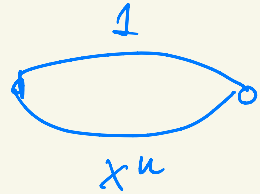
~~Fist~~ First class



$r = 1$

Claim if $\mathcal{C}$ = max d degree polinomials $\sum_{i=0}^{n} a_i x^i \quad a_i \geq 0 \ \forall i$

$\alpha(\mathcal{C})$ = PoA on above 2-node network

Proof sketch:

Recall PoA depends on PoA on worst edge.
See notes from 1st lecture for value

Overprovisioning
optimizing flow or adding capacity

~ Then adding capacity is better

f Equilibrium flow ($s_i - t_i$ rate r_i)

$f_P \geq 0$ flow on path P

$$\sum_{P\, s_i \to t_i} f_P = r_i \; ; \; f(e) = \sum_{P: e \in P} f_P$$

all P s.t. $f_P > 0$ $s_i - t_i$

$\Longrightarrow$ P min cost path s_i to t_i

f^* min cost flow $s_i - t_i$ rate $\boxed{2 r_i}$

Thm: $\text{cost}(f) \leq \text{cost}(f^*)$ cost

mon increasing, cont, ≥ 0

Proof:

$$\text{cost}(f) = \sum_e f(e)\, c_e(f(e))$$

$$= \sum_P c_P(f)\, f_P$$

$\bar{c}_e = c_e(f(e))$ fixed cost

(1) cost (f^* with $\bar{c}$ costs) $\geq 2 \text{cost}(f)$

f carries all flow min cost

& f^* has double amount

& costs are fixed

Claim

$$f^*(e)\, c_e(f^*(e)) \geq f^*(e)\, c_e(f(e)) - f(e)\, c_e(f(e))$$

Proof:

if $f^*(e) \geq f(e)$

c_e monotone, true with last - term

if $f^*(e) < f(e)$

r.h.s negative

Add for all e

$$\sum_e f^*(e)\, c_e(f^*(e)) \geq \sum_e f^*(e)\, c_e(f(e)) - \sum_e f(e)\, c_e(f(e))$$

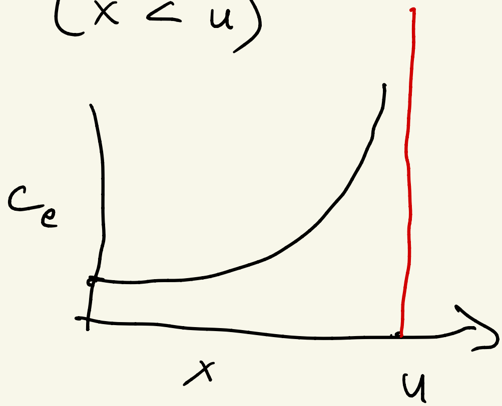
$$\mathrm{cost}(f^*) \geq [\mathrm{cost}(f^* \text{ with } c)] - \mathrm{cost}(f)$$

using claim 1

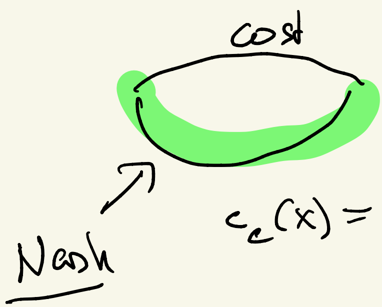
$$\geq 2\,\mathrm{cost}(f) - \mathrm{cost}(f) = \mathrm{cost}(f)$$

Favorite cost function:

$$c_e(x) = \frac{1}{u-x} \qquad (x < u)$$



Price anarchy for this class



$$c_e(x) = \frac{1}{u-x}$$

$$r = u - y$$

$$\text{cost} = \frac{1}{y}$$

PoA comes out to be $\sim \dfrac{1}{2\sqrt{y/u}}$