Priority Inversion and Its Prevention*

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Abstract

A priority inversion occurs when a low-priority task causes execution of a higher-priority task to be delayed. The possibility of priority inversions complicates the analysis of systems that use priority-based schedulers because priority inversions invalidate the assumption that a task can be delayed only by higher-priority tasks. This paper formalizes priority inversion and gives sufficient conditions as well as some new protocols for preventing priority inversions.

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1 Introduction

Task specifications in real-time systems usually involve bounds on completion times. Use of a priority scheduler can simplify implementing such specifications [8,14]. Each task is assigned a priority; whenever there is contention for a resource, access is granted to the task with the highest priority among those competing. Thus, the system designer controls the order in which resources are granted to tasks by assigning priorities. This, in turn, allows control over how resource contention affects task completion times.

A priority inversion occurs when a lower-priority task delays execution of a higher-priority task [7,11]. For example, a task holding a lock for some data will cause any task attempting to acquire a conflicting lock to be delayed. If the task holding the lock has a lower priority than the task attempting to acquire the conflicting lock, then a lower-priority task is delaying a higher-priority one and a priority inversion has occurred.

The possibility of priority inversions complicates the analysis of a system that uses a priority scheduler. Not only must a task compete for resources with higher-priority tasks, but during a priority inversion, it competes with lower-priority tasks as well. If a high-priority task $\tau_H$ can be delayed by a lower-priority task $\tau_L$, then $\tau_H$ effectively competes with all tasks assigned priorities at least that of $\tau_L$, rather than with only those tasks assigned priorities at least that of $\tau_H$. Since $\tau_L$ can be an arbitrary task, establishing that $\tau_H$ will meet a response-time goal involves reasoning about all tasks in the system rather than just the subset having priority at least that of $\tau_H$.

Priority inversions in a system controlled by a priority scheduler do not always lead to timing-constraint violations nor to more complicated analysis. If the duration of priority inversions can be bounded, then delays by lower-priority tasks can be considered part of the time required to allocate a resource. For example, in priority inheritance protocols [7,12], the duration of priority inversions is bounded and task deadlines are met by dynamically modifying task priorities. A task's priority is elevated to a level that is the maximum of its original priority and the priority of any task that is being delayed by it. Thus, priority inversions are permitted, but only in a carefully controlled way.

In this paper, we give a formal framework for investigating resource allo-
cation strategies that avoid priority inversions completely. We then use this framework to help understand techniques to avoid—rather than control—priority inversions. In Sections 2 and 3, we formalize priority inversion and give sufficient conditions for its prevention; we then illustrate these ideas using some simple reservation-based protocols. To further illustrate the use of our framework, in Section 5 we develop a new concurrency control scheduler for avoiding priority inversions in database systems. Of note is how we model transactions which, unlike ordinary tasks, can be aborted. In Section 6, we consider conditions for avoiding priority inversions in systems where there are multiple independent schedulers, each making allocation decisions for some subset of the resources. Section 7 puts our work in context and discusses some unsolved problems.

2 System Model

Formalizing priority inversion requires that we formalize the notions of priority assignment and delay. To do this, we model a system as a set of tasks \( T = \{ \tau_1, \tau_2, \ldots, \tau_n \} \) where a task is any computation that can be scheduled. Thus, our use of the term task is synonymous with alternatives such as process, job, and transaction.

2.1 Task Priorities

A priority assignment is an irreflexive, partial order\(^1\) on \( T \) where \( \tau \prec \tau' \) means that task \( \tau \) has lower priority than \( \tau' \). Observe that this definition allows tasks to have incomparable priorities. Therefore, it is possible that neither \( \tau \prec \tau' \) nor \( \tau' \prec \tau \) holds for some pair of tasks \( \tau \) and \( \tau' \). By assigning incomparable priorities to tasks, the number of constraints imposed by a priority assignment is reduced, avoiding the introduction of extraneous priority inversions in our model of a system.

Define the peer group of a task \( \tau \) to be the set of tasks \( \tau' \) such that either \( \tau \prec \tau' \) or \( \tau' \prec \tau \) is incomparable to \( \tau \). In the absence of priority inversions, we need only consider \( \tau \) and tasks that are in its peer group when analyzing

\(^1\)An irreflexive partial order \( \prec \) on a set is an asymmetric, irreflexive, transitive relation on pairs of elements from that set.
whether $\tau$ will satisfy given response-time constraints. This is because only tasks in the peer group of $\tau$ can cause it to be delayed.

2.2 Resources

Tasks can cause each other to be delayed in a variety of ways. Some of these causes are explicit, such as when one task awaits a message sent by another or when a lock held by one task prevents another from acquiring that lock. Other causes of delay are implicit. For example, the presence of finite-capacity, time-multiplexed resources, such as memory, processors, and I/O devices, can lead to the (implicit) delay of a task requiring use of a resource by the task using that resource. Notice that we do not distinguish between consumable and serially-reusable resources [3], as has been traditional when considering deadlock. In formalizing priority inversion, our concern is with the existence of a delay and not the details of the system activity to terminate that delay.

We postulate that a system comprises a set of resources and a scheduler\(^2\). A task obtains access to a single unit of a resource $r$ by invoking the

\[
\text{request}(r)
\]

operation. If the resource is not available then the task invoking the request is delayed. A delayed task may later be granted the resource by the scheduler when another task invokes the

\[
\text{release}(r)
\]

operation. Observe that a task may have only one outstanding request at a time. Thus, in our model, if multiple units of a resource or several different resources are needed for execution, a task must request and acquire them sequentially. We return to the possibility of a task requesting multiple resources through a single operation in Section 7.3.

Request and release operations are not always explicitly invoked by tasks. Sometimes, these operations are invoked implicitly as part of some

\(^2\)Most real systems have multiple schedulers, but postulating a single scheduler is not a limitation. It is always possible to model the effect of a collection of schedulers by using a single scheduler that makes allocation decisions using only information that would be available to the relevant scheduler. We return to the subject of multiple schedulers in Section 6.
other system operation. For example, an operation to receive a message will implicitly invoke a request operation that is granted when the message becomes available for receipt. In other cases, request or release might not be invoked by tasks at all, instead being invoked due to other activity in the system. Consider a multiprogrammed processor that uses an interval timer to force task switches. An execution of the interval-timer interrupt handler can be regarded as (i) performing a release and a subsequent request on behalf of the task that was executing when the timer-interrupt occurred and then (ii) granting the request to the task that is next selected for execution on the processor.

