Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad Hoc Networks

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Abstract— **The topology of wireless multihop ad hoc networks can be controlled by varying the transmission power of each node. We propose a simple distributed algorithm where each node makes local decisions about its transmission power and these local decisions collectively guarantee global connectivity. Specifically, based on the directional information, a node grows it transmission power until it finds a neighbor node in every direction. The resulting network topology increases network lifetime by reducing transmission power and reduces traffic interference by having low node degrees. Moreover, we show that the routes in the multihop network are efficient in power consumption. We give an approximation scheme in which the power consumption of each route can be made arbitrarily close to the optimal by carefully choosing the parameters. Simulation results demonstrate significant performance improvements.**

I. Introduction

The lifetime of a wireless network that is operating on battery power is limited by the capacity of its energy source. For increasing longevity of such networks and thus increasing their usefulness, it is imperative that we find ways of either increasing battery power or alternative tether-less sources of energy that nodes in a wireless network can use. A complementary approach to tackling the network longevity problem is to develop energy-efficient algorithms and mechanisms that optimize the use of the battery power while maintaining network connectivity.

Generally speaking, a node in a wireless network independently explores its surrounding region and establishes connections with other neighboring nodes that are within its transmission and reception range. In establishing these local connections, it is desirable to choose only those local connections that will guarantee overall global network connectivity while satisfying different and often contradictory performance metrics such as overall throughput, network utilization, and power dissipation. Unlike wired networks, each node in a multihop wireless network can potentially change its set of one-hop neighbors and consequently the overall network topology by simply changing its transmission and receive power. Without proper topology control algorithms in place a randomly connected multihop wireless ad hoc network may suffer from poor network utilization, high end-to-end delays, and short network lifetime.

Although the problem domain is fairly clear, there has been only a limited amount of work in the general area of topology control and network design for increasing network longevity. Hu [1] describes a distributed, Delaunay triangulation-based algorithm for choosing logical links and as a consequence carrying out topology control. In choosing these links he follows a few heuristic guidelines such as not exceeding an upper bound on the degree of each node and choosing links that create a regular and uniform graph structure. He does not take advantage of adaptive transmission power control. Ramanathan and Rosales-Hain [2] describe a centralized spanning tree algorithm for creating connected and bi-connected static networks with the objective of minimizing the maximum transmission power for each node. Additionally, they describe two distributed algorithms, that adjust the node transmit power to maintain network connectivity. Their reasoning and algorithms are based on simple heuristics and consequently do not guarantee network connectivity in all cases. Rodoplu and Meng [3] propose an ingenious distributed topology control algorithm that guarantees connectivity of the entire network. Their algorithm relies on a simple radio propagation model for transmit power roll-off as $1/d^n, n \geq 2$. Using this they achieve the minimum power topology, which contains the minimum-power paths from each node to a designated master-site node.

Other researchers working in the field of packet radio networks, wireless ad hoc networks, and sensor networks have also considered the issue of power efficiency and network lifetime but have taken different approaches. For example, Hou and Li [4] analyze the effect of adjusting transmission power to reduce interference and hence achieve higher throughput as compared to schemes that use fixed transmission power [5]. Heinzelman et al. [6] describe an adaptive clustering-based routing protocol that maximizes network lifetime by randomly rotating the role of per-cluster local base stations (cluster-head) among nodes with higher energy reserves.

In this paper we describe our approach to tackling the network longevity problem. Specifically, we describe a novel distributed *c*one-based topology control algorithm that increases network lifetime while maintaining global connectivity with reasonable throughput in a multihop wireless ad hoc network. Network lifetime is increased by determining the minimal operational power requirement for each node in the network while guarantying the same maximum connected node set as when all nodes are transmitting with full power. In contrast to previous approaches that rely on knowing and sharing the global position information of the nodes in the network, our algorithm is a distributed algorithm that relies solely on local information, using directional information of incoming signals from neighboring nodes. We show the validity of our algorithm both theoretically and via simulation. We show that the routes in the multihop network are efficient in power consumption. We give an approximation scheme in which the power consumption of each route can be made arbitrarily close to the optimal by carefully choosing the parameters.

Our work is similar to [3], in that we have the same goal as them - of designing location-based, distributed topology-control algorithm that increases network lifetime. We designed our algorithm with the following objectives in mind: (1) Each node in the multihop wireless network must use local information only for determining its transmission radius and hence its operational power. The local decisions must be made in such a way that they collectively guarantee the node connectivity in the global topology just as if all nodes were operating at full power; (2) As in [3] our algorithm must minimize power consumption by finding minimum power paths, and thus indirectly increases network lifetime; (3) Our algorithm must find a topology with small node degree, so that interference is minimal and hence throughput is sufficient. (4) Our algorithm must be simple and efficient so that it is suitable for small and mobile (sensor) nodes. (5) Finally, our algorithm must make very few assumptions about the radio propagation model and/or on the hardware of each node (e.g. non-availability of Global Positioning System).

We describe and analyze our *c*one-based topology control algorithm, which meets these objectives. Our algorithm is designed specifically for multihop wireless ad hoc networks deployed on a 2-dimensional surface. It consists of two phases, which are summarized as follows: Starting with a small radius, each node (denoted by Node u) broadcasts a neighbor-discovery message. Each receiving node acknowledges this broadcast message. Node u records all acknowledgments and the directions they came from. (We assume that the node can determine the direction of the sender when receiving a message.) It then determines whether there is at least one neighbor in every cone of α degrees, centered on Node u. In this first phase, Node u continues the neighbor discovering process by increasing its transmission radius (operational power) until either the above condition is met or the maximum transmission power P is reached. We prove that, for α smaller than or equal to $2\pi/3$, the algorithm guarantees maximum connected node set. For smaller angles we also can guarantee good minimum power routes. In the second phase, the algorithm performs a redundant edge removal process without impacting the connectivity. This phase is designed to reduce the node degrees, which helps in reducing interference and enhancing throughput [4]. Redundant edge removal is carried out without deteriorating the minimum power routes of the network.

