Strategizing against no-regret learners

~~seller~~ is strategizing
buyers are no-regret learners

Buyer values

$\frac{1}{4}$ prob $\frac{1}{2}$

$\frac{1}{2}$ —"— $\frac{1}{4}$

1 —"— $\frac{1}{4}$

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best price, repeat T times

$p \sim 1$ revenue $\sim T/4 = \frac{T}{4} \cdot 1$

$p \sim \frac{1}{2}$ —"— $T/4 = \frac{T}{2} \cdot \frac{1}{2}$

$p \sim \frac{1}{4}$ —"— $T/4 = T \cdot \frac{1}{4}$

If buyer are mean based
no-regret player seller can do better

"Auction" — buyer ask for item yes/no

yes → get item price $[0,1]$ will
be given later

time $1, \ldots, T$

first $T/2$ item price $= 0$

then price $= 1$

follow the leader learners after $\frac{T}{2} + x$ step

value	no	yes
1	0	$\frac{T}{2} + 0 \cdot x$
$1/2$	0	$\frac{T}{2} \cdot \frac{1}{2} - x \cdot \frac{1}{2} \geq 0$ if $x \leq \frac{T}{2}$
$1/4$	0	$\frac{T}{2} \cdot \frac{1}{4} - x \frac{3}{4} \geq 0$ if $x \leq \frac{T}{6}$

Revenue

$$\frac{T}{2} \cdot 1 \cdot \frac{1}{2} + \frac{T}{6} \cdot \frac{1}{2} = \frac{T}{4} + \underbrace{\frac{T}{12}}_{\text{extra}}$$

	L	M	R
U	ε	-1	0
D	-1	$+1$	0

no-regret players $\sim$ L, ... L, R, R ... R

Recall CE & CCE

correlated eq — coarse c.E.

advisor = correlator tells you what to play

no-regret learning outcome

playing $\frac{1}{2}$-$\frac{1}{2}$ on (U,L) & (D,R) is no-regret

but <u>not</u> a CE

instead of R you prefer M

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better learning guarantee

swap-regret:

$$\text{S-Reg} = \max_{b,b'} \sum_{t:\, b^t=b} u^t(b') - \sum_{t:\, b^t=b} u^t(b)$$

Made with Goodnotes

Theorem: player has no-swap regret in 2-person game & opponent can only get Stackelberg value

Proof:

$a_1 \ldots a_T \qquad b_1 \ldots b_T$

say player 2 has no swap regret.

We know $\forall$ b

$$\sum_{t: b_t = b} u_2(a_t, b_t) \geq \sum_{t: b_t = b} u_2(a_t, b') \quad \forall b'$$

prob player 1:

$$\alpha^b(a) \; \# \text{prob} = \frac{\#\text{ time } a_t = a \,\&\, b_t = b}{\#\text{ times } b_t = b}$$

player 2's best response is b

as leader player 1 can get

$$V^{*} \geq \sum_{t: b_t = b} u_1(a_t, b) \cdot \frac{T}{\#(\text{times } b_t = b)}$$

V = max Stackelberg value

to bound total value for player 1

$$\sum_{t=1}^{T} u_1(a_t, b_t) = \sum_{b} \sum_{t:\, b^t = b} u_1(a_t, b_t)$$

$$\leq \frac{V}{T} \sum_{b} \#(\text{times } b_t = b) = V$$