Sept 20:

hiarchy of equilibrium concept

finite games

pure Nash $\subseteq$ mixed Nash

may not exist

exist as proven by Nash

mixed Nash $\subseteq$ Correlated eq = CE

CE $\subseteq$ Coarse correlated equilibrium = CCE

learning algorithm find it approximately

Hotelling game: N $x_1 \ldots x_n$

$w_x \geq 0$ # people at location x

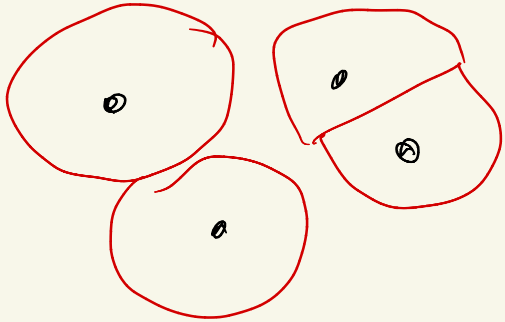
k players, choose location each

~~di~~ people go to closest if dist $\leq d$

utility for player i

= # customers choosing it

$$SW(s) = \sum_i u_i(s)$$



s solution $\quad$ s^* social optimal one

① if s Nash

$$u_i(s) \geq u_i(s_i^*, s_{-i})$$

② we proved:

$$\sum_i u_i(s_i^*, s_{-i}) \geq SW(s^*) - SW(s)$$

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① + ② combines $\quad$ $SW(s) \geq \frac{1}{2} SW(s^*)$

s – Nash

New version: $s^1 \dots s^t \dots$

no-regret learning outcome

instead of ①

①' $\displaystyle\sum_t u_i(s^t) \geq \sum_t u_i(s_i^*, s_{-i}^t) - \text{reg}$

combine ① & ②

$$\sum_t sw(s^t) = \sum_t \sum_i u_i(s^t)$$

no reg. $\geq \sum_i \left(\sum_t u_i(s_i^*, s_{-i}^t) - reg \right)$

② $\geq \sum_{t=1}^{T} \left[sw(s^*) - sw(s^t) \right] - k\, reg$

$$\Rightarrow \frac{1}{T} \sum_t sw(s^t) \geq \frac{1}{2} sw(s^*) - \frac{k\, reg}{2T}$$

General recepie:

s^* strategy vector we used

main fact all vectors s

$$\sum u_i(s_i^*, s_{-i}) \geq \lambda\, Opt\, sw - \mu\, sw(s)$$

called (λ, μ) smooth

$\Rightarrow$ Price of anarchy proof

s Nash

$$sw(s) \geq \frac{\lambda}{1+\mu} sw\, Opt$$

& bound extends to all CCE

First price auction:

$v_1 \ldots v_k$ value of bidders

notational convenience

$v_1 > v_2 > \ldots v_k$

Nash $b_1 = v_2$ $b_i = v_i$ $\forall i > 2$

& person 1 wins

$SW = (v_1 - p) + 0 \ldots + 0 + p = v_1$

utilities for bidder

auctioneer's revenue

Monday: learning outcome