Our work is different from Rodoplu and Meng [3] in the following way: First, our algorithm guarantees that the maximum connected set of nodes for the network will always be found. Second, our algorithm is computationally less demanding, and we do not need to specify a deployment region, which is an important consideration for the case when nodes regularly change deployment region. Third, our algorithm does not need exact location information but only directional information. This can be a factor when cost of nodes is a consideration. Forth, our algorithm is not coupled with any radio propagation model. Due to the large influence of environmental factor on radio frequency communications radio propagation models can be notoriously inaccurate. Finally, fifth our algorithm is able to give a worst-case analysis for both, the minimum power routes and the maximum node degrees in the network.

The rest of our paper is organized as follows. In section II we describe our network model and the assumptions we make about the environment. In Section III we describe the cone-based topology control algorithm in detail. In Section IV we prove the correctness of our algorithm. In Section V we demonstrates that our algorithm is competitive with respect to minimum energy path metric. In Section VI we present the results from our performance evaluation of the algorithm. Finally we conclude in Section VII.

II. Model

We are given a set V of n nodes (points in the plane). A node consists of a power supply entity, a processor and local memory to perform simple local computations, and a radio communication unit to send and receive messages. A node does not know its position.

A node is able to send a *broadcast* message with arbitrary power p. It is called broadcast because the sending node has no control over the direction in which the message is transmitted. Nodes can vary their broadcast power, but not beyond a maximum power P, that is $0 \leq p \leq P$. We assume the existence of an underlying MAC layer that resolves interference problems. For example, if node u broadcasts with power p, the nodes that can receive node u's broadcast message (the set N) will acknowledge (with another broadcast message) to node u. After having received acknowledgments of all nodes in N, node u knows the set N. The assumption to have a reliable broadcast is not needed for the correctness of our algorithm, but it simplifies the presentation.

We assume that the radio communication unit is able to determine the direction of the sender when receiving a message. Thus, if two nodes u, v exchange a broadcast/acknowledge message pair, both of them know which direction the other node is, that is, node u knows that node v is in direction ρ, and node v knows that node u is in direction $\rho + \pi$, with with $0 \leq \rho < 2\pi$. Techniques to estimate direction without positioning information are available, and discussed in the IEEE antenna and propagation community as the Angle-of-Arrival (AOA) problem. It can be accomplished by using more than one directional antenna. We refer to [7]. If the radio communication unit is not capable to conclude the direction of a message, we can alternatively supply a node with a more abundant global positioning unit, and calculate directions from positions piggybacked to messages.

Compared with [3], we have a weak physical radio propagation model. We assume that the environment is not obstructed, and that the nodes are homogeneous. More formally, we assume that the power p is a uniform and non-decreasing, but *unknown* function of the distance d. Due to uniformity, if a node u can reach node v with power $p' \geq p$, then node v can also reach node u with power $p'' \geq p$. In other words, a node u can figure out how much power is needed to communicate with node v but cannot deduce the distance of v. Power models like Rician are intuitively appealing, but it is very difficult to determine the model parameters such as the local mean of the scattered power and the power of dominant component precisely as this requires physically isolating the direct wave from the scattered components. In order to keep our system simple and easy to deploy, we decided against models that are unduly complex. For an excellent discussion on the applicability of other power models we refer to Section 3 of [3].

III. Algorithm

Our algorithm has two phases. In the first phase we describe a decentralized scheme that builds a connected graph upon our node network by letting nodes find close neighbor nodes in different directions. The second phase improves the performance by eliminating non-efficient edges in the communication graph. The algorithm is simple and does not need any complicated operations. The algorithm is also distributed and without synchronization. The two phases are merely for the ease of description.

The first phase of the algorithm: Each node u beacons with growing power p, initially $p = \epsilon$. If node u discovers a new *neighbor* node v, node u will put v into its local set of neighbors $N(u)$. Node u will continue to grow the transmission power until the neighbor set $N(u)$ is big enough such that, for any *cone* with angle α there is at least one neighbor $v \in N(u)$, or until node u hits the maximum transmission power P. The termination criterion can be easily determined. For a given node u, each neighbor $v \in N(u)$ covers a cone, as in Figure 1. If the union of these cones cover the whole 2π angle, node u goes to phase 2.

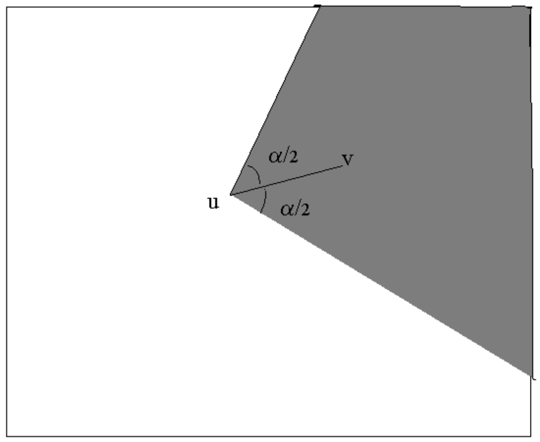


Fig. 1. Coverage determination

Node u might use heuristics in order to optimize conflicts on the lower MAC layer. For example, node u will grow the transmission power so that exactly one new neighbor is expected to acknowledge, given the probability distribution of the nodes in the plane. Moreover, node u might include meta-information in its broadcast, in order to prevent already established neighbors to answer again, or in order to find new neighbors in a specific direction, where no neighbor has been found yet. These optimizations are not essential for the correctness of our algorithm;

they are subject of future work.

For node u, let $p(u)$ be the minimum power to find a neighbor in every cone with angle α, or $p(u) = P$. Depending on our goal we will later specify α to be at most $2\pi/3$ (correctness only) or at most $\pi/2$ (competitive performance). If a node u with maximum transmission power P has a cone $C = [\rho, \rho + \alpha]$ without any node in $N(u)$, then node u will decrease its transmission power, back to the minimum power p such that there is no cone without a neighbor that has a neighbor when transmitting with maximum power P.

