2,02} - C_n^2 \alpha_{n-1,01}^2}{(C_n^1)^2 \beta_{n-2,02} - C_n^2 \beta_{n-1,01}^2} = \frac{(C_n^1)^2 \alpha_{n-2,11} - 2 C_n^2 \alpha_{n-1,10} \alpha_{n-1,01}}{(C_n^1)^2 \beta_{n-2,11} - 2 C_n^2 \beta_{n-1,10} \alpha_{n-1,01}} = k \tag{6}$$

$$\begin{cases} r = 1 \pm \sqrt{k} \\ s = \frac{1}{n}(\alpha_{n-1,10} \pm \beta_{n-1,10}\sqrt{k}) \\ t = \frac{1}{n}(\alpha_{n-1,01} \pm \beta_{n-1,01}\sqrt{k}) \end{cases} \quad (7)$$

We see that the relationship for conics proposed in [14] is only a special case of the general high degree planar-curved feature.

Equation (6) generally establishes correspondence between different high degree planar-curved features. We propose the following correspondence criterion.

$$\sigma_{pq}^2 = \sum_{i,j=1,2,3,i\neq j} \frac{(k_{ip}-k_{jq})^2}{3} \quad (8)$$

Where

$p,q=1,2,\cdots,m$.

$$k_{1p} = \frac{(C_n^1)^2 \alpha_{n-2,20p} - C_n^2 \alpha_{n-1,10p}^2}{(C_n^1)^2 \alpha_{n-2,02p} - C_n^2 \alpha_{n-1,01p}^2};$$

$$k_{2p} = \frac{(C_n^1)^2 \alpha_{n-2,20p} - C_n^2 \alpha_{n-1,10p}^2}{(C_n^1)^2 \alpha_{n-2,11p} - 2C_n^2 \alpha_{n-1,10p}\alpha_{n-1,01p}};$$

$$k_{3p} = \frac{(C_n^1)^2 \alpha_{n-2,02p} - C_n^2 \alpha_{n-1,01p}^2}{(C_n^1)^2 \alpha_{n-2,11p} - 2C_n^2 \alpha_{n-1,10p}\alpha_{n-1,01p}};$$

The expressions of k_{1q}, k_{2q}, k_{3q} are similar to the expressions of k_{1p}, k_{2p}, k_{3p}. m is the number of high degree planar-curved features. For the left p-th curved-feature, the q-th curved feature in the right image which minimizes the above criterion is its correspondence. In case of ambiguity, the other $(n+1)(n+2)/2-5$ equations should be considered.

Equation (7) gives two solutions. one solution can be obtained by imposing a visibility constraint.

Suppose we have obtained two planes (P_1, P_2 denote their surface norms). Let M be an arbitrary point on the plane. The one that satisfies the following constraint is the actual physical planar surface on which the high degree planar-curved feature lies.

$$(\overrightarrow{O_1M} \bullet \overrightarrow{P_i}) \bullet (\overrightarrow{O_2M} \bullet \overrightarrow{P_i}) > 0, \quad (i=1,2) \quad (9)$$

In our real experiment, the plane thus obtained is used as an initial solution. A much more accurate solution is obtained by Newton iteration method which takes the rest of the redundant equations into account.

2.3 Solution in the Reference Camera Coordinate System

Once the parameters r, s, t in the canonical coordinate system is obtained, a simple coordinate transformation can bring the solution back to the original camera coordinate system.

3 Experiments

In our paper, without loss of generality, we take cubic planar-curved features as an example of high degree planar-curved features. In the canonical coordinate systems, the hypersurfaces and the plane on which the high degree planar-curved feature lies are expressed as follows:

$$\begin{cases} x_l^3 + C_1 y^3 + C_2 z^3 + C_3 x_l^2 y + C_4 x_l y^2 + C_5 x_l^2 z + \\ C_6 x_l z^2 + C_7 y^2 z + C_8 y z^2 + C_9 x_l y z = 0 \\ x_r^3 + C_1 y^3 + C_2 z^3 + C_3 x_r^2 y + C_4 x_r y^2 + \\ C_5 x_r^2 z + C_6 x_r z^2 + C_7 y^2 z + C_8 y z^2 + C_9 x_r y z = 0 \end{cases} \quad (10)$$

$$r x_r + s y + t z + d = 0 \quad (11)$$

Applying equations (6) and (7), we have:

$$\frac{3C_4 - C_3^2}{3D_4 - D_3^2} = \frac{3C_6 - C_5^2}{3D_6 - D_5^2} = \frac{3C_9 - 2C_3C_5}{3D_9 - 2D_3D_5} = k \quad (12)$$

$$\begin{cases} r = 1 \pm \sqrt{k} \\ s = \frac{C_3 \pm D_3\sqrt{k}}{3} \\ t = \frac{C_5 \pm D_5\sqrt{k}}{3} \end{cases} \quad (13)$$

Let the following equation represents the plane in the first camera frame:

$$a x_l + b y_l + c z_l + d = 0 \quad (14)$$

$[a \;\; b \;\; c]$ relates $[r \;\; s \;\; t]$ by the following equation:

$$[a \;\; b \;\; c] = \frac{1}{1-r}[r \;\; s \;\; t] R_l \quad (15)$$

Where R_l is the rotation matrix which transforms the coordinates of the first camera frame into the first canonical frame.

We obtain two images (see Fig. 1) from our two CCD cameras mounted on our mobile vehicle. The calibration result is obtained by the method proposed in [15]. The cubic planar-curved feature is fitted by a least mean square method. In our real experiment, we directly fit the hypersurface in the canonical coordinate system. The points used for fitting are obtained by transforming the points in the retinal plane (assuming f=1) into the canonical coordinate systems. Table 1&2 are the results for correspondence and 3D orientation estimation respectively. Comparing the results of the estimated 3D orientation of the planar surface with those derived directly from the calibration results, we see our estimation results are quite accurate.

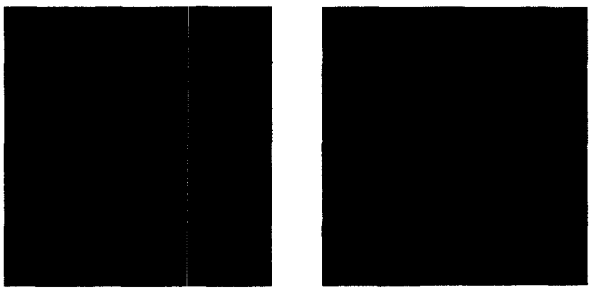

Figure 1. Two real images

σ_{ij}^2 i, j = 1,2	1st Curved Feature in Left Image	2nd Curved Feature in Left Image
1st Curve in Right Image	6.633×10^{-7}	3.051×10^{3}
2nd Curve in Right Image	3.052×10^{3}	2.021×10^{-4}

Table 1. Values of the Correspondence Criterion

3D Orientation	Horizontal Plane	Vertical Plane
Calibration Results	0.439,–0.0125,0.364	0.306,–0.401,–0.382
Estimated Results	0.440,–0.0112,0.364	0.311,–0.404,–0.382

Table 2. 3D Orientation of the Planar Surfaces

4 Conclusion

We have presented a pose estimation method based on planar-curved features on the surface of an object. This method is linear and generally applicable to any high degree(≥ 2) planar-curved features. It's soundness has also been verified by our experimental results.

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