Our request/release model turns out to be quite general. It can even be used to describe situations in which tasks are delayed because of some application-dependent aspect of the system state. For example, it is not unusual for a concurrent program to contain some form of conditional wait statement that delays a task until the program variables satisfy some Boolean expression. (Most synchronization primitives are instances of such conditional wait statements.) We can model a conditional wait by regarding it as a request on a resource. The request is granted if the Boolean expression is true; otherwise, the request is delayed. Execution of any assignment statement in another task that makes the Boolean expression true is considered a release on the resource.

2.3 Delay

We model task delays using a binary relation. A task \( \tau_i \) waits for task \( \tau_j \) at time \( t \), denoted by \( \tau_i \to^t \tau_j \), if and only if \( \tau_i \) is delayed at time \( t \) in its request for some resource and \( \tau_j \) can release that resource. Note that more than one task might be able to release the resource being requested. We can model this using a conjunction of waits-for relations. For example, if there are multiple units of the resource available, then the delay is attributed to the set of all tasks \( T \) that have been granted but not yet released the resource:

\[
\bigwedge_{\tau \in T} (\tau_i \to^t \tau).
\]
3 Characterizing Priority Inversion

In order to characterize priority inversion, we must reason about the transitive closure $\rightarrow^+_t$ of the waits–for relation. Priority inversion has occurred at time $t$ when progress of a task is blocked by the actions of a lower priority task. Thus, a system contains a priority inversion at time $t$ if and only if there exist tasks $\tau_i$ and $\tau_j$ such that $(\tau_i \rightarrow^+_t \tau_j) \land (\tau_j \prec \tau_i)$.

It will be convenient to represent the priority assignment and waits–for relations in effect at a given time $t$ with directed graphs. The directed graph corresponding to a priority assignment $\prec$ on a set of tasks $T$ is $P = (T, E_P)$ where $E_P = \{(u, v) \mid u \prec v\}$. Thus, the nodes of $P$ represent tasks and an edge is drawn from a task to all higher priority tasks. Similarly, the waits–for relation at time $t$ can be represented as the directed graph $W = (T, E_W)$ where $E_W = \{(u, v) \mid u \rightarrow^t v\}$. Here, edges are drawn from a task to all other tasks that it waits for.

Given these graphs, the system state at a time $t$ can be represented by a composite system graph, $G = (T, E_P \cup E_W)$. Figure 1 depicts such a graph for a system of four tasks. Single–arrow edges represent waits–for

---

\[\text{Figure 1: Four–Task Composite System Graph.}\]
relations and double-arrow edges represent priority relations. Thus, the graph depicts the situation where there are four tasks such that \( r_3 \prec r_1 \prec r_2 \) and \((r_1 \xrightarrow{t} r_2), (r_2 \xrightarrow{t} r_4), (r_4 \xrightarrow{t} r_3)\). Note that there can be multiple edges between a pair of nodes.

Composite system graphs can be used to characterize priority inversions. In a composite system graph, a cycle containing exactly one priority edge is equivalent to the existence of a priority inversion.\(^4\) We call such cycles \( \pi \)-cycles to distinguish them from cycles containing no priority edges (which happen to model deadlocks). Thus, the system depicted in Figure 1 contains two priority inversions, corresponding to the two \( \pi \)-cycles \((r_3 \prec r_1 \xrightarrow{t} r_2 \xrightarrow{t} r_4 \xrightarrow{t} r_3)\) and \((r_3 \prec r_2 \xrightarrow{t} r_4 \xrightarrow{t} r_3)\).

Any condition that prevents a composite system graph from having a \( \pi \)-cycle is a sufficient condition for avoiding priority inversions. Examples of such conditions are the following:

1. **No Priority Assignment.** If all tasks were incomparable, then the priority graph would be empty and a \( \pi \)-cycle could never form.

2. **No Delay.** Without delay, the waits-for graph would be empty, guaranteeing the absence of the \( \pi \)-cycles.

3. **Preemption.** A waits-for relation persists until a task relinquishes a resource. Preemption causes a task to relinquish a resource, so preemption can change the waits-for relation in a way that prevents a \( \pi \)-cycle from forming.

4. **No \( \pi \)-Cycle.** The waits-for and the priority relations must form in a particular fashion for priority inversion to exist. The relevant property is a \( \pi \)-cycle.

Note that preemption both removes and adds elements to the waits-for relation. By preempting a resource \( r \) that had been granted to a task \( r_i \) and granting \( r \) to \( r_j \), all waits-for relations from \( r_j \) to other tasks are removed and waits-for relations from \( r_i \) to all tasks that have access to the resource are added.

---

\(^4\)Cycles containing more than one priority edge do not seem to correspond to any interesting property.
Conditions 2, 3, and 4 correspond to the three necessary and sufficient conditions for deadlock of [5]. Our conditions 2 and 3 correspond exactly to the Mutual Exclusion and Nonpreemption conditions of [5]; our condition 4 is a refinement of the Resource Waiting condition of [5].

Any strategy for preventing priority inversions ultimately must be based on avoiding \( \pi \)-cycles. In the strategies that follow, we do just this by ensuring that at least one of the sufficient conditions above holds. First, in Section 4, we show how by eliminating the possibility of certain waits–for relations, a \( \pi \)-cycle is avoided. Thus, this priority inversion strategy is based on condition 4, No \( \pi \)-Cycle. Then, in Section 5, we show an application of preemption by giving a timestamp–based concurrency controller. This strategy is based on condition 3, Preemption.

4 Understanding Reservation Protocols

In non-preemptive schedulers, \( \pi \)-cycles can be prevented by having each task reserve in advance the interval during which it will hold a resource and using that information to prevent certain waits–for relations from forming. If reservations from a low–priority task never overlap with reservations from a higher–priority task, then priority inversions do not occur. In this section, we describe two protocols that exploit this insight for avoiding priority inversions. The protocols themselves are not complicated, although getting all of their details right is difficult without a suitable formal characterization for priority inversions. We illustrate below that our theory is just such a characterization.