The algorithm is symmetric, that is, if node u wants node v to be in its neighbor-set, then node v also needs to put node u in its neighbor-set.

From the algorithm description of phase 1 we conclude:

Fact III.1: For each node u and for each angle ρ ($0 \leq \rho < 2\pi$), if there is a node v in the cone $C = [\rho, \rho + \alpha]$ when sending with maximum power P, then there is a node v' in the cone C when sending with minimum power $p(u)$.

Because of the simple nature of the first phase of the algorithm there is room for improvement.

The second phase of the algorithm: If node u has two neighbor nodes $v, w \in N(u)$, such that the power needed to send from u to w directly is not less than the total power to send via v, we can remove w from $N(u)$. More formally, if there are two nodes v, w with $v, w \in N(u)$ and $w \in N(v)$, and $p(u, v) + p(v, w) \leq p(u, w)$, then we remove node w from $N(u)$.

This improvement gives us less neighbors, while keeping all the best routes. We can determine two neighbors v, w for which this basic power inequality holds by some simple local exchange of the transmission powers, or, if distances and power model are known, by a simple local computation step without any message exchange.

It is believed that, from a performance point of view, a node should have as few neighbors as possible. Thus we might consider removing nodes from our neighborhood even though a direct transmission uses less power than an indirect. One good candidate for removal is a neighbor node v that is in great distance of the sending node u, since, whenever u transmits to distant neighbor v, many other nodes experience interference.

We extend the first idea in the following way: If there are two nodes v, w with $v, w \in N(u)$ and $w \in N(v)$ and $p(u, v) \leq p(u, w)$, and $p(u, v) + p(v, w) \leq q \cdot p(u, w)$, then we remove w from $N(u)$ (and by symmetry also u from $N(w)$). If there is more than one node v that satisfies the power inequality for node w, we chose the node with minimum $p(u, v)$. By traversing the neighbor nodes with increasing power distance (with ties broken by identifier), we make sure that the edge (u, v) will stay. Note that if constant $q = 1$ we only remove edges that use more power than an indirect path.

From the algorithm description of phase 2 we conclude:

Fact III.2: For each node u, if there was a neighbor node $w \in N(u)$ after the first phase of the algorithm, there is a neighbor $v \in N(u)$ after the second phase of the algorithm such that $p(u, v) + p(v, w) \leq q \cdot p(u, w)$, for a constant $q \geq 1$.

Note that after phase 2 of the algorithm Fact III.1 is not necessarily true anymore.

Let us sum up this section. We have presented an algorithm that, starting from a set of nodes V, builds an undirected graph $G = (V, E)$ such that there is an edge $e = (u, v)$ if and only if $v \in N(u)$ (and because of symmetry also $u \in N(v)$). This graph G has several advantageous properties, which will be proven in the next two sections of this paper:

- If $\alpha \leq 2\pi/3$, the graph G will be connected if it was connected when all nodes broadcast with maximum power P.
- For a reasonable class of power cost functions and for $\alpha \leq \pi/2$ we will show that the graph G has very good power consumption, in fact within an arbitrarily small constant factor of the optimal (achieved by a much more complicated algorithm).
- The degree of any node can be bounded by a constant, for $q \geq 2$.

IV. Correctness

In this section we will prove that an angle $\alpha \leq 2\pi/3$ is sufficient to make the graph G connected.

Definition IV.1: A *path* p of nodes is an ordered set $(u_1, u_2, \ldots, u_k)$ of nodes such that there is an edge between consecutive nodes: $e = (u_i, u_{i+1})$ for $i = 1, \ldots, k-1$ with $e \in E$. A graph is *connected* if there is a path from any node to any other node in the graph.

Definition IV.2: The *distance* of two nodes is their Euclidean distance in the plane. Let $G = (V, E)$ be the graph constructed by our algorithm. On the other hand, let $G' = (V, E')$ be the connection graph when all nodes always beacon with maximum power P.

Theorem IV.3: We have $\alpha \leq 2\pi/3$. Let G' be connected. Then graph G will be connected.

Proof: We prove the first phase of the algorithm by contradiction. Assume that graph G is not connected, while G' is. Then there exists a least a pair of nodes such that there is no path between the pair. Let the nodes u,v be the pair with minimum power to beacon each other, that is $p(u, v) \leq p(u', v')$ for any pair of nodes u',v' without a path. Since G' is connected we know that $p(u, v) \leq P$.

Let $d := p^{-1}(u,v)$, that is with power $p(u,v)$ one can reach distance d. The algorithm has given node u minimum transmission power $p(u)$. Since there is no edge $e=(u,v)$ we have $p^{-1}(u) < d$.

The remainder of the proof is geometric. Let w be a neighbor node of u. We construct a triangle of the nodes u, v, w, such as in Figure 2.

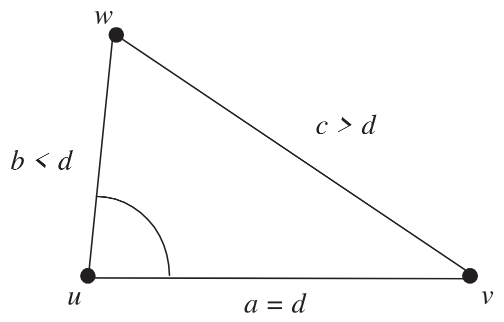


Fig. 2. Triangle u, v, w

A basic triangle result is $c^2 = a^2 + b^2 - 2ab\cos\gamma$. We have $a = d(u,v) = p^{-1}(u,v) = d$, $b = d(u,w) = p^{-1}(u,w) \leq p^{-1}(u) < d$, and $c = d(v,w) = p^{-1}(v,w) \geq p^{-1}(u,v) \geq d$. We are interested in the angle γ which is on the opposite of side c.

