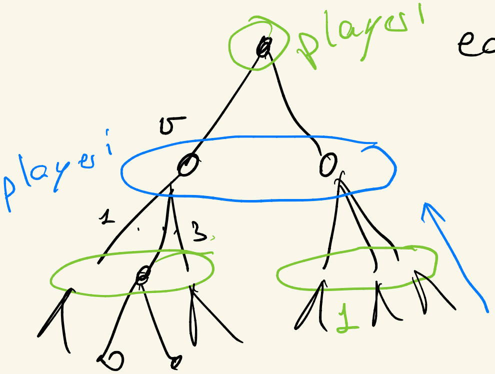
Extensive form games



each node a player move or random

payoff on bottom

information set

set i knows he/she is in

perfect recall: all player remember all moves they made.

pure strategies k^n if n_i = # decision nodes for player i

k = # options

strategy:

each info set v for player i

give prob distr on choices

$x_v^i(j)$ = prob of choosing option j at node v

variables

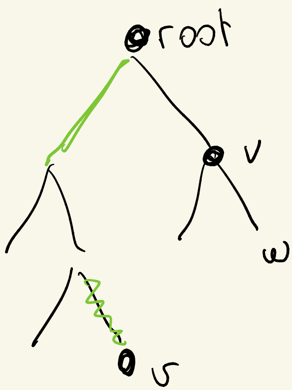
$n \cdot k$ only

OK if game tree is the input

New variables $z(v)$ info set v

for player i

product of x probabilities on the path from root to node v



$\boxed{z_i(r)=1}$ all player

- v is not ith decision

$\boxed{z_i(v)=z_i(w)}$

- v is ith decision

w_j children of i

$\boxed{\sum_j z_i(w_j) = z_i(v)}$

proof: $z_i(w_j) = z_i(v)\cdot x^i_v(j)$

$\boxed{z_i(v) \geq 0}$ all i all v

also need: $\boxed{z_i(v)=z_i(w)}$ if v & w in same info set

<u>Claim:</u>

$x^i_v(j)$ & $z_i(v)$ variables are in one-to-one correspondence

<u>Claim:</u> z-formulation of 2-person 0-sum games useful to solve for Nash.

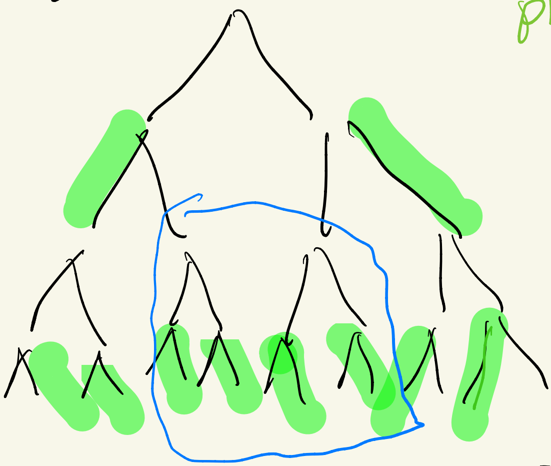
Proof: z_1 is a Nash if

(1) satisfies $z_1(v) \geq 0$ &
satisfies $\boxed{}$

(2) given z_1 best responding player 2

all possible pure strategies Γ
expected payoff $\geq \lambda$
maximize λ

given Γ



player 2

Γ

final tree nodes
some not reachable

payoff

not reachable

$$\sum_{v \text{ reachable by } \Gamma} z_1(v)\, a_1(v) \geq \lambda$$

linear inequality

Fact: if given a proposed z

we can find Γ such that

$$\min_{\Gamma} \sum_{v \text{ reachable}} z_1(v)\, a_1(v)$$

$\Rightarrow$ ellipsoid method can solve the linear program

We need subroutine to get best response

Fact: this is solveble bottom up

using same alg we have seen of

fall into games