Let \( H_i = \{ \tau \mid \tau_i \prec \tau \} \) be the set of tasks that have higher priority than \( \tau_i \), and let \( I_i = \{ \tau \mid \neg(\tau_i \prec \tau) \land \neg(\tau \prec \tau_i) \} \) be the set of tasks incomparable to \( \tau_i \). Thus, the peer group for a task \( \tau_i \) is \( PG_i = H_i \cup I_i \). For each resource \( r \), assume that each task \( \tau_i \) is able to compute \( hold_i^r(t) \), the upper bound on the amount of time that \( \tau_i \) will hold \( r \) the next time (with respect to \( t \)) it is granted \( r \), and \( next_i^r(t) \), the lower bound on the next time (with respect to \( t \)) \( \tau_i \) will request \( r \). During the interval between when task \( \tau_i \) requests resource \( r \) and when it releases \( r \), define \( next_i^r(t) \) to equal \( t \). And, if \( \tau_i \) holds \( r \) at time \( t \) then define \( hold_i^r(t) \) to be an upper bound on the remaining amount of time \( \tau_i \) will hold \( r \).

\[5\] Instead of Resource Waiting, other authors have called this condition Circular Waiting.
Our assumptions about the availability of $\text{next}_i^r(t)$ and $\text{hold}_i^r(t)$ are satisfied in systems where tasks are periodic and can be analyzed before execution commences — systems in which priority inversions can be avoided by using a priori static scheduling of tasks. However, it is not hard to imagine systems for which static scheduling is not possible, but $\text{next}_i^r(t)$ and $\text{hold}_i^r(t)$ are still computable.

Two simple allocation policies based on knowledge of $\text{next}_i^r(t)$ and $\text{hold}_i^r(t)$ are:

**Policy 1** A request by task $\tau_i$ for resource $r$ at time $t$ is granted if

$$\text{hold}_i^r(t) \leq \min_{\tau_j \in \mathcal{P}_i} (\text{next}_j^r(t) - t).$$

*Otherwise, the request is delayed.*

**Policy 2** Let $R$ be the set of all shared resources. A request by task $\tau_i$ for resource $r$ is granted if

$$\text{hold}_i^r(t) \leq \min_{\tau_j \in \mathcal{P}_i} \min_{r_k \in R} (\text{next}_j^{r_k}(t) - t).$$

*Otherwise, the request is delayed.*

While it is probably clear that these policies prevent priority inversions, it may not be clear whether they are overly restrictive. For example, must incomparable tasks be included in the test of Policy 1? By giving a formal derivation of Policy 1—as we now do—the need for this part of the test can be understood.

In general, an allocation policy can be derived from an invariant that precludes formation of $\pi$-cycles. For a non-preemptive scheduler, the invariant must imply that there is no $\pi$-cycle present in the current state and that the current state is not one from which formation of a $\pi$-cycle is inevitable. Therefore, it suffices that the invariant imply the stronger condition that there is no $\pi$-cycle present in the current state and that the current state is not one from which formation of a $\pi$-cycle is possible. Although such a stronger invariant could rule out safe states, it is usually easier to construct and maintain.

---

6A *invariant* is an assertion about the program state that is not invalidated by program execution.
We now derive Policy 1. In a system where resources are shared infrequently, the most common \( \pi \)-cycle would involve just two tasks \( \tau_i \) and \( \tau_j \) (say) such that \( \tau_j \rightarrow_t \tau_i \land \tau_i \prec \tau_j \). An obvious invariant to choose is the negation of this predicate:\(^7\)

\[
(\forall \tau_i, \tau_j: \tau_j \rightarrow_t \tau_i \supset \neg(\tau_i \prec \tau_j)) \tag{1}
\]

However, (1) does not imply that there can be no \( \pi \)-cycle in a composite system graph since a \( \pi \)-cycle might involve a chain of \( \rightarrow_t \) edges. The following predicate does:

\[
(\forall \tau_i, \tau_j: \tau_j \rightarrow^+_t \tau_i \supset \neg(\tau_i \prec \tau_j)) \tag{2}
\]

So, we strengthen (1). First, note that because \( \prec \) is asymmetric, we have:

\[
(\forall \tau_i, \tau_j: \tau_j \prec \tau_i \supset \neg(\tau_i \prec \tau_j)) \tag{3}
\]

Thus,

\[
(\forall \tau_i, \tau_j: \tau_j \rightarrow_t \tau_i \supset \tau_j \prec \tau_i) \tag{4}
\]

implies (1), and so if (4) also implied (2) then (4) would imply there can be no \( \pi \)-cycle in the composite system graph.

We now show that (4) implies (2). Assume (4) holds. Since \( \prec \) is a transitive relation, we have from (4) and the definition of \( \rightarrow^+_t \) that:

\[
(\forall \tau_i, \tau_j: \tau_j \rightarrow^+_t \tau_i \supset \tau_j \prec \tau_i)
\]

This last equation together with (3) implies (2). Thus, (4) implies that there can be no \( \pi \)-cycle in the composite system graph and can be used as the basis for the invariant.

Unfortunately, (4) does not imply that the current state is one from which later formation of \( \pi \)-cycles is impossible—only that no \( \pi \)-cycle currently exists. For example, suppose a task \( \tau_i \) can allocate a resource \( r \) that a task \( \tau_j \) will later request and \( \tau_i \prec \tau_j \) holds. If \( \tau_i \) allocates \( r \), then (4) is not invalidated. However, if \( \tau_j \) subsequently attempts to allocate \( r \), then (4) is invalidated because the request cannot be granted (without preempting \( r \)

\(^7\)We write \( A \supset B \) to denote the logical implication, \( A \) implies \( B \).
from \( \tau_i \). So, (4) is not strong enough to prevent subsequent formation of a \( \pi \)-cycle.