We get immediately:

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab} \leq \frac{b^2}{2db} < \frac{1}{2}.$$

and thus $\gamma > \pi/3 \geq \alpha/2$. Therefore there is no node $v' \in N(u)$ in the cone $C = [-\alpha/2, +\alpha/2]$; this contradicts Fact III.1. By symmetry the same holds for v.

The second phase of the algorithm does not destroy connectivity since we only remove an edge (u,w) when we made sure that there are edges $(u,v) \in E$ and $(v,w) \in E$. ■

V. Competitive Analysis

In this section we will show that our algorithm is not only correct (results in a connected graph), but also that the routes that can be found in the graph are very power efficient. In this section, we need to make stronger assumptions on the power model.

Definition V.1: A direct transmission from node u to node v costs power $p = p(u,v)$, where p is a function of the distance $d = d(u,v)$. Any function $p(d)$ is eligible as long as $cd^x \leq p(d) \leq czd^x$, for parameters c, x, z, all independent of d, and with $z \geq 1$, and $x \geq 2$.

Definition V.2: The *power consumption* of route $r = (s = u_1, u_2, \ldots, u_k = t)$ is $C(r) = \sum_{i=1}^{k-1} p(u_i, u_{i+1})$. Let G' be the graph when all nodes transmit with maximum power P, as defined in Definition IV.2. For given source node s and sink node t, let r^* be a route such that $C(r^*) \leq C(r)$, for any eligible route r in G'. Then route r^* is a *minimum power route* in G'.

After our algorithm has done the neighborhood detection as described in Facts III.1 and III.2, a routing algorithm is applied that finds minimum power routes in the graph G. In other words, nodes keep tables that tell them to which neighbor they should send in order to route a message to a given destination node. These tables are generally small since the geometry of the plane can be used [8]. (Usually nodes in a destination region will be sent to the same neighbor.) We directly get:

Definition V.3: Let G be the graph constructed by our algorithm, as defined in Definition IV.2. For given source node s and sink node t, let $\hat{r}$ be a route such that $C(\hat{r}) \leq C(r)$, for any eligible route r in G. Then route $\hat{r}$ is a *minimum power route* in G.

Lemma V.4: We are given a triangle where angle $\gamma \geq \pi/2$. Then $a^x + b^x \leq c^x$ for $x \geq 2$.

Proof: We have $\cos\gamma \leq 0$. With $c^2 = a^2 + b^2 - 2ab\cos\gamma$, we get $a^2 + b^2 \leq c^2$. From $a/\sin\alpha = b/\sin\beta = c/\sin\gamma$ we know that $0 \leq a, b \leq c$ and we directly get $a^x + b^x \leq a^2c^{x-2} + b^2c^{x-2} = (a^2+b^2)c^{x-2} \leq c^x$. ■

Lemma V.5: We have a triangle with nodes A, B, C, edges a, b, c and angles α, β, γ. Let $b < c$ and $b < a$ and $\alpha \leq \pi/4$. Then $\gamma \geq \pi/2$, and $a < c$.

Proof: With $a/\sin\alpha = b/\sin\beta$ and $b < a$ we know that $\beta < \alpha$. With $\alpha \leq \pi/4$ we get $\gamma = \pi - \alpha - \beta \geq \pi/2$, and with $a/\sin\alpha = c/\sin\gamma$ we get $a < c$. ■

Lemma V.6: We have a triangle with nodes A, B, C, edges a, b, c and angles α, β, γ. Let $a \leq b < c$ and $\alpha \leq \pi/4$. Then $a^x + b^x \leq c^x(1 + 2\sin(\alpha/2))$ for $x \geq 2$.

Proof: We know that $\gamma \geq (\pi - \alpha)/2$. With $c^2 = a^2 + b^2 - 2ab\cos\gamma$ and $a \leq b < c$, we get

$$a^2 + b^2 < c^2 + 2c^2\cos\gamma \leq c^2(1 + 2\sin(\alpha/2)).$$

We use the same method as in Lemma V.4 to extend this result for $x \geq 2$. ■

Theorem V.7: We have $\alpha \leq \pi/2$. Let s be a source node and t be a sink node. Let $C(\hat{r})$ resp. $C(r^*)$ be the minimum power routes in G resp. G', as in Definitions V.2 and V.3. Then $C(\hat{r}) \leq C(r^*)zq(1 + 2\sin(\alpha/2))$, for z from the radio model V.1 and q from Fact III.2.

Proof: First we consider phase 1 of the algorithm:

The minimum power route r^* is an ordered set of nodes $r^* = (s = u_1, u_2, \ldots, u_k = t)$, where $p(u_i, u_{i+1}) \leq P$ for all $i = 1, \ldots, k-1$.

In this proof we will show that our algorithm finds a path

$$\begin{aligned} r &= (s = u_1, u_1^1, u_1^2, \dots, u_1^{l_1}, u_2, u_2^1, u_2^2, \dots, u_2^{l_2}, \\ &\quad u_3, \dots, u_{k-1}, u_{k-1}^1, u_{k-1}^2, \dots, u_{k-1}^{l_{k-1}}, u_k = t), \end{aligned}$$

where u_i in r is the same node as u_i in r^*.

We focus on the path between u_i and u_{i+1}. Let $l = l_i$, and for convenience $u_i^0 = u_i$, for any i. Let us construct our path $(u_i^0, u_i^1, \dots, u_i^l, u_{i+1})$. We distinguish the following cases.

Case 1: Nodes u_i^j and u_{i+1} are neighbors in the graph G. Then $l = j$.

Case 2: Nodes u_i^j and u_{i+1} are not neighbors. Since there is a neighbor in each cone $[\rho - \alpha/2, \rho + \alpha/2]$ we know that node u_i^j has a neighbor node u_i^{j+1} such that the angle at node u_i^j (in the triangle $u_i^j, u_i^{j+1}, j_{i+1}$) is less than $\alpha/2$.

