two auctions last time
- first price & all pay

today add uncertainty of opponent

Reminder: $s_i^* = v_i/2$

do not regret bidding $s_i^*$

We have seen:

$$\boxed{\sum_i u_i(s_i^*, s_{-i}) \geq \frac{1}{2} \, OptSW - Rev(s)}$$

$\sigma$ mixed Nash

$$\sum_i \underset{s \sim \sigma}{E}(u_i(s)) \geq \sum_i \underset{s \sim \sigma}{E}(u_i(s_i^*, s_{-i}))$$

combining the two

$$E(SW(s)) \geq \frac{1}{2} \, OptSW$$

---

Bayes version:

$\mathcal{F}$ distribution of participants

$v \sim \mathcal{F}$ ( ~~with~~ fixed large population)
subset shows up

Bayes Nash equilibrium

~~stragy~~ strategy for each player

(OK to randomize)

$$\mathop{E}_{v\sim F} \mathop{E}_{s\sim v} \left[ u_i(v,s) \right] \geq \mathop{E}_{v\sim F} \mathop{E}_{s\sim v} \left( u_i(v, s_i', s_{-i}) \right)$$

combine with green box

$$\sum_i \mathop{E}_{\substack{v\in F \\ s\sim v}} \left( u_i(v,s) \right) \geq \sum_i \mathop{E}_{\substack{v\sim F \\ s\sim v}} u_i(v, s_i^*, s_{-i}))$$

green & take EXP

$$\geq \frac{1}{2} \mathop{E}_{v\sim F} [\mathrm{OptSW}(v)] - \mathop{E}_{\substack{v\in F \\ s\sim v}} [\mathrm{Rev}(s)]$$

$$\Rightarrow \mathop{E}_{\substack{v\in F \\ s\sim v}} \left( SW(v,s) \right) \geq \frac{1}{2} \mathop{E}_{v\sim F} \left( \mathrm{OptSW}(v) \right)$$

---

learning outcomes

$$\sum_t u_i(v,s) \geq \sum_t u_i\left(\frac{v_i^*}{2}, s_{-i}\right) - \mathrm{reg}$$

$v, s$ from distr

same result extends with small loss for regret

All pay

recall $s^*$ for all pay

$i \neq \arg\max_j v_j$ $\quad s_i^* = 0$

$u_i(s_i^*, s_{-i}) \geq 0$

$i = \arg\max_j v_j$

$s_i^*$ unif $[0, v_i]$

no-regret guarantee bounds for all $x \in [0, v_i]$

$u_i(s) \geq u_i(x, s_{-i}) - \text{reg}$ all $x \in [0, v_i]$

$\Rightarrow u_i(s) \geq E_{x \sim [0, v_i]} [u_i(x, s_{-i})] - \text{reg}$

We had:

$$\boxed{E_x \left[\sum_i u_i(s_i^*, s_{-i})\right] \geq \frac{1}{2} \max_j v_j - \text{Rev}(s)}$$

trouble $s_i^*$ depends on other people's values.

We would like subset of iterations

to use different $s_i^*$ as comparator

We don't know how to guarantee this