We can strengthen (4) by weakening its antecedent. The antecedent asserts that \( \tau_j \) is waiting for \( \tau_i \); a weaker requirement is that \( \tau_j \) may wait for \( \tau_i \) at some future point. This weaker antecedent can be formalized in terms of \textit{hold} and \textit{next} as

\[
(\exists t': t \leq t': (\exists r: r \in R_\iota(t): (\text{hold}_i^r(t) + t > t') \land (\text{next}_j^r(t) \leq t'))) \]

where \( R_\iota(t) \) is the set of resources that \( \tau_i \) has allocated at time \( t \). After some manipulation using Predicate Logic, the strengthened (4) is:

\[
I: (\forall \tau_i, \tau_j: \neg(\tau_j < \tau_i) \supset (\forall r: r \in R_\iota(t): \text{hold}_i^r(t) + t \leq \text{next}_j^r(t)))
\]

To ensure that \( I \) holds throughout execution, we must show that it is true initially and guarantee that execution does not invalidate it. Assuming that tasks are initially started with no resources allocated to them, \( R_\iota(0) \) will be empty for all tasks \( \tau_i \), and so \( I \) is initially true. To guarantee that \( I \) is not invalidated by execution, assume that it is true and some task \( \tau_i \) requests a resource \( r' \). Let \( \text{ok}(r', I) \) denote those system states in which \( r' \) could be added to \( R_\iota(t) \) without invalidating \( I \). Then, any policy that ensures a condition \( C(r', \tau_i) \) holds before \( r' \) is granted to \( \tau_i \), such that

\[
C(r', \tau_i) \land I \supset \text{ok}(r', I)
\]

is valid, will ensure that \( I \) holds throughout execution (hence the formation of \( \pi \)-cycles is precluded).

The allocation of a resource \( r' \) to a task \( \tau_i \) can be modeled by an assignment statement \( R_\iota(t) := R_\iota(t) \cup \{r'\} \). The set of states that must hold before execution of an assignment \( x := E \) in order to ensure that some predicate \( Q \) holds is given by \( \text{wp}(x := E, Q) \), Dijkstra’s weakest precondition predicate transformer [6].\(^8\) Thus, we compute

\[
\text{ok}(r', \mathcal{I}) = \text{wp}(R_\iota(t) := R_\iota(t) \cup \{r'\}, \mathcal{I})
\]

\[
= \mathcal{I}^{R_\iota(t) \cup \{r'\}}
\]

\[
= (\forall \tau_i, \tau_j: \neg(\tau_j < \tau_i) \supset
(\forall r: r \in R_\iota(t) \cup \{r'\}: \text{hold}_i^r(t) + t \leq \text{next}_j^r(t)))
\]

Thus, for

\(^8\)For an assignment statement \( x := E \), [6] defines \( \text{wp}(x := E, Q) \) to be \( Q^{x \leftarrow E} \), the result of replacing in \( Q \) every free occurrence of \( x \) by \( E \).
\[
C(r', \tau_i) \overset{\text{def}}{=} (\forall \tau_j: \neg(\tau_j < \tau_i) \supset hold_t'(t) + t \leq next_t'(t))
\]
we have
\[
C(r', \tau_i) \land I \supset ok(r', I).
\]
Therefore, provided \(C(r', \tau_i)\) holds before \(r'\) is granted to \(\tau_i\), we can conclude that \(I\) will not be invalidated by program execution. This can be done by ensuring that the scheduler never grants \(r'\) to \(\tau_i\), unless \(C(r', \tau_i)\) holds, which is exactly Policy 1.

Policy 1 is rather conservative. It does not allow waits—for edges to develop between incomparable tasks in fear of a cycle developing indirectly between two comparable tasks. Policy 2 is conservative in a different way. It avoids instances of Figure 2. And, it can be derived in a manner similar to Policy 1. In a system with a total order priority assignment, Policy 1 is superior, while in a system where there are many incomparable tasks and few shared resources, Policy 2 is appropriate. Depending on the application, there may be other properties of the possible composite systems graphs that can be exploited in order to derive a better resource allocation policy.
5 A New Concurrency Control Protocol

We now consider how \( \pi \)-cycles can be prevented in a database system by a timestamp-based concurrency controller. Although the utility of transactions in a real-time system is debatable, deriving a concurrency controller for avoiding priority inversions does provide another illustration of our theory. Moreover, our concurrency controller appears to be the first to use timestamps for avoiding priority inversions. All other database concurrency controllers that we know of avoid priority inversions use locking [1,12].

5.1 Serializability and Priority Inversion

A task accesses a database by encapsulating its operations on the database within a transaction. A concurrency controller schedules these operations so that their execution is serializable—that is, each transaction either commits or aborts, and execution of the committed transactions is equivalent to executing them in some serial order. A transaction \( \tau_i \) precedes a transaction \( \tau_j \) in a history \( h \), written \( \tau_i \prec \tau_j \), if in all serial executions equivalent to \( h \), \( \tau_i \) executes before \( \tau_j \). Serializability of a set of transactions \( T \) can be characterized in terms of a serialization graph, \( G = (T, E_C) \) where \( E_C = \{(u, v) | u \prec v \} \). By definition, \( \prec \) is an irreflexive partial order, so \( G \) is acyclic.

By allowing only serializable executions, a concurrency controller ensures that the serialization graph \( G \) never contains cycles [2]. For example, suppose some operation \( \alpha_i \) of \( \tau_i \) was executed before a conflicting operation \( \beta_j \) of \( \tau_j \), where two operations conflict if their execution order cannot be interchanged without altering the effects of one or the other. In this case, a concurrency controller effectively adds an edge from \( \tau_i \) to \( \tau_j \) in \( G \). If two more conflicting operations, \( \gamma_i \) of \( \tau_i \) and \( \delta_j \) of \( \tau_j \), are to be executed, then \( \gamma_i \) must execute before \( \delta_j \), since to do otherwise would add an edge in \( G \) from \( \tau_j \) to \( \tau_i \), creating a cycle.

A concurrency controller can take one of three actions for each operation that a transaction submits. It can execute the operation; it can reject the operation and abort the transaction; or else, it can postpone performing the operation. The appropriate action is chosen to avoid cycles in the serialization graph.

Using the formalism of Section 2.2, we model transactions as tasks and
a concurrency controller as a scheduler. Executing an operation from a
transaction is equivalent to granting a resource to the corresponding task.
Rejecting or postponing an operation from a transaction is equivalent to
delaying the corresponding task. We next discuss these two methods of
delaying a task.