Case 2a: If $d(u_i^j, u_{i+1}) < d(u_i^{j+1}, u_{i+1})$, we know by Lemma V.5 that the angle at u_i^{j+1} is at least $\pi/2$, and that node u_i^{j+1} is strictly closer to u_{i+1} than node u_i^j was. Since there are only a finite number of nodes we will eventually arrive at node u_{i+1}, or get into one of the other cases.

Case 2b: If $d(u_i^j, u_{i+1}) \geq d(u_i^{j+1}, u_{i+1})$, we know that node u_{i+1} is a neighbor of node u_i^{j+1}. Thus $l = j + 1$.

Figure 3 shows an example of a path from u_i to u_{i+1}, where we have a series of cases 2a, followed by a single case 2b.

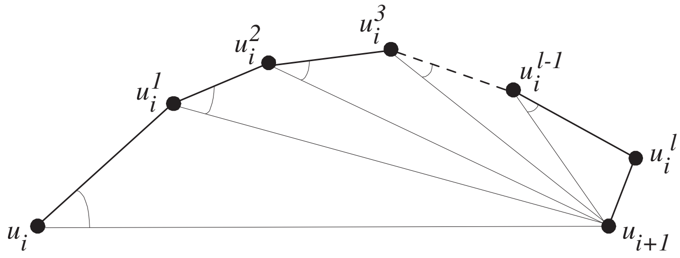


Fig. 3. Path from u_i to u_{i+1}

Let us calculate the cost of our path from u_i to u_{i+1}.

$$\begin{aligned} C(u_i, u_{i+1}) &= p(u_i, u_i^1) + p(u_i^1, u_i^2) + \dots \\ &\quad + p(u_i^{l-1}, u_i^l) + p(u_i^l, u_{i+1}). \end{aligned}$$

By using our radio model (Definition V.1) we know that $p(d) \leq czd^x$, thus

$$\begin{aligned} C(u_i, u_{i+1}) &\leq czd^x(u_i, u_i^1) + czd^x(u_i^1, u_i^2) + \dots \\ &\quad + czd^x(u_i^{l-1}, u_i^l) + czd^x(u_i^l, u_{i+1}). \end{aligned}$$

We know that all nodes except u_1^l are of case 2a, therefore we can apply Lemma V.4 repeatedly, and get

$$C(u_i, u_{i+1}) \leq czd^x(u_i, u_i^l) + czd^x(u_i^l, u_{i+1}).$$

By Lemma V.6 we know that

$$C(u_i, u_{i+1}) \leq czd^x(u_i, u_{i+1})(1 + 2\sin(\alpha/2)).$$

From the radio model (Definition V.1) we know that $cd^x \leq p(d)$, thus

$$C(u_i, u_{i+1}) \leq p(u_i, u_{i+1}) z (1 + 2\sin(\alpha/2)).$$

We can use the same analysis for all the pieces of the optimal path. With fact V.3 we get

$$C(\hat{r}) \leq C(r) \leq C(r^*) z (1 + 2\sin(\alpha/2)).$$

With Fact III.2, phase 2 of the algorithm might replace an edge with two edges such that the total power consumption is at most multiplied with a factor q. The Theorem follows directly. ∎

Corollary V.8: Let $z = q = 1$. In order to guarantee paths that use at most $1 + \epsilon$ of the power of the optimal paths we need $\alpha \leq 2\arcsin(\epsilon/2)$, which is roughly $\alpha \leq \epsilon$.

The second phase of the algorithm already helps to arrive at a sparse graph, as you can see in the simulation section. More formally:

Theorem V.9: Let q of phase 2 (confer Fact III.2) be not less than 2. Then the degree at any node is at most 6.

Proof: For node u, let v, w be two nodes in the neighborhood of u. Because of symmetry, either (i) $v \in N(w)$ and $w \in N(v)$, or (ii) both $v \notin N(w)$ and $w \notin N(v)$.

Case (i): If there are three nodes u, v, w such that they all are in each other's neighborhood, then phase 2 will at least remove the edge with maximum power between them. The largest angle γ in the triangle u, v, w is at least $\pi/3$. Therefore the edge that uses most power is at least the same size as the other two, and $q \geq 2$ would remove that edge.

Case (ii): We have $p(u, v) \leq p(v) < p(v, w) > p(w) \geq p(u, w)$. Therefore the side opposite of node u is the largest in the triangle u, v, w, and the angle at node u is the largest, i.e. larger than $\pi/3$.

In both cases any two nodes v, w in the neighborhood of u have at least angle $\pi/3$. There cannot be more than 6 neighbors. ∎

VI. Simulation Results and Evaluation

We measure the impact of our topology control on the network through simulations. To compare the performance of our algorithm with prior work in topology control, we would also want to simulate topology control algorithms in the literature. In a multihop wireless network, each node is expected to potentially send and receive messages from many nodes. Therefore an important requirement of such network is strong connectivity. Besides strong connectivity, the most important design metric of multihop wireless networks is perhaps energy efficiency. As it directly impact the network lifetime. As far as we know, among the topology control algorithms in the literature [5], [4], [1], [2], [3], only Rodoplu and Meng's algorithm [3] attempts to optimize for energy efficiency subject to maintaining strong network connectivity. The work in [5], [4], [1] tries to maximize network throughput. Their algorithms do not guarantee strong connectivity. Ramanathan and Rosales-Hain [2] has considered optimizing for the min-max transmission power in centralized algorithms, however their distributed heuristic algorithms do not guarantee strong connectivity. Therefore, we only compare with [3]. We refer to their algorithm as R&M. We refer to our basic algorithm as Phase1Only, and to our complete algorithm with ConeBased. Sometimes we give the parameter α, the size of the angle of the cone. As a reference, we also compare with the no topology control case where each node always uses the maximum transmission radius for broadcasting a packet (MaxPower). For example, the AODV [9] route request packet is sent using neighbor broadcast. Unicast packet only needs to use the minimum power to reach a given next hop. The use of maximum transmission radius for broadcast packets is the only way to avoid unnecessary partition if no topology control is used.

