Sept 9: auctions

Last week's learning: K experts

$\ell_t(i)$ loss time t expert i

in games loss $\sim$ cost or $-$utility
experts $\sim$ strategies
loss = cost for you had you choosen
given strategy

$$\ell_t(i) \sim c_i(s_i, s^t_{-i}) = \ell_t(s_i)$$

Wed: ~~the~~ learners in auctions

today: single item auction
private value
N players, player i has value v_i

Step 1: ask all player a bid b_i
$\sim$ willingness to pay

Step 2: select winner
– today $\arg\max_i b_i$

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Step 3: announce required payment

(a) first price $= \max_i b_i$

(b) second price

$$i^* = \arg\max b_i$$

$$\text{price} = \max_{i \neq i^*} b_i$$

(c) all pay

Thm: 2^{nd} price it is best strategy to use $b_i = v_i$

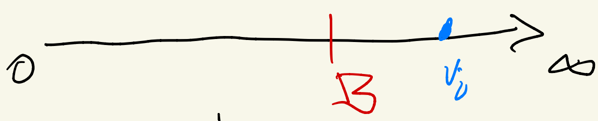
= dominant strategy = no matter what others do you as well off as possible with $b_i = v_i$

user i wants to maximize

$$u_i = v_i - p \quad \text{if winning}$$

price

Proof:



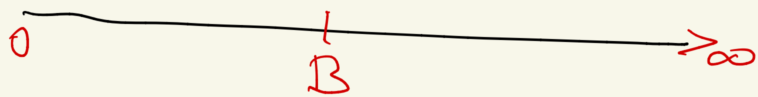
$$B = \max_{j \neq i} b_j$$

will be the price if you win

if $B > v_i$ $\Longrightarrow$ don't want to ~~le~~ win
$b_i = v_i$ does this

if $B < v_i$ $\Longrightarrow$ want to win & pay B
$b_i = v_i$ does that

First price:
with lots of knowledge:



$$B = \max_{j \neq i} b_j$$

bid just above B if $v_i > B$
otherwise bid $< B$

All pay:
example: 2 players values
$v_1 = v_2 = 1$

no pure strategy Nash

if b_1, b_2 both want to change

Proposed mixed strategy Nash

$b_i \in [0,1]$ uniformly at random

value for player 1 on pure strategy

$b_i = x$

$$\text{utility} = \underbrace{-x}_{\text{payment}} + \underbrace{x}_{\text{prob of winning}} \cdot 1 = 0$$

utility for randomization also 0

Example 2: $v_1 = 1$, $v_2 = 2$

Nash: $b_2 \in [0,1]$ unif random

$$b_1 = \begin{cases} 0 & \text{prob } 1/2 \\ [0,1] \text{ unif} & \text{prob } 1/2 \end{cases}$$

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