Postponing Operations

A concurrency controller postpones an operation $\alpha_i$ of a transaction $\tau_i$ if
executing $\alpha_i$ would introduce a cycle in the serialization graph. If some
transaction $\tau_j$ would be part of that cycle then $\tau_i$ waits for $\tau_j$ and we have
that $\tau_i \not\leftarrow \tau_j$ holds. Henceforth, we assume that if $\tau_i \not\leftarrow \tau_j$ because an op-
eration of $\tau_i$ is postponed, then $\tau_j \ll \tau_i$. This is a reasonable assumption
because were it not true, then there would exist an equivalent serial ex-
ecution in which $\tau_i$ precedes $\tau_j$ yet the concurrency controller postpones
operation $\alpha_i$ of $\tau_i$ until some operation $\beta_j$ of $\tau_j$ completes. Such a delay
would be capricious, since all the operations of $\tau_i$ could execute before any
operation of $\tau_j$ and yield the same result as when $\alpha_i$ is delayed. It is,
therefore, not surprising that all the concurrency control algorithms that
we know of satisfy this assumption.

By postponing operations, a concurrency controller can cause priority
inversions. Define a priority-ordered concurrency controller to be one that
ensures

POCC$_1$: For all transactions $\tau_i$ and $\tau_j$, if $\tau_j$ starts before $\tau_i$ completes and
$\tau_i \prec \tau_j$, then $\tau_j \ll \tau_i$.

POCC$_1$ ensures that higher-priority transactions are completed before concur-
currently executed lower-priority ones.

We now show that a priority-ordered concurrency controller cannot in-
troduce priority inversions: By assumption, a delay edge in a composite
system graph that can be attributed to the postponement of an operation
has a corresponding conflict edge in the serialization graph. By POCC$_1$, for
every priority edge in the composite system graph there is a corresponding
conflict edge in the serialization graph. Thus, there is a $\pi$-cycle in the com-
posite system graph only if there is a cycle in the serialization graph. Since
a concurrency controller ensures the absence of cycles in the serialization
graph, there can be no $\pi$-cycle in the composite system graph.
Rejecting Operations

In terms of our model, rejecting an operation submitted by a task $\tau_a$ (and thereby aborting $\tau_a$) can be regarded as having $\tau_a$ wait for some (virtual) resource held by other tasks. This is because a transaction that is aborted does no useful work prior to its restart, and in our model, a task whose transaction can do no useful work is considered blocked.

Aborting a transaction $\tau_a$ can cause a priority inversion, depending on the priorities of transactions holding the virtual resource being requested by (and blocking) $\tau_a$. The set $B_a$ of transactions holding this virtual resource is the smallest set such that had none of these executed at all, then $\tau_a$ would not have been aborted. Thus, if each transaction in $B_a$ is in the peer group of $\tau_a$, then aborting $\tau_a$ to avoid a cycle in the serialization graph does not create a priority inversion. Let $C$ be the set of transactions involved in cycles in the serialization graph, and let $A$ be the subset of $C$ that are aborted in order to remove those cycles. Then, $B_a \subseteq C - A$ holds, and we have:

POCC$_2$: A concurrency controller that aborts a transaction $\tau_a$ will avoid priority inversions provided $B_a$ is a subset of the peer group of $\tau_a$.

5.2 Timestamp-based Concurrency Controllers

Timestamp-based concurrency controllers work by assigning a unique timestamp $ts(\tau_i)$ to each transaction $\tau_i$. These timestamps are used to totally order transactions. The ordering is such that if $ts(\tau_i) < ts(\tau_j)$, then there is an edge in the serialization graph from $\tau_i$ to $\tau_j$.

Timestamp-based concurrency controllers both postpone operations and reject operations. In order to ensure that such a concurrency controller does not introduce priority inversions, two conditions suffice. The first condition is that the concurrency controller be priority-ordered, since being priority-ordered (i.e. satisfies POCC$_1$) implies that postponing operations will not introduce priority inversions. The second is that rejecting operations does not introduce priority inversions (i.e. satisfies POCC$_2$). We now consider each of these conditions in detail.

A timestamp-based concurrency controller can be made priority-ordered by suitable assignment of timestamps. This is because POCC$_1$ requires that
certain edges exist in a serialization graph. Since the concurrency controller
adds such edges by assigning timestamps, it suffices to ensure that

POTS: If $\tau_j$ starts before $\tau_i$ completes and $\tau_i \prec \tau_j$, then $ts(\tau_j) < ts(\tau_i)$.

It is not hard to assign such timestamps. For simplicity, assume integer
priorities; extension to general priorities is straightforward. Also assume
there exist $P$ priority levels $1, 2, \ldots, P$, where level $l_i \prec l_j$ if and only if
$l_i < l_j$, and assume that timestamps are real numbers. Let

$$\text{max}^x$$

be the largest timestamp of all committed transactions,

$$\text{min}^a_i$$

be the smallest timestamp of all active transactions of
priority $i$, and

$$\text{max}^a_i$$

be the largest timestamp of all active transactions of
priority $i$.

If no transactions of priority $i$ are active, then the value of $\text{min}^a_i$ and $\text{max}^a_i$
is defined to be $\bot$, where $\text{min}(x, \bot) = \text{max}(x, \bot) = x$. Initially, $\text{max}^x = 0.0$
and for all priority levels $\ell$, $\text{min}^a = \text{max}^a = \bot$. A timestamp $s$ for a new
transaction with priority $p$ can be computed by finding upper and lower
bounds for its value and selecting a unique value in that interval.\textsuperscript{9} To be
able to commit, the value of $s$ must be larger than $\text{max}^x$; to satisfy POTS,
it must be larger than the timestamps assigned to all transactions with
higher-priority and smaller than the timestamps assigned to all transactions
with lower priority. This is implemented by the code in Figure 3.

Having ensured that the concurrency controller is priority-ordered, it
only remains to ensure that POCC\textsubscript{2} holds and so aborts do not introduce
priority inversions. Suppose operation $\alpha_i$ from transaction $\tau_i$ is submitted
before a conflicting operation $\beta_j$ from $\tau_j$. If $ts(\tau_i) > ts(\tau_j)$, then executing
these operations in the order they are submitted would create a cycle in the
serialization graph\textsuperscript{10}. Thus, the concurrency controller must abort either $\tau_i$;

\textsuperscript{9}Such a value will exist because we have assumed that timestamps are real numbers.
If timestamps are implemented using finite-precision numbers, then such a value may not
exist. To avoid this problem, we can modify the code in Figure 3 to expand the range
when necessary by aborting the transaction whose timestamp is the lower-bound of the
interval.