A. Simulation Environment

Our topology control algorithm is implemented in ns-2 [10], using the wireless extension developed at Carnegie Mellon [11]. Our simulation is done for a network of 100 nodes with WaveLAN-I radios. The nodes are placed uniformly at random in a rectangular region of 1500 by 1500 meters. There has been some work on realistic topology generation such as [12], [13]. However, their work has the Internet in mind. Since large multihop wireless networks such as sensor networks are deployed automatically, we believe uniform random assumption is valid in most such networks.

We assume the two-ray propagation model for terrestrial communications. It has a $1/d^4$ transmit roll-off [14]. The model has been shown to be close to reality in many environment settings [14]. The carrier frequency is $914MHz$, and the transmission raw bandwidth $2MHz$. We assume omni-directional antennas with $0dB$ gain ,and the antenna is placed 1.5 meter above a node. The receive threshold is $94dBW$. The carrier sense threshold is $108dBW$ and the capture threshold is $10dB$. These parameters simulate the 914MHz Lucent WaveLAN DSSS radio interface.

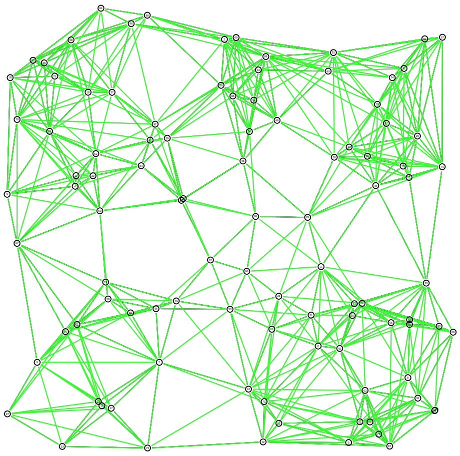
In order to simulate the effect of power control, we made changes to the physical layer of the ns-2 simulation code. Specifically, for every neighbor broadcast packet, a node's transmission power uses the final transmission power of its neighbor discovery process of each topology control algorithm. For every unicast packet, a node's transmission power uses the minimum power for the source to reach the destination, as determined during the neighbor discovery process. A node's energy reserve is then subtracted by the appropriate amount for any transmission and reception.

To simulate interference and collision, we choose the WaveLAN-I CSMA/CA MAC protocol. Since topology control is independent of routing, a routing protocol is needed. We choose AODV in our simulation. Other protocols to disseminate application data without an explicit routing protocol in sensor network can also be used [15], [16]. Since [3] optimizes for minimum energy path metric, we modify the ns-2 AODV implementation with the minimum energy path metric instead of using the current shortest path metric.

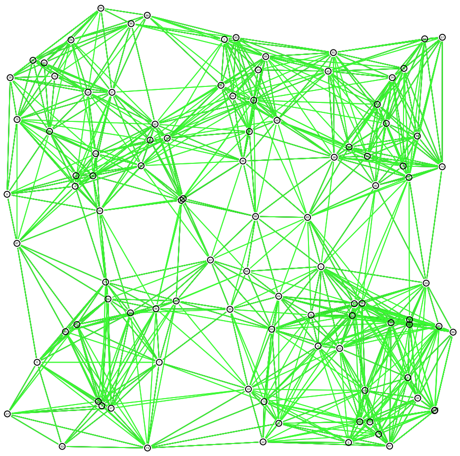
To simulate the network application traffic, we use the following application scenario: All nodes periodically send UDP traffic to the master data collection site situated at the boundary of the network. This application scenario has also been used in [6]. Network traffic characteristics has been studied extensively in the telephony network and the Internet [17], [18]. Although our application traffic scenario is not valid in those settings, it does represent a set of environment monitoring sensor applications. In this setting, sensors periodically transmit data to the data collection site. The data collection site will analyze the data for interesting events.

B. Analysis of the Resulting Topology of Different Topology Control Algorithms

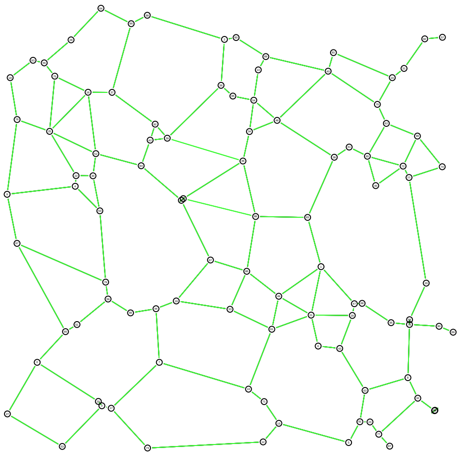
Before we move on to simulate different topology control algorithms, we would like to understand the characteristics of the resulting topology of different topology control algorithms. Figure 4 shows the topology generated by different topology control algorithms. The average node degree of each topology is shown in Table I. The average degree $\bar{d}$ of the multihop wireless networks should not be too large because a large $\bar{d}$ typically implies that a node has to communicate with other distant nodes directly. This increases interference and collision, and would waste energy. The average degree $\bar{d}$ should not be too small either because that tends to increase the overall network energy consumption as longer paths have to be taken. So we believe the average node degree is an important performance metric for multihop wireless network topology. Other metrics like k-connectivity and regular structure are also important. Those metrics will be our future research. The average node degree of our Phase1Only increases as the α



