6840 Sept 27

multi-item auction

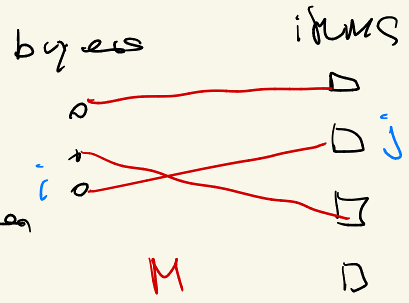
each sold seperately on first price

player i has value v_{ij} for item j

A set of items $v_i(A) = \max_{j \in A} v_{ij}$

"value for set"

Optimal SW



matching

(at most one item per person & at most one person per item)

$$OPTSW = \max_{M} \sum_{(i,j) \in M} v_{ij}$$

s = vector of arbitrary strategies

s_i^* if i matched to j in optimal matching

$$s_i^* = (0..,0, \frac{v_{ij}}{2}, 0...0)$$

↑ item j

What can we say

$$u_i(s_i^*, s_{-i}) \geq \frac{1}{2} v_{ij} - p_j(s)$$

if s_i^* wins j to person i or looses

$p_j(s)$ = price of item j in s

i has no item in Opt $s_i^* = (0 \dots 0)$

$$u_i(s_i^*, s_{-i}) \geq 0$$

Sum over all players

$$\boxed{\sum_i u_i(s_i^*, s_{-i})} \geq \frac{1}{2} \sum_{(i,j) \in M^*} v_{ij} - \sum_{j \text{ matched in } M^*} p_j$$

M^* = max value matching

$$\boxed{\geq \frac{1}{2} \text{OptSW} - \text{Rev}(s)}$$

σ mixed strategy Nash, or CCE

$$\mathop{E}_{s \sim \sigma}(u_i(s)) \geq \mathop{E}_{s \sim \sigma}(u_i(s_i^*, s_{-i}))$$

sum over i & use inequality

$$\sum_i \mathop{E}_{s \sim \sigma}(u_i(s)) \geq \sum_i \mathop{E}_{s \sim \sigma} u_i(s_i^*, s_{-i}))$$

$$\geq \mathop{E}_{s \sim \sigma}\left(\frac{1}{2} \text{Opt SW}\right) - \mathop{E}_{s \sim \sigma}(\text{Rev}(s))$$

$$\Rightarrow \boxed{\mathop{E}_{s \sim \sigma}(SW(s)) \geq \frac{1}{2} \text{Opt SW}.}$$ theorem

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Can players learn to bid with no regret?

distretize bids $\{\varepsilon v_{ij}, 2\varepsilon v_{ij} \ldots \quad v_{ij}\}$

possible strategies $= \left(\frac{1}{\varepsilon}\right)^m$

m items

exponentially many options...

If bidding on one item only

strategies $m \cdot \frac{1}{\varepsilon}$

Regret error $\sqrt{2T \ln K}$

$K = $ # strategies

Note: s_i^* depends on optimal matching, so depends on all player values, not only i

$\Longrightarrow$ not good for our Bayesian Nash result!

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