\textsuperscript{10}Actually, if both operations are writes, the second write can be ignored and no conflict
occurs [13].

16
\[ \text{low} := \max\{\max^e, \max^q; 1 \leq \ell \leq p\}; \]
\[ \text{high} := \min\{\min^q; p < \ell \leq P\}; \]
\[ \text{if (high = \bot) then } s := \text{low} + 1 \]
\[ \text{else } s := (\text{low} + \text{high})/2; \]
\[ \min^q := \min\{\min^q, s\}; \]
\[ \max^q := \max\{\max^q, s\}; \]

Figure 3: Timestamp Allocator

or \(\tau_j\) to avoid this cycle. From POTS, if \(\tau_i \prec \tau_j\) then \(ts(\tau_j) < ts(\tau_i)\) holds, so, according to POCC\(_2\), no priority inversion can result by aborting \(ts(\tau_i)\). The following rule, therefore, describes a rule for aborting transactions without introducing priority inversions.

**Priority Abort Rule:** If \(\alpha_i\) from transaction \(\tau_i\) is submitted before conflicting operation \(\beta_j\) from \(\tau_j\) and \(ts(\tau_i) > ts(\tau_j)\), then \(\tau_i\) is aborted to avoid a cycle in the serialization graph.

This rule is the opposite of what is traditionally used in timestamp-based concurrency controllers [2]. Traditionally, the transaction that submitted the last operation (e.g. \(\tau_j\) above) would be aborted because this eliminates all cycles introduced by that operation. For example, suppose that \(ts(\tau_i) = i\) and the following sequence of operations are submitted, where \(r_i[x]\) denotes an operation by transaction \(\tau_i\) to read \(x\) and \(w_i[x]\) denotes an operation by transaction \(\tau_i\) to write \(x\):

\[ r_2[x] \quad r_3[x] \quad w_1[x]. \]

Performing \(w_1[x]\) would introduce two cycles in the serialization graph—one involving \(\tau_1\) and \(\tau_2\), the other involving \(\tau_1\) and \(\tau_3\). The traditional abort rule would abort \(\tau_1\), but would create a priority inversion if \(\tau_2 < \tau_1\) or \(\tau_3 < \tau_1\). The Priority Abort Rule would abort both \(\tau_2\) and \(\tau_3\) and cannot cause a priority inversion because of the way timestamps are assigned.

Implementing the Priority Abort Rule is not completely straightforward. See [9] for a detailed explanation of such an implementation.
6 Systems with Multiple Schedulers

It is not uncommon for system resources to be partitioned into disjoint subsets, where each subset is controlled by an independent scheduler. For example, the processors of a system are scheduled by a CPU scheduler, disk drives by some device driver module, and database accesses by the concurrency control module of a database manager. A database transaction would interact with all three separate schedulers during its execution.

Although using multiple, independent schedulers simplifies construction of a system, it complicates the avoidance of priority inversions. This is because a scheduler's decision to delay granting some resource to a task must be made using only partial information about the system state, and a delay caused by one scheduler might cause a priority inversion with tasks being delayed by other schedulers. To see this, consider a system with schedulers $S_1$ and $S_2$ and tasks $\tau_i$, $\tau_j$, and $\tau_k$, where $\tau_k \prec \tau_i$. Further, suppose that an allocation decision by $S_1$ leads to $\tau_i \not\rightarrow^+ \tau_j$ and an allocation decision by $S_2$ leads to $\tau_j \not\rightarrow \tau_k$. This is a priority inversion since $(\tau_i \not\rightarrow^+ \tau_k) \land (\tau_k \prec \tau_i)$. Notice that neither $S_1$ nor $S_2$ maintains sufficient local information to detect or prevent this priority inversion.

We can enjoy the benefits of having separate schedulers if freedom from priority inversions can be ensured by analyzing each scheduler in isolation. An obvious local criterion for correctness of a scheduler $S$ is that $S$ prevent priority inversions among tasks that have requested but not released resources from $S$. The two-scheduler example of the previous paragraph illustrates that this criterion by itself is not sufficient to ensure freedom from priority inversion—both $S_1$ and $S_2$ avoided such local priority inversions. We, therefore, now investigate useful conditions to ensure that avoiding local priority inversions is sufficient for avoiding all priority inversion.

Consider a system in which there is a set of independent schedulers and a single priority assignment that is known to all.\(^{11}\) Define $\tau_i \not\rightarrow^+ \tau_j$ to hold if and only if $\tau_i \not\rightarrow \tau_j$ and scheduler $S$ is delaying $\tau_i$'s request for some resource. The *local state* of a scheduler $S$ can be characterized by a directed graph $G_S = (T, E_S)$ where $E_S = \{(u, v) \mid u \prec v \lor u \not\rightarrow v\}$. Thus,

\(^{11}\)The case where schedulers do not share a common priority assignment is discussed in Section 7.2.
$G_S$ includes all of the priority edges of the composite system graph but only a subset of the waits-for edges. A local priority inversion exists for scheduler $S$ if and only if $G_S$ contains a $\pi$-cycle.

Since the composite system graph for a system with multiple schedulers is given by $G = (T, \cup_S E_S)$, a system contains a priority inversion at time $t$ if and only if $\cup_S G_S$ contains a $\pi$-cycle. However, as illustrated in the two-scheduler example above, existence of a $\pi$-cycle in $\cup_S G_S$ does not imply a $\pi$-cycle in $G_S$ for some scheduler $S$. The following theorem shows that this discrepancy is linked to specifying priority assignments with partial orders.

**Theorem 1** A system with multiple schedulers is free from priority inversion if the priority assignment is a total order and each scheduler avoids local priority inversions.

**Proof:** By contradiction. Assume that the system has a priority inversion characterized by the $\pi$-cycle

$$\tau_i \prec \tau_j \xrightarrow{t} \cdots \xrightarrow{t} \tau_k \xrightarrow{t} \tau_l \xrightarrow{t} \cdots \xrightarrow{t} \tau_i$$

in the composite system graph. For each $\tau_k \xrightarrow{t} \tau_l$ in the $\pi$-cycle, we conclude $\tau_k \prec \tau_l$ because every pair of tasks is related by the priority assignment and no scheduler allows a local priority inversion. By the transitivity of $\prec$, we have $\tau_j \prec \tau_i$. Thus, from $\tau_i \prec \tau_j$ we obtain a contradiction. \qed

Even in systems where the priority assignment is a partial order, it is possible to design schedulers that use only local information yet still manage to avoid all priority inversions. The strategy is for schedulers to be conservative and never permit a local configuration necessary for a $\pi$-cycle to develop. An example of such a strategy is:

No task $\tau_i$ is allowed to wait when there exists $\tau_j$ and $\tau_j \prec \tau_i$. \hfill (5)

(This is because the stated allocation policy prevents executions that lead to $\tau_j \prec \tau_i \xrightarrow{t} \tau$ for any $\tau$, a necessary configuration for a $\pi$-cycle.) A symmetric policy, which prevents $\tau \xrightarrow{t} \tau_j \prec \tau_i$ configurations, also works. However, both policies may degenerate to "do not allocate any resource to any task," a policy that is not very useful. The simple system with three tasks $\tau_i, \tau_j, \tau_k$ and a priority assignment where $\tau_i \prec \tau_j$ and $\tau_i \prec \tau_k$
illuminates the problem with the first policy. Suppose all three tasks share a single processor. Task \( \tau_i \) cannot be scheduled since this allocation decision would lead to \((\tau_j \rightarrow \tau_i) \wedge (\tau_i \prec \tau_j)\), a priority inversion. Task \( \tau_j \) cannot be scheduled since it would lead to \( \tau_k \rightarrow \tau_j \), which violates the scheduling policy that a task \((\tau_k)\) is never allowed to wait when there exists a task \((\tau_i)\) with lower priority. Finally, task \( \tau_k \) cannot be scheduled for the symmetric argument. In other words, none of the three tasks can run even though two have incomparable priorities!

In many systems, the priority assignment is encoded by associating an integer priority \( \Pi(\tau) \) with each task \( \tau \). If each task is assigned a unique priority, then the result is a total order and Theorem 1 implies that avoiding local priority inversions is sufficient for avoiding all priority inversions. However, constructing a total order from a partial order can require introduction of fictitious priority relations—avoiding priority inversions that involve these fictitious relations is unnecessary. Thus, we now consider the case where a unique priority is not assigned to each task, but \( \Pi \) does satisfy the following less-restrictive conditions, which define a partial order (as opposed to an irreflexive partial order).

**P1.** \( \Pi(\tau) < \Pi(\tau') \) if and only if \( \tau \prec \tau' \)

**P2.** \( \Pi(\tau) = \Pi(\tau') \) implies \( \neg(\tau \prec \tau') \wedge \neg(\tau' \prec \tau) \).

Observe that for a given priority assignment, a mapping that satisfies P1 and P2 might require adding some fictitious priority relations, but would require adding fewer priority relations than if \( \Pi \) defined a total order. The following theorem asserts that even though \( \Pi \) defines a partial order, avoiding local priority-inversions suffices to avoid all priority inversions.

**Theorem 2** If a task \( \tau \) is only allowed to wait for a task \( \tau' \) for which \( \Pi(\tau) \leq \Pi(\tau') \) then the system is guaranteed to be free from all priority inversions.

**Proof:** By contradiction. Assume a priority inversion characterized by the \( \pi \)-cycle

\[
\tau_i \prec \tau_j \rightarrow_t \tau_k \rightarrow_t \cdots \rightarrow_t \tau_i
\]
in the composite system graph. From the allocation policy, $\tau_k \rightarrow^t \tau_l$ implies $\Pi(\tau_l) \geq \Pi(\tau_k)$ for any two tasks $\tau_k$ and $\tau_l$ in the $\pi$-cycle. By transitivity, we have $\Pi(\tau_i) \geq \Pi(\tau_j)$. By the hypothesis, $\tau_i \prec \tau_j$ and thus by P1 we have $\Pi(\tau_i) < \Pi(\tau_j)$, a contradiction. 

Note that the problem illustrated by strategy (5) above no longer exists. One possible mapping that corresponds to the priority assignment of the example is $\Pi(\tau_i) = 1, \Pi(\tau_j) = \Pi(\tau_k) = 2$. Since tasks $\tau_j$ and $\tau_k$ have equal priority levels, either one could be scheduled without risking a priority inversion in the global system state.

## 7 Discussion

A characterization of priority inversion is useful only to the extent that the formal model on which it is based correctly captures the relevant aspects of reality. We, therefore, now discuss the suitability of our model and the relaxation of certain of its restrictions.

### 7.1 Priority Assignments as Partial Orders

We have elected to formalize priority assignments using irreflexive partial orders rather than mappings from tasks to integers (as is done in many operating systems). This selection was made because irreflexive partial orders are more expressive. For example, there is no mapping of tasks to integers to state that a task $\tau_1$ has the same priority as both $\tau_2$ and $\tau_3$ but $\tau_3$ has priority over $\tau_2$. Such a mapping $\Psi$ would have to satisfy $\Psi(\tau_1) = \Psi(\tau_2), \Psi(\tau_1) = \Psi(\tau_3)$ and $\Psi(\tau_3) > \Psi(\tau_2)$. Also, using an irreflexive partial order avoids introducing fictitious priority relations, which, in turn, avoid fictitious priority inversions.

When irreflexive partial orders are used to specify priority assignments, there are two possible interpretations for the case where two tasks have incomparable priorities. One is that these tasks do not compete for resources; the other is that these tasks compete for resources but on an equal footing. In either case, if two tasks are incomparable then, by definition (see Section 2.1) each will be in the peer group of the other. At first, including in the peer group for $\tau$ tasks that do not compete for resources with $\tau$ might
seem troubling. However, if two tasks do not compete for resources, then it
doesn’t matter whether one is in the peer group of the other—any analysis
based on that peer group will be no more complicated by the presence of
the non-competing task.