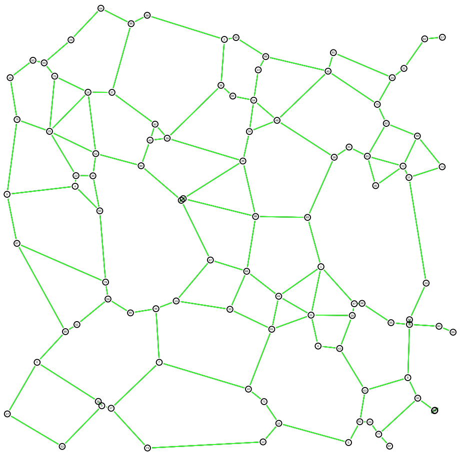
(a) Phase 1 Only $\alpha = 2\pi/3$



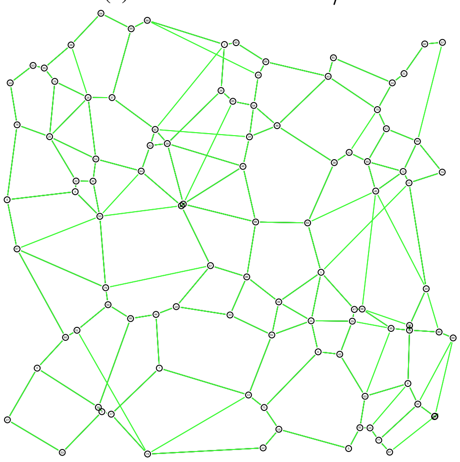
(b) Phase 1 Only $\alpha = \pi/2$



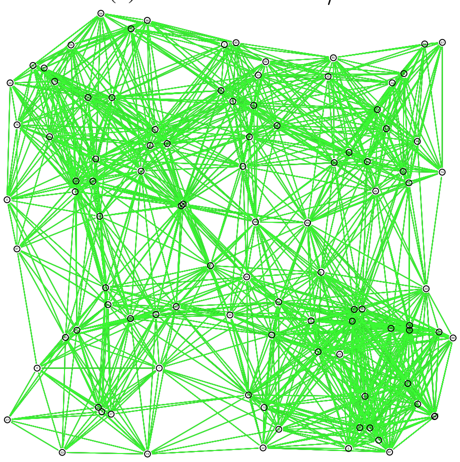
(c) Cone Based $\alpha = 2\pi/3$



(d) Cone Based $\alpha = \pi/2$



(e) R&M [3]



(f) Max Power

Fig. 4. Topology Graph of different topology control algorithms

parameter decreases. We remark that the boundary node contributes more to the average degree statistics. This is because, in trying to cover the maximum angle, it tends to involve more distant nodes. The average node degree of the inner nodes are much less than the average (as shown in Figure 4). We remark that the average node degree can be reduced if we know the boundary of the network. It is true that our Phase1Only topology has a much higher $\bar{d}$ than R&M. However, in the environment where there is only directional information, Phase1Only works while R&M does not.

Our ConeBased algorithm implements the redundant edge removal as described in Section III with $q = 1$. As shown in both Table I and Figure 4, it generates similar low degree topology graph as R&M algorithm.

C. Network Performance Analysis of Different Topology Control Algorithms

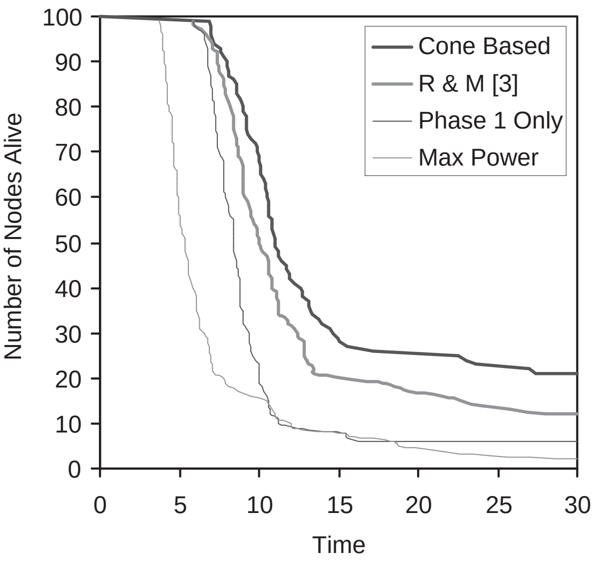


Fig. 5. Network lifetime

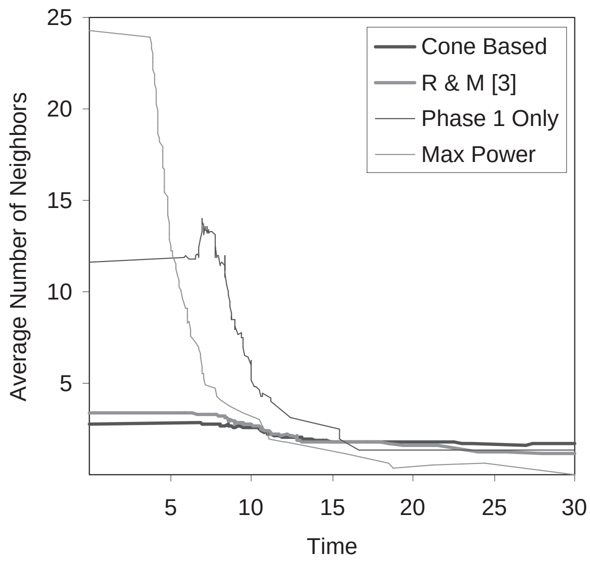


Fig. 6. Average node degree over time

We would like to measure the network performance using different topology control algorithms. We particularly care about network lifetime in the multihop wireless networks environment. We measure the network lifetime as the number of nodes still alive over time. We also want to understand how the network topology evolves over time.

We only simulated a static network. If mobility is low, a proactive approach to reconfigure the network topology may be used. If the mobility is high, an on-demand approach to reconfigure the network topology may be the only viable way to keep the reconfiguration control traffic low. How to make the topology control algorithm deal with mobility efficiently is our future research. In our simulation, we do not simulate the process of adjusting to the right transmission radius. It is adjusted to the right transmission radius immediately after AODV detects a node that has failed.

