Follow the perturbed leader

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time t ect

$$s^* = \arg\max_s \sum_{\tau=1}^{t-1} u_i(s, s_{-i}^{\tau})$$

our framework:

adversary sets $u^t(s)$ $s \in S$

$$s^t = \arg\max_s \sum_{\tau=1}^{t-1} u^{\tau}(s)$$

Robinson '56 : 2-person 0-sum games

follow the leader converges to Nash

in marginal dist

Cheat algorithm: be the leader

$$s^t = \arg\max \sum_{\tau=1}^{t} u^{\tau}(s)$$

Claim: sequence has no-regret:

player has strategy set S

adversary sets $u^t : S \to \mathbb{R}$

Proof: induction

$t=1$ by choice

$t>1$

$$\sum_{\tau=1}^{t} u^\tau(s^\tau) = \sum_{\tau=1}^{t-1} u^\tau(s^\tau) + u^t(s^t)$$

$$\overset{\text{i.h}}{\geq} \max_s \sum_{\tau=1}^{t-1} u^\tau(s) + u^t(s^t)$$

$$\geq \sum_{\tau=1}^{t} u^\tau(s^t) \geq \max_s \sum_{\tau=1}^{t} u^\tau(s)$$

by choice of s^t

Add noise:

$$s^t = \arg\max_s \left(\sum_{\tau=1}^{t} u^\tau(s) + \xi_s \right)$$

indep random

Claim: regret of this sequence in exp

$$\leq E(\max_s \xi_s)$$

Proof: base $t=0$ ✓ ✓

$$E\left(\sum_{\tau=1}^{t} u^\tau(s^\tau)\right) = E\left(\sum_{\tau=1}^{t-1} u^\tau(s^\tau)\right) + E\left(u^t(s^t)\right)$$

$$\overset{\text{i.h}}{\geq} \max_s E\left(\sum_{\tau=1}^{t-1} u^\tau(s) + \xi_s\right) - E\left(\max_s \xi_s\right) + E\left(u^t(s^t)\right)$$

$$\geq E\left(\sum_{\tau=1}^{t} u^\tau(s^t) + \xi_{s^t}\right) - E\left(\max_s \xi_s\right)$$

$$\geq E\left(\sum_{\tau=1}^{t} u^\tau(s) + \xi_s - \cancel{\xi_{s^t}}\right) - E\left(\max_s \xi_s\right)$$

stronger i.h

$$E\left(\sum_{\tau=1}^{t} u^\tau(s^\tau)\right) + E\left(\max_s \xi_s\right) \geq E\left(\sum_{\tau=1}^{t} u^\tau(s) + \xi_s\right)$$

for all s

implies desired statement if $\xi_s \geq 0$

Next step:

1. define ξ_s need $E\left(\max_s \xi_s\right)$

2. Prob $\left(\operatorname{argmax} \sum_{\tau=1}^{t} u^\tau(s) + \xi_s = \operatorname{argmax} \sum_{\tau=1}^{t-1} u^\tau(s) + \xi_s \right) \geq 1 - \varepsilon$