Using irreflexive partial orders to specify priority assignments does have
drawbacks. As shown in Section 6, using individual schedulers that ensure
freedom from local priority inversion does not by itself guarantee that a
system will be free of priority inversions. However, Theorem 2 shows that
if the priority assignment is restricted to one that could be represented by
a mapping from tasks to integers, guaranteeing freedom from local priority
inversions does guarantee freedom from all priority inversions. Thus, in
systems with multiple, independent schedulers, there are advantages to
employing the less-expressive formulation of priority assignment.

7.2 Static and Global Priority Assignment

One limitation of our model is the assumption of a single, static priority
assignment. This rules out systems where a task’s priority is a function
of the system state. It also rules out systems with multiple independent

A time-varying or dynamic priority structure can be modeled as a se-
quence of priority assignments,

\[ \prec_1, \prec_2, \ldots, \prec_t, \ldots \]

where \( \prec_t \) is the priority assignment in effect at time \( t \). The formal character-
ization of priority inversion in Section 3 remains valid with this extension,
but the protocols of Sections 4 and 5 require modification. This is because if
the priority assignment is not static, priority inversions can be caused sim-
ply by changing the priority assignment; with a static priority assignment,
a priority inversion can only occur from an (ungranted) request operation.
Avoiding priority inversions for a dynamic priority structure, therefore, re-
quires that changes to the priority assignment be coupled with the waits-for
relation.

Our definition of priority inversion does not work for the case where
different schedulers can assign different priorities to tasks. To understand
the problem, consider a system with two resources—a processor and a com-
munications channel—and two tasks \( r_1 \) and \( r_2 \). Suppose the priority assign-
ment \( \prec_P \) used by the processor has \( \tau_1 \prec_P \tau_2 \) and the priority assignment \( \prec_C \) used by the channel has \( \tau_2 \prec_C \tau_1 \). It is possible for \( \tau_1 \rightarrow \tau_2 \) due to an allocation decision by the processor and for \( \tau_2 \rightarrow \tau_1 \) due to an allocation decision by the communications channel. We believe that this scenario should not be considered a priority inversion because no task is being prevented from using a resource by a task that has a lower priority for the use of that resource. However, \((\tau_2 \prec \tau_1) \land (\tau_1 \rightarrow \tau_2)\) holds, which means there is a priority inversion according to the definition of Section 3.

### 7.3 Multiple-unit Resource Requests

Another limitation of our model is the requirement that tasks request resources sequentially. As a result of this limitation, we have not had to define what constitutes a priority inversion when a task can request multiple resources simultaneously. It is not clear what the correct definition should be. Consider a system consisting of two processors \( P_1 \) and \( P_2 \) and three tasks \( \tau_1, \tau_2 \) and \( \tau_3 \). Further, suppose \( \tau_2 \prec \tau_1, \tau_1 \) requires two processors, and \( \tau_2 \) and \( \tau_3 \) each require a single processor. It seems reasonable to claim that \( \tau_2 \) executing on \( P_1 \) while \( P_2 \) is idle constitutes a priority inversion, since \( \tau_2 \) holds a resource required by higher-priority task \( \tau_1 \). What is not clear is whether \( \tau_3 \) executing on \( P_1 \) while \( P_2 \) is idle should also be considered a priority inversion. On the one hand, no task holds resources required by a higher-priority task, suggesting that this should not be considered a priority inversion. On the other hand, if this is not considered a priority inversion then the seemingly harmless act of putting idle processor \( P_2 \) to work executing \( \tau_2 \) should not be considered a priority inversion, either. Yet, \( \tau_2 \) now holds a resource required by \( \tau_1 \), implying that a priority inversion exists.

### 7.4 Bounded Priority Inversions

This paper has been concerned with avoiding priority inversions completely. However, it is sometimes acceptable for priority inversions to occur, provided they are bounded in duration. Suppose a task is being delayed by some lower priority task. If the duration of this delay has a known bound, then we could view the situation as if the delay were included in the fixed
overhead associated with allocating the resource. So, by adding to the
cost of a request operation any delay due to a priority inversion, an anal-
ysis using peer groups would remain valid (despite the priority inversion).
The priority inheritance protocols in [12], for example, provide a way to
bound priority inversions under certain circumstances; the protocols of [10]
prevent priority inversions of unbounded duration.

Define a $D$-bounded priority inversion to be a priority inversion whose
duration is not longer than $D$ and an unbounded priority inversion to be
one whose duration cannot be so bounded. Avoiding all but $D$-bounded
priority inversions can be easier and less costly than avoiding all prior-
ity inversions.\footnote{Analyzing a scheduling protocol to determine a bound $D$ can be a hard problem, however [12].} For example, detection can be used to eliminate all but
$D$-bounded priority inversions—tasks run their course and periodically a
protocol is executed to detect and eliminate priority inversions that may
have formed.\footnote{Elimination of the priority inversion requires either that resource allocations to tasks
be preemptable or that task executions be abortable.} The frequency with which the detector runs determines the
upper bound $D$ on the duration of priority inversions.

In theory, it is easy to build a detector for priority inversions. The
detector must construct the composite system graph from the information
available to schedulers. In a distributed system with multiple independent
schedulers, a distributed snapshot algorithm [4] would have to be employed
for this purpose. Priority inversion detection is then simply a matter of
checking for $\pi$-cycles in the composite system graph of a snapshot. Note
that a priority inversion might have vanished by the time it is detected
because occurrence of a $\pi$-cycle is not a stable property [4]. Such “ghost”
priority inversions do not cause problems, however, because they are not
unbounded priority inversions.

8 Conclusions

This paper gives a formal characterization of priority inversion and gives
a set of sufficient conditions for its avoidance. Based on these conditions,
we have been able to derive new protocols to avoid priority inversion. We
have also been able to give conditions to avoid priority inversion in systems
with multiple schedulers that do not communicate.

The existence of a theory characterizing priority inversions makes it possible to design both general and application-specific protocols to avoid priority inversions. The theory also permits the consequences of system design decisions to be better understood. For example, we were surprised to find that choosing between an irreflexive partial order and an integer mapping representation of priority assignment can be significant. We were also surprised that the definition of priority inversion is elusive for systems in which multiple resource requests are possible or in which independent schedulers use different priority assignments.

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