As can be seen from Figure 5, our ConeBased algorithm performs as good as the R&M algorithm, while using only directional information. . They both perform significantly better than MaxPower. From Figure 5, we see that when 80% of the MaxPower nodes are dead, both ConeBased and R&M still have around 90% percent of nodes alive. Our Phase1Only algorithm performs not as good as our ConeBased algorithm and the R&M algorithm, but it performs much better than no topology control case. When 80% of the MaxPower nodes are dead, Phase1Only still has more than 60% of nodes alive. It is interesting to see that some constant number of nodes stay alive for all the topology control algorithms except MaxPower. The reason is that, when a node is partitioned from the rest of the network, if its lower layer receives an AODV route request packet which is a broadcast packet, it will be sent with zero transmission range due to topology control. However, MaxPower will still be broadcasting with maximum radius since it has a pre-configured transmission power.

Figure 6 shows how the network topology evolves over time. It is interesting to note that the topology control algorithms tend to maintain the same average node degree for the remaining alive nodes as nodes die over time. The average node degree decreases noticeably only when the network has less than 40% nodes alive. Since MaxPower do not respond to topological changes, the average node degree will decrease quickly over time.

We also collected throughput statistics at the end of our simulation. Our ConeBased algorithm and the R&M algorithm achieve 4 times the throughput of the MaxPower. Our basic Phase1Only algorithm achieves 3 times the throughput of the MaxPower. The throughput statistics show that it is undesirable to transmit over large radius. This will increase energy consumption and also cause unnecessary interference. Increased interference will result in decreased throughput.

VII. Conclusion

The lifetime of a wireless network operating on battery power is critical to its usefulness. Network lifetime can be

	Phase 1 Only		Cone Based		R&M [3]	Max Power
	$\alpha = 2\pi/3$	$\alpha = \pi/2$	$\alpha = 2\pi/3$	$\alpha = \pi/2$		
Average Node Degree	11.6	15.6	2.8	2.8	3.4	24.3

TABLE I

AVERAGE DEGREE OF DIFFERENT TOPOLOGY CONTROL ALGORITHMS

increased by efficiently managing the power-consumption in each individual node belonging to the network. In this paper we describe a distributed cone-based topology control algorithm that determines the minimal power consumption operating point for each node in a multihop wireless ad hoc network. Our algorithm is unique in that it requires only local reachability information to determine the node power-consumption that guarantees a maximum connected node set. Running on every node in the wireless mode, our algorithm uses in-exact direction information about the location of neighboring nodes for making operating point decisions. The result is an approximation scheme that is able to bring the total power consumed for each route arbitrarily close to optimal.

We prove our algorithm theoretically and present results obtained via extensive ns-2 based simulations that show its validity. We focus primarily on the static ad hoc multihop network topology case, leaving the case of mobile nodes and changing network topology to future research.

REFERENCES

[1] L. Hu, "Topology control for multihop packet radio networks," *IEEE Trans. on Communications*, vol. 41, no. 10, October 1993.

[2] R. Ramanathan and R. Rosales-Hain, "Topology control of multihop wireless networks using transmit power adjustment," in *Proc. IEEE Infocom 2000*, March 2000.

[3] V. Rodoplu and T. H. Meng, "Minimum energy mobile wireless networks," *IEEE J. Selected Areas in Communications*, vol. 17, no. 8, August 1999.

[4] T.-C. Hou and V. O. K. Li, "transmission range control in multihop radio networks," *IEEE Trans. on Communications*, vol. 34, no. 1, pp. 38–44, January 1986.

[5] H. Takagi and L. Kleinrock, "Optimal transmission ranges for randomly distributed packet radio terminals," *IEEE Trans. on Communications*, vol. 32, March 1984.

[6] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *Proc. IEEE Hawaii Int. Conf. on System Sciences*, January 2000, pp. 4–7.

[7] K. Krizman, T. E. Biedka, and T.S. Rappaport, "Wireless position location: fundamentals, implementation strategies, and source of error," in *IEEE 47th Vehicular Technology Conference*, 1997, vol. 2.

[8] B. Karp and H.T. Kung, "Greedy perimeter stateless routing (gpsr) for wireless networks," in *Proc. ACM/IEEE MOBICOM*, 2000.

[9] C. Perkins, "Ad hoc on demand distance vector(aodv) routing," Internet Draft, draft-ietf-manet-aodv-04.txt, October 1999.

[10] THE VINT PROJECT, "The ucb/lbnl/vint network simulator-ns (version 2)," http://mash.cs.berkeley.edu/ns.

[11] THE CMU MONARCH GROUP, "Wireless and mobility extensions to ns-2," http://www.monarch.cs.cmu.edu/cmu-ns.html, October 1999.

[12] E.W. Zegura, K. Calvert, and S. Bhattacharjee, "How to model and internetwork," in *Proc. IEEE Infocom*, 1996.

[13] K. Calvert, M. Doar, and E.W. Zegura, "Modeling internet topology," *IEEE Communications*, June 1997.

[14] T.S. Rappaport, *Wireless communications: principles and practice*, Prentice Hall, 1996.

[15] C. Intanagonwiwat, R. Govindan, and D. Estrin, "Directed diffusion: a scalable and robust communication paradigm for sensor networks," in *Proc. ACM/IEEE MOBICOM*, 2000.

[16] W. Heinzelman, J. Kulik, and H. Balakrishnan, "Adaptive protocols for information dissemination in wireless sensor networks," in *Proc. ACM/IEEE MOBICOM*, 1999.

[17] V. Paxson and S. Floyd, "Wide-area traffic: the failure of Poisson modeling," *IEEE/ACM Transactions on Networking*, vol. 3, no. 3, pp. 226–244, 1995.

[18] V. Paxson and S. Floyd, "Why we don't know how to simulate the internet," *Proceedings of the 1997 Winter Simulation Conference*, pp. 1037